# Machine Learning

## Appendix 1: Linear Algebra - Basics

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# Linear Algebra - Basics

Matrix $\mathbf{X}_{m \times n}$ : m rows, n columns

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ & \dots & & \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

Column vector: a matrix with only one column

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix}$$

Row vector: a matrix with only one row

$$\mathbf{w} = [w_1 \; w_2 \; \dots \; w_n]$$

# Linear Algebra - Basics

Matrix $\mathbf{X}_{m \times n}$: m rows, n columns

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In all lectures, we will use ***column vector*** as default representation for "vector"

Column vector: a matrix with only one column

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix}$$

Row vector: a matrix with only one row

$$\mathbf{w} = [w_1 \; w_2 \; \dots \; w_n]$$

# Linear Algebra - Basics

## Transpose

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ & \dots & & \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \quad \Rightarrow \quad \mathbf{X}^T = \begin{bmatrix} x_{11} & x_{21} & \dots & x_{m1} \\ x_{12} & x_{22} & \dots & x_{m2} \\ & \dots & & \\ x_{1n} & x_{2n} & \dots & x_{mn} \end{bmatrix}$$

an $m \times n$ matrix

an $n \times m$ matrix

$$\mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{bmatrix} = [w_1 \; w_2 \; \dots \; w_n]^T$$

# Linear Algebra - Basics

## Addition and Subtraction

$$\mathbf{X}+\mathbf{Y}=\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix}+\begin{bmatrix} y_{11} & y_{12} & x_{13} \\ y_{21} & y_{22} & y_{23} \end{bmatrix}=\begin{bmatrix} (x_{11}+y_{11}) & (x_{12}+y_{12}) & (x_{13}+y_{13}) \\ (x_{21}+y_{21}) & (x_{22}+y_{22}) & (x_{23}+y_{23}) \end{bmatrix}$$

$$\mathbf{X}-\mathbf{Y}=\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix}-\begin{bmatrix} y_{11} & y_{12} & x_{13} \\ y_{21} & y_{22} & y_{23} \end{bmatrix}=\begin{bmatrix} (x_{11}-y_{11}) & (x_{12}-y_{12}) & (x_{13}-y_{13}) \\ (x_{21}-y_{21}) & (x_{22}-y_{22}) & (x_{23}-y_{23}) \end{bmatrix}$$

**X**, **Y**: 2×3 matrices

## Scalar multiplication

$$\alpha\mathbf{X}=\alpha\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix}=\begin{bmatrix} \alpha x_{11} & \alpha x_{12} & \alpha x_{13} \\ \alpha x_{21} & \alpha x_{22} & \alpha x_{23} \end{bmatrix}$$

**X**: 2×3 matrices
□: a number

# Linear Algebra - Basics

**Matrix multiplication**

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} (1 \cdot 7 + 2 \cdot 9 + 3 \cdot 11) & (1 \cdot 8 + 2 \cdot 10 + 3 \cdot 12) \\ (4 \cdot 7 + 5 \cdot 9 + 6 \cdot 11) & (4 \cdot 8 + 5 \cdot 10 + 6 \cdot 12) \end{bmatrix}$$

# Linear Algebra - Basics

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# Linear Algebra - Basics

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# Linear Algebra - Basics

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**X** : m×p matrix
**Y** : p ×n matrix
**XY** : m×n matrix

# Linear Algebra - Basics

**Element-wise matrix multiplication (Hadamard product)**

$$\mathbf{X} \odot \mathbf{Y} = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \end{bmatrix} \odot \begin{bmatrix} y_{11} & y_{12} & x_{13} \\ y_{21} & y_{22} & y_{23} \end{bmatrix} = \begin{bmatrix} x_{11}\,y_{11} & x_{12}\,y_{12} & x_{13}\,y_{13} \\ x_{21}\,y_{21} & x_{22}\,y_{22} & x_{23}\,y_{23} \end{bmatrix}$$