ECE 7600: Introduction to Digital Signal Processing
Assignment #1
Due June 27, 2022

This assignment is meant to introduce you to the use of Matlab as it relates to the course topics, and also help with understanding/visualizing the course material. Matlab (or a similar computational analysis package) is essential for practical digital signal processing – while calculating convolution sums and Fourier series coefficients by hand is very important for understanding underlying concepts, it's not going to get you very far in most real-world applications.

I have posted some material on Matlab basics and some example code to help you. You may also find the help files very….well, helpful. When creating plots, make sure to label them clearly using the "title", "xlabel", and "ylabel" functions. You may also find the "sprintf" function useful when doing such labelling.

Assignments must be done individually, and you must submit your own work.

Instructions

You must submit your assignment as a .pdf file to the appropriate Assignment folder (under Assessments) in Brightspace. For each question, please make sure to include in the .pdf file i) your Matlab code (copy and paste the code into the .pdf file for each question), ii) any plots or other required Matlab outputs (again, copy and paste them into the .pdf file), and iii) written responses to any questions that require it. Title any plots with the question number, as well as an informative description (e.g., "Question 1c: h_A[n] found using the 'filter' function"). Questions should appear in order in the .pdf file. For "by hand" questions, you can type your responses, or hand write and take a photo/scan if that's easier…just make sure everything is clear/readable or it will not be marked. You should ***also submit your .m file*** via the appropriate Assignment folder in Brightspace. Please include code for all questions in a single .m file that can be run in Matlab to produce all your plots and other outputs. Again note that your code and plots and any other Matlab outputs should be pasted into the .pdf file for each question – if it is not you will not get credit for it.

Problems

1. System A is a causal LTI system described by the linear constant coefficient difference equation

System A: $$y[n] - y[n-1] + y[n-2] = x[n]$$

a) Compute (by hand) the system's impulse response function, $h_A[n]$, directly from the difference equation. Compute $h_A[n]$ over a sufficient interval of n to capture its behaviour for all n.
b) Use Matlab to compute the system's impulse response function, $h_A[n]$, directly from the difference equation. Plot $h_A[n]$ vs. n for $-5 \le n \le 60$.
c) Use the "filter" function in Matlab to find the system's impulse response function, $h_A[n]$, using the coefficients of the difference equation. Plot $h_A[n]$ vs. n for $-5 \le n \le 60$.
d) Is the length of $h_A[n]$ what you would have expected based on the difference equation? Why?
e) Use the "filter" function to find the system's response to the input $x[n]$ below. Plot both $x[n]$ vs. n and $y[n]$ vs. n for $-5 \le n \le 60$ in the same figure using the subplot command.

$$x[n] = \cos({2\pi}/_{6}\,n)u[n]$$

f) Is this a stable system? Justify your answer in two ways: using $y[n]$ from part e, and using just $h_A[n]$.

2. Consider a second causal LTI system, System B, with impulse response equal to:

$$\text{System B:} \qquad h_B[n] = h_A[n](u[n] - u[n-5])$$

where $h_A[n]$ is the impulse response of System A from Question 1.

a) In Matlab, compute and plot $h_B[n]$ vs. n for $-5 \leq n \leq 60$.

b) Since this is an FIR system, the linear constant coefficient difference equation can be found directly from the convolution sum. Find (by hand) the difference equation for System B.

c) Using the "filter" function and the coefficients from the difference equation found in b), find the system's response to the input $x[n]$ below (same as from Question 1). Plot both $x[n]$ vs. n and $y[n]$ vs. n for $-5 \leq n \leq 60$ in the same figure using the subplot command.

$$x[n] = \cos({}^{2\pi}/_{6}\, n)u[n]$$

d) Is this a stable system? Justify your answer.

e) Now find System B's response to $x[n]$ above through convolution using the conv function in Matlab. Be careful to define $x[n]$ and $h[n]$ starting at $n = 0$ (and note what happens when you define them starting at $n = -5$ as in the other questions). Plot both $x[n]$ vs. n and $y[n]$ vs. n for $0 \leq n \leq 60$ in the same figure using the subplot command. How do your results compare to those from part c?

f) What is the total length of the output, $y[n]$, from part e. Why is it this length? Is it valid for that whole length? Why or why not?

3. Load the file named 'Assignment1_Q3_DTsignal.mat' available in the Course Content->Assignments->Assignment #1 folder in Brightspace. The variable "xn" contains the signal $x[n]$, which is a rectangular pulse buried in random white noise. Plot $x[n]$ using the "plot" function (rather than the "stem" function). A simple moving average filter is a very effective way to reduce the unwanted noise in this scenario, by "smoothing" the signal in the time domain. Determine the appropriate difference equation or impulse response, $h[n]$, for an N-point moving average FIR filter, and use either the "filter" or the "conv" function to find the output $y_1[n]$ when the filter is applied to $x[n]$. Plot $y_1[n]$, overlaid onto the plot of $x[n]$. Explore different filters with different values of N between 5 and 50, plotting the output of each filter (call them $y_2[n]$, $y_3[n]$, etc.). Describe the effect of using a lower value of N versus a higher value. What would you say is a good value of N to use in this case, and why?

4. The signal $x[n]$ is a discrete-time, even periodic square wave with $N = 32$ and $N_1 = 4$.

a) In Matlab, compute and plot the Fourier series coefficients for $x[n]$. Find the coefficients through direct computation via the Fourier series analysis equation, rather than through a built-in Matlab function like FFT.

b) Now, using the Fourier series coefficients you found in a), use the synthesis equation to generate the original signal $x[n]$ (do this in Matlab). Plot $x[n]$ including just the first i) 8, ii) then 16, iii) then 24, and finally iv) all 32 harmonic frequencies in the sum. Have we managed to represent our signal exactly as a sum of complex exponentials?

5. Load the file named 'Assignment1_Q5_DTsignal.mat' available in the Course Content->Assignments-> Assignment #1 folder in Brightspace.

a) The variable "xa" contains a discrete-time signal, $x_a[n]$. $x_a[n]$ is a linear combination of three different discrete-time periodic signals of the form $x_k[n] = \sin(\frac{2\pi M_k n}{N})$ or $x_k[n] = \cos(\frac{2\pi M_k n}{N})$, where M_k and N are positive integers. Specifically, $x_a[n] = x_1[n] + x_2[n] + x_3[n]$. Use Matlab to help you determine $x_1[n]$, $x_2[n]$, and $x_3[n]$. Show how you arrived at your answer, including any Matlab outputs.

b) Repeat the above for the signal in the variable "xb".