

SampleSizePlanner: A Tool to Estimate and Justify Sample Size for Two-Group Studies

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Abstract

Planning sample size often requires researchers to identify a statistical technique and to make several choices during their calculations. Currently, there is a lack of clear guidelines for researchers to find and use the applicable procedure. In the present tutorial, we introduce a web app and R package that offer nine different procedures to determine and justify the sample size for independent two-group study designs. The application highlights the most important decision points for each procedure and suggests example justifications for them. The resulting sample size report can serve as a template for preregistrations and manuscripts.

Keywords: sample size determination; power analysis; study design

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Introduction

Social and behavioral sciences are known to be plagued by undersampling (Ioannidis, 2005). In the traditional statistical framework, even when the effect exists, undersampled studies yield either nonsignificant results or significant results due to overestimating the size of the effect. Because nonsignificant results are less likely to reach publications than significant ones, results of undersampled studies either remain unpublished or impose a substantial bias on our body of published empirical findings. In addition, the low informational value of undersampled studies may not justify the cost or potential risk they induce (Halpern, Karlawish, & Berlin, 2002). To mitigate these issues, authors are increasingly expected to plan and justify the sample size of their study (Maxwell, 2004). However, such sample size justifications are only meaningful if they provide sufficient information to the readers to judge the adequacy of the author's decisions.

In the statistical literature, a few methods have been proposed to determine and justify sample size. In practice, however, authors are short of practical guides on how to navigate among the different sample size methods. The aim of our tutorial is to point out for each method the essential decision points that a researcher has to face during this process. We provide a short description of each method and the corresponding parameters, but we avoid listing their advantages and disadvantages. As there are disagreements between the experts of the field regarding the correct use of some of the methods, we intentionally try to remain impartial and do not favor any of the presented methods. Researchers who want to know more about each method can find a number of useful references in the description of the methods. We also provide a collection of ready-to-use analysis code and a ShinyApp that helps researchers use and report the main sample size estimation techniques for different scenarios. The tutorial is focused exclusively on the scenario of the comparison of two independent groups (i.e., the independent t -test design)

with a one-sided test.

Sample Size Determination and Justification

A lot of factors go into the determination of the sample size for an independent two group study design. In this section, we will first provide a birds-eye view of the most important decisions. Next, we will go into more detail on the specific inference tool that results from the combination of the larger choices.

It is crucial to not just state how we determined our planned sample size but to also give the reader insight into the reasons behind our choices. In a recent overview, Daniel Lakens (2021) lists six types of general approaches to justify sample size in quantitative empirical studies: (1) Measure entire population; (2) Resource constraints; (3) A-priori power analysis; (4) Accuracy; (5) Heuristics; and (6) No justification. For the first approach, no quantitative justification is necessary; and for the second approach, the researcher has no freedom to increase the sample size. Power analysis, or more generally the estimation of true positive rate, is used when one plans to conduct hypothesis testing; accuracy justifications are used when one plans to conduct parameter estimation. Our tutorial mainly focuses on approaches two, three, and four, and is aimed at providing a hands-on approach for the mechanical part of the sample size determination (i.e., the calculation). For a deeper discussion of justification of these approaches, or for other approaches (i.e., using heuristics or not providing justification), we refer the reader to Daniel Lakens (2021).

Choosing a method in case of sample size justification

In an ideal world, the choice for the number of participants would be solely determined by scientific considerations, and depending on the chosen technique the collection of data would continue until either the desired sample size or a desired outcome

has been reached. In practice, researchers are limited by time (collecting data is quite demanding), money (participants or people collecting the data may be paid, and the same may hold for renting space or equipment), or availability of participants (the population may be relatively small, and/or the participation rate quite low).

When constrained by limited resources, it is important to be transparent about those limitations. It is also important to be open about scientific considerations. Depending on the nature of the study (perhaps it is an initial exploration?), small sample sizes need not be a dealbreaker. So although more data are always preferred from an informational point of view, by owning the limitations of our study, we improve future readers' understanding of the process leading up to the eventual paper, and we also answer in advance to those who think the chosen sample size was insufficient.

Whether or not authors have limited resources, two important choices need to be made: (1) whether they are interested in *statistical testing* or in *parameter estimation*; and (2) whether they want to conduct their statistical inference within the *frequentist* framework or within the *Bayesian* framework. Starting with the first decision, statistical testing is the primary framework when one is interested in establishing whether an underlying population effect is equal to, different from, larger than, or smaller than a certain value. In essence, statistical testing lends itself to binary decision making. Typically, testing is concerned with a fixed point null hypothesis (e.g., there is no difference between two groups), although using intervals for testing is also possible. Alternatively, one might be interested in parameter estimation that is less interested in establishing the existence of a difference and instead is concerned with establishing the magnitude of the difference.

The second important decision concerns the statistical framework. Choosing to conduct statistical tests within a frequentist framework, one is usually interested in balancing the type I (false positive) and type II (false negative) error rates. Practitioners

choosing to conduct statistical tests within a Bayesian framework are typically interested in being able to quantify the relative probability of hypotheses or models being true given the data and in including prior information.

Within the realm of statistical testing, there are some other factors that affect the preferred inference tool: Do you prefer to test for equivalence (no difference in mean) or for superiority (mean of one group larger than mean of other group), are you interested in calculating a required sample size for a specific hypothetical effect size or for a range of possible values, and do you wish to employ sequential testing (applicable to Bayesian testing)? In case of testing, some of the methods are designed to find support for the null hypothesis (e.g., TOST, ROPE), while others are designed to find support for the alternative hypothesis (e.g., traditional null hypothesis testing), and some methods are designed to find support for either (e.g., BFDA). For frequentist estimation, the preferred inference tool might differ depending on whether we evaluate uncertainty for each group separately or jointly. We will describe these specific factors when we go into detail about each of the preferred methods. A flow-chart representing all of these choices is given in Figure 1.

How to use this guide

In the next section, we will illustrate the specific inference tools and resulting sample size calculations in more detail using a ShinyApp and an R package we have developed. Throughout this section, we recurrently use two terms that have different meanings for different techniques. These are the *true positive rate* (TPR), and the *equivalence band* (EqBand). The TPR reflects the long-run probability of concluding there is an effect, given that it does exist. For traditional null hypothesis testing, this is typically referred to as power, but related concepts exist for different inference tools. The EqBand refers to an effect size region, typically around zero, that is deemed clinically insignificant or irrelevant. Different names are given to this region depending on the technique that employs them,

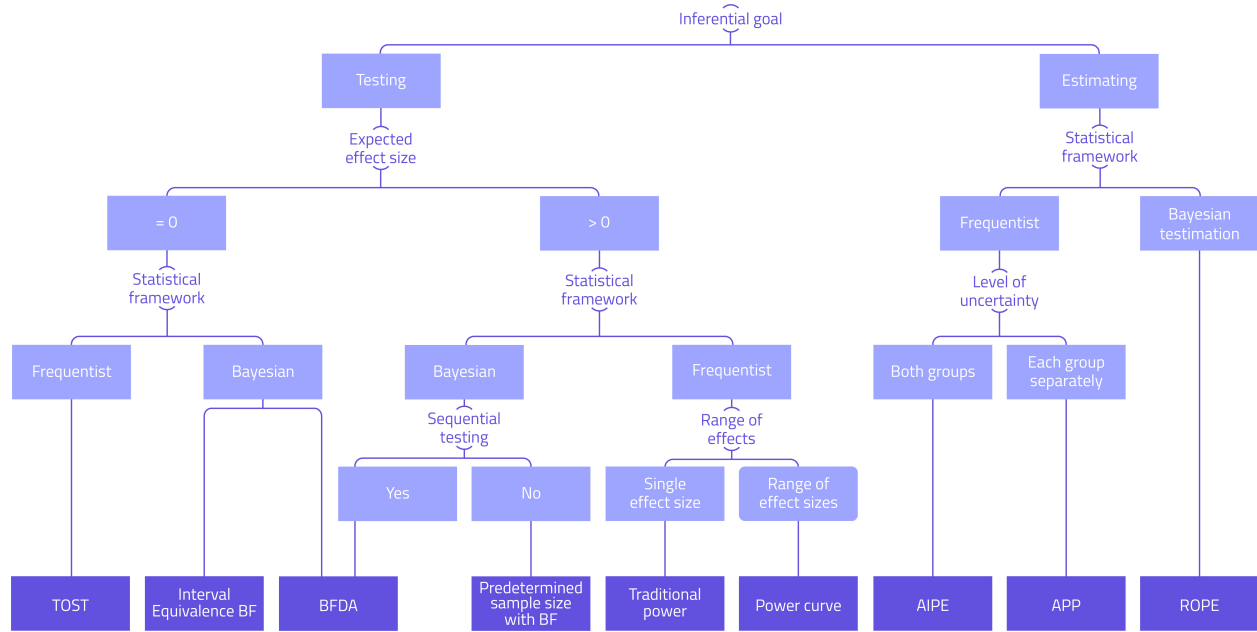


Figure 1. The figure depicts the decisions that one faces when choosing among sample size estimation methods. The nine sample size estimation methods discussed in this paper are listed in the bottom row. Some decisions are determined by the investigated question and the design of the study while others are based on the preferred statistical framework.

such as statistical effect size of interest (SESOI) or region of practical equivalence (ROPE). For both TPR and EqBand, we explain the specific meaning in context of the relevant inference tool below.

For each method, only the main parameters can be adjusted with a certain range of values in the ShinyApp by using a slider. These parameters are presented in the text in bold. Other parameters are set to preset values in the application but can be adjusted in the accompanying R package to any sensible value. These parameters are highlighted in italics in the tutorial. Both the app and the package allow the users to save or copy a text template with the results of the sample size determination. We offer a list of possible justifications at the decision points for each method (indicated between square brackets), but users are able to provide their own justification as free-text. It is important to note that the listed justifications are meant to provide guidance for the user, and they are not

sufficient without further details provided by the researcher in the context of the given study. For example, previously reported values should always be accompanied by a theoretical justification of why these values make sense. The provided justification text could serve as a stub for the description of the chosen sample size in a paper, a preregistration or registered report, or a grant proposal.

Throughout, we will use the example story of Mary the educational psychologist. Mary has come up with a new set of games that challenge spatial insight. She would like to test whether distributed and targeted engagement with these games for a period of six months for children in the age range of 8 to 12 will lead to lasting improvements on their IQ score as measured through Raven's progressive matrices test (population mean 100, population SD 15). Mary collects data for a control sample that gets regular education and for an experimental sample and plans to compare those samples. Mary has good reason to be skeptical about the effectiveness of training on increasing performance as there are several studies questioning the existence of such effects (Owen et al., 2010; Simons et al., 2016). For illustrative purposes, in some of the upcoming examples Mary expects a null effect and in others Mary expects a positive effect in order to highlight the different research scenarios for each sample size planning method. We will also present a justification text for each sample size planning method based on Mary's choices described in the example research scenario for the given method.

The ShinyApp is available on <https://martonbalazskovacs.shinyapps.io/SampleSizePlanner> and the R package can be installed by running the following command in R `devtools::install_github("marton-balazs-kovacs/SampleSizePlanner")`. There is more information about the R package and the ShinyApp on the projects' Github page <https://github.com/marton-balazs-kovacs/SampleSizePlanner>, or on the website <https://marton-balazs-kovacs.github.io/SampleSizePlanner/>.

1. Testing

1.1. Effect size = 0.

1.1.1. Two One-Sided Tests (TOST).

Study context.

Mary would like to know what sample size she needs for a power of .80 to study whether the mean IQ score of the experimental group's population is practically equivalent to the mean IQ score of the control group. She tests this assumption in a frequentist framework, and considers a population effect size between -0.2 and 0.2 to be 'practically equivalent' to no difference. This would correspond to IQ scores between 97 ($100+15 \cdot -.2$) and 103 ($100+15 \cdot .2$).

Description.

TOST is a frequentist equivalence testing approach that adopts two one-sided hypotheses to designate an interval hypothesis (Schuirmann, 1987). The lower and upper boundaries of the interval are determined by the equivalence band (i.e. SESOI) around the expected population effect size (e.g., 0). Daniël Lakens, Scheel, and Isager (2018) lists several methods that can be used to determine the SESOI. In case of TOST, the two null hypotheses state that the effect size is equal to the lower and upper equivalence band values, whereas the alternative hypotheses state that the effect size is significantly smaller than the upper equivalence band value and significantly larger than the lower equivalence band value. In case both one-sided tests reject the null-hypothesis at a given significance level, the group means are considered to be practically equivalent. See Daniël Lakens, Scheel, and Isager (2018), for further reading.

Parameters.

Delta: The expected population effect size. In most cases, this value will be zero.

TPR: The desired long run probability of obtaining a significant result with TOST, given Delta.

EqBand: The chosen width of the region for practical equivalence, i.e. the SESOI.

Alpha: The level of significance. The alpha level in the application is preset to 0.05.

How to use the package.

```
SampleSizePlanner::ssp_tost(tpr = 0.8, eq_band = 0.2, delta = 0)
```

How to report your sample size estimation.

In order to calculate an appropriate sample size for testing whether the two groups are practically equivalent, we used the Two One-Sided Tests of Equivalence [TOST; Schuirmann (1987)] method. We used an alpha of 0.05. We set the aimed TPR to be 0.8, because [1) it is the common standard in the field; 2) it is the journal publishing requirement]. We consider all effect sizes below 0.2 equivalent to zero, because [1) previous studies reported the choice of a similar equivalence band; 2) of the following substantive reasons: ...]. The expected delta was 0 because [1) we expected no difference between the groups]. Based on these parameters, a sample size of 429 per group was estimated in order to reach a TPR of 0.8 with our design.

1.1.2. Equivalence interval Bayes factor.

Study context.

Mary would like to know what sample size she needs to have a long-run probability of .80 of obtaining a Bayes factor larger than 10. Mary would like to test whether the mean IQ score of the experimental group's population is practically equivalent to the mean IQ score of the control group. Mary hypothesizes that there is no difference (i.e., H_0 is true). Mary tests this assumption in a Bayesian framework. Mary considers a population effect size between -0.2 and under 0.2 to be 'practically equivalent.' This would correspond to IQ scores between 97 ($100+15*(-.2)$) and 103 ($100+15*(.2)$).

Description.

Equivalence interval Bayes factors contrast an equivalence hypothesis to a non-equivalence hypothesis and quantify the evidence with Bayes factors. Typically, H_0 constitutes the equivalence interval (comparable to SESOI in the TOST framework), and H_a constitutes the complementary non-equivalence regions. Formally, the Bayes factor is calculated by dividing the fraction *posterior area inside the interval/posterior area outside the interval* (i.e., the posterior odds) by the fraction *prior area inside the interval/prior area outside the interval* (i.e., the prior odds). The resulting value quantifies how much more likely it is that the data occurred under a population effect size deemed ‘equivalent’ relative to the data having occurred under a population effect size deemed non-equivalent. The current implementation uses a default Cauchy prior on effect size with the possible scale parameters of medium ($r = 1/\sqrt{2}$), wide ($r = 1$), or ultra-wide ($r = \sqrt{2}$). For further reading, see Morey and Rouder (2011), Ravenzwaaij, Monden, Tendeiro, and Ioannidis (2019), and Linde, Tendeiro, Selker, Wagenmakers, and Ravenzwaaij (2020).

Parameters.

Delta: The expected population effect size.

TPR: The desired long-run probability of obtaining a Bayes factor at least as high as the Threshold, given Delta.

EqBand: The chosen width of the equivalence region.

PriorScale: The scale of the Cauchy prior distribution. The PriorScale in the application can be set to: $1/\sqrt{2}$, 1, and $\sqrt{2}$.

Threshold: Critical threshold for the Bayes factor. The threshold level in the application can be set to 10, 6, or 3.

How to use the package.

```
SampleSizePlanner::ssp_eq_bf(tpr = 0.8, delta = 0, eq_band = 0.2,
    thresh = 10, prior_scale = 1/sqrt(2))
```

How to report your sample size estimation.

In order to estimate the sample size, we used the interval equivalent Bayes factor (Morey & Rouder, 2011; Ravenzwaaij, Monden, Tendeiro, & Ioannidis, 2019) method. We used a Cauchy prior distribution centered on 0 with a scale parameter of $1/\sqrt{2}$. We set the aimed TPR at 0.8, because [1) it is the common standard in the field; 2) it is the journal publishing requirement]. We consider all effect sizes below 0.2 equivalent to zero, because [1) previous studies reported the choice of a similar equivalence region; 2) of the following substantive reasons: ...]. The expected delta was 0 because [1) we expected no difference between the groups]. Our Bayes factor threshold for concluding equivalence was 10. Based on these parameters, a minimal sample size of 144 per group was estimated in order to reach 0.8 TPR for our design.

1.2. Effect size >0.

1.2.1. Frequentist.

1.2.1.1. Classical power analysis.

Study context

Mary would like to know what sample size she needs for a power of .80 to study whether the mean IQ score of the experimental group's population is significantly higher than the mean IQ score of the control group. She tests this assumption in a frequentist framework for a hypothetical population effect size of 0.5. This corresponds to a mean IQ score of 107.5 in the experimental group ($100 + 15 \cdot 0.5$), assuming a mean IQ score of 100 in the control group.

Description

The classical power analysis approach allows one to calculate the required sample size in order to obtain a significant result for the null hypothesis test a certain proportion of times in the long run given an assumed population effect size.

Parameters

Delta: The expected population effect size.

TPR: The desired long-run probability of obtaining a significant result with a one-sided t -test, given Delta.

Maximum N: The maximum number of participants per group (both groups are assumed to have equal sample size).

Alpha: The level of significance. Alpha is preset to 0.05 in the application.

How to use the package

```
SampleSizePlanner::ssp_power_traditional(tpr = 0.8, delta = 0.5,  
max_n = 5000, alpha = 0.05)
```

How to report your sample size estimation

We used a power analysis to estimate the sample size. We used an alpha of 0.05. We set the aimed TPR at 0.8, because [1) it is the common standard in the field; 2) it is the journal publishing requirement]. The expected delta was 0.5 because [1) previous results published in ...; 2) of the following substantive reasons: ...]. Based on these parameters, a minimal sample size of 51 per group was estimated in order to reach 0.8 TPR for our design.

1.2.1.2. Power curve.

Study context

Mary would like to know what sample size she needs for a power of .80 to study whether the mean IQ score of the experimental group's population is significantly higher than the mean IQ score of the control group. She tests this assumption in a frequentist

framework. However, she is reluctant to commit to a single hypothetical population effect size a-priori, preferring to calculate required sample size for a range of hypothetical deltas between 0.1 and 0.9.

Description

The power curve method is similar to a classical power analysis but instead of calculating the appropriate sample size for one hypothesized population effect size, the method calculates the required sample size for a range of plausible population effect sizes.

Parameters

Delta: A range of hypothetical population effect sizes.

TPR: The desired long-run probabilities of obtaining a significant result with a one-sided *t*-test, given each value of Delta.

Maximum N: The maximum number of participants per group (both groups are assumed to have equal sample size).

Alpha: The level of significance. Alpha is preset to 0.05 in the application.

How to use the package

```
# Determine the sample sizes for each delta
curve_data <- SampleSizePlanner::ssp_power_curve(tpr = 0.8, delta = seq(0.1,
  0.9, 0.01), max_n = 5000)

# Plot the power curve
SampleSizePlanner::plot_power_curve(delta = curve_data$delta,
  n1 = curve_data$n1, animated = FALSE)
```

How to report your sample size estimation

We used a power analysis to estimate the sample size. We used an alpha of 0.05. We set the aimed TPR at 0.8, because [1) it is the common standard in the field; 2) it is the

journal publishing requirement]. Because [1) we have no clear expectation of the magnitude of delta 2) we expected the delta to be around...], we include power calculations for delta ranging from 0.1 to 0.9. Based on these parameters, minimal sample sizes per group for different hypothetical effect sizes to reach 0.8 TPR can be found in Figure 2.

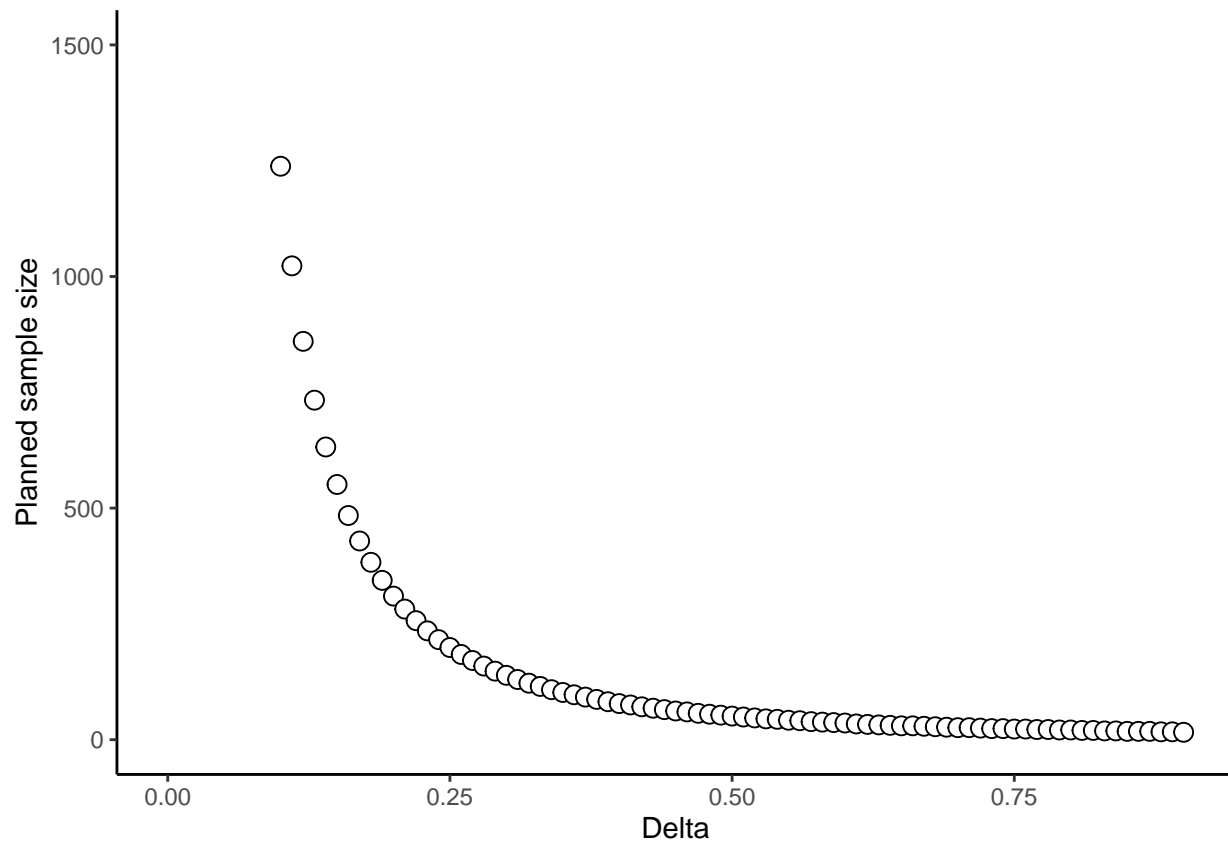


Figure 2. The figure shows the resulting power curve created by the application. The X-axis shows the range of deltas from the example, while the Y-axis shows the corresponding sample sizes determined by the power curve method.

1.2.2. Bayesian.

1.2.2.1. Predetermined sample size with Bayes factor.

Study context

Mary would like to test whether the mean IQ score of the experimental group's population is higher than the mean IQ score of the control group. She'd like to know what

sample size she needs to have for a long-run probability of .80 of obtaining a Bayes factor larger than 10. Mary plans to collect all her data in one batch without testing sequentially. Mary expects the population effect size to be 0.5. This corresponds to a mean IQ score of 107.5 ($100 + 15 \cdot .5$) in the experimental group, assuming a mean IQ score of 100 in the control group.

Description

The present method calculates the corresponding default Bayes factor for a t -test statistic with Cauchy prior distribution centered on 0 with scale parameter of either $1/\sqrt{2}$, 1, or $\sqrt{2}$ for several sample sizes (the so-called Jeffrey-Zellner-Siow Bayes factor, see e.g., Rouder, Speckman, Sun, Morey, & Iverson, 2009). The function returns the optimal sample size needed to reach the TPR for a given Bayes factor threshold to detect an expected population effect size. If a range of possible population effect sizes are plausible under the given hypothesis, the function can calculate the optimal sample sizes for the given range of effect sizes and present the results in a figure (analogous to the Power Curve method). This method is designed to determine the sample sizes for the existence of an effect (i.e., $\Delta > 0$).

Parameters

Delta: The expected population effect size or a range of expected effect sizes.

TPR: The long-run probability of obtaining a Bayes factor at least as high as the critical threshold favoring superiority, given Delta.

Maximum N: The maximum number of participants per group (both groups are assumed to have equal sample size).

PriorScale: The scale of the Cauchy prior distribution. The PriorScale in the application can be set to: $1/\sqrt{2}$, 1, and $\sqrt{2}$.

Threshold: Critical threshold for the Bayes factor. Three threshold levels are available in the app: 3, 6, and 10.

How to use the package

```
SampleSizePlanner::ssp_bf_predetermined(tpr = 0.8, delta = 0.5,  
    thresh = 10, max_n = 5000, prior_scale = 1/sqrt(2))
```

How to report your sample size estimation

We used the Jeffrey-Zellner-Siow Bayes factor method to estimate the sample size.

We used a Cauchy prior distribution centered on 0 with a scale parameter of $1/\sqrt{2}$. We set the aimed TPR at 0.8, because [1) it is the common standard in the field; 2) it is the journal publishing requirement]. The expected delta was 0.5 because [1) previous results published in ...; of the following substantive reasons: ...]. Our evidence threshold was 10. Based on these parameters, a minimal sample size of 105 per group was estimated in order to reach a 0.8 TPR for our design.

1.2.2.2. Bayes Factor Design Analysis (BFDA).

Study context

Mary would like to know what sample size she needs to have a long-run probability of .80 of obtaining a Bayes factor larger than 10. Mary would like to test whether the mean IQ score of the experimental group's population is higher than the mean IQ score of the control group in a Bayesian framework. Mary plans to collect all her data incrementally and as such is interested in using the advantage of not testing more than strictly necessary offered by sequential testing in her Bayesian analysis. Mary expects the population effect size to be 0.5. This corresponds to a mean IQ score of 107.5 in the experimental group ($100+15*.5$), assuming a mean IQ score of 100 in the control group.

Description

The description of the BFDA method is functionally identical to the one provided in section 'Predetermined sample size with Bayes factor,' but gains in TPR due to the addition of sequential testing. In the app, H_0 and H_a indicate the proportion of times

sequential testing leads to Bayes factors providing evidence with the given threshold for the null hypothesis and for the alternative hypothesis, respectively. Users of the Shiny app and R package should set Delta to 0 if they wish to determine the sufficient sample size for rejecting an effect, and use Delta > 0 if they wish to find support for the existence of an effect. For further reading, see Schönbrodt and Wagenmakers (2018) and Schönbrodt, Wagenmakers, Zehetleitner, and Perugini (2017).

Parameters

Delta: The expected population effect size.

TPR: The long run probability of obtaining a Bayes factor at least as high as the critical threshold favoring superiority, given Delta.

PriorScale: The scale of the Cauchy prior distribution. The PriorScale in the application can be set to: $1/\sqrt{2}$, 1, and $\sqrt{2}$.

Threshold: Critical threshold for the Bayes factor. Three threshold levels are available in the app: 3, 6, and 10.

How to use the package

```
SampleSizePlanner::ssp_bfda(tpr = 0.8, delta = 0.5, thresh = 10,  
  n_rep = 10000, prior_scale = 1/sqrt(2))
```

How to report your sample size estimation

We used the BFDA method to estimate the sample size. We used a Cauchy prior distribution centered on 0 with a scale parameter of $1/\sqrt{2}$. We set the aimed TPR at 0.8, because [1) it is the common standard in the field; 2) it is the journal publishing requirement]. The expected delta was 0.5 because [1) previous results published in ...; 2) of the following substantive reasons: ...]. Our evidence threshold was 10. Based on these parameters, a minimal sample size of 81 per group was estimated in order to reach a 0.8 TPR for our design.

2. Estimation

2.1. Frequentist.

2.1.1. Accuracy In Parameter Estimation (AIPE).

Study context.

Mary would like to know what sample size she needs, such that the 95% confidence interval for the population effect size has an expected width of 0.4. She estimates the population effect size to be 0.2.

Description.

Accuracy in parameter estimation aims to determine the sufficient sample size to obtain a confidence interval with a desired width (precision) around the expected effect size (Kelley & Rausch, 2006). Note that the width of the calculated confidence interval will depend on the sample variance. As a result, it is possible that for a given sample the variance is relatively large, leading to a resulting confidence interval that is larger than the width of the desired interval for a given sample. Thus, the AIPE method aims to establish the expected value of the calculated confidence interval, which can be thought of as the 50% long-run probability of obtaining a confidence interval no wider than the provided width.

Parameters.

Delta: The expected population effect size.

Width: The desired width of the confidence interval, given Delta.

Confidence level: The desired level of confidence.

How to use the package.

```
SampleSizePlanner::ssp_aipe(delta = 0.5, width = 0.2, confidence_level = 0.8)
```

How to report your sample size estimation.

In order to estimate the sample size, we used the accuracy in parameter estimation [AIPE; Kelley and Rausch (2006)] method. We aimed for a 95% confidence level, because [1) it is the common standard in the field; 2) it is the journal publishing requirement]. The desired width was 0.4 because [1) previous studies reported the choice of a similar region of practical equivalence; 2) of the following substantive reasons: ...]. We expected an underlying population effect size of 0.3, because [1) previous results published in ...; 2) of the following substantive reasons: ...]. Based on these parameters, a minimal sample size of 195 per group was estimated for our design.

2.1.2. A Priori Precision (APP).

Study context

Mary would like to know the sample size for which she will have a 95% long-run probability that the sample means in both the experimental and the control group lie within 0.2 standard deviations (3 IQ points) of the true population mean.

Description

APP aims to determine the sample size needed to have a certain long-run probability of both sample means being within a certain range of their respective population means, expressed in terms of standard deviations (Trafimow & MacDonald, 2017). As a result, APP is not reliant on the expected effect size.

Parameters.

Closeness: The desired closeness of the sample mean to the population mean defined in standard deviation.

Confidence: The desired probability of obtaining the sample mean with the desired closeness to the population mean.

How to use the package

```
SampleSizePlanner::ssp_app(closeness = 0.2, confidence = 0.95)
```

How to report your sample size estimation

In order to estimate the sample size, we used the a-priori precision [APP; Trafimow and MacDonald (2017)] method. Before data collection, we wanted to be 95% confident that both sample means lie within 0.2 SD of the true population means. Based on these parameters, the resulting minimum sample size was 126 per group for our design.

2.2. Bayesian testimation.

2.2.1. Region of Practical Equivalence (ROPE).

Study context

Mary would like to conduct parameter estimation to see whether the mean IQ score of her experimental group's population is practically equivalent to 100. She would like to know what sample size she needs to have a long-run probability of .80 of obtaining a 95% highest density interval that is contained within her predefined region of practical equivalence (ROPE). Mary hypothesizes that there is no difference (i.e., H_0 is true). She considers a population effect size between -0.2 and under 0.2 to be 'practically equivalent.' This would correspond to IQ scores between 97 ($100+15*(-.2)$) and 103 ($100+15*.2$).

Description

The highest density interval region of practical equivalence technique (HDI-ROPE, often just referred to as ROPE) shares some features with the equivalence interval Bayes factor procedure. Both define an equivalence interval, construct a prior for the population effect size, and update to a posterior after the data comes in. The equivalence interval Bayes factor procedure then focuses on the posterior and prior odds under complementary hypotheses. The ROPE procedure, on the other hand, identifies the 95% highest density interval (HDI; other percentages are permissible as well) and determines whether or not

the HDI is fully contained within the equivalence interval. For further reading, see Kruschke (2018) and Kruschke (2011).

Parameters

Delta: The expected population effect size.

TPR: The desired long run probability of having the HDI fully contained within the ROPE interval, given Delta.

EqBand: The chosen ROPE interval.

PriorScale: The scale of the Cauchy prior distribution. The PriorScale in the application can be set to: $1/\sqrt{2}$, 1, and $\sqrt{2}$.

How to use the package

```
SampleSizePlanner::ssp_rope(tpr = 0.8, delta = 0.5, eq_band = 0.2,  
                             prior_scale = 1/sqrt(2))
```

How to report your sample size estimation

In order to estimate the sample size, we used the Region of Practical Equivalence (Kruschke, 2018) method. We used a Cauchy prior distribution centered on 0 with a scale parameter of $1/\sqrt{2}$. We set the aimed TPR at 0.8, because [1) it is the common standard in the field; 2) it is the journal publishing requirement]. We consider all effect sizes below 0.2 equivalent to zero, because [1) previous studies reported the choice of a similar region of practical equivalence; 2) of the following substantive reasons: ...]. The expected delta was 0 because [1) we expected no difference between the groups]. Based on these parameters, a minimal sample size of 517 per group was estimated in order to reach a 0.8 TPR for our design.

Summary

Justifying the decisions made during the sample size planning process presents valuable information when one evaluates the inferences drawn from a study. The Shiny app and R package presented in this paper aim to help researchers to choose and employ their sample size estimation method. In addition, the tool provides assistance in reporting the process and justification behind sample size choices. We encourage users and experts of the field to provide feedback and recommendations towards further developments.

Authors contribution

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Project Administration: Marton Kovacs.

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Notes

Glossary

Accuracy in Parameter Estimation (AIPE): A sample size estimation method used for parameter estimation. The approach aims to find the required sample size, such that the confidence interval has a certain expected width.

Priori Procedure (APP): The approach aims to plan a sample size based on how close the researcher wishes both sample means to be to their respective population parameter, and how confident the researcher wants to be in this.

Bayesian inference: A general framework for updating one's prior beliefs in light of new data.

Bayes Factor Design Analysis (BFDA): This technique provides an expected sample size such that compelling evidence in the form of a Bayes factor can be collected for a given effect size with a certain long-run probability when allowing for sequential testing.

Testing vs. Estimation: Two schools of inference, focusing on establishing whether or not an effect exists versus establishing the magnitude of an effect, respectively.

Equivalence band (EqBand): The region of effect sizes considered practically equivalent to zero. In our paper, SESOI and ROPE are subsumed under EqBand.

Frequentist inference: A general framework in which probabilities are defined as frequencies in hypothetical repeated events. In the context of statistical testing, frequentist inference is concerned with long-run error rates of rejecting the null hypothesis for the observed or more extreme parameters in a given design when the model assumptions (e.g., independence of observations) are true.

Statistical power: The long-run probability of finding a significant effect given a certain population effect size.

True positive rate (TPR): The long-run probability of finding evidence for an effect, given that it exists. In our paper, statistical power is subsumed under TPR.

Classical power analysis: This method is used to estimate the minimum sample size that a design needs to reach a certain level of statistical power, given a desired significance level and expected effect size.

Power-curve: This curve shows how changes in effect size modify the statistical power of a test.

Region Of Practical Equivalence (ROPE): The region of effect sizes considered

515 practically equivalent to zero under the HDI-ROPE method.

516 *Smallest Effect Size Of Interest (SESOI)*: The region of effect sizes considered

517 practically equivalent to zero under the TOST method.

518 *Sequential testing*: The practice of incrementally testing as data comes in, typically

519 until some pre-determined level of evidence is obtained.

520 *Two One-Sided Tests (TOST)*: A frequentist statistical testing approach aimed at

521 establishing equivalence between two groups.

522 *Equivalence interval BF*: A Bayesian statistical testing approach aimed at

523 establishing equivalence between two groups.

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