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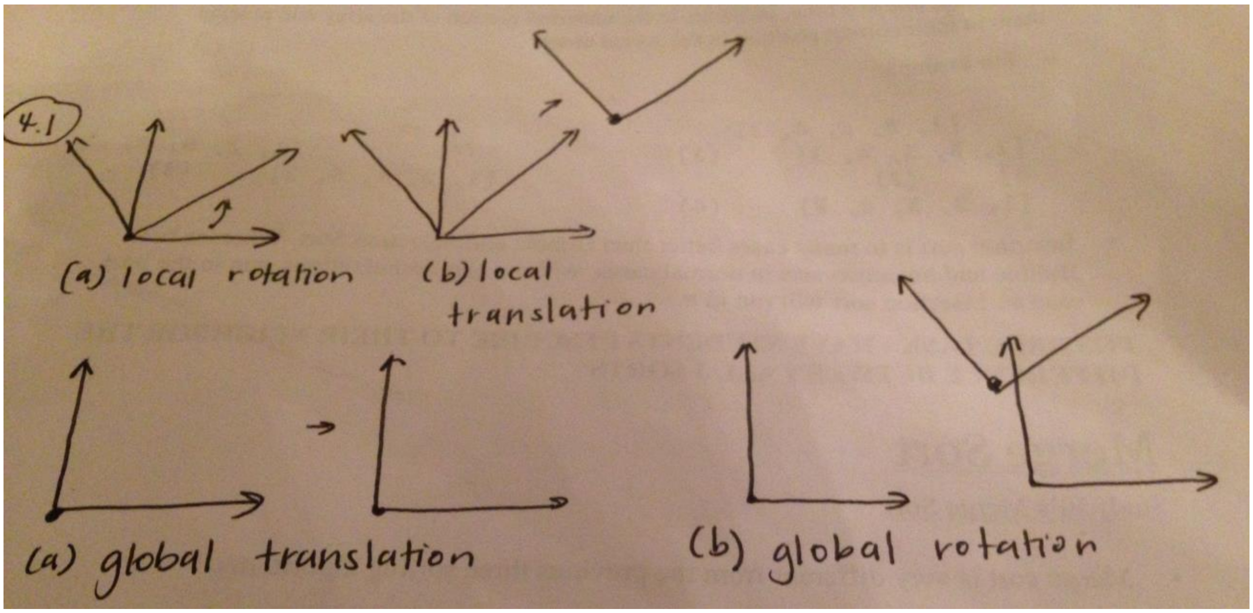
Pset 1.5 Written Exercises

4.1)

(The figures are illustrated in the diagrams below, not to scale)



4.2)

The equation $\vec{f}^{\,t} = \vec{f}^{\,t}ST$ indicates a scale by two with respect to the original frame, then translates it by one unit with respect to the new frame, so the origin moves by **two units** of the original frame.

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4.3)

- ✓ R is Rotation by Ø, T is translation by d_1 in the x direction and then d_2 in the y direction
- ✓ A generic rotation in 2D can be represented as:

$$\begin{bmatrix} cos\emptyset & -sin\emptyset \\ sin\emptyset & cos\emptyset \end{bmatrix}$$

- ✓ In both cases, where $\underset{b}{\rightarrow}^{t} = \underset{a}{\rightarrow}^{t}$RT and $\underset{b}{\rightarrow}^{t} = \underset{a}{\rightarrow}^{t}$TR, then the rotation is the same. The matrix of rotation should be:

$$\begin{bmatrix} cos\emptyset & -sin\emptyset & 0 \\ sin\emptyset & cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ If we look at the equation $\underset{\boldsymbol{b}}{\rightarrow}^{\boldsymbol{t}} = \underset{\boldsymbol{a}}{\rightarrow}^{\boldsymbol{t}}$RT and we do the rotation first, then we should do then translation with respect to the *local frame*, which means that we should translate d_3 units in the x direction and d_4 in the y direction. This matrix can be represented as:

$$\begin{bmatrix} 1 & 0 & d_3 \\ 0 & 1 & d_4 \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ If instead we look at the equation $\underset{\boldsymbol{b}}{\rightarrow}^{\boldsymbol{t}} = \underset{\boldsymbol{a}}{\rightarrow}^{\boldsymbol{t}}$TR and we do the rotation first, then we should do then translation with respect to the *global frame*, which means that we should translate $d_1$units in the x direction and d_2 in the y direction. This matrix can be represented as:

$$\begin{bmatrix} 1 & 0 & d_1 \\ 0 & 1 & d_2 \\ 0 & 0 & 1 \end{bmatrix}$$

- ✓ So to summarize, for the equation $\underset{\boldsymbol{b}}{\rightarrow}^{\boldsymbol{t}} = \underset{\boldsymbol{a}}{\rightarrow}^{\boldsymbol{t}}$RT

$$R = \begin{bmatrix} cos\emptyset & -sin\emptyset & 0 \\ sin\emptyset & cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 & d_3 \\ 0 & 1 & d_4 \\ 0 & 0 & 1 \end{bmatrix}$$

for the equation $\underset{\boldsymbol{b}}{\rightarrow}^{\boldsymbol{t}} = \underset{\boldsymbol{a}}{\rightarrow}^{\boldsymbol{t}}$TR

$$R = \begin{bmatrix} cos\emptyset & -sin\emptyset & 0 \\ sin\emptyset & cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad T = \begin{bmatrix} 1 & 0 & d_1 \\ 0 & 1 & d_2 \\ 0 & 0 & 1 \end{bmatrix}$$

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4.4)

M can be thought of as the composition of two transformations: the transformation that takes $\underset{\boldsymbol{a}}{\rightarrow}^{\boldsymbol{t}}$ to $\underset{\boldsymbol{b}}{\rightarrow}^{\boldsymbol{t}}$, and then the transformation that takes $\underset{\boldsymbol{b}}{\rightarrow}^{\boldsymbol{t}}$ to $\underset{\boldsymbol{c}}{\rightarrow}^{\boldsymbol{t}}$**.** We know that the first transformation is expressed by the matrix N, which we can think of as a translation up and to the right of distance *d* followed by a rotation by the angle β. We see that the second transformation can also be expressed as a translation of distance *d* and a rotation by an undetermined angle. Because it translates down and to the left, we can think of the second transformation as N^{-1}. However, M is clearly not as simple as NN^{-1} (the identity matrix), and we can see that there is an intermediate transformation, which is simply rotation by ∅, since the β rotations cancel out.

Therefore M = $N\begin{bmatrix} cos\emptyset & -sin\emptyset & 0 \\ sin\emptyset & cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix}N^{-1}$