5 POINTS AVAILABLE

INSTRUCTIONS

Please write your **Name and Student Number** at the top of this page.
Remember: you have to write quizzes in your **registered** tutorial.

Make sure to show as many steps of your work as possible, justify as much and annotate any interesting steps or features of your work. ***Do not just give the final answer.***

Consider the following new definition:

DEFINITION 1

An edge $e = (a, b)$ in a connected graph G is called a **bridge** if the graph $G - e$ is disconnected.

QUESTION 1

Show that if a graph has a bridge, then it cannot have a Hamilton cycle.

Solution: Let G be a graph with a bridge $e \in G$. Suppose that there exists a Hamilton cylce, $H \subseteq G$. Since G admits a Hamilton cycle, then we can remove any edge, $\epsilon \in G$, from G, and the graph $G - \epsilon$ will still be connected, however, $G - e$ is disconnected, a contradiction.

ASIDE: Intuitively, to see why removing an edge from a graph with a Hamilton cycle means that the resulting graph is still connected, notice that we can "use" the Hamilton cycle to get from any vertex to any other vertex (so we have connectedness of this graph), and removing any particular edge either:

(a) doesn't change the Hamilton cycle embedded in the graph, or

(b) removes one edge from the Hamilton cycle embedded in the graph

In both cases we can still get from any vertex to any other vertex in the graph. (We just might have to "go the other way" in the Hamilton cycle.)