COMP4500 - Assignment 2

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1 Question 1

1.1 Part a

Provide an efficient algorithm that allows you to collect all the flags and minimises the distance travelled.

```
FLAG_FETCH(flags) // flags is an array with f1 through to fn (n flags)
distance = 0
flags.sort() // sort in order of shortest distance
// Priority queue with shortest distance having highest priority
flag_queue = enqueue(flags)
distance_array = [] // empty array to store each distance
while not flag_queue.isEmpty()
    flag_distance = flag_queue.next()
    distance += 2*flag_distance
    foreach flag in distance_array
        distance += flag
    distance_array.add(flag_distance)
```

1.2 Part b

Prove that your algorithm in part a is optimal.

In order for a greedy algorithm to be considered optimal, it must meet the following two criteria:

- Optimal Substructure.
- Greedy Choice Property - A locally optimal solution is a globally optimal solution.

The problem is a shortest path problem, with the addition of travelling to previously collected flags as well. In this situation, the problem is still a greedy problem. Since the flags are sorted by distance (ascending), the shortest path globally is to travel to the flags in order. This behaviour exhibits the optimal substructure property, because finding the shortest global distance can be done by solving the shortest distance of each individual flag.

Proof of Greedy Choice Property:

Let S_j be a nonempty subproblem containing the set of flags that have distance greater than f_j. Let f_m be the flag in S_j with the shortest travel distance, then f_m is a part of a maximum-size subset, F_j, that consists of mutually compatible flag distances that exist in S_j. Define the shortest distance flag in F_j to be f_k. If $f_k = f_m$ then the global shortest distance is in a maximum-size subset of mutually compatible flag distances in S_j and the proof is finished.

Assume $f_k \neq f_m$, then define the set $G_j = F_j - f_k \cup f_m$ i.e. the set of mutually compatible flag distances in S_j not including f_k or f_m. This means that the flags in G_j are disjoint because:

- The flags in F_j are disjoint
- f_k is the shortest distance flag in F_j
- f_m has shorter distance than f_j

Here we have $|G_j| = |F_j|$ so we can conclude that G_j is also a maximum-size subset of mutually compatible flags in S_j and that it includes f_m. From this we know that the globally shortest distance exists in G_j and due to it being a maximum-size subset of the original problem set, the algorithm satisfies the Greedy Choice Property. Therefore the algorithm is optimal.

References

[1] Professor Cliff Stein, *Greedy Algorithms*. Analysis of Algorithms 1, Columbia University, `http://www.columbia.edu/~cs2035/courses/csor4231.F11/greedy.pdf`, Fall 2011.