Name: Grade:____

Problem 1. Implement an algorithm for computing the least squares problem for the linear system of equations,

$$Ax = b$$

for a (possibly) nonsquare matrix A. The algorithm will need to solve the Normal Equations

$$A^T A x = A^T b$$

as discussed in class.

Problem 2. Implement an algorithm to compute the QR-factorization of a square matrix. Use the Modified Gram-Schmidt algorithm to compute the orthogonal columns of the matrix. You should test the routine on relatively small systems of equations due to roundoff and orthogonality constraints. I would suggest at least one test case where the matrix is a 5×5 .

Problem 3. Describe in detail how you tested your QR-factorization method.

Problem 4. Implement a code that solves a square system of equations using QR-factorization. Use the system in your test problem from Problem 2 and 3 of this assignment. Use the ideas you have seen in previous homework to test your solution method.

Problem 5. Use the QR-factorization solution method from Problem 4 on systems of equations of size n = 10, 20, 40, 80, 160 to fully test your code. Plot out the l_2 -error in the solution as compared to the exact solution of "ones".

Problem 6. Implement a version of the code on page 149 of the textbook that generates a set of data to test the least squares solution of a linear system of equations.

Problem 7. Use the codes you have developed in this homework to solve textbook problem 1 at the end of Chapter 6.

Problem 8. Use the codes you have developed in this homework to solve textbook problem 2 at the end of Chapter 6.

Problem 9. Use the codes you have developed in this homework to solve textbook problem 3 at the end of Chapter 6.