5 POINTS AVAILABLE

INSTRUCTIONS

Please write your **Name and Student Number** at the top of this page.
Remember: you have to write quizzes in your **registered** tutorial.

Make sure to show as many steps of your work as possible, justify as much and annotate any interesting steps or features of your work. ***Do not just give the final answer.***

QUESTION 1

Find a simple, closed form for the generating function of the sequence $a_n = n^2$.

Hint: $n^2 = 2\binom{n}{2} + n$.

Solution: Let $A(x)$ denote the generating function for the sequence $(a_n)_{n\geqslant 0}$. Then using the definition and employing the hint we have

$$\begin{aligned} A(x) &= \sum_{n\geqslant 1} n^2 x^n \\ &= \sum_{n\geqslant 1} \left(2\binom{n}{2} + n\right) x^n \\ &= 2\sum_{n\geqslant 2} \binom{n}{2} x^n + \sum_{n\geqslant 1} n x^n \\ &= 2\frac{x^2}{(1-x)^3} + \frac{x}{(1-x)^2} \qquad \text{(cf. The solution to Example 8.6)} \end{aligned}$$

(NOTE: we are losing terms in the sums along the way because these terms are 0, so we can drop them.)