Ad auctions

As we've discussed:

<u>Generation 1</u> of Web Ads were "traditional": large contracts negotiated to show a large number of impressions. Contracts were negotiated case-by-case ... just like magazine advertising.

<u>Generation 2</u> was started by Overture in 1997 and changed the game completely:

- advertisers bid for positions on particular keywords
- and they paid per click

→ Ads were targeted and payment was only when ad was "successful"

Now with Basic Auction theory under our belts, we can discuss what ad auctions actually implement.

<u>The Ad auction framework</u>

<u>Multiple goods</u> (ad slots) up for sale, for each search.

Bidder valuations unknown → hence the auction each search.

Bidder valuations unknown → hence the auction

→ So this is a more general setting than we talked about last time.

☆ note submit a single bid even though slots are of different value → done to simplify advertiser task.

Ad auctions circa 1997

Goto/Overture's first auctions were designed as

1st price auctions.

top k bids are assigned to slots 1 → k and pay their bids

→ we've seen already that this is not a great design.

And here it can easily lead to cyclic behavior.

ex:

Bidders	Slots
v_1 = \$10 /click	1 200 clicks/hr
v_2 = \$4	2 175 clicks/hr
v_3 = \$2	

Suppose $b_2 = 2.01$ ...to ensure he gets a slot
Then $b_1 = 2.02$ ...to beat b_1 & get slot 1
then $b_2 = 2.03$
and so on, until $b_1 = 4.01$
then $b_2 = 2.01$
and we cycle.

→ no pure strategy equilibrium

& so bidders will want to constantly revise their bids.

☆ this was actually observed in Overture

★ this was actually observed in Overture

(see ppt)

Despite this issue, Overture was successful, and was used by most search engines until 2002.

Note: GoTo/overture started at IdeaLab ...a pasadena company... that also launched Roomba and lots of other cool stuff.

It wasn't until 2002 that Google introduced its own system — Adwords — to address this

They introduced:

Generalized Second price Auctions

... and then Yahoo/overture followed suit quickly

This generalizes the single item 2^{nd} price auction in the "natural" way

for $i <$ # positions bidder with ith largest bid, b_i, pays b_{i+1}

ex: In our earlier example, if we suppose all bidders bid truthfully, this gives
slot 1 → Bidder 1 for \$4/click
& slot 2 → Bidder 2 for \$2/click

Q: Is truthtelling a dominant strategy always?

A: No!

Consider our example...

payoff to 1 in truthful setting is

$$(10-4)200 = 1200$$

if he deviates to a \$3 bid he gets 2nd position for \$2/click and has payoff

$$(10-2)175 = 1400$$

★ Further there can be multiple equilibria!

$(10-2)\,1/3 - 1/00$

★ Further there can be multiple equilibria!
... we'll come back to this and see an example later.

So GSP doesn't have the nice properties that 2nd price had for a single item....

but it's what Yahoo & Google use today (to a first approx)

... though for some settings 1st price is still used!

because the infrastructure is hard to change.

But, there's an important issue left to discuss:

Different ads have different click through rates for different terms.
r_i

Q: How should this change the auction?

A (Google): $b_i \mapsto r_i b_i$

A (Yahoo): $b_i \mapsto b_i$ (ignore Clickthrough rates)

Q: Why the difference?

A: Google's maximizes it's own revenue
Yahoo's maximizes the "relevance" of the ads shown.

So, you now have a good idea of what Yahoo & Google use for their auctions.

... but we've also seen that GSP has some problems.

Q: Can we design a truthful auction for this setting?

A: Yes ... it turns out Google & Yahoo just didn't know their economics.

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just didn't know their economics.

VCG Auctions

It turns out that google choose the "wrong" generalization of the 2^{nd} price auction.

The "right" one was given in 60s by Economists.

VCG ⇒ Vickery, Clark, Groves

All three independently discovered the design in different contexts

V - 61
C - 71 } → tell story
G - 73

The GSP auction isn't truthful
The VCG auction is...

So, what is the "right" generalization of the 2^{nd} price auction?

☆ Charge each winner a price equal to the total amount better off everyone else would be if the indivual weren't there.

(same principle as marginal cost pricing for routing games)

ex: Returing to

Bidders	Slots
$v_1 = \$10$/click	1 200 clicks/hr
$v_2 = \$4$	2 175 clicks/hr
$v_3 = \$2$	

We have:

- Bidder 1 pays: $\underbrace{\$100}_{\text{extra 2 would get w/o 1 there}} + \underbrace{\$350}_{\text{extra 3 would get w/o 1 there}} = \$450 \Rightarrow \$2.50$/click
- Bidder 2 pays: $\underbrace{\$350}_{\text{extra 3 would get w/o 2 there}}$

extra $ would get
w/o 2 there

Formally:

The top bids get assigned to slots in order

& $p_{ij} = V^{S}_{B-j} - V^{S-i}_{B-j}$ $\longrightarrow$ $p_{ij} = (r_j - r_{j+1}) b_{i+1} + p_{i+1, j+1}$

price for advertiser j to get slot i

max value of an assignment given slots $S-i$, bidders $B-j$

Observe: this reduces to 2nd price in case of a single item

$V^{S-i}_{B-j} = 0$

$V^{S}_{B-j} = $ 2nd highest value

Now, time to prove the main result for VCG:

> **Thm:** VCG results in
> (1) truthfulness is the dominant strategy
> (2) assignment maximizes total value

pf: (2) is immediate given (1).

If all bid their true valuation then assigning them in that order maximizes total value.

→ this is a key benefit of truthful auctions

(1) takes some more work

Suppose j announces true valuation v_{ij}, she gets position i &

her payoff is $v_{ij} - p_{ij}$

We now want to show that she has no incentive to deviate.

Suppose j lies about her valuations, and gets slot h instead.

Then we need to show that

$$v_{ij} - p_{ij} \overset{!}{\geq} v_{hj} - p_{hj}$$

$$\Leftrightarrow \quad v_{ij} - \left[V^{S}_{B-j} - V^{S-i}_{B-j}\right] \overset{!}{\geq} v_{hj} - \left[V^{S}_{B-j} - V^{S-h}_{B-j}\right]$$

$$\Leftrightarrow \quad \underbrace{v_{ij} + V^{S-i}_{B-j}} \overset{!}{\geq} \underbrace{v_{hj} + V^{S-h}_{B-j}}$$

$= V_B^S$

since i is matched to j when true valuation is given

$\leq V_B^S$

since h is forced to be matched to j, which is not what happens when true valuation is used.

Nothing in this argument depends on decisions made by other players

e.g. they may not announce their true valuations

(V_{B-j}^{S-i} is the maximal valuation assignment using announced values other than yours ... same with V_{B-j}^{S-h})

$\square$

Comparing VCG & GSP

Q: Why haven't Google & Yahoo switched from GSP to VCG?

A1: They have the GSP architecture already built and switching might be (i) expensive & (ii) confusing to advertisers

A2: Maybe they get more revenue from GSP?

↓

Is this true?

ex:

Click through rate	Slots	advertisers	revenue/click
10	a	x	\$7
4	b	y	\$6
0	c	z	\$1

VCG: Assigns $x \to a$
$y \to b$
$z \to c$

valuations of all 3 are

	a	b	c
x	70	28	0
y	60	24	0
z	10	4	0

So x pays "harm to y & z"

$$40 = \underbrace{(60-24)}_{\text{for } y} + \underbrace{(4-0)}_{\text{for } z}$$

& y pays

$$4 = \underbrace{(70-70)}_{\text{for } x} + \underbrace{(4-0)}_{\text{for } z}$$

revenue = 44

GSP: Has 2 equilibria

1) $b_1 = 5$ $\quad P_1 = \$4$: 1st slot
$b_2 = 4$ $\quad P_2 = \$2$: 2nd slot
$b_3 = 2$ $\quad$ ||

v_3 <

$\Downarrow$

revenue of $48 = 40 + 8$

To check that this is a Nash:

1 gets payoff $70 - 40 = 30$
would get $28 - 8 = 20$ if dropped price

2 gets payoff $24 - 8 = 16$
would get $60 - 50 = 10$ if raised price

2) $b_1 = 3$
$b_2 = 5$
$b_3 = 1$

$P_1 = 1$: 2^{nd} slot
$P_2 = 3$: top slot

$\Downarrow$

revenue of $34 = 30 + 4$

$\Downarrow$

To check that this is a Nash:

1 gets payoff $28 - 4 = 24$
would get $70 - 50 = 20$ if raised price

2 gets payoff $60 - 30 = 30$
would get $24 - 4 = 20$ if lowered price

So VCG collects more than 2^{nd} GSP equilibrium but less than the 1^{st}.

$\Rightarrow$ GSP may collect more revenue depending on which equilibria bidders converge to.

... lots of interesting current research on this topic.