Problem 3

- Degree & closeness

We present the following counterexample:

From degree centrality, both B and C have the same order of difference (both have $C_D(B) = C_D(C) = 2/4 = 1/2$. However, if we look at closeness, we have that

$$C_C(B) = \frac{4}{1+1+2+3} = \frac{4}{7} \quad \text{but} \quad C_C(C) = \frac{4}{2+1+1+2} = \frac{4}{6} = \frac{2}{3}.$$

- Degree & Betweenness. We use the same counterexample as above. Both B and C have the same degree centralities. Now, we calculate their betweenness centralities.

$$C_B(B) = \frac{1+1+1}{6} = \frac{1}{2} \quad \text{but} \quad C_B(C) = \frac{1+1+1+1}{6} = \frac{2}{3}.$$

- Degree & PageRank

To show proportionality, we introduce a scalar constant $c = \frac{1}{2|E|}$.

such that $r_i = \frac{1}{2|E|} d_i$.

Thus, we get that

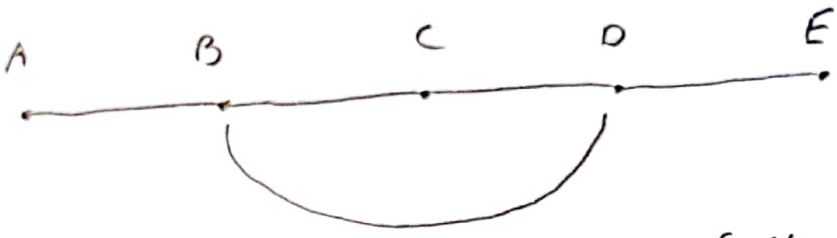
(1) $r_i \geq 0$ for all i

(2) $\sum_{i=1}^{n} r_i = \frac{1}{2|E|} \sum_{i=1}^{n} d_i = 1$ (total degree = 2 · number of edges)

(3) $\sum_{j \in N(i)} \frac{r_j}{d_j} = \sum_{j \in N(i)} \frac{1}{2|E|} = \frac{1}{2|E|} d_i = r_i$

Thus we have shown that degree is proportional to PageRank

- Closeness ≠ Betweenness. We use the following graph as a counterexample and focus on nodes B and C.



Let's calculate closeness centrality first.

$$C_c(B) = \frac{4}{1+1+1+2} = \frac{4}{7}, \qquad C_c(C) = \frac{4}{2+1+1+2} = \frac{2}{3}$$

Now, let's calculate betweenness centrality.

$$C_B(B) = \frac{1+1+1+1}{6} = \frac{2}{3}, \qquad C_B(C) = \frac{0}{6} = 0.$$

From above, based on closeness, we have that C ranks higher than B, but based on betweenness, C ranks lower than B.

- Closeness ≠ PageRank: B/C degree ≠ PageRank and degree ≠ closeness, PageRank ≠ closeness.

- Betweenness ≠ PageRank: B/C degree ≠ PageRank and degree ≠ betweenness, PageRank ≠ betweenness.

b. Degree centrality: Should use this when you care about direct connections. If you want to see who is popular on a social network where edges denote friendships, then you could use this.

closeness centrality: Useful for identifying "broadcasters" or people that can quickly influence the entire network. This might be useful for placing post office like post offices have to be able to route information throughout the entire network efficiently.

Betweenness centrality: Useful for finding individuals who can influence the flow around a system. Potentially useful for identifying potential traffic bottlenecks on certain roads (intersections that connect other routes together.

PageRank: This identifies nodes whose influence goes beyond their direct connections into the larger network. In addition to the application of search engines, PageRank can also be used to identify very important publications/authors of research literature where an edge means a citation.