CSCE 222 [Sections 503, 504] Discrete Structures for Computing
Fall 2019 – Hyunyoung Lee

Problem Set 8

Due dates: Electronic submission of *yourLastName-yourFirstName-hw8.tex* and *yourLastName-yourFirstName-hw8.pdf* files of this homework is due on **Friday, 11/8/2019 before 10:00 p.m.** on `http://ecampus.tamu.edu`. You will see two separate links to turn in the .tex file and the .pdf file separately. Please do not archive or compress the files. **If any of the two submissions are missing, you will likely receive zero points for this homework.**

Name: Andrew Han **UIN: 227009495**

Resources. Discrete Mathematics and its Applications 8th Edition

On my honor, as an Aggie, I have neither given nor received any unauthorized aid on any portion of the academic work included in this assignment. Furthermore, I have disclosed all resources (people, books, web sites, etc.) that have been used to prepare this homework.

Electronic Signature: __Andrew Han__

Total 100 points.

Problem 1. (2.5 points $\times$ 4 = 10 points) Section 6.3, Exercise 20, page 435

Solution. a) $\binom{10}{3} = 120$
b) At least 6 0s. $\binom{10}{6} + \binom{10}{7} + \binom{10}{8} + \binom{10}{9} + \binom{10}{10} = 386$
c) At least seven 1s is the same as at most 3 0s. $\binom{10}{0} + \binom{10}{1} + \binom{10}{2} + \binom{10}{3} = 176$
d) At least three 1s is the same as the total combinations subtracted by the strings with at most 2 1s. $2^{10} = 1024$ - $\binom{10}{0}$ - $\binom{10}{1}$ - $\binom{10}{2} = 968$

Problem 2. (2 points $\times$ 5 = 10 points) Section 6.3, Exercise 22 b), c), d), e), and f), page 435

Solution. b) $6! = 720$
c) $5! = 120$
d) $5! = 120$
e) $4! = 24$
f) Not possible to have both. If BCA exists, then ABF cannot exist and vice versa.

Problem 3. (5 points $\times$ 2 = 10 points) Section 6.4, Exercise 12 a) and b), page 444

Solution. a) $\binom{6}{3} * (5x^2)^{(6-3)} * (2y^3)^3$
$\binom{6}{3} * (5^3)(2^3)x^6y^9$
Coefficient is $20 * 125 * 8 = 20000$

b) $\binom{6}{5} * (5x^2)^{(6-5)} * (2y^3)^5$
$\binom{6}{5} * (5^1)(2^5)x^2y^{15}$
Coefficient is $6 * 5 * 32 = 960$

Problem 4. $(10 + 3 + 7 = 20$ points) Section 8.1, Exercise 10, page 537. For a) and c), explain and show your work.

Solution. a) Looking at a bit string a of length n, if you look at one number, the total number of bit strings containing the string 01 is going to be a_{n-1}, which we should already know through recursion. Now we can look at two numbers and the remaining bit string is going to be length of n-2. Looking at three, the remaining bit string is going to be length of n-3 and so on. Using this info, you can make a geometric sequence as follows.

$$\sum_{k=0}^{n-2} 2^k$$

By using the definition of the sum of a geometric sequence, we can write this as

$$2^{n-1} - 1$$

To get our closed form for a_n, we need to go back and add the a_{n-1} that was found recursively to get

$$a_n = a_{n-1} + 2^{n-1} - 1$$

b) A string needs to have at least a length of 2 to contain 01. That means the initial conditions are $a_0 = 0$ and $a_1 = 0$

c)

$$a_0 = 0$$

$$a_1 = 0$$

$$a_2 = a_1 + 2^1 - 1 = 1$$

$$a_3 = a_2 + 2^2 - 1 = 4$$

$$a_4 = a_3 + 2^3 - 1 = 11$$

$$a_5 = a_4 + 2^4 - 1 = 26$$

$$a_6 = a_5 + 2^5 - 1 = 57$$

$$a_7 = a_6 + 2^6 - 1 = 120$$

Problem 5. (20 points) Section 8.1, Exercise 28, page 538. This problem has two parts as below.

Solution.
a) (10 points) *Show that the Fibonacci numbers satisfy ...*

The Fibonacci sequence is defined as $f_n = f_{n-1} + f_{n-2}$ We can expand this out to see if the recurrence relation given will satisfy this.

$$f_n = f_{n-1} + f_{n-2}$$

$$= (f_{n-2} + f_{n-3}) + (f_{n-3} + f_{n-4}) = f_{n-2} + 2f_{n-3} + f_{n-4}$$

$$= (f_{n-3} + f_{n-4}) + 2 * (f_{n-4} + f_{n-5}) + f_{n-4} = f_{n-3} + 4f_{n-4} + 2f_{n-5}$$

$$= f_{n-4} + f_{n-5} + 4f_{n-4} + 2f_{n-5} = 5f_{n-4} + 3f_{n-5}$$

b) (10 points) *Use this recurrence relation to show that ...* (Prove by induction on n.)
Induction base:
P(1): $f_5 = 5$. Since 5 is divisble by 5, so P(1) holds.
Induction Step: As induction hypothesis, assume that P(n) holds. Then show that P(n+1) holds.

$$f_{5(n+1)} = f_{5n} + f_{5n-1} \text{ by def. of Fibonacci numbers}$$

$$= 5f_{5(n-4)} + 3f_{5(n-5)} + 5f_{5((n-1)-4)} + 3f_{5((n-1)-5} \text{ by IH}$$

Because the two terms that can be formed from the expression above can be divided by 5 from the induction hypothesis, adding the two together will still result in a multiple of 5. Therefore, the claim holds by induction on n.

For Problems 6 and 7, use Table 1 on page 568.

Problem 6. (5 points $\times$ 3 = 15 points) Section 8.4, Exercise 6 b)–d), page 575

Solution. b) $\frac{2x}{1-2x}$
c) $\frac{2x-1}{(1-x)^2}$
d) $\frac{e^x-1}{x}$

Problem 7. (5 points $\times$ 3 = 15 points) Section 8.4, Exercise 8 b)–d), page 575.

Solution. b) $\binom{3}{n} * -(-3)^n$
c) If n is an even integer, 2^n. If n is an odd integer, 0.
d) $\frac{n(n-1)}{2}$ for $n \geq 2$ and initial conditions $a_0 = 0$ $a_1 = 0$

Checklist:
☐ Did you type in your name and UIN?
☐ Did you disclose all resources that you have used?
(This includes all people, books, websites, etc. that you have consulted.)
☐ Did you electronically sign that you followed the Aggie Honor Code?
☐ Did you solve all problems?
☐ Did you submit both of the .tex and .pdf files of your homework to the correct link on eCampus?