ECE 7600: Introduction to Digital Signal Processing
Assignment #2
Due August 1, 2022

Instructions

You must submit your assignment as a .pdf file to the appropriate Assignment folder (under Assessments) in Brightspace. For each question, please make sure to include i) your Matlab code (copied and pasted into the .pdf file), ii) any plots or other required Matlab outputs, and iii) written responses to any questions that require it. Title any plots with the question number, as well as an informative description. Questions should appear in order. For any "by hand" questions, you can type your responses, or hand write and scan if that's easier…just make sure everything is clear/readable or it will not be marked. You should ***also submit your .m file*** via the appropriate Assignment folder in Brightspace. Please include code for all questions in a single .m file that can be run in Matlab to produce all your plots and other outputs.

Assignments must be done individually, and you must submit your own work.

Problems

1. For each of the finite-length signals below

$$\text{i) } x[n] = \begin{cases} 3, n = 0 \\ 2, n = 1 \\ 1, n = 2 \\ 0, \ else \end{cases} \qquad\qquad \text{ii) } x[n] = \begin{cases} 1, & 0 \le n \le 6 \\ 0, & else \end{cases}$$

   a) In Matlab, compute the N-point DFT, directly from the transform equation, for four different cases: $N = 32, N = 64, N = 128, N = 256$. Recall that for $N > L$ (where L is the length of the non-zero extent of the signal), you need to use zero-padding to achieve the desired N. Compute and plot the magnitudes of the four transforms, $|X_N[k]|$, above using the stem function. Include all four plots in the same figure using the subplot function. Use appropriate labels for your plots.
   b) What is the expression for the DTFT, $X(e^{j\omega})$, of the signal? Find it by hand via the transform equation or by tables of basic transforms and properties. Next, compute the magnitude of the signal's DTFT, $|X(e^{j\omega})|$, for $\omega = {}^{2\pi}/_{32}$ and $\omega = {}^{9\pi}/_{8}$ (you can use Matlab). What value of k do these frequencies correspond to in each of the N-point DFTs from a)? Do the values for the DFTs and DTFT align as you expect them to? Explain.

2. Repeat Question 1a, this time using the fft function in Matlab. The fft function computes the DFT using the very efficient fast Fourier transform algorithm. Compare the results of the fft with your computation in Question 1.

3. Assume that the following discrete-time signals were obtained by sampling continuous-time signals at the sampling rate indicated. When visualizing a DT signal obtained through sampling a CT signal, we usually want to retain the original time information and plot it as though it was still the original continuous signal, rather than plotting it versus the sample number. In each case below, find the expression for the original signal, $x(t)$, and plot it against the continuous variable t. Use the "plot" function rather than "stem".
   a) $x[n] = \sin({}^{\pi}/_{6}\, n)\, u[n]$, obtained from sampling at $f_s = 100Hz$. Plot from 0 to 0.5 seconds.
   b) $x[n] = 0.9^n \cos({}^{\pi}/_{8}\, n)\, u[n]$, obtained from sampling every 0.25 seconds. Plot from 0 to 15 seconds.
   c) $x[n] = n$, obtained from sampling at $f_s = 20\ samples/second$. Plot from 0 to 2.5 seconds.

4. In Matlab, load the file Assign2_EEG.mat which can be found in the Assignments folder on the D2L website. Use the command EEG=load('Assign2_eeg.mat') to load the file. This is a segment of EEG data that was recorded at a sampling rate of 256 Hz.
   a) What was the precise duration of the recording (to the nearest millisecond)? Plot the signal as a continuous function of time.
   b) A lowpass filter has been applied to this signal to greatly attenuate frequencies above a certain frequency. Use what you've learned about the DFT to find out what the cutoff frequency of this lowpass filter was in radians per sample, and what frequency that relates to in Hz (i.e., cycles per second).

5. Consider the system function, $H(z)$, given below.

$$H(z) = \frac{0.0001201z^8 + 0.0009608z^7 + 0.003363z^6 + 0.006725z^5 + 0.008407z^4 + 0.006725z^3 + 0.003363z^2 + 0.0009608z + 0.0001201}{z^8 - 3.919z^7 + 7.325z^6 - 8.275z^5 + 6.106z^4 - 2.989z^3 + 0.9423z^2 - 0.1742z + 0.01442}$$

a) Using the Filter Visualization Tool in Matlab (fvtool), plot the system's i) frequency response (magnitude and phase), ii) impulse response, $h[n]$, and iii) pole-zero map. Note that you can right click on the plot and select "Analysis Parameters" to change the units of the magnitude plot from dB.

b) This system is a filter. i) Is it an IIR or FIR filter? ii) Is it lowpass, highpass or bandstop? Iii) What is the approximate cut-off frequency of the filter?

c) Demonstrate the functionality of the filter by creating an input signal, $x[n]$, with different frequency components, and showing that after applying the filter to $x[n]$ some of the frequencies are attenuated.