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1. There are no dominant strategies. We see that if player B plays L, then player A will play T, but if player B plays R, then player A will play B. Similarly, if player A plays T, then player B will play L and if player A plays B, then player B will play R.

2. The pure Nash equilibria are (T, L) and (B, R). For (T, L) the expected payoff is 5 for player A and 2 for player B. For (B, R), the expected payoff is 3 for player A and 4 for player B. In both cases, we see that once one of the players chooses their part of the Nash equilibrium strategy, then the other player has no reason to deviate (deviation would cause a payout of 0).

3. Suppose player B plays L w/ p probability and R w/ $(1-p)$ probability. Player A's payoff is:

$$p(5) + 0(1-p) = 5p \text{ if A plays T and}$$

$$0(p) + 3(1-p) = 3-3p \text{ if A plays B.}$$

Player B will benefit from minimizing the payoff of player A so setting $5p = 3-3p$, we get that $p = 3/8$ and that the expected payoff for A will be $15/8$.

Carrying out similar analysis for player A: let's assume player A plays T w/ probability q and B w/ probability (1-q).

The expected payoff for player B is:

$2q$ if player B plays L and

$4(1-q)$ if player B plays R.

Minimizing Player B's payout:

$$2q = 4-4q$$
$$q = 2/3$$

and thus the expected payout for player B is $4/3$.

Hence, player A plays the mixed strategy of $(T, B) = (\frac{2}{3}, \frac{1}{3})$.
Player A has an expected payout of $15/8$.

Player B plays the mixed strategy of $(L, R) = (3/8, 5/8)$.
Player B has an expected payout of $4/3$.

4. The two pure Nash equilibria are (5,2) and (3,4).
B/c the randomizing device picks each w/ equal probability (50%), the expected payoffs are:

player A: $\frac{5}{2} + \frac{3}{2} = 4.$

Player B: $\frac{2}{2} + \frac{4}{2} = 3.$

5. No, neither players have incentives to disobey the randoming device. B/c the randoming device is only picking from Nash equilibria, by definition, given that one player will always obey the device, the other player will not choose to deviate.

(2) The final payout should be:

Capofamiglia (Boss): $9,999

Sotto capo (underboss): $0

Caporegime: $1

Soldier: $0.

To see why this is, we work backwards and build up our scenario. First, let's just say there are only two member in the mafia. Clearly, the allocation would be:

Caporegime: $10,000

Soldier: $1.

Now, let's add sotto capo into the mix. The allocation would now be:

Sotto capo: $9,999

Caporegime: $0

Soldier: $1

Note that b/c the soldier is now earning more than what he will earn w/o sotto capo, the soldier will vote for the allocation.

Now, when we add the final member: capofamiglia, we note that this new member only needs one other person to vote for whatever allocation he puts forth. Thus, he offers Caporegime $1 which is more than what he would be getting w/o the addition of a new member. Hence, we get our final allocation:

Boss: $9,999

Underboss: $0

Caporegime $1.

Soldier: $0.

Note that it must be Caporegime that gets the $1 not the soldier b/c if the soldier was offerred $1, he would be making the same amount as the case w/ just 3 people in the mafia, so the soldier would not vote and thus get the boss killed.

This does go against intuition as one may expect the boss to have to give up a fair share of the earnings to appease the rest of the mafia and stay alive. However, a greedy approach turns out to instead be the optimal strategy for the boss.

(3)

a. When N=88 (or any even number), it is Irene's turn to offer. Her strategy would be to only offer \$1. The rationale behind this is very simple; Irene knows that Carly will accept any non-zero amount b/c if Irene declines the offer, then she would make \$0. Thus, Irene will offer the minimum amount of money to ensure that Carly will accept. Offering the minimum amount necessary also means that Irene will maximize her earnings.

b. When N=99, it is Irene's turn to receive an offer. Her best strategy is to accept any non-zero amount {\$1, \$2, \$3, \$4, \$5} and reject \$0. By rejecting \$0, Irene knows that Carly is forced to offer a non-zero amount (b/c otherwise Carly would also make \$0), and by accepting any non-zero amount, Irene is guaranteeing that she will make more than the case where she rejects and makes \$0.