5 POINTS AVAILABLE

INSTRUCTIONS

Please write your **Name and Student Number** at the top of this page.
Remember: you have to write quizzes in your **registered** tutorial.

Make sure to show as many steps of your work as possible, justify as much and annotate any interesting steps or features of your work. ***Do not just give the final answer.***

Consider the following new definition:

DEFINITION 1

We say that a connected graph G is **2-connected** if there is no vertex v in G so that $G - v$ is disconnected.

QUESTION 1

Prove that if G is a graph with at least three vertices and with the property that ($\star$) for any three distinct vertices u, v, w in G there is a path between u and v which does not go through w, then G is *2-connected*.

Solution: [Notice that this is the converse of the statement in Quiz 4, TUT101]

For contradiction, suppose that G has property ($\star$) and that G is *not* 2-connected. Since G is not 2-connected there exists $w \in G$ such that $G - w$ *is* disconnected.

Let u and v be vertices in $G - w$ for which there exists no path between between them (these vertices exist because G has at least 3 vertices and $G - w$ is disconnected). By property ($\star$) there exists a path in G from u to v which does not pass through w, however, this must also be a path connecting u to v in the subgraph $G - w$, a contradiction.