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1) See attached code

2) 300 shares at \$40 ⇒ \$12,000

initial margin: 50 % initial ⇒ \$6000

Assets = \$12,000 + \$6000 = \$18,000

Liabilities = \$12,000 (borrowed stocks)

Equity = \$18000 - \$12000 = \$6000

b) Must maintain at least 25% of liability in equity, x = total new shares value

⇒ Equity = \$18000 - x

$0.25x = \$18000 - x$

$1.25x = \$18000$

$x = \$14400$ ⇒ $x = 300 \cdot$ (new share value) $= \$14,400$ $\boxed{\therefore \text{ new share value} = \$48}$

c) Stock price = \$35 ⇒ liabilities = 300 · \$35 = \$10500 and Equity = \$18000 - \$10500 = \$7500

$$\text{Leverage}_{\text{Before}} = \frac{\text{Total liabilities}}{\text{Equity}} = \frac{\$12000}{\$6000} = 2$$

$$\text{Leverage}_{\text{After}} = \frac{\text{Total liabilities}}{\text{Equity}} = \frac{\$10500}{\$7500} = 1.4$$

$$\text{ROE} = \frac{\text{Earnings}}{\text{Initial Equity}} = \frac{\$12000 - \$10500}{\$6000} = 0.25$$

$$\text{Margin Ratio} = \frac{\text{Equity}}{\text{Liability}} = \frac{\$7500}{\$10500} = 0.714$$

3) $R_1 = a_1 + b_1 R_m + \epsilon_1$, $R_2 = a_2 + b_2 R_m + \epsilon_2$

$$R_{Tot} = w_1 R_1 + w_2 R_2 = w_1(a_1 + b_1 R_m + \epsilon_1) + w_2(a_2 + b_2 R_m + \epsilon_2)$$

$$= R_m(b_1 w_1 + b_2 w_2) + w_1(a_1 + \epsilon_1) + w_2(a_2 + \epsilon_2)$$

$R_m, \epsilon_1, \epsilon_2$ uncorrelated ⇒ $var(R_{Tot}) = var(R_m(b_1 w_1 + b_2 w_2)) + var(w_1(a_1 + \epsilon_1)) + var(w_2(a_2 + \epsilon_2))$

$$\Rightarrow var(R_{Tot}) = (b_1 w_1 + b_2 w_2)^2 \sigma_m^2 + w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2$$

$$\mu_{tot} = \mu\left(R_m(b_1w_1 + b_2w_2)\right) + \mu\left(w_1(a_1+\epsilon_1)\right) + \mu\left(w_2(a_2+\epsilon_2)\right)$$

$$= b_1w_1\mu_m + b_2w_2\mu_m + w_1a_1 + w_1\mu_1 + w_2a_2 + w_2\mu_2$$

$$P(R_{tot} < 0) = P\left(\frac{R_{tot} - \mu_{tot}}{\sigma_{tot}} < \frac{0-\mu_{tot}}{\sigma_{tot}}\right) = \phi\left(\frac{0-\mu_{tot}}{\sigma_{tot}}\right)$$

$$\Rightarrow A = 0,\quad B = 1,\quad \beta = \left(\frac{-\left((b_1w_1 + b_2w_2)\mu_m + w_1a_1 + w_1\mu_1 + w_2a_2 + w_2\mu_2\right)}{(b_1w_1+b_2w_2)^2\sigma_m^2 + w_1^2\sigma_1^2 + w_2^2\sigma_2^2}\right)$$

4) Loan A: $\$120{,}000 \cdot 0.92 = \$110400$ so percent annual rate is $\frac{\$9600}{\$110400} = 8.70\%$

Loan B: $\$110{,}000 \cdot 0.94 = \$103400$ so percent annual rate is $\frac{\$6600}{\$103400} = 6.38\%$

Loan C: $\$130{,}000 \cdot 0.935 = \$121550$ so percent annual rate is $\frac{\$8450}{\$121550} = 6.95\%$

Never take loan A as the interest rate is much higher than the other two loans, especially considering loan C provides more cash and a smaller rate. Between loan B and C, one might consider B if they do not wish to borrow as much cash for a cheaper loan, or one might choose C for the extra-cash at a slightly higher cost

5) $S_n = \begin{cases} uS_{n-1} & \text{w/ prob } p_u \\ dS_{n-1} & \text{w/ prob } p_d \end{cases}$ no arbitrage since $d < e^r < u$

Initial Portfolio at $t=0$: $X_0 = \Delta_0 S_0 + \Psi_0$

at $t=1$: $X_1 = \Delta_1 S_1 + \Psi_1 e^r \Rightarrow X_1 = \Delta_1(uS_0) + \Psi_0 e^r$

$$X_1 = \Delta_1(dS_0) + \Psi_0 e^r$$

at $t=2$: $X_2 = \Delta_2 S_2 + \Psi_2 e^r \Rightarrow X_2 = \Delta_2(u^2S_0) + \Psi_1(uS_0)e^r$

$$X_2 = \Delta_2(udS_0) + \Psi_1(uS_0)e^r$$

$$X_2 = \Delta_2(duS_0) + \Psi_1(dS_0)e^r$$

pick one since $X_2 = V_2(S_2)$ is path independent

$$X_2 = \Delta_2(d^2S_0) + \Psi_1(dS_0)e^r$$

$\therefore$ 6 equations w/ variables dependent on each other so we have a system

b) No p_d or p_u in these constraints

c) 6 equations w/: 6 unknowns $(S_0, \Delta_0, \Psi_0, \Psi_1(uS_0), \Psi(dS_0), \Delta_1(uS_0), \Delta_1(dS_0))$

d) It is plausible for a unique solution to exist since number of unknown = number of equations so coefficient matrix of the system will be square. This means it is plausible for coefficient matrix to be invertible $\Rightarrow$ unique solution to system.

e) If derivative payout is path-dependent, then:

$$\Delta_2 (udS_0) + \Psi_1 (uS_0) e^r \neq \Delta_2 (duS_0) + \Psi(dS_0) e^r$$

Which would mean now we have 7 equations and 6 unknowns, and assuming all the constraints are linearly independent, then it is not possible to get a unique solution since coefficient matrix is no longer square (thus not invertible).