CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 9

Due dates: Electronic submission of *yourLastName-yourFirstName-hw9.tex* and *yourLastName-yourFirstName-hw9.pdf* files of this homework is due on **Friday, 11/15/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

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Resources. Discrete Mathematics and its Applications 8th Edition

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: **Andrew Han**

Total 100 + 10 (extra credit) points.

Problem 1. (10 pts $\times$ 3 = 30 points) Section 8.2, Exercise 4 b), c) and d) page 551. For each subproblem, find the closed form solution for a_n by answering the following step by step:

1. (2 points) What is the characteristic equation of the recurrence relation?
2. $(2 + 2$ points) What are the roots of the characteristic equation? Express a_n in a generic form in terms of the roots you found. For (d), you will get only one root; refer Theorem 2 in page 544 for the generic form in this case.
3. (4 points) Find the closed form solution for a_n using the initial conditions. Show your work.

Solution. b) 1. $x^2 - 7x + 10$
2. $x_1 = 5, x_2 = 2$
3.

$$a_n = \alpha 5^n + \beta 2^n$$

$$a_0 = \alpha + \beta = 2$$

$$a_1 = 5\alpha + 2\beta = 1$$

$$\alpha = -1, \beta = 3$$

$$a_n = -(5)^n + 3 * (2)^n$$

c) 1. $x^2 - 6x + 8$
2. $x_1 = 4, x_2 = 2$
3.

$$a_n = \alpha 4^n + \beta 2^n$$

$$a_0 = \alpha + \beta = 4$$

$$a_1 = 4\alpha + 2\beta = 10$$

$$\alpha = 1, \beta = 3$$

$$a_n = 4^n + 3 * 2^n$$

d) 1. $x^2 - 2x + 1$
2. $x_1 = 1$
3.

$$a_n = \alpha(1)^n + n\beta(1)^n$$

$$a_0 = \alpha = 4$$

$$a_1 = \alpha + \beta = 1$$

$$\alpha = 4, \beta = -3$$

$$a_n = 4 - 3n$$

Problem 2. (5 points $\times$ 3 = 15 points) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 1, 2, 3, 4\}$.

a) List all the ordered pairs in the relation $R = \{(a, b) \mid a + b = 5\}$ on A.

b) List all the ordered pairs in the relation $R = \{(a, b) \mid a < b\}$ on A.

c) List all the ordered pairs in the relation $R = \{(a, b) \mid a < b\}$ from A to B.

Solution. a) (1,4), (2,3), (3,2), (4,1)
b) (1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)
c) (1,2), (1,3), (1,4), (2,3), (2,4), (3,4)

Problem 3. (8 points $\times$ 3 = 24 points) Section 9.1, Exercise 6 a), b) and d), page 608

Solution. a) Not reflexive because if $1, 1 \in \mathbf{R}, 1 + 1 \neq 0$
Symmetric because x + y = 0 is the same as y + x =0
Not anti-symmetric because if $1, -1 \in \mathbf{R}, 1 - 1 = 0$ and $-1 + 1 = 0$, but $1 \neq -1$
Not transitive because if $1, -1 \in \mathbf{R}$ and $-1, 1 \in \mathbf{R}, 1, 1 \in \mathbf{R}$ is NOT true as shown above.
b) Reflexive because if $x, x \in \mathbf{R}$, then $x = \pm x$ will have to be true.
Symmetric because if $x, y \in \mathbf{R}$ and $x = \pm y$, then $y, x \in \mathbf{R}$ and $y = \pm x$
Not anti-symmetric because if $1, -1 \in \mathbf{R}$ and $-1, 1 \in \mathbf{R}$, $1 \neq -1$ which means the implication that 1 = -1 is not true.
Transitive because if $x = \pm y$ and $y = \pm z$, then $x = \pm z$

Problem 4. (5 points $\times$ 2 = 10 points) Let A be the set of all people and $(x, y) \in A \times A$. Is each of the following an equivalence relation? For each subproblem, explain which of the three properties of an equivalence relation – reflexivity, symmetry, and transitivity – are satisfied and which are not, by explaining why or why not.Then, answer whether it is an equivalence relation.

a) $R_1 = \{(x, y) \mid x \text{ and } y \text{ have the same parents}\}$

b) $R_2 = \{(x, y) \mid x \text{ and } y \text{ have a common grandparent}\}$

Solution. a) Reflexive because if $x, x \in \mathbf{R}$, then x will have the same parents as x.
Symmetric because if $x, y \in \mathbf{R}$ and $y, x \in \mathbf{R}$, x has the same parents as y and y has the same parents as x.
Transitive because if $x, y \in \mathbf{R}$ and x has the same parents as y and $y, z \in \mathbf{R}$ amd y has the same parents as z, then $x, z \in \mathbf{R}$ and x will have the same parents as z. All three properties are satisfied, so this is an equivalence relation.
b) Reflexive because if $x, x \in \mathbf{R}$ then x will have a common grandparent with x.
Symmetric because if $x, y \in \mathbf{R}$ and x and y have a common grandparent, then if $y, x \in \mathbf{R}$, y and x will have a common grandparent.
Not transitive because if $x, y \in \mathbf{R}$ and x and y have a common grandparent, an $y, z \in \mathbf{R}$ and y and z have a common grandparent, x and z don't necessraily have a common grandparent. Because not all three properties are met, this is not an equivalence relation.

Problem 5. (10 points) We define on the set $\mathbf{N}_1 = \{1, 2, 3, \cdots\}$ of positive integers a relation $\sim$ such that two positive integers x and y satisfy $x \sim y$ if and only if $x/y = 2^k$ for some integer k. Show that $\sim$ is an equivalence relation.

Solution. Reflexive because for $x, x \in \mathbf{N}_1, x/x = 1 = 2^k = 2^0$.
Symmetric because for $x, y \in \mathbf{N}_1, \frac{x}{y} = 2^k$ and $y, x \in \mathbf{N}_1 = \frac{y}{n} = 2^{-k}$ because $\frac{y}{x}$ is the inverse of $\frac{x}{y}$
Transitive because if $x, y \in \mathbf{N}_1$ and $y, z \in \mathbf{N}_1$, then $\frac{x}{y} = 2^k$ and $\frac{y}{z} = 2^l$. Then for $x, z \in \mathbf{N}_1, \frac{x}{z} = \frac{x}{y} * \frac{y}{z} = 2^k * 2l = 2^{k+l}$
Therefore, $\sim$ is an equivalence relation.

Problem 6. (7 points $\times$ 3 = 21 points) Section 9.6, Exercise 2 b), c) and e), page 662. For each subproblem, explain which of the three properties of a partial ordering – reflexivity, antisymmetry, and transitivity – are satisfied and which are not, by explaining why or why not. Then, answer whether it is a partial ordering.

Solution. b) Reflexive because for each element, a, there exists a pair on the relation where $a, a \in \mathbf{R}$
Anti-symmetric because for $2, 0 \in \mathbf{R}, 2, 3 \in \mathbf{R}$, the condition that $a, b \in \mathbf{R} \wedge b, a \in \mathbf{R}$ is false, meaning the whole statement is vacuously true. The rest of the pairs satisfies the condition $a, b \in \mathbf{R} \wedge b, a \in \mathbf{R}$ and implies $a = b$

Transitive because for all the pairs where $a, b \in \mathbf{R}$ and $b, c \in \mathbf{R}, a, c \in \mathbf{R}$ also exists in the relation. Thus, the relation has all the properties of a partial ordering.
c) Reflexive because for each element, a, there exists a pair on the relation where $a, a \in \mathbf{R}$
Anti-symmetric because for the two pairs, (1,2) and (3,1), $a, b \in \mathbf{R}$ exists but $b, a \in \mathbf{R}$ does not hold true, so the claim holds vacuously true. For all the other pairs, both $a, b \in \mathbf{R}$ and $\underline{,}a \in \mathbf{R}$ hold true, making the claim true.
Not transitive because $3, 1 \in \mathbf{R}$ and $1, 2 \in \mathbf{R}$ exist, but $3, 2 \in \mathbf{R}$ is not true. Thus, the relation does not have all the properties of a partial ordering.

Checklist:
- ☐ Did you type in your name and UIN?
- ☐ Did you disclose all resources that you have used?
  (This includes all people, books, websites, etc. that you have consulted.)
- ☐ Did you electronically sign that you followed the Aggie Honor Code?
- ☐ Did you solve all problems?
- ☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?