5 POINTS AVAILABLE

INSTRUCTIONS

Please write your **Name and Student Number** at the top of this page.
Remember: you have to write quizzes in your **registered** tutorial.

Make sure to show as many steps of your work as possible, justify as much and annotate any interesting steps or features of your work. ***Do not just give the final answer.***

QUESTION 1

Suppose that G is a graph with *exactly* one odd-length cycle C. Prove that G can be 5-coloured.

Hint: Consider the graph $G - C$ (where we remove all the vertices in the cycle C).

Solution: We do as the hint says and consider the graph $H := G - C$. Notice that since C was the *only* odd cycle in G, this implies that H has no odd cycles. By a theorem in the book (Theorem 11.5) we know that H containing no odd cycles implies that H is, in fact, bipartite (i.e. 2-colourable).

Thus, we colour H with two colours (say, red and blue), and we can colour the cycle C with three different colours (say, green, yellow, and purple). Combining these two colourings (to get a colouring of G) means that *at worst* G is 5-colourable, as desired.