...Intro on slides....

Herding experiment:

I have a bag of 3 marbles.

{ w/prob 1/2 1 is red & 2 are blue
{ w/prob 1/2 2 are red & 1 is blue

Your task: determine if the bag is majority red (MR) or majority blue (MB)

Rules: you come up 1 at a time and pull a marble out, look at it without showing it to anyone else, and the put it back it the bag. At which point you make a guess of whether the bag is MB or MR.

Note: you hear all the guesses of those ahead of you, but don't see their marbles.

Let's try it a few times

Q: What was the decision rule used by

— the first person?

... the second person?

... etc.?

<u>Q:</u> Does the class learn the correct answer?

Let's do some analysis...

To maximize chance of winning (being right) each person chooses

$$MB \iff Pr(MB \mid \text{what has been announced}) > \tfrac{1}{2}$$

For the 1^{st} person

$$Pr(MB) = Pr(MR) = \tfrac{1}{2} \quad \text{a priori}$$

$$\& \quad Pr(\underbrace{\text{pull Blue}}_{B} \mid MB) = Pr(\underbrace{\text{pull Red}}_{R} \mid MR) = 2/3$$

$$\Rightarrow \quad Pr(B) = Pr(MB)\, Pr(B \mid MB) + Pr(MR)\, Pr(B \mid MR) = \tfrac{1}{2}$$

$$\text{so} \quad Pr(MB \mid B) = \frac{Pr(MB \,\&\, B)}{Pr(B)} = \frac{Pr(MB)\, Pr(B \mid MB)}{Pr(B)}$$

<u>Q:</u> What's this called?

<u>A:</u> Bayes' Rule

$$= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\ } = \frac{2}{\ }$$

$$\frac{\quad}{\frac{1}{2}} = \overline{3}$$

That's nothing unexpected, but what if we look at the 3rd student.

Suppose they saw B, B, and then pulled a R... what should they guess?

$$Pr(MB \mid B,B,R) = \frac{Pr(MB)\, Pr(B,B,R \mid MB)}{Pr(B,B,R)}$$

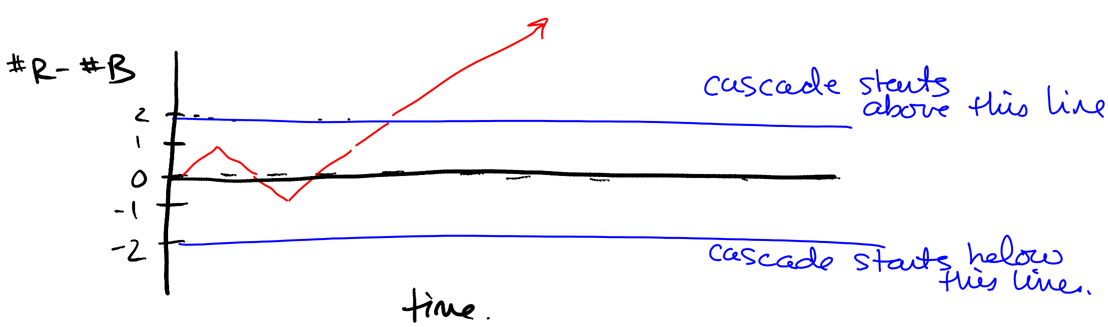
$$= \frac{\frac{1}{2} \cdot \left(\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}\right)}{Pr(MB)\, Pr(B,B,R \mid MB) + Pr(MR)\, Pr(B,B,R \mid MR)}$$

$$= \frac{\frac{1}{2} \cdot \frac{4}{27}}{\frac{1}{2} \cdot \frac{4}{27} + \frac{1}{2} \cdot \left(\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3}\right)} = \frac{\frac{4}{27} \cdot \frac{1}{2}}{\frac{1}{9}}$$

$$= \frac{2}{3}$$

$\Rightarrow$ say's B despite seeing R!

$\Rightarrow$ Everyone in the future says B regardless of what they see!!



That was a very simple example of a

cascade.... you can look in the book for some generalizations of it.

What we're going to do, since our focus is on networks is to investigate cascading behavior in a network setting.

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see ppt for examples

As usual, we'll look at a very simple model of this to see what we can learn...

A Network Coordination Game

Let's consider a scenario with 2 products (A&B) and a population of n people connected with some social network graph. G.

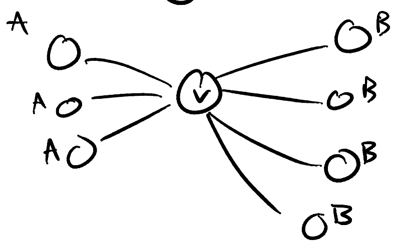
We'll look at 1 component, so that we can assume it's connected..

At time 0 everyone is using product B. then at time one a few people switch to a new product, A.

The question is, will this change cause a cascade?

i.e. cause everyone to switch from $B \to A$?

<u>Decision Making Model</u>:



v has $e(v)$ neighbors.

$p\,e(v)$ are using A

$(1-p)e(v)$ are using B

A has "value" a

B has "value" b

<u>def</u>: v choose A $\iff p\,e(v)\,a \geq (1-p)\,e(v)\,b$

i.e. "value" of choosing A is larger than "value" of choosing b

(think VHS vs DVD)

Notice $p\,e(v)\,a \geq (1-p)\,e(v)\,b$

$\iff p\,e(v)(a+b) \geq e(v)\,b$

$\iff p \geq \frac{b}{a+b} := q$

q small $\Rightarrow$ A is "better" so you're convinced by only a small # of friends using A

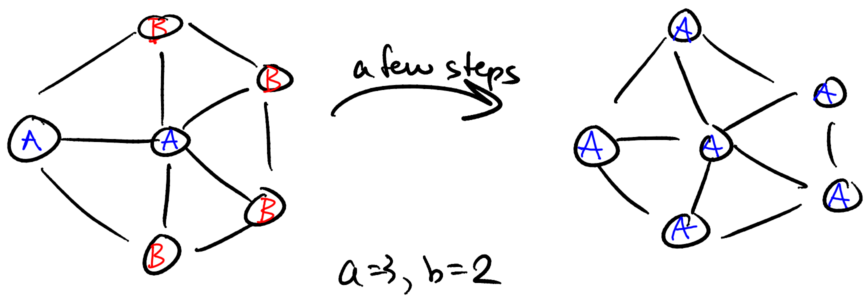
q large $\Rightarrow$ B is "better" so you need a lot of friends to choose B before you do.

Now, our question is:

Q: What are the "equilibria" choices for product adoption.

A: Two obvious ones: all A or all B but others are also possible...

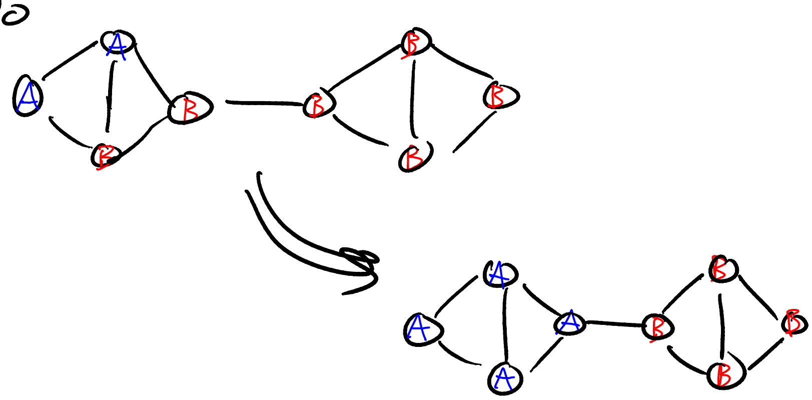
Ex:



Idea: as soon as two nodes fix decisions as A (maybe they are completely convinced A is better) a cascade follows.

Q: Will cascades always take over the whole graph?

A: No



Clusters limit the spread!

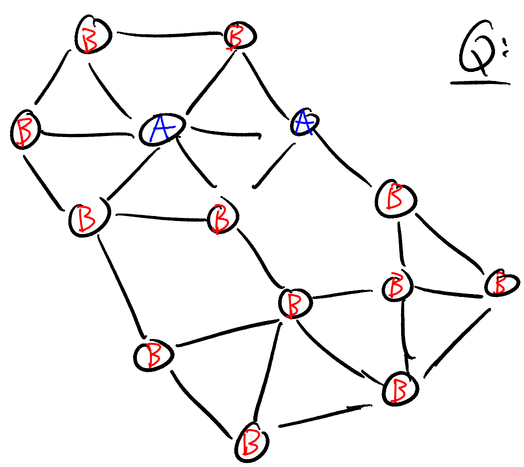
Thm: Consider a set of adopters of A w/ threshold q for nodes to adopt A.

A complete cascade will not follow $\iff$ the remaing network contains a cluster of density greater than $1-q$

(cluster has density $= d$ iff each node has d fraction of its neighbors in the set.)

(in example above Thm, clusters have density $2/3$)

ex:



Q: will the cascade become complete?

A: No.

Pf: The proof has 2 parts

i) Clusters prevent cascades

ii) Only clusters prevent cascades.

Part i) Consider a cluster w/ density $\geq 1-q$.

We will argue that the cascade will

not lead to any node in the cluster switching from $B \to A$.

Q: Why?

A: Consider the first one to change...
by definition it has $\geq 1-q$ fraction of nodes in the cluster and by assumption none of these are using A. So, it must have $< q$ fraction using A... and so it can never swap to A.

part ii) Suppose a cascade stopped before everyone switches to A.
Then the set of remaining nodes using B must be a set of nodes w/ density $> 1-q$ because every node has $\geq 1-q$ fraction of its neighbors in the set (otherwise it would have switched to A).

□

So, that was 1 simple "network cascade" example. There are many other variations that are interesting to consider.

In the book, you'll see discussion of
node dependent thresholds
& a few other interesting modifications

& a few other interesting modifications

One key limitation of all of this though is our deterministic model of choice...

clearly that's not realistic

It turns out that its much harder (but not impossible) to study probabilistic models of cascades/epidemics.

Thanks!