5 Points Available

Instructions

Please write your **Name and Student Number** at the top of this page.
Remember: you have to write quizzes in your **registered** tutorial.

Make sure to show as many steps of your work as possible, justify as much and annotate any interesting steps or features of your work. ***Do not just give the final answer.***

Question 1

Recall that $K_{n,m}$ is the bipartite graph with vertices split between two sets X and Y with $|X| = n$ and $|Y| = m$, and with the additional property that every vertex in X is adjacent to every vertex in Y and vice-versa.

Assuming that $n \leqslant m$ are fixed positive integers, how many X-matchings (matchings which pair-off every vertex in X) are there in $K_{n,m}$? (Justify your answer.)

Solution: *tl;dr* — The answer is:

$$m \cdot (m-1) \cdot (m-2) \cdot \cdots \cdot (m-n+1).$$

To see this begin with the first vertex in X, say $x_1 \in X$. Since $K_{n,m}$ is fully connected, there are edges between x_1 and all the vertices of $Y = \{y_1, \ldots, y_m\}$, thus pick any of the m vertices in Y to match x_1. (i.e., there are m choices here.)

Proceed to x_2, there are $m-1$ remaining vertices which x_2 can be matched with. (i.e., there are $m-1$ choices here.) Continue on this way until we reach x_n. From this we see that there are $m-n+1$ remaining vertices which x_n can be matched with. Hence we see that there are $m \cdot (m-1) \cdot (m-2) \cdot \cdots \cdot (m-n+1)$ many X-matchings, as desired.