Statement: $-1 = 0$

Proof:

Let $f(n) = \frac{1}{\sqrt{n}+\sqrt{n+1}}$

$f(1) = \frac{1}{\sqrt{1}+\sqrt{2}}$ $f(2) = \frac{1}{\sqrt{2}+\sqrt{3}}$ $f(3) = \frac{1}{\sqrt{3}+\sqrt{4}}$

$f(1) + f(2) + f(3) + \cdots$

$= \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots$

$$f(n) = \frac{1}{\sqrt{n} + \sqrt{n+1}} \times \frac{\sqrt{n} - \sqrt{n+1}}{\sqrt{n} - \sqrt{n+1}}$$

$= \frac{\sqrt{n}-\sqrt{n+1}}{n-(n+1)}$

$= -\sqrt{n} + \sqrt{n+1}$

$f(1) + f(2) + f(3) + \cdots$

$= -\sqrt{1} + \cancel{\sqrt{2}} - \cancel{\sqrt{2}} + \cancel{\sqrt{3}} - \cancel{\sqrt{3}} + \cancel{\sqrt{4}} - \cdots$

$= -\sqrt{1} = -1$

Let $k = f(1) + f(2) + f(3) + \cdots$

$k = -1 < 0,\ k = \frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots > 0 (\frac{1}{\sqrt{1}+\sqrt{2}} > 0, \frac{1}{\sqrt{2}+\sqrt{3}} > 0, \frac{1}{\sqrt{3}+\sqrt{4}} > 0 \ldots)$

$k = -k$

$k = 0$

$-1 = 0$

Q.E.D.