512 PS5

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Based on the χ^2 value, are the parameters dialed into my test script an acceptable fit?
The chisq value was calculated using the following formula:

$$\chi^2 = \sum_i^N \left(\frac{data_i \text{ - } model_i}{\sigma_i} \right)^2$$

where σ_i is the associated error with a data point $model_i$.

The χ^2 value I get from the script for the parameters dialed into the test script is:

$$\chi^2 = 15267.937150261654$$

Using the degrees of freedom, which are given as 2501, we can compute the reduced chi squared statistic as follows

$$\chi^2_{reduced} = \frac{15267.937}{2501} = 6.10473296692$$

Which, since the value is much larger than 1, suggests that the model is a poor fit to the data.

What do you get for χ^2 for parameters equal to [69,0.022,0.12,0.06,2.1e-9,0.95?]

For the chi squared value, I got

$$\chi^2 = 3272.2053559$$

Would you consider these values an acceptable fit?
From this, we can once again calculate the reduced chi square value:

$$\chi^2_{reduced} = \frac{3272.2053559}{2501} = 1.30835879884$$

Since the reduced chi squared value is around 1, we can say that this is approaching a better fit to the data. It is not perfect, but it is far superior than the earlier parameters.

2

Use newton's method to find the best-fit parameters
I used the newton's method solver I wrote for problem set 4. The errors in the parameters were calculated the regular way,

$$\sigma_m = \sqrt{(A^T N^{-1} A)^{-1}}$$

Where A is the curvature matrix given by newtons method and N^{-1} is given by the diagonal $\frac{1}{\sigma_i{**}2}$, assuming that errors are not correlated.

I have stored the best fit parameters and their associated errors in but for quick reference their values are.

$$p = [68.02486080899662, 0.022297145855993544, 0.11755522241214612,$$

$$0.03374469033405603, 2.001337623107444e-09, 0.9704951233926593]$$

$$\delta p = [7.10392710e-01, 2.28932207e-04, 1.28267891e-03,$$
$$1.15521756e-02, 4.44550143e-11, 4.26260148e-03]$$

The associated chi squared value of this parameters is: 2580
Bonus
What are the best-fit parameters when the dark–matter density is set to zero?
From very very slowly stepping my dark matter density down to 0 in around 200 steps (1% every 2 steps), I got the following parameters for my space:

$$H0 = 2.24803238e+02$$

$$\text{Baryon Density} = 3.16886099e-02$$

$$\text{Dark matter density} = 0$$

$$\text{Optical Depth} = 7.78530924e-01$$

$$\text{Primordial Amplitude of the spectrum} = 6.91900415e-09$$

and

$$\text{Primordial tilt of the spectrum} = 1.47215710e+00$$

$$\chi^2_{\text{No Dark Matter}} = 10931.341595868582$$

Which indicates a very poor fit. This is best shown in the following figure:

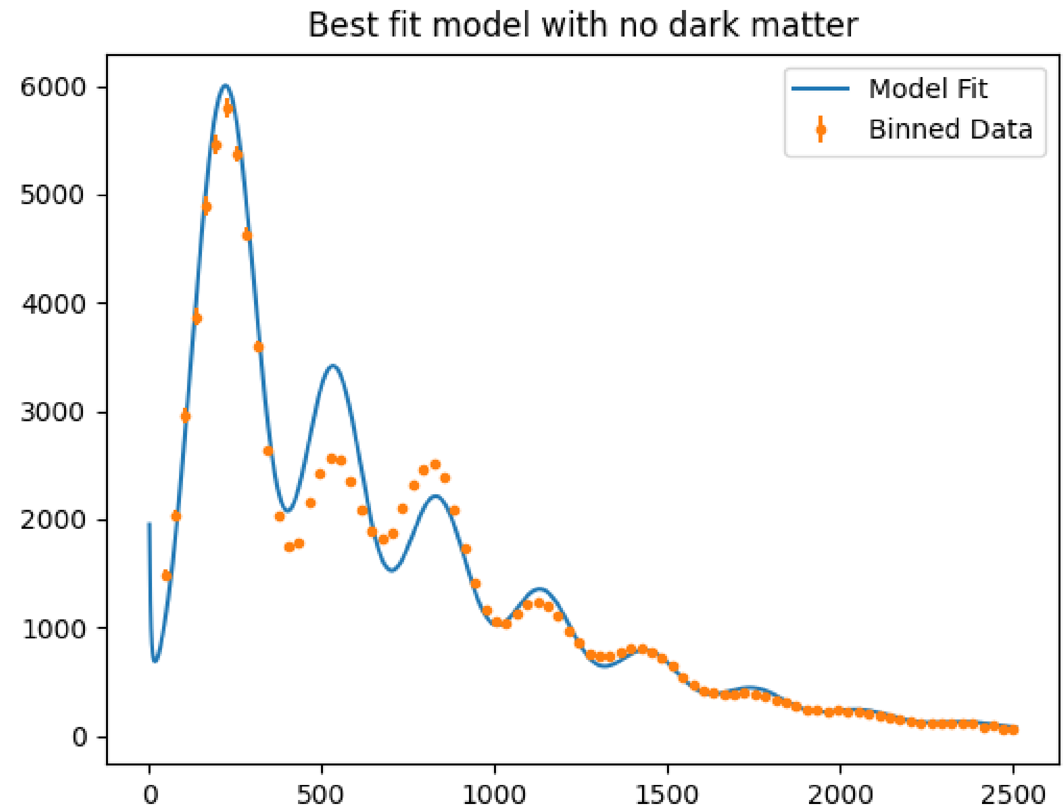

Figure 1: Best fit model with no dark matter. As can be seen, the model attempts to capture the peaks in the power spectrum but just can't quite get them quite right...

How does the χ^2 value compare to the standard value?
This chi square value of 10931 is approximately four times larger than the standard chi square value of 2580 from the improved parameters Jon gave. This shows that a model not accounting for dark matter in the universe will give a poor description of the CMB data.

3

Estimate parameter values and uncertainties using an MCMC sampler
I ran an mcmc chain using a guess for the parameters that Jon gave us and the errors in my converged newtons method times a constant for my steps. I did not change the temperature on the chains. Before cutting off the burn in the chains looked as follows:

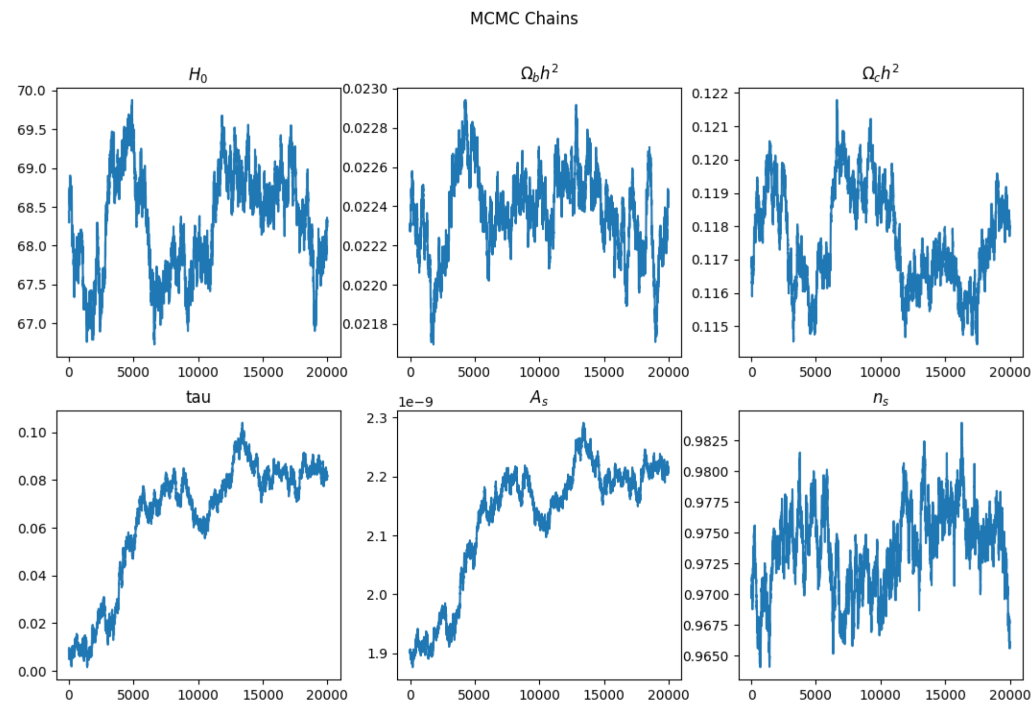

Figure 2: Chain before cutting off the burn in. As can be seen the tau and primordial amplitude values needed to burn in before converging

The mean and uncertainty of cosmological values was calculated using the mean and standard deviation of converged chain values after the burn in was cut off. The parameters and their uncertainties (rounded) were as follows

$$H_0 = 68.4 \pm .6$$

$$\Omega_b h^2 = 0.0224 \pm 0002$$

$$\Omega_c h^2 = 0.118 \pm 0.001$$

$$\tau = 0.079 \pm .009$$

$$A_s = 2.19e - 9 \pm 4e - 11$$

$$n_s = .974 \pm .003$$

The chi square given by these parameter values was: 2580.490 *Save your chain in planck_chain.txt* You will find the raw data for the chains in this file.

Why do you think your chains are converged (if they have converged)
So if we plot the chain after cutting out the chain

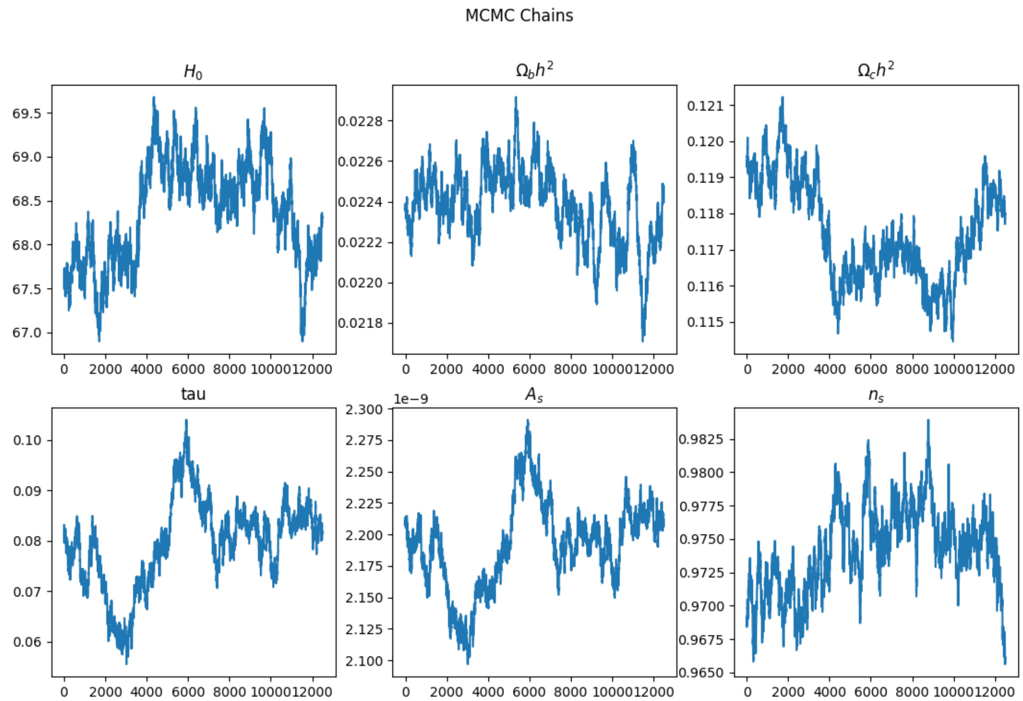

Figure 3: Plot of the chains after cutting out the burn in

and then the power spectrum

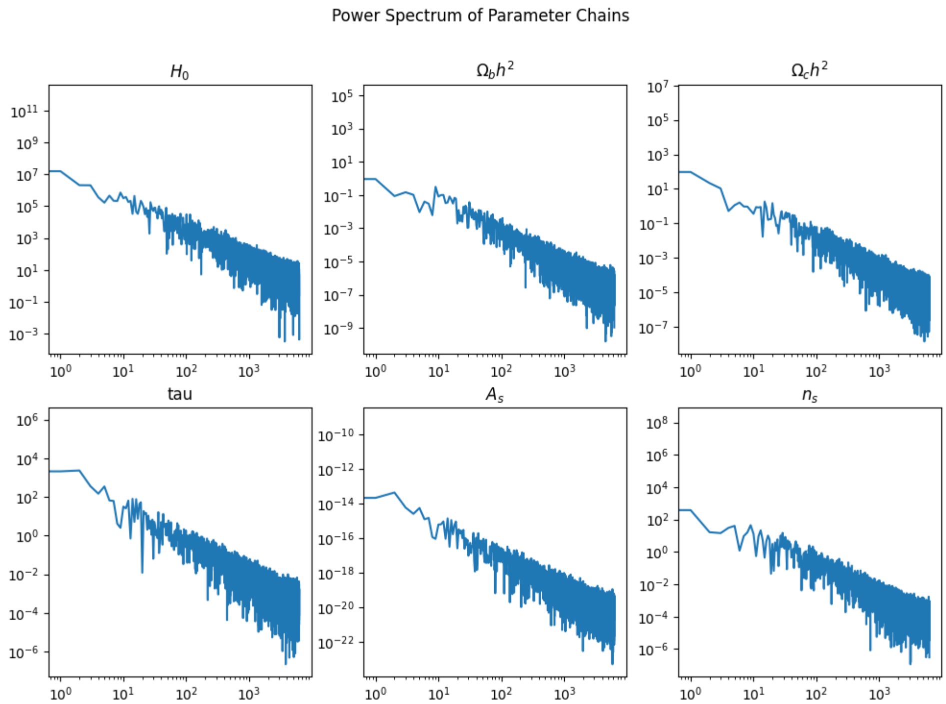

Figure 4: Power spectrum of the chains after cutting out the burn in. As can been seen, the chains start out relatively flat, but quickly angle downward, implying a lack of independent samples.

As can be seen by the power spectrums of the parameter chains, some of the samples have converged, more specifically $\Omega_b h^2$,H_0, and n_s. The other chains have more correlation implying they haven't quite converged.

However, when we plot the parameters we got from our MCMC vs. our data we can see

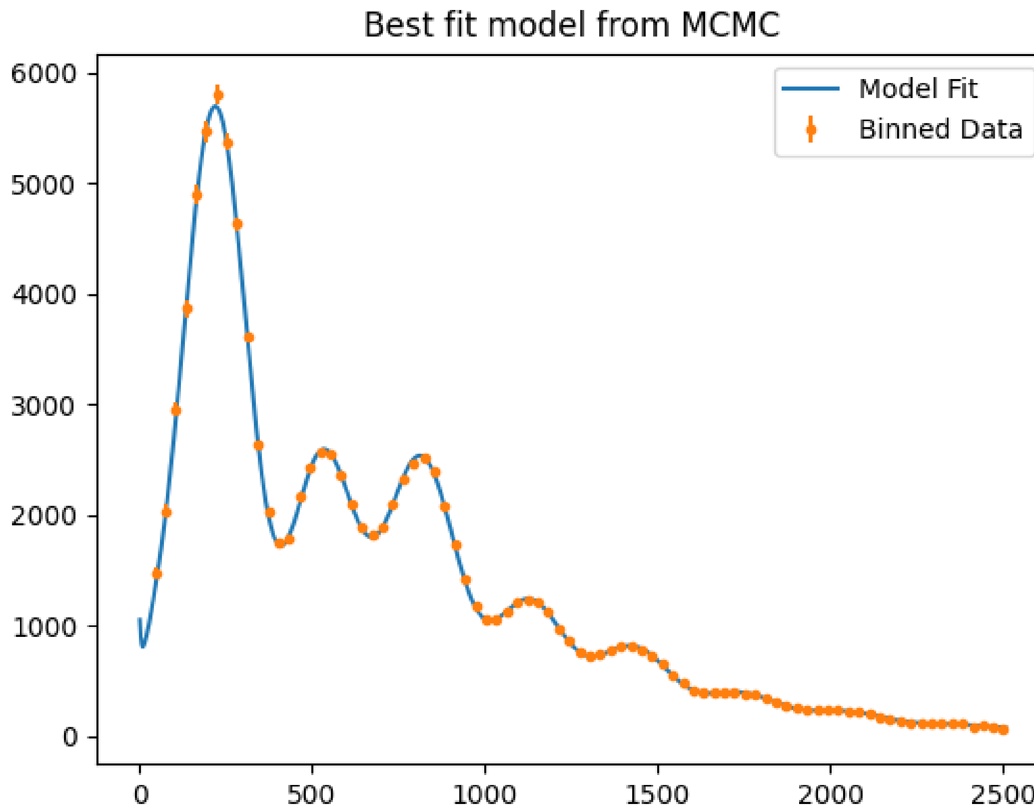

Figure 5: Plot of our model vs. the data

It gets quite close to capturing the intricacies of the data.

What is your estimate on the mean value of dark energy and its uncertainty?
Given that

$$h = H_0/100$$

our value of h given our parameters is:

$$h = 68.4/100 = 0.684$$

and our value squared is:

$$h^2 = 0.467856$$

Using this value we can now get values for Ω_b, Ω_c.

$$\Omega_b = \Omega_b h^2/h^2$$

$$\Omega_b = 0.0224/.467856 = 0.04787797954$$

and

$$\Omega_c = \Omega_c h^2/h^2$$

$$\Omega_c = 0.118/.467856 = 0.2522$$

Using the relation from the assignment

$$\Omega_b + \Omega_c + \Omega_\Lambda = 1$$

$$\Omega_\Lambda = 1 - 0.04787797954 - 0.2522$$

Therefore,

$$\Omega_\Lambda = 0.6999$$

4

Run a new chain where we include the constraint on τ. Save the results

I ran a chain where I include the constraint on tau. The way the constraint was included was by looking at the range of accepted tau values. If the the tau value was outside of the accepted range, we rejected the step. The chain is saved as: *planck_chain_tauprior.txt*.

What about importance sampling?
From the notes, importance sampling can be implemented in the following way: by adjusting the phase space density to the new variable

$$L'(p)/L(p)$$

Which since we're just using chi square can be written as:

$$e^{-\frac{1}{2}\delta\chi^2}$$

Therefore we can right our liklihood sample as:

$$e^{-\frac{1}{2}(chisqNewParam - chisqTrial)}$$

by normalizing over the sum of all the ratios and then multiplying these values over the entire chain, more heavily weighing values where tau is in the constrained limits:

The weighted sampling gives parameters of:

[6.82175744e+01 2.22497674e-02 1.17661477e-01 2.19281089e-02
1.95976301e-09 9.72395791e-01]

Which are practically indistinguishable from our mcmc priors. This makes sense as our tau value in our unconstrained chain was nowhere close to the tau value in our constrained chain.

What did we get from our chain The results we got from our constrained chain were the following:

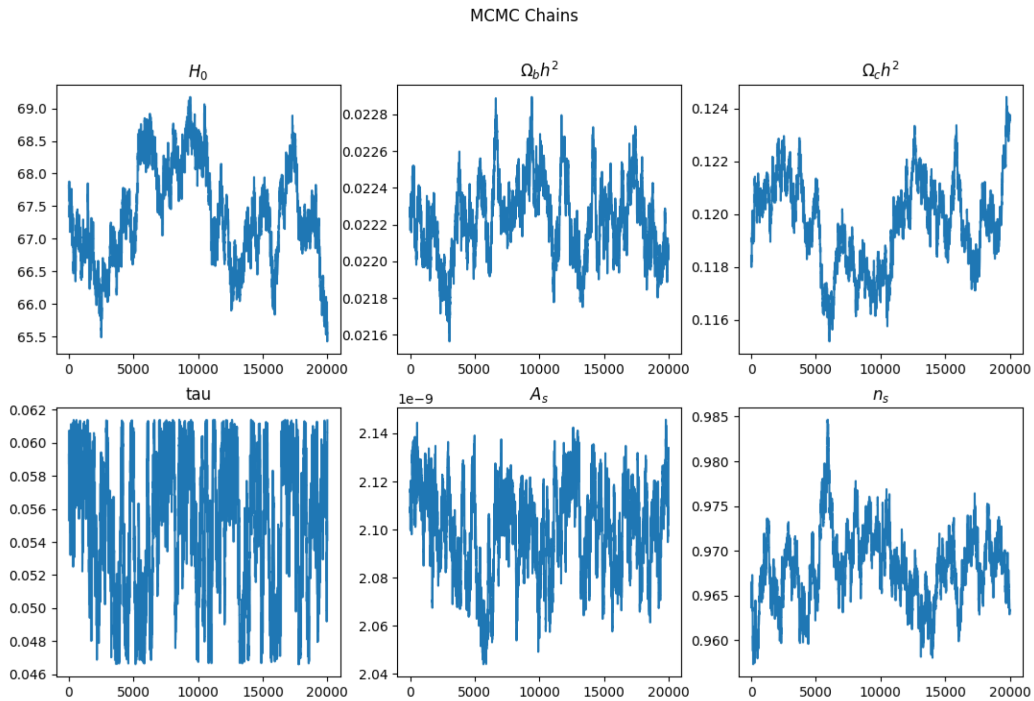

Figure 6: MCMC chains of our constrained tau mcmc

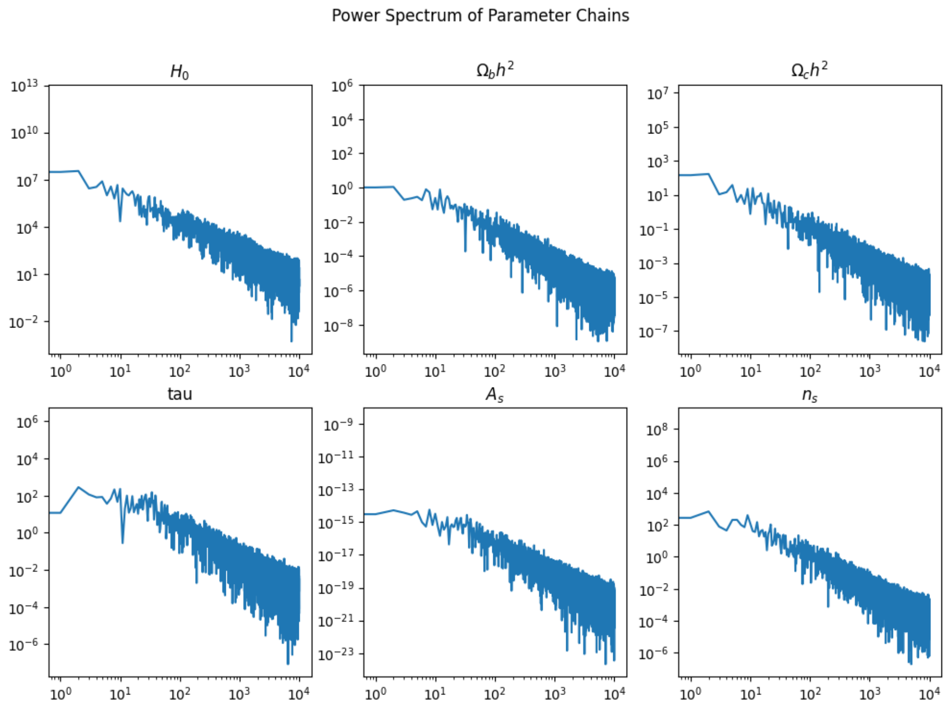

Figure 7: Power spectrum of our constrained tau plot.

As can be seen by both the chains and the power spectrum plots, the data looks a lot more like white noise. This corresponds to a more converged looking chain a flatter power spectrum.
The corner plot looks as follows

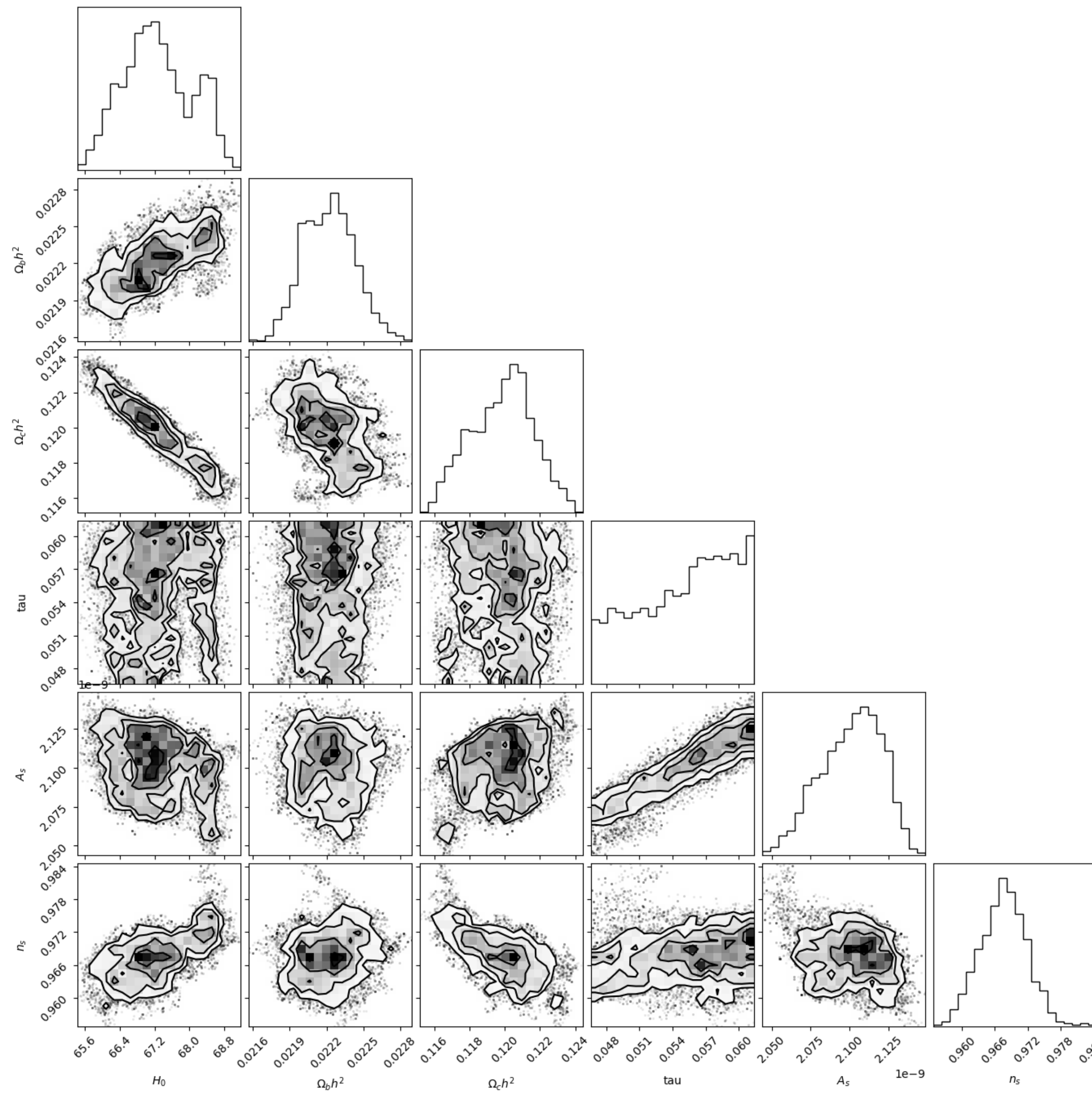

Figure 8: Corner plot of the tau variables. As can be seen the value τ and A_s are highly correlated as well as the values H_0 and $\Omega_b h^2$

These corner plots show mostly blobby behabior with some smearing wrt tau. This was probably because the tau values were taken almost uniformly across the entire bin instead of converging like the rest of the chains. Possibly by taking smaller steps. The parameter values were as follows:

$$H_0 = 67.4 \pm 0.8$$

$$\Omega_b h^2 = 0.0222 \pm 2$$

$$\Omega_c h^2 = 0.119 \pm 2$$

$$\tau = 0.055 \pm 0.004$$

$$A_s = 2.098e - 9, 2e - 11$$

$$n_s = 0.969 \pm .004$$

Compare those results to what your get from importance sampling
The values from the importance sampling was closer to accepted values for the hubbles constant, however all the values were actually pretty close to each other except for the optical depth.

Adendum: What went wrong

I realize that these MCMC chains did not really converge, unfortunately. Therefore most of the error in my method was that I did not have converged chains to work with around accepted values. For instance, on some of my chains my tau values went negative, an unphysical phenomena. I suspect that this mostly had to do with the size of my steps, or their ratio. By having one or two values converge and then letting the other (often) correlated parameters cancel each other out with smaller steps it was possible to get a wandering behavior.