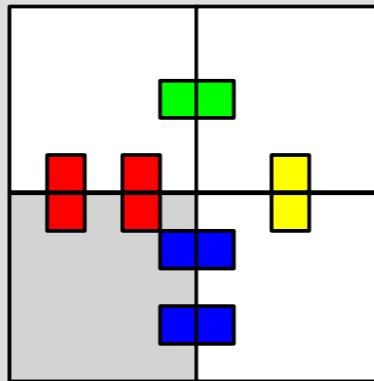


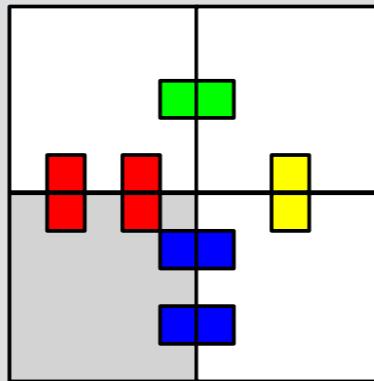
# A Tour of DNA Tile Self-Assembly



Andrew Winslow

Department of Computer Science, UTRGV

# A Tour of DNA Tile Self-Assembly

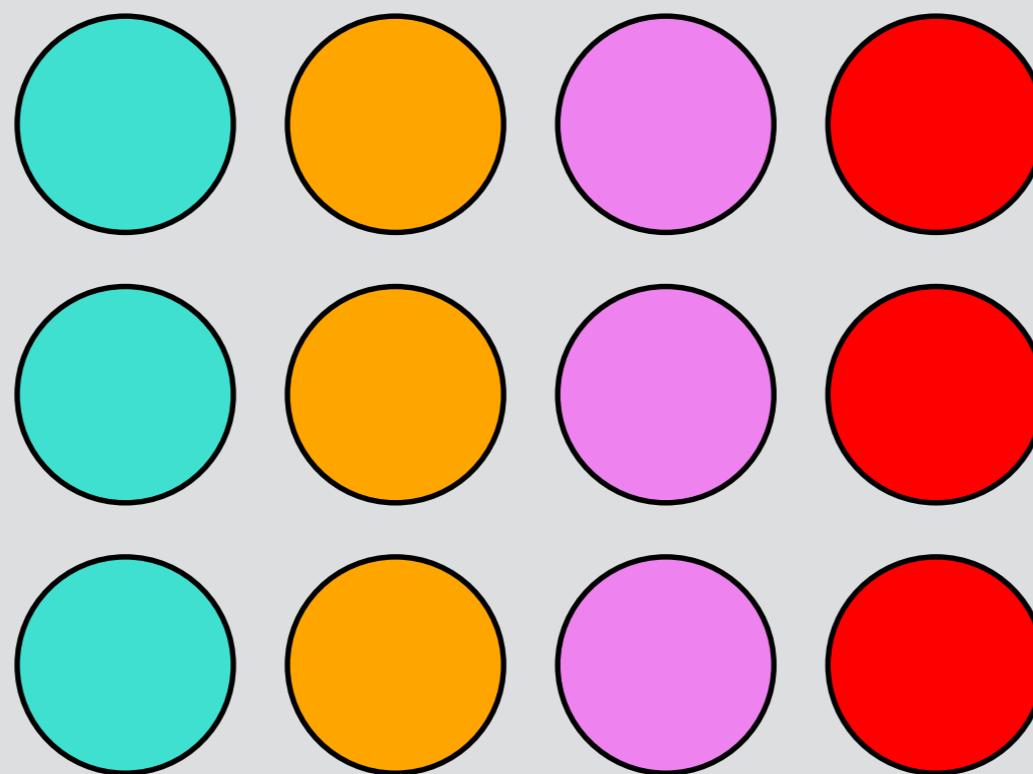


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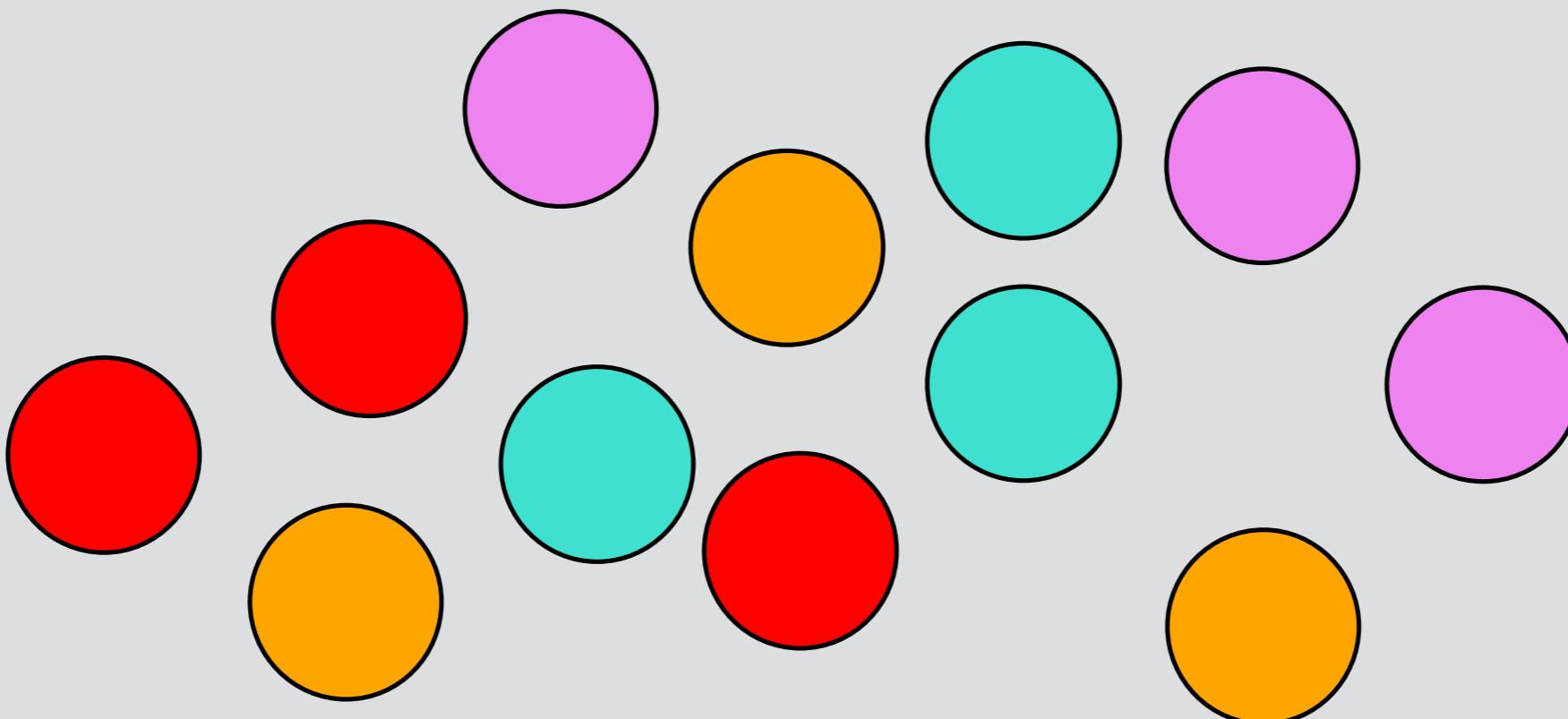
# Self-Assembly

Simple particles coalescing into complex superstructures.



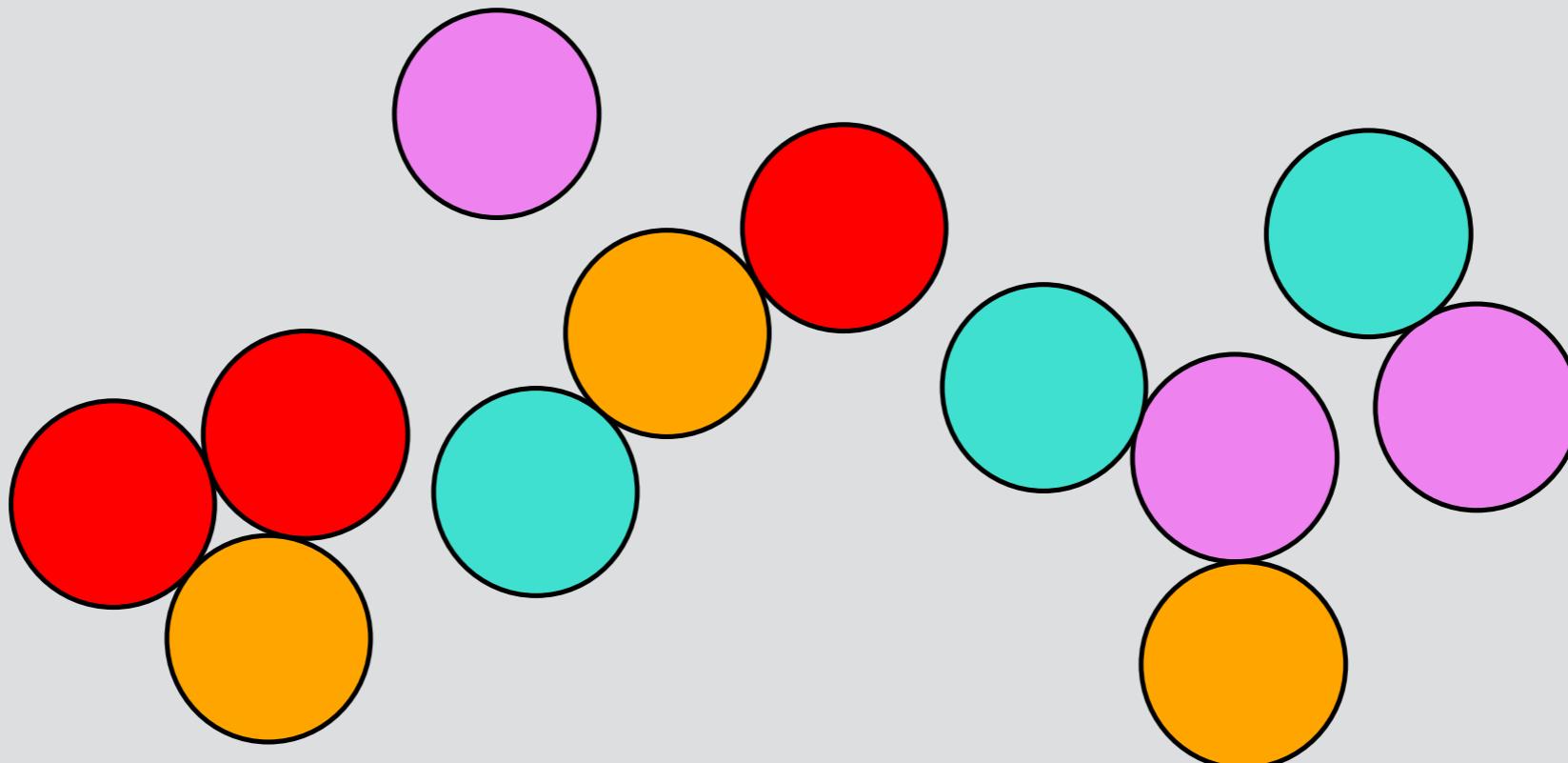
# Self-Assembly

Simple particles coalescing into complex superstructures.



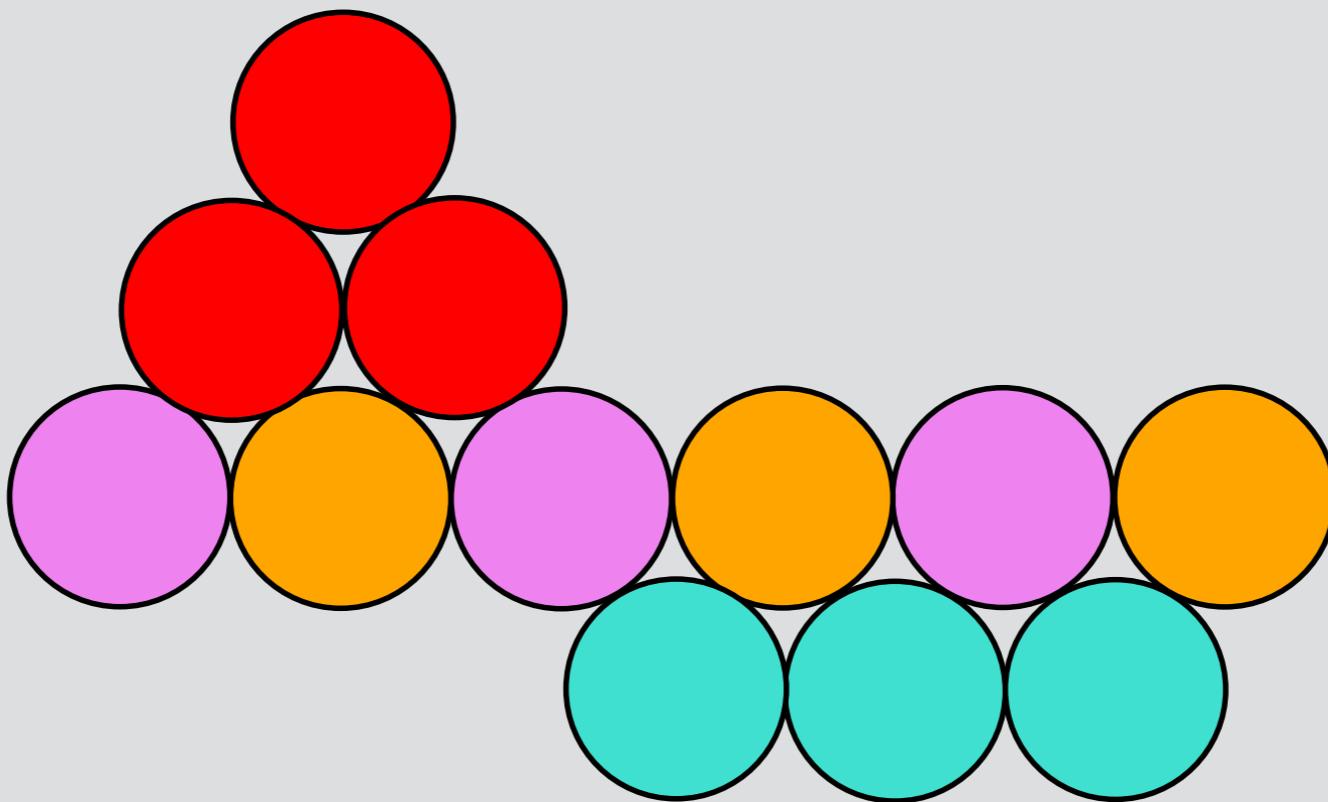
# Self-Assembly

Simple particles coalescing into complex superstructures.

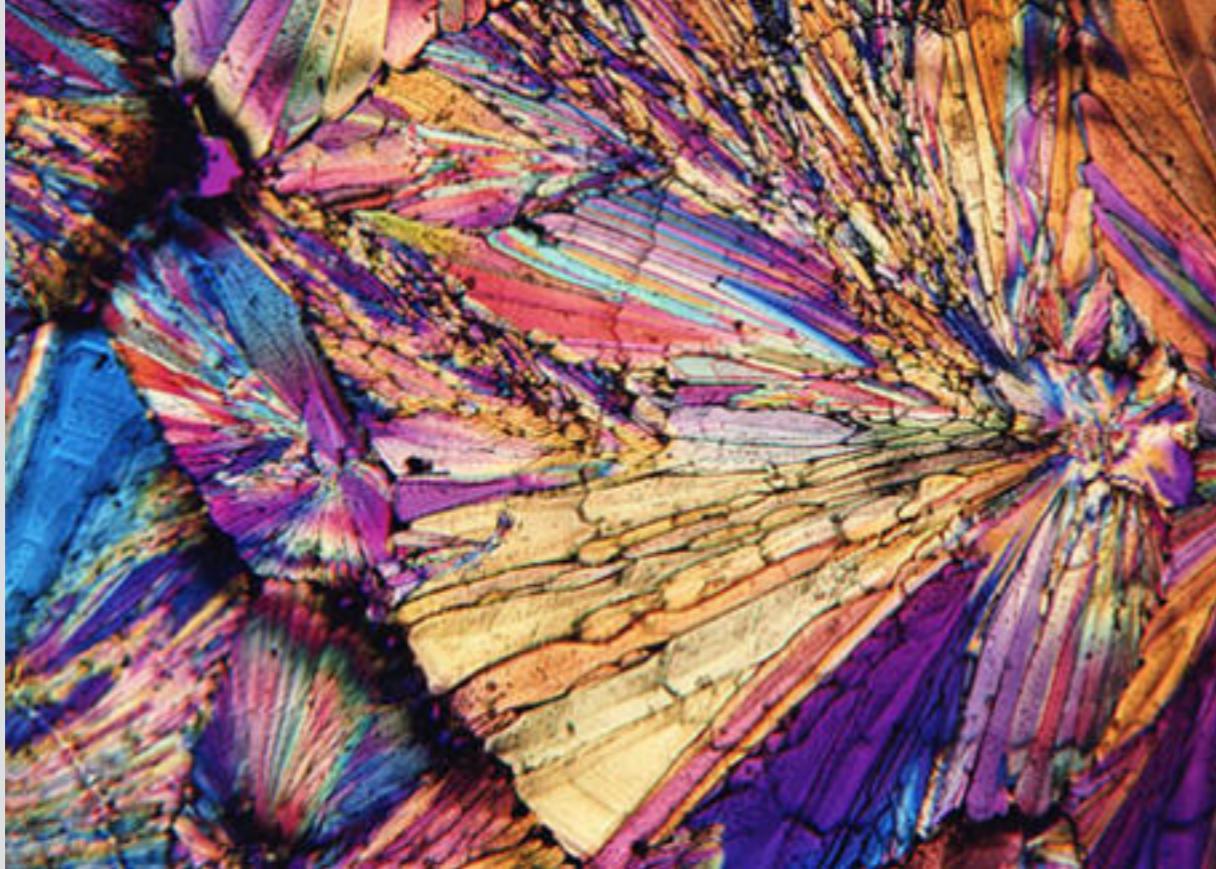
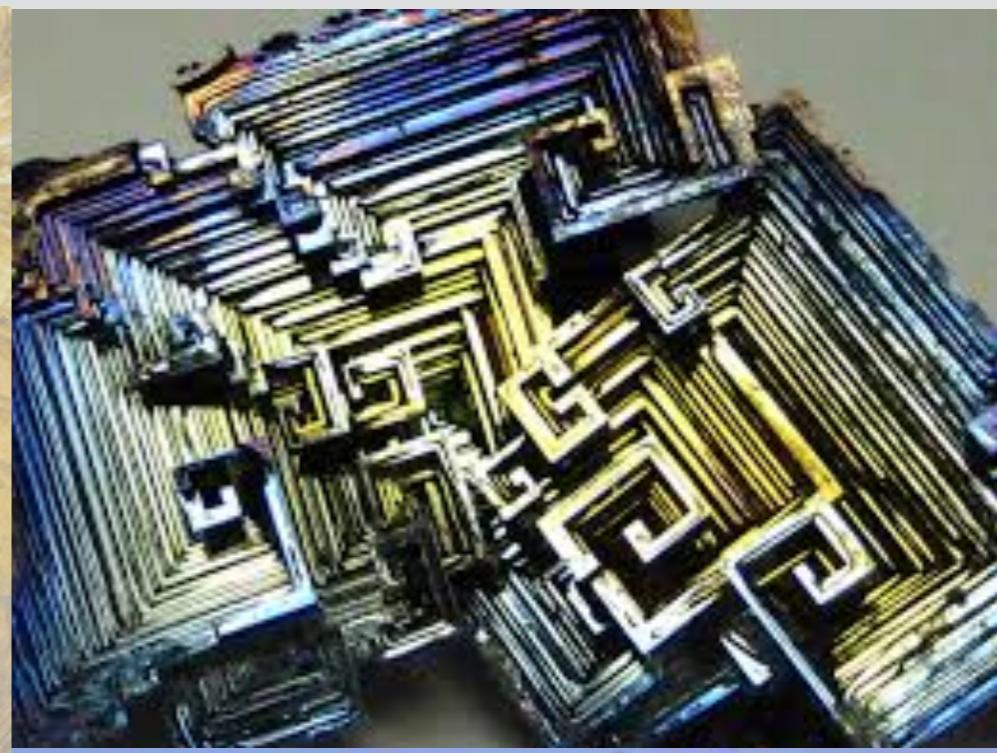


# Self-Assembly

Simple particles coalescing into complex superstructures.



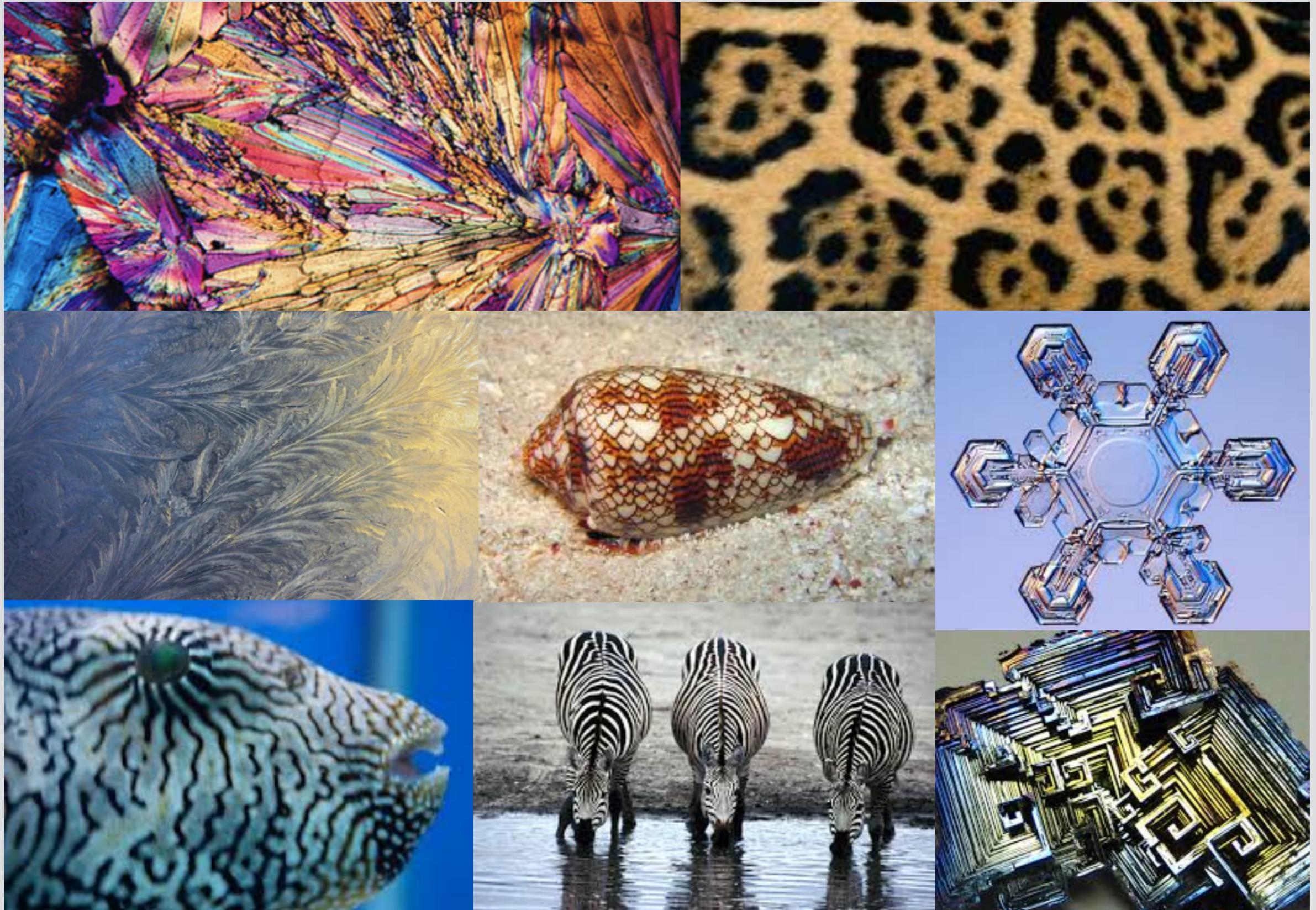
# Crystallization



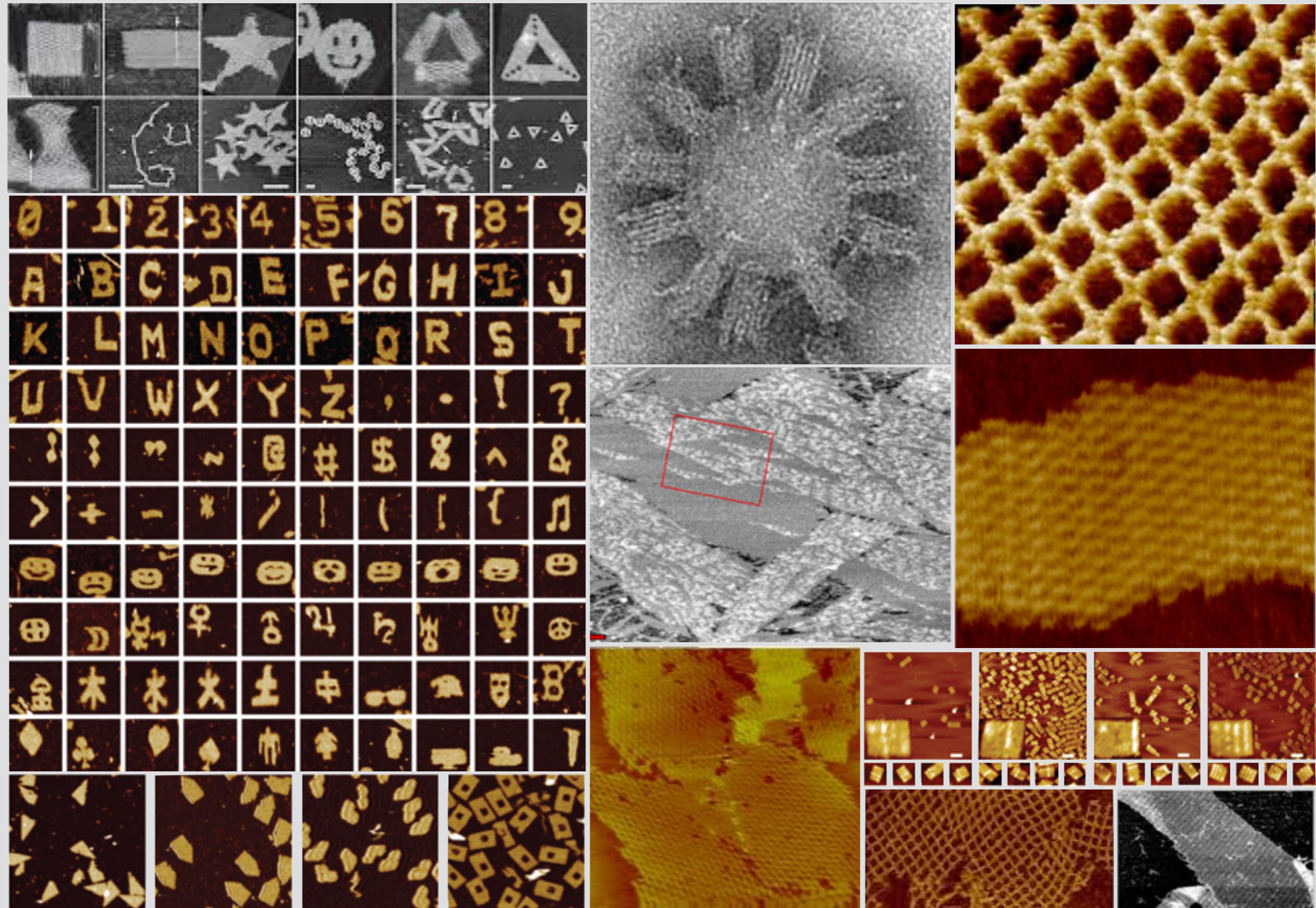
# Morphogenesis



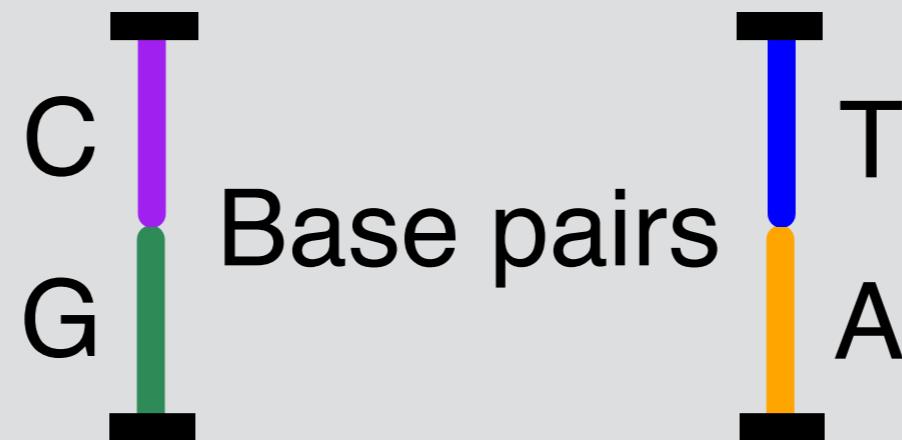
# Natural self-assembly



# Synthetic DNA self-assembly



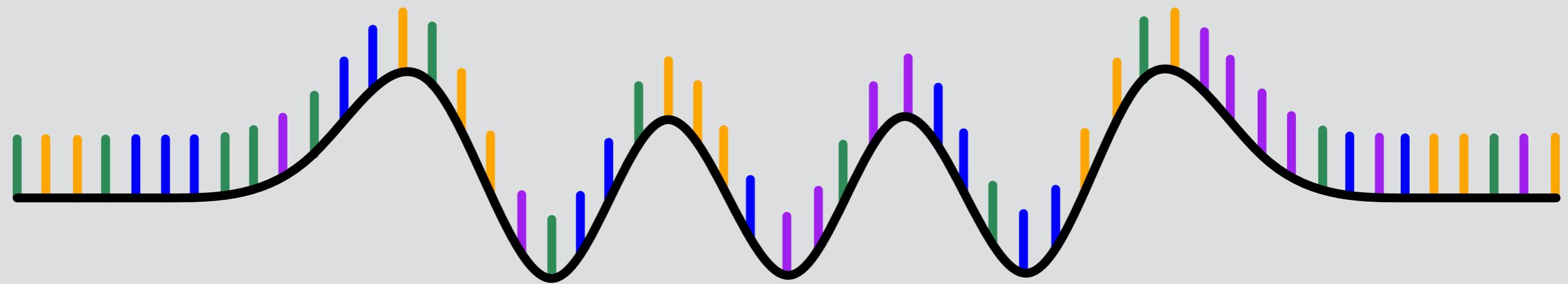
# DNA

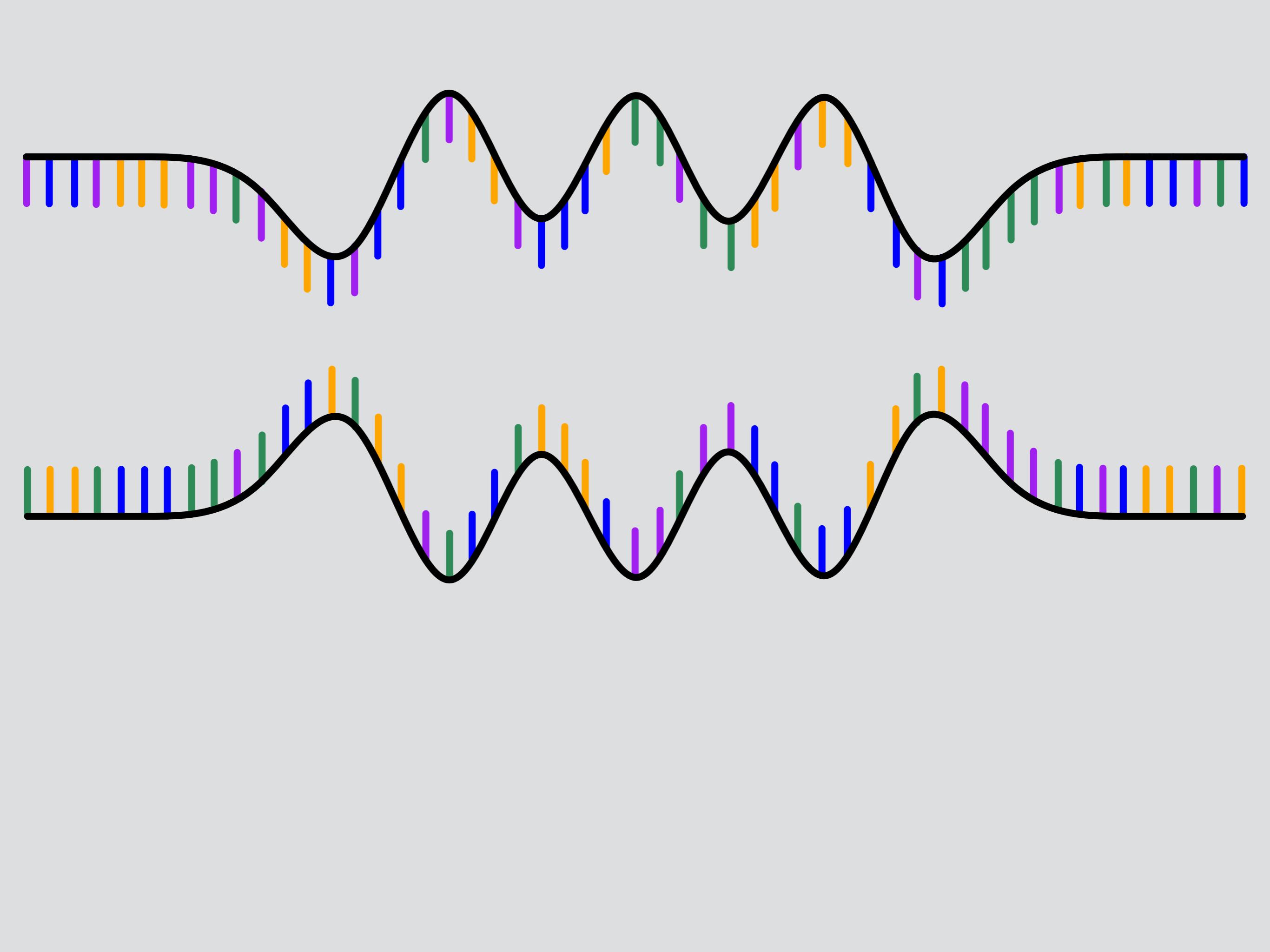


GAAGTTTGGCGTTAGAACGTGAAATCCGCCCTTGTTAAGACCCCGTCATAAGCA  

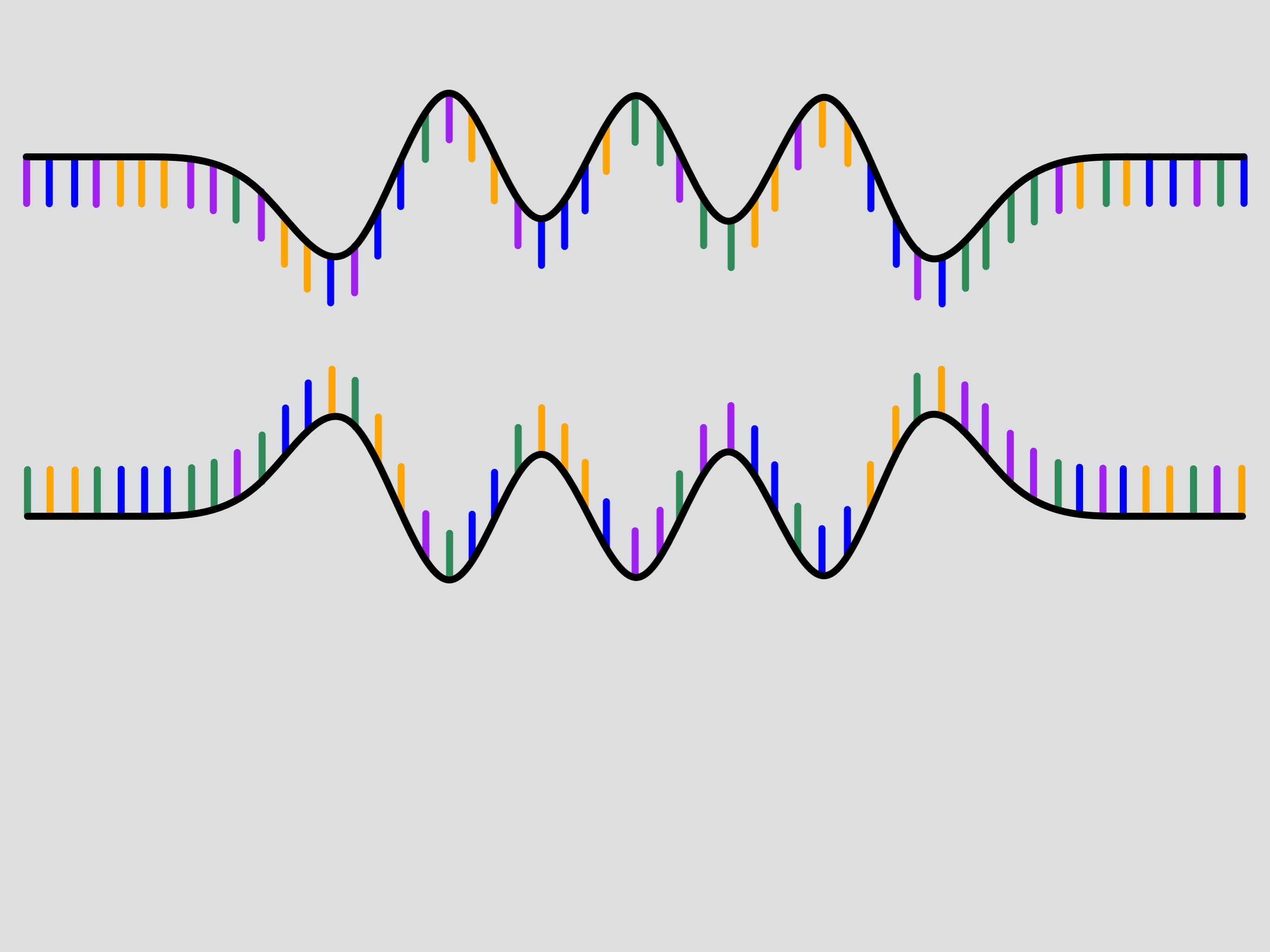
A sequence of DNA bases is shown above a horizontal line. Below the line, vertical colored bars (green, orange, blue, purple) represent the bases: G, T, A, C respectively. The sequence is: G, A, A, G, T, T, T, G, G, C, G, T, T, A, G, A, A, C, C, C, G, T, C, T, A, A, G, C, A. The bars are color-coded according to the base-pairing diagram above, showing how the sequence is composed of alternating green/orange and blue/purple segments.

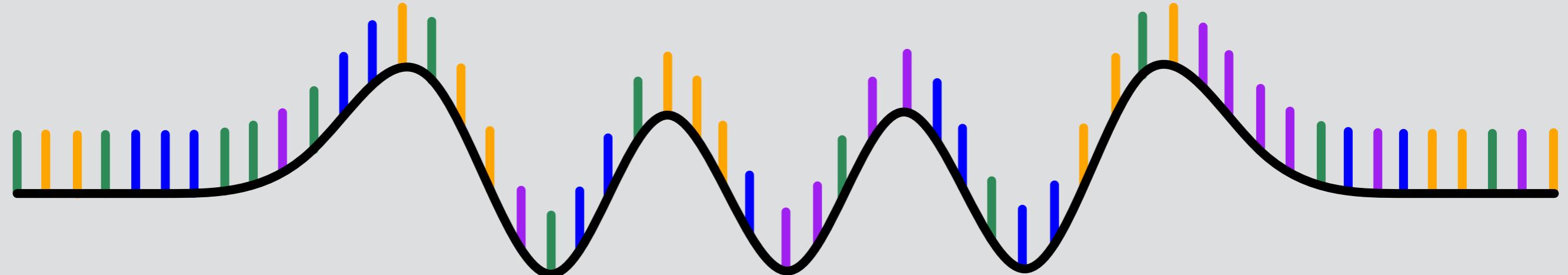
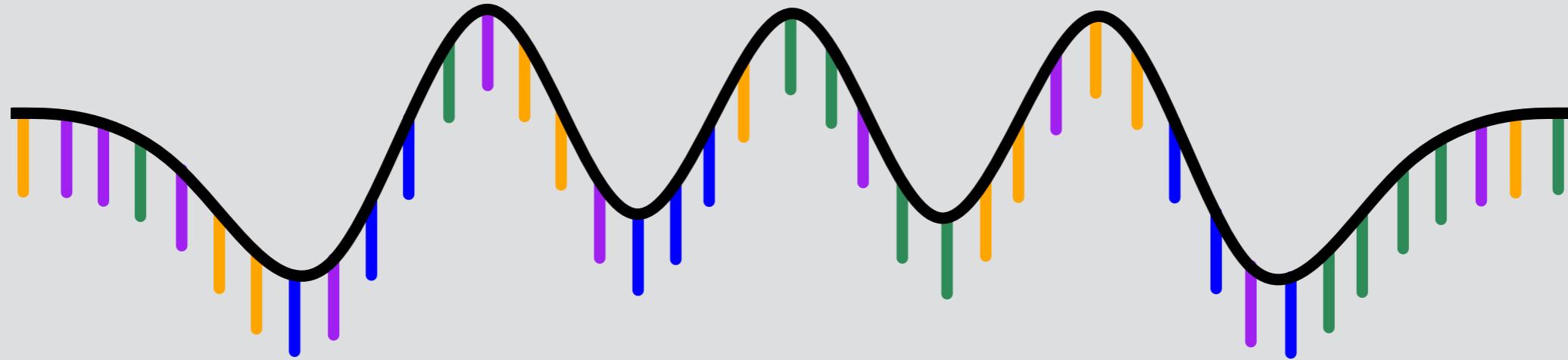
Single strand





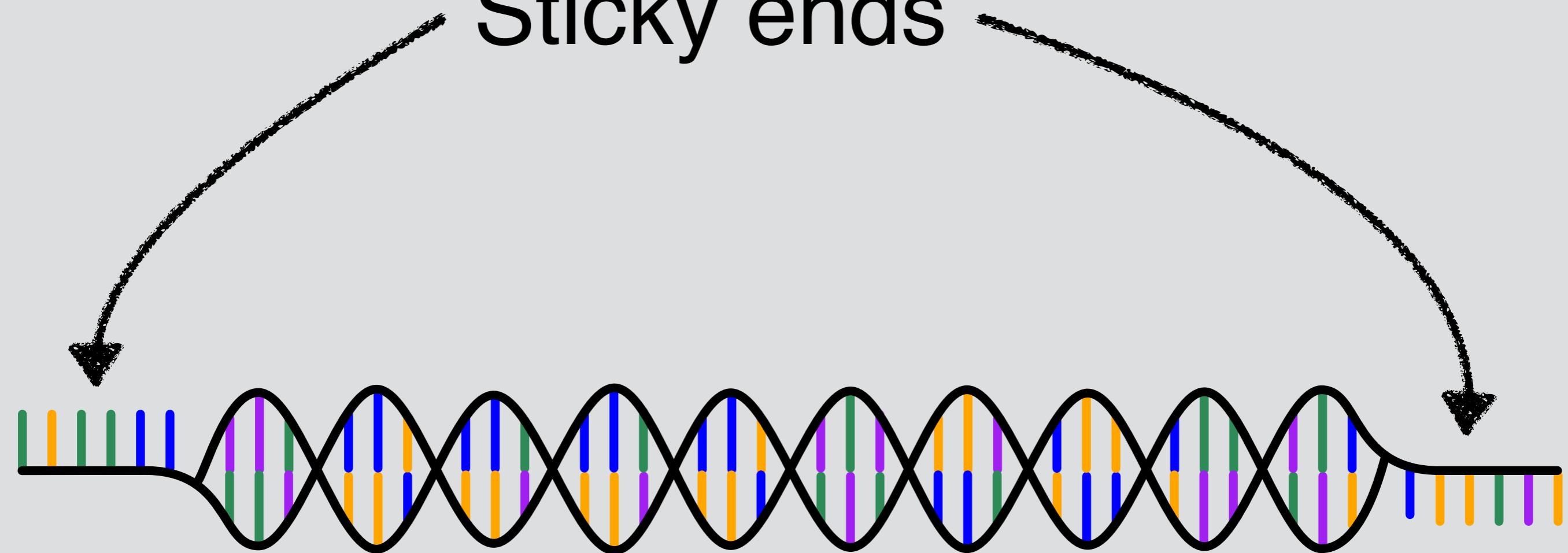




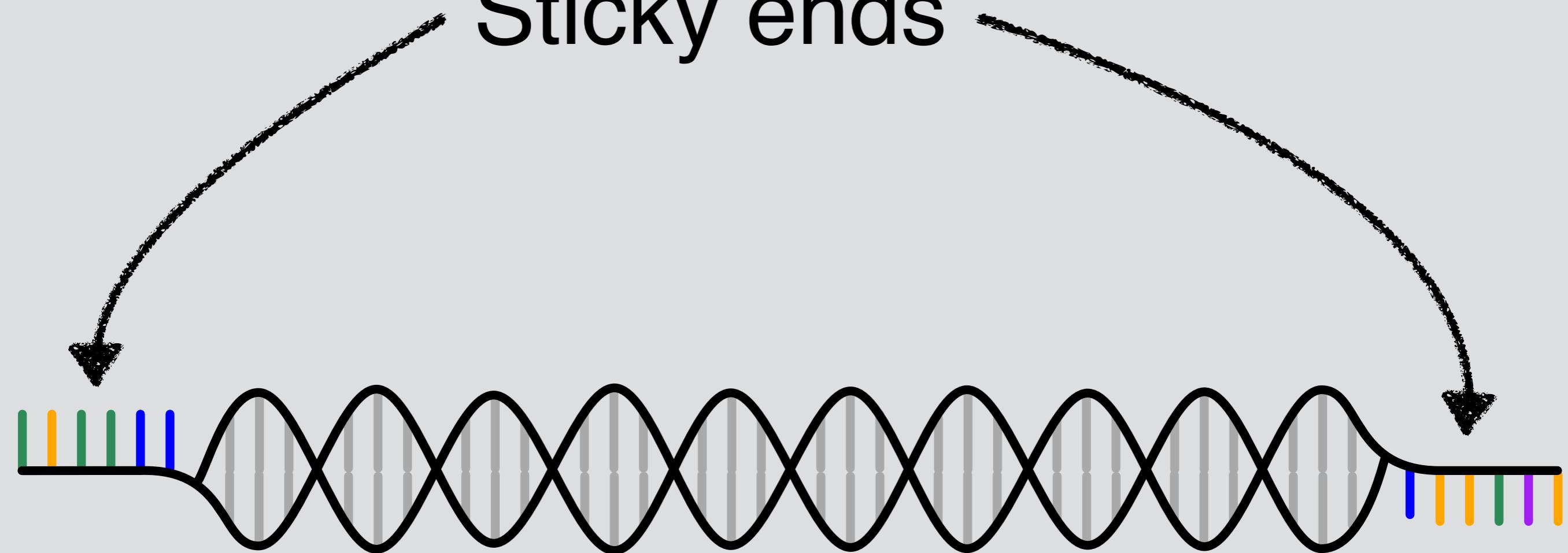


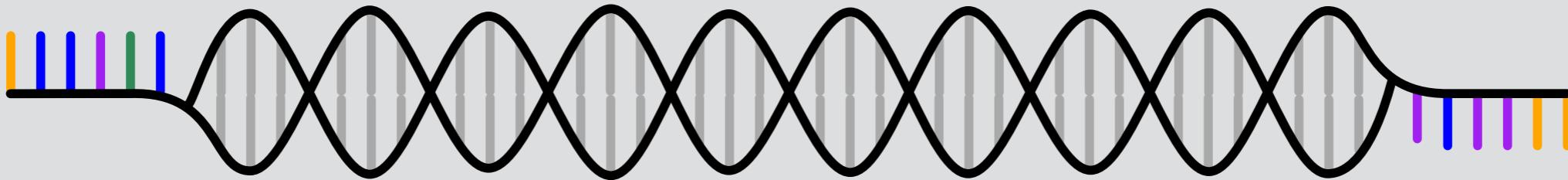


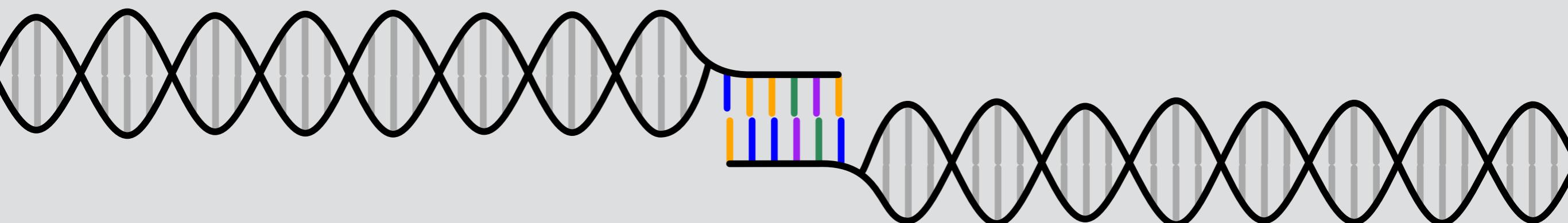
# Sticky ends

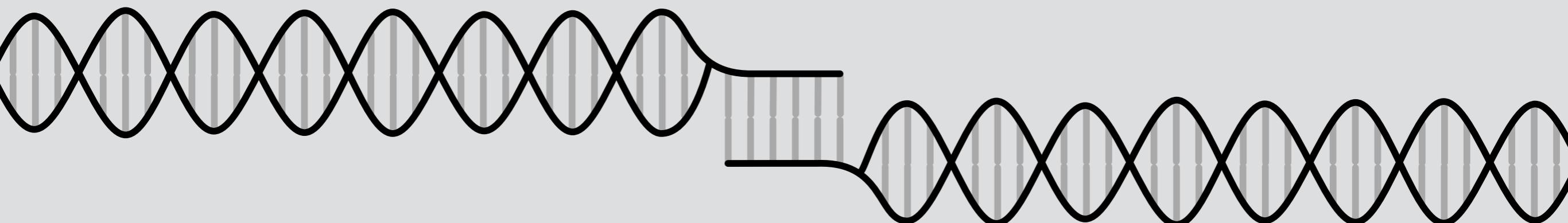


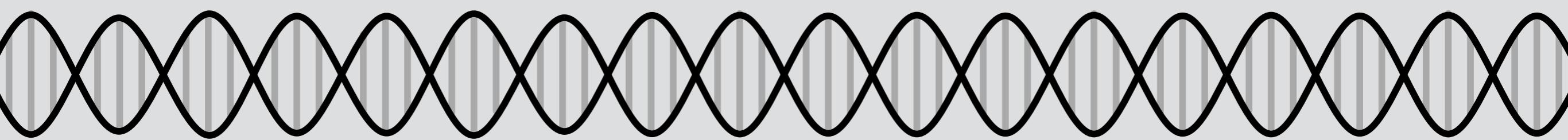
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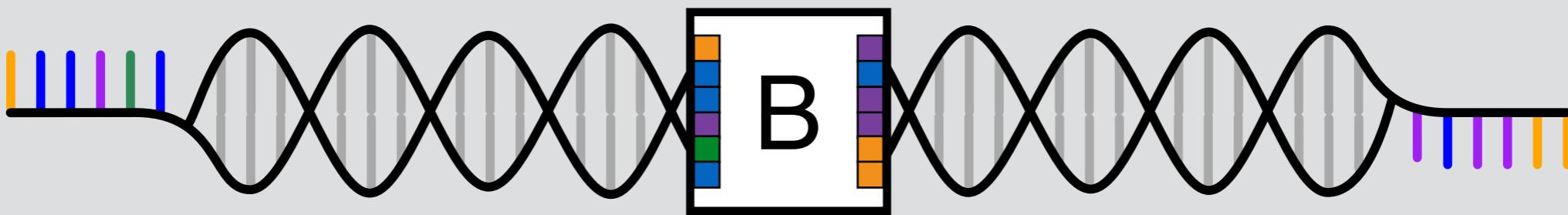
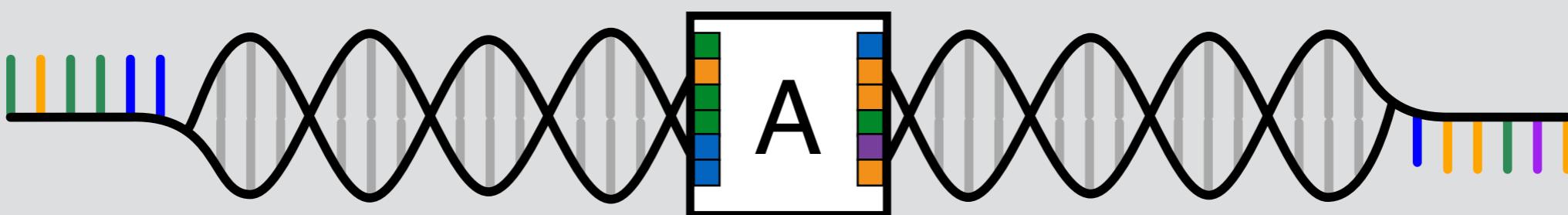




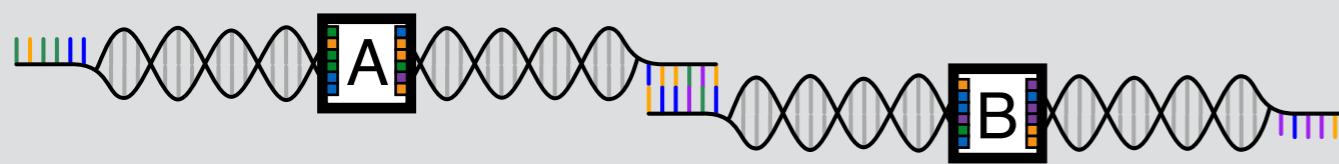
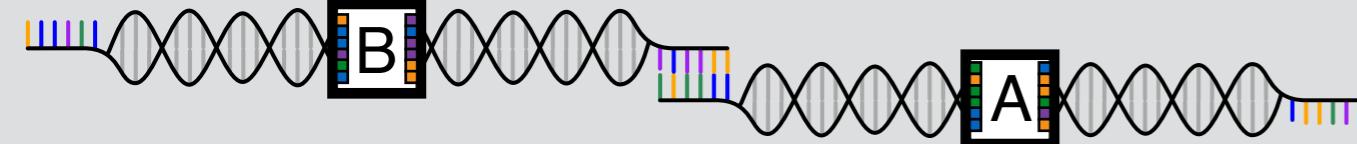
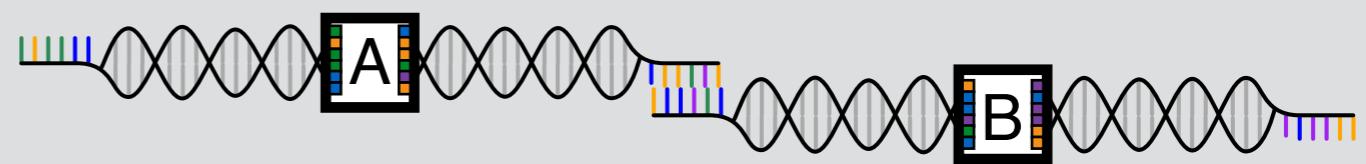
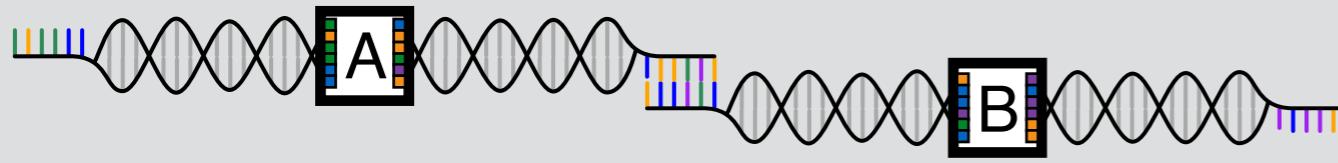
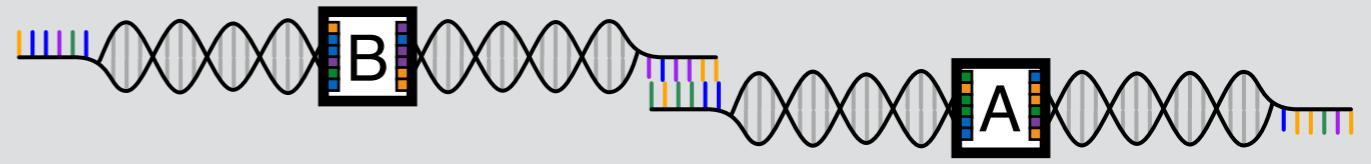


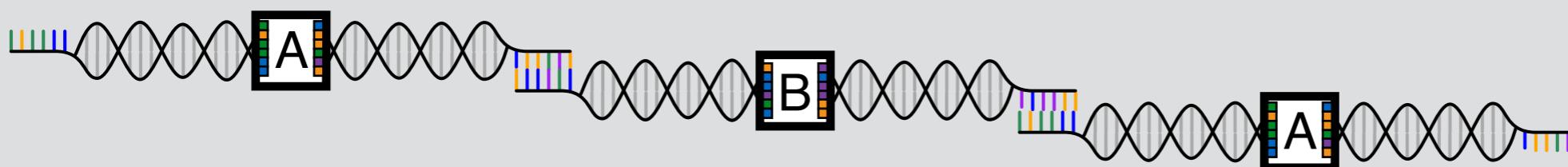
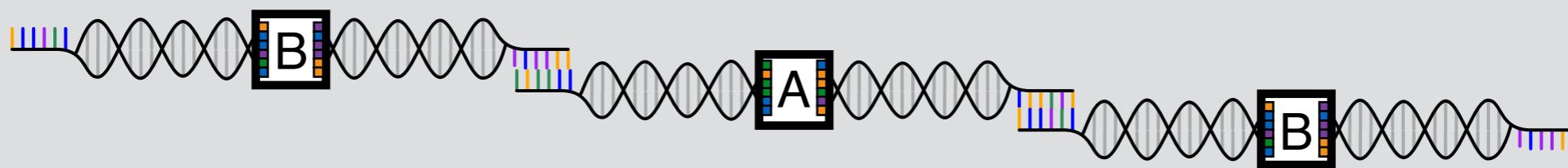
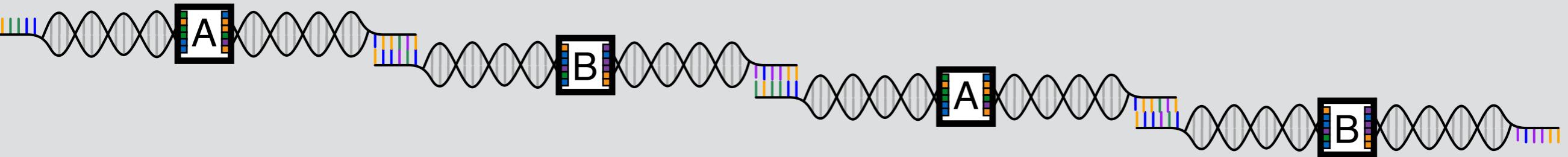
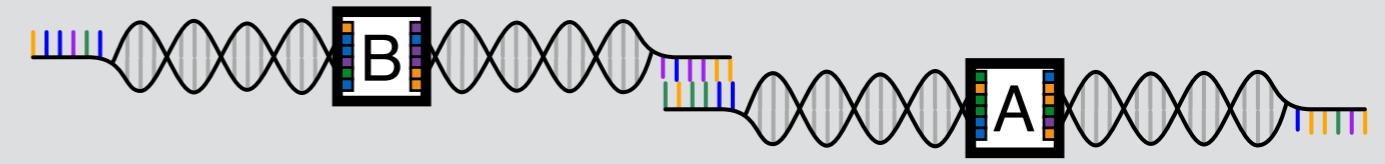


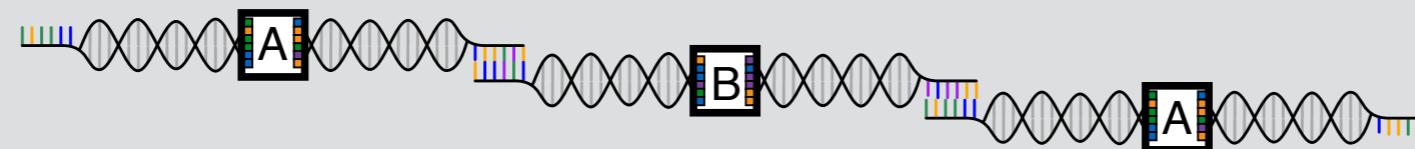
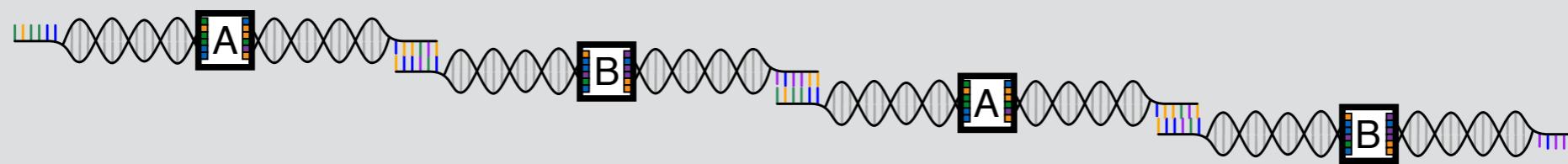
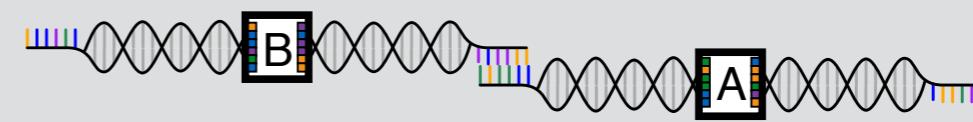


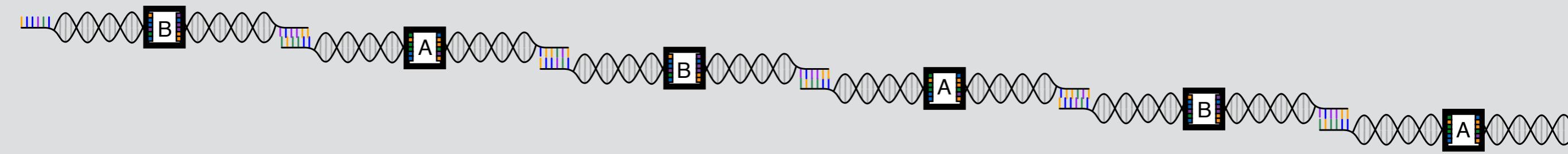
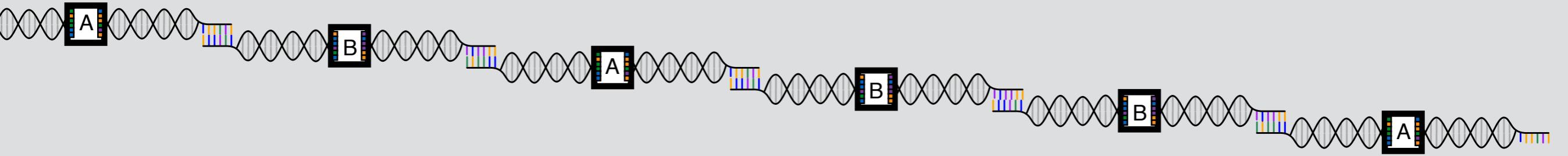


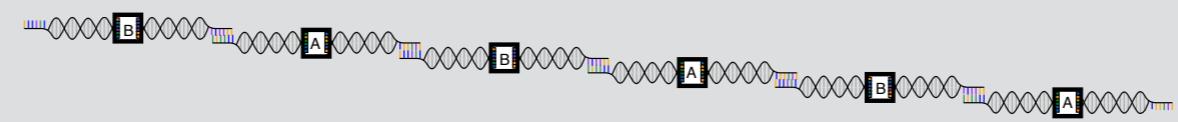


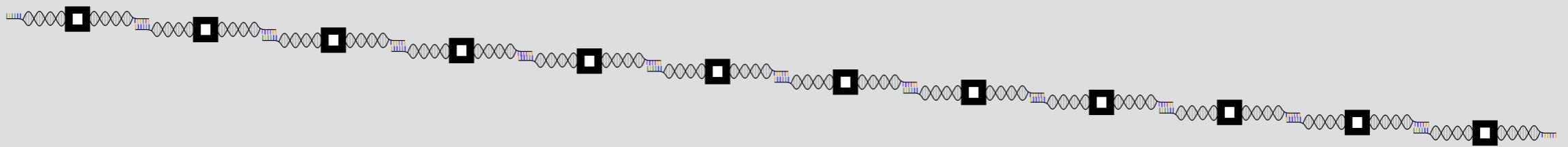






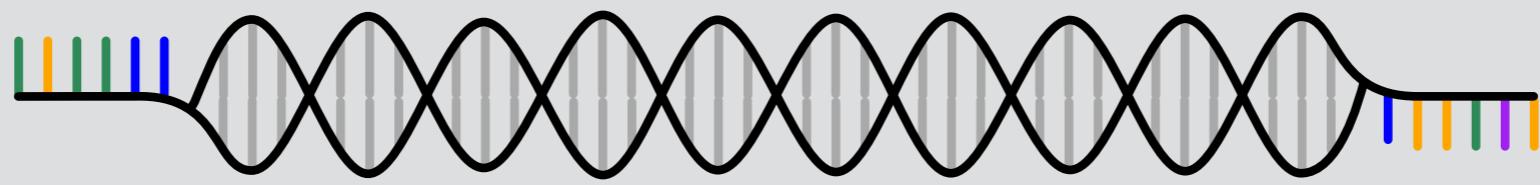


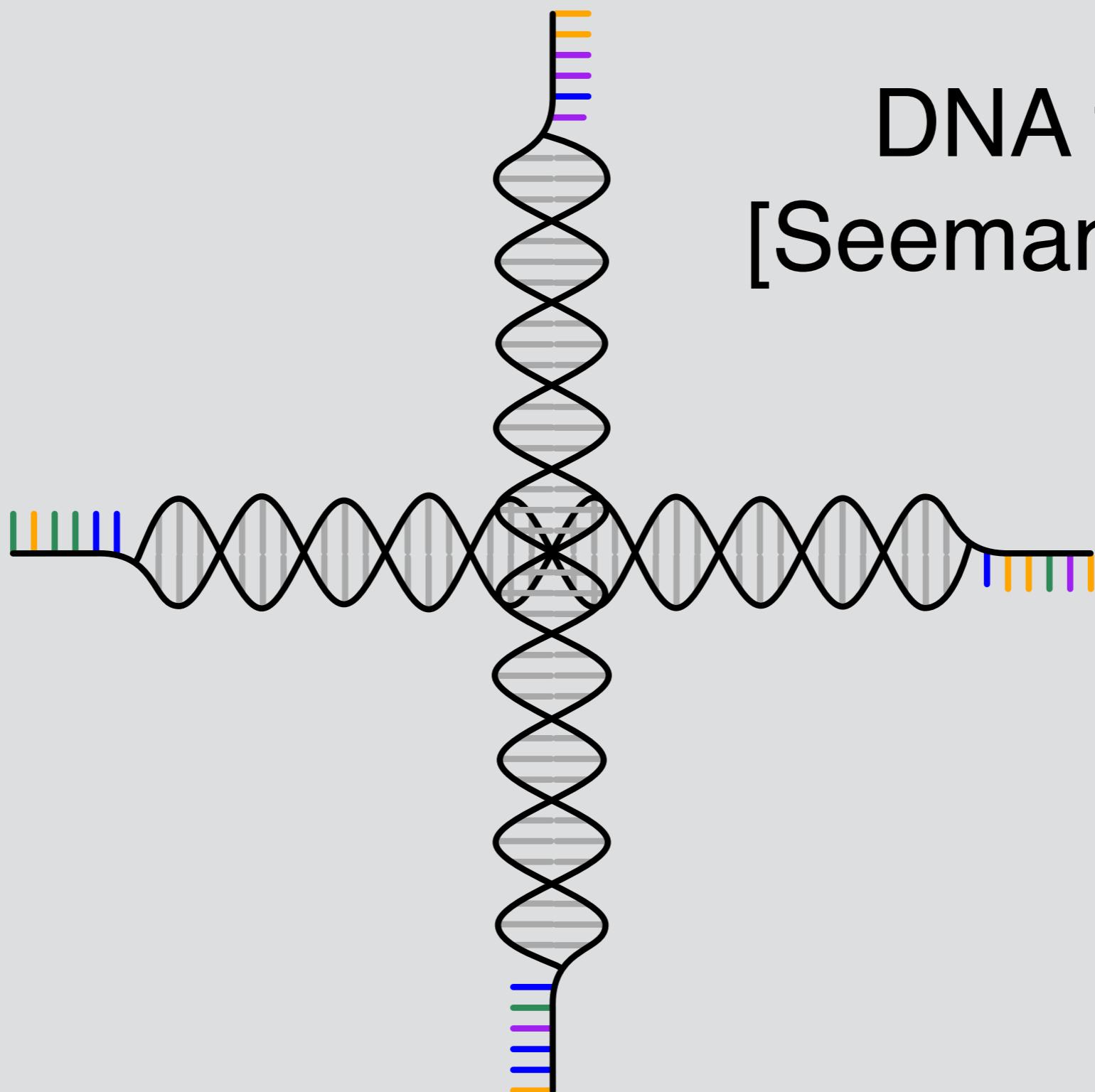






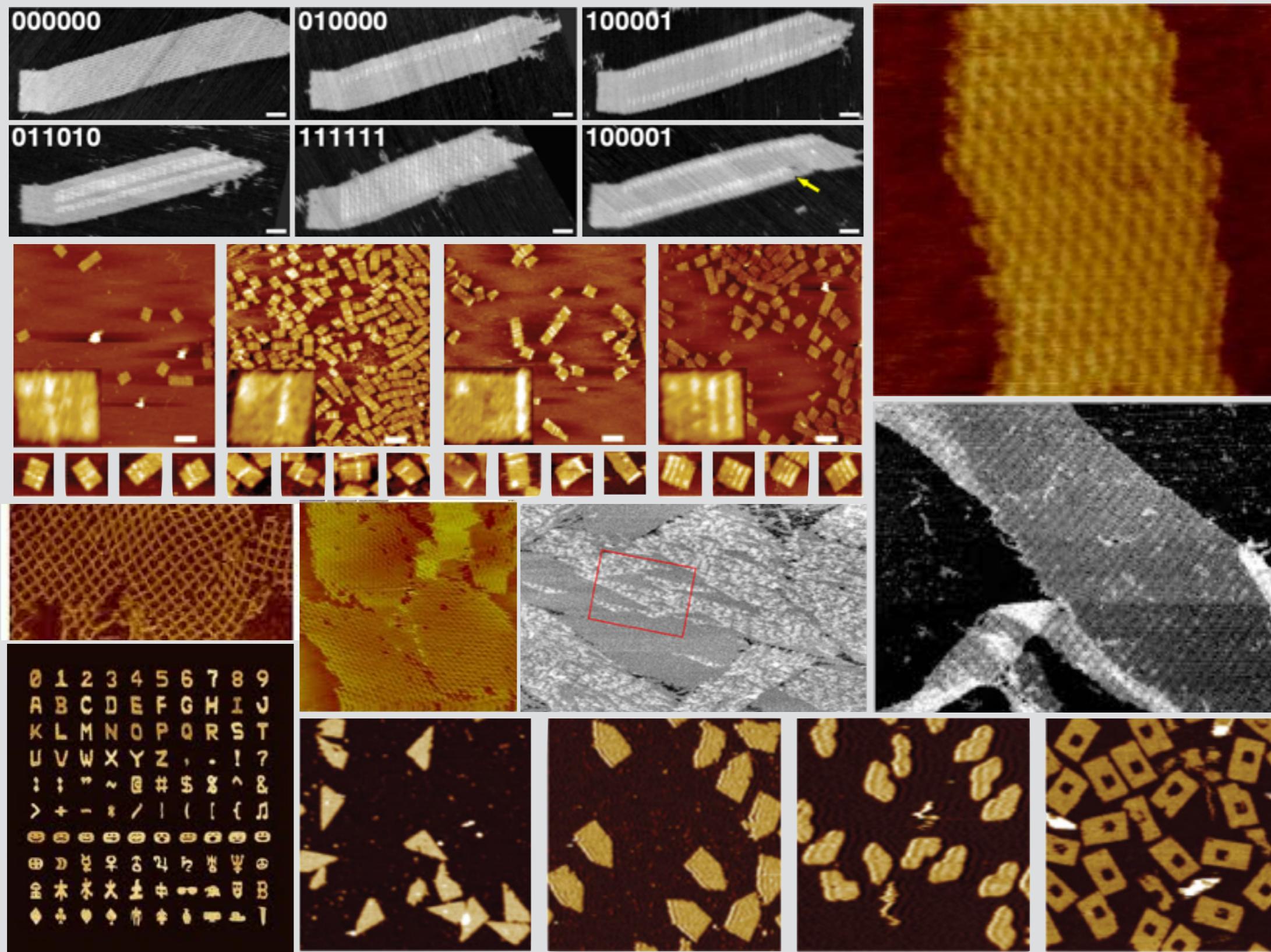
# [Wei, Dai, Yin 2012]

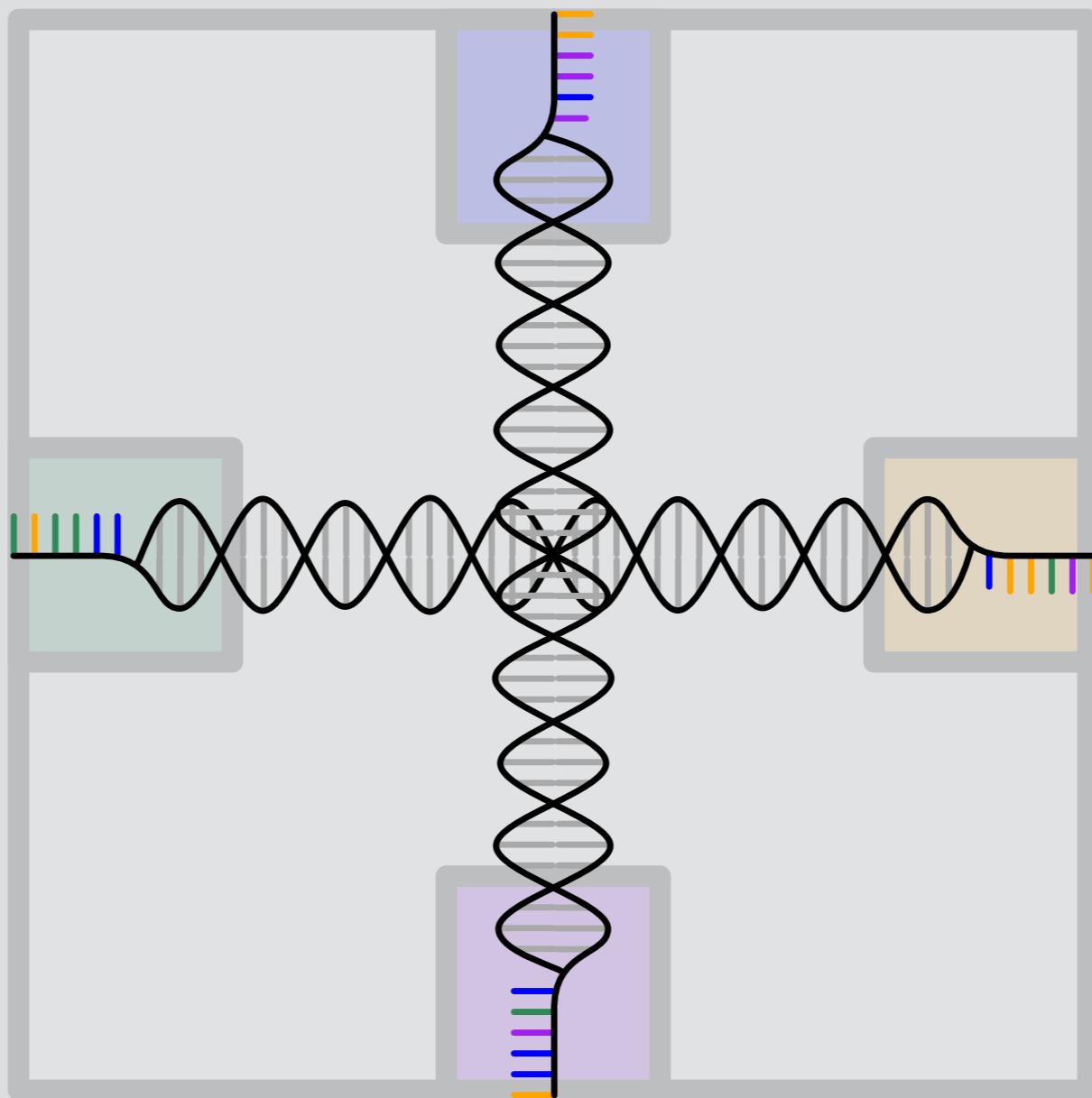


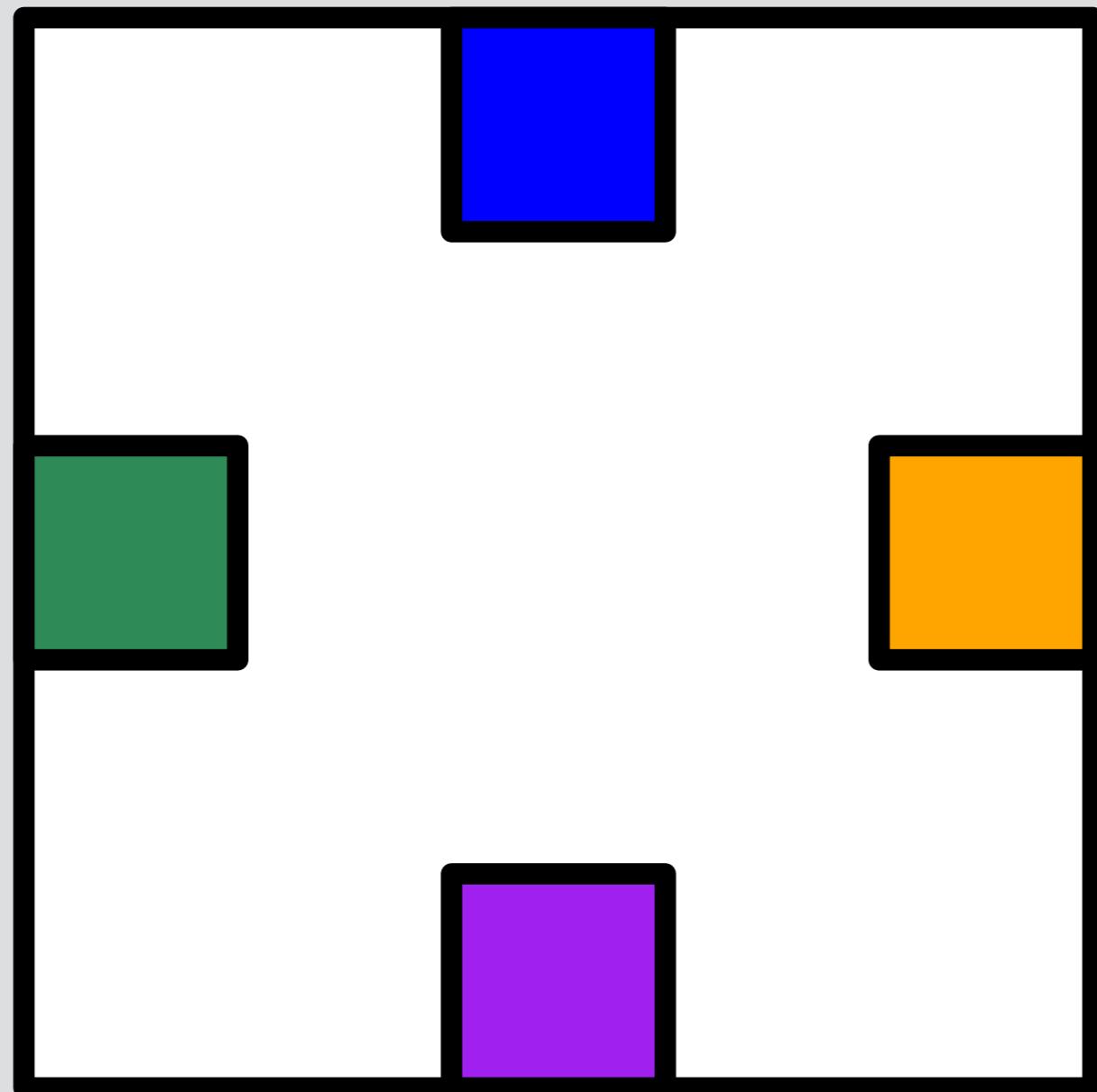


DNA tiles  
[Seeman 1982]

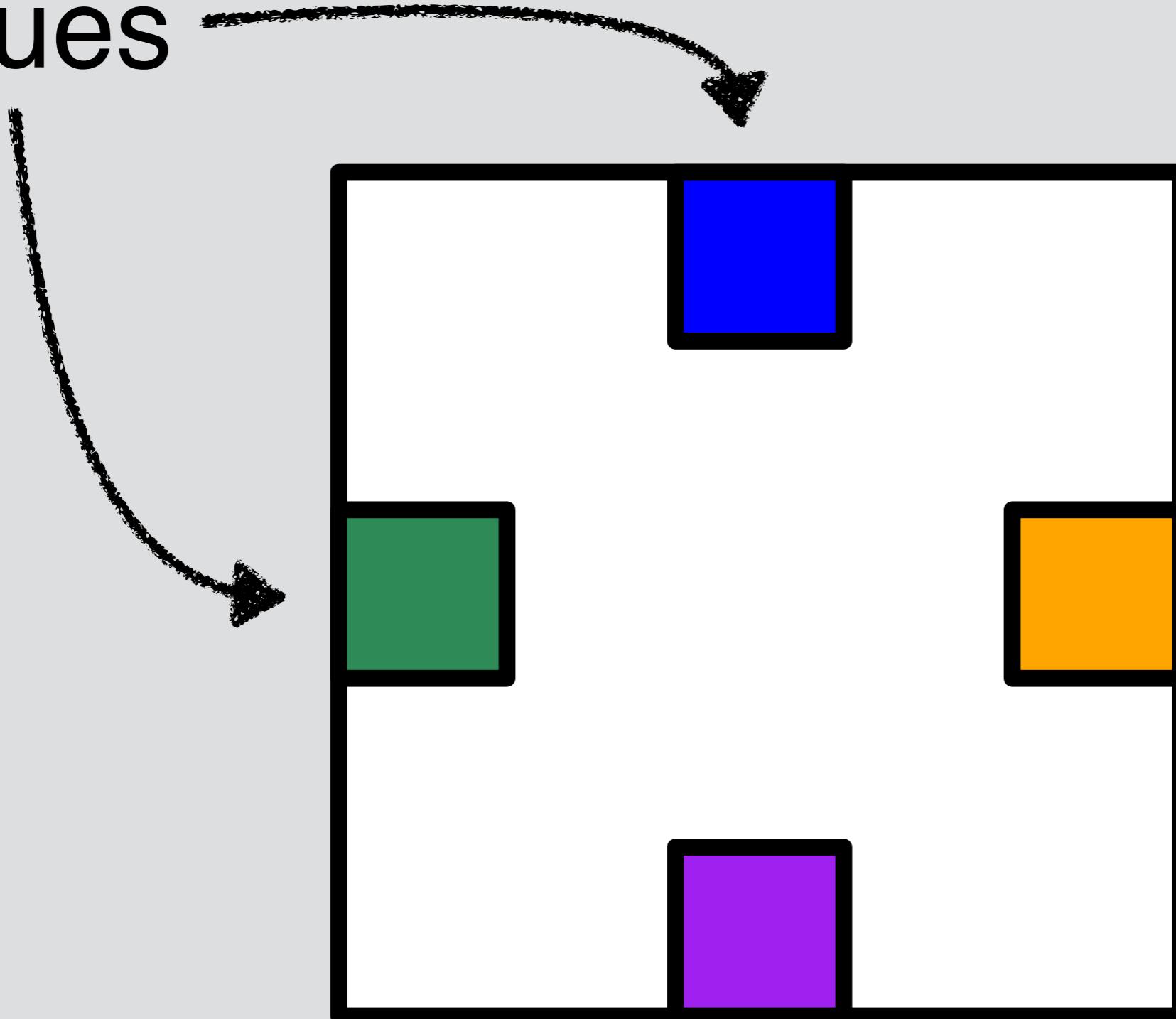
# DNA tile self-assembly

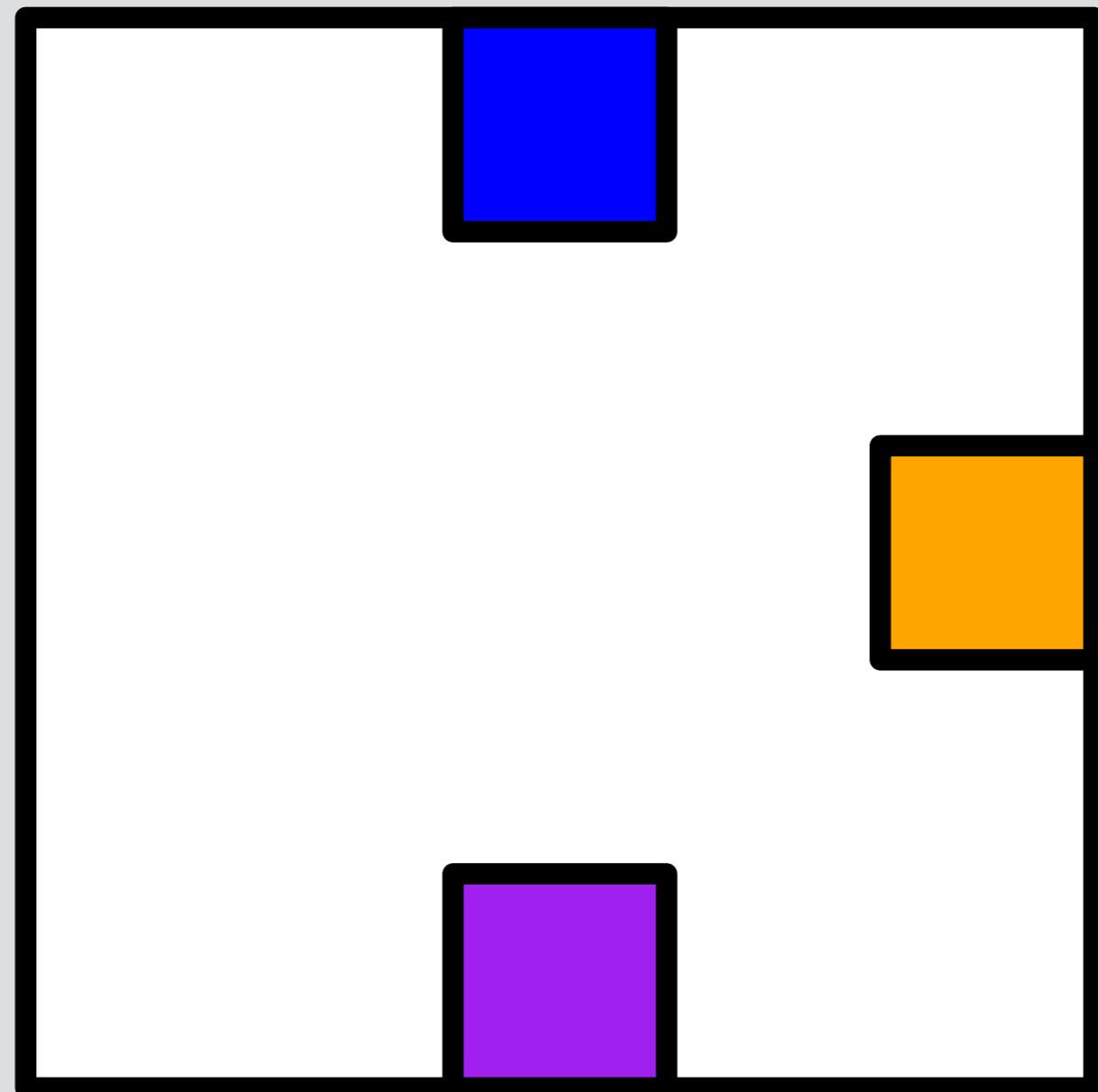


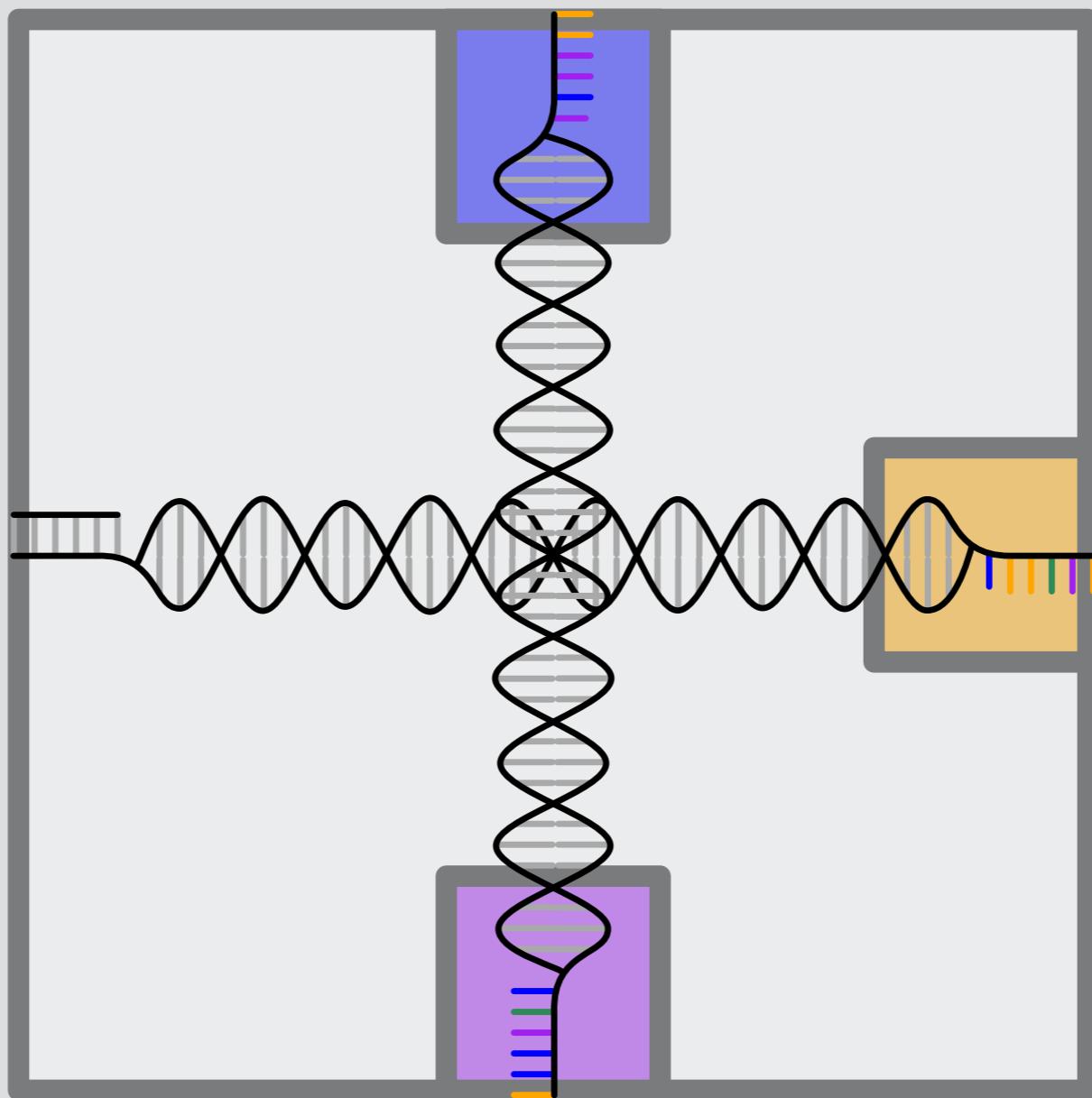




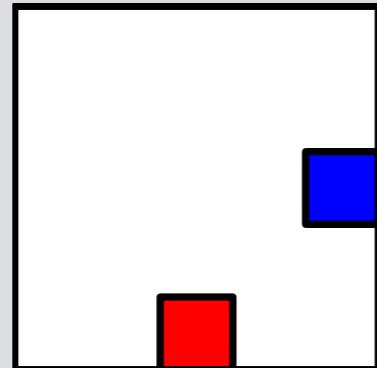
# Glues



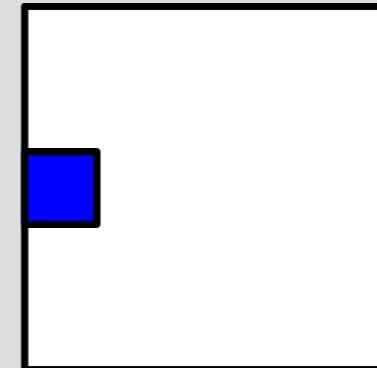




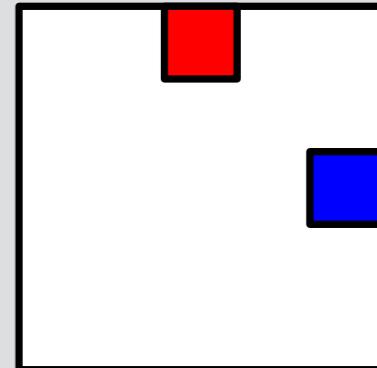
# Tile types



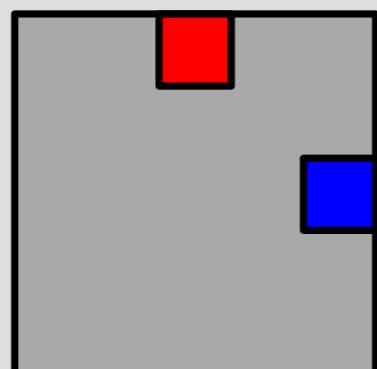
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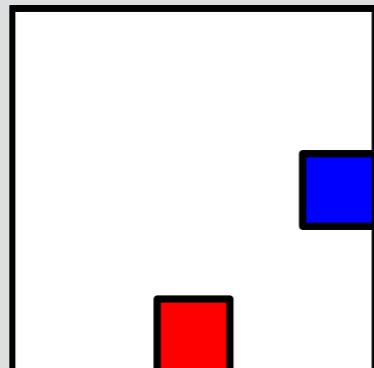


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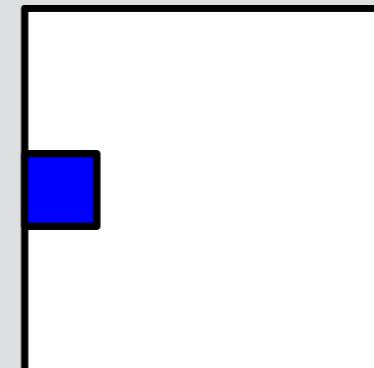


Seed tile

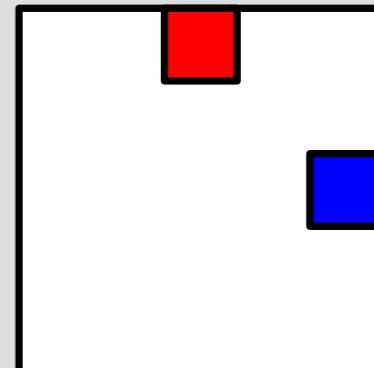
# Tile types



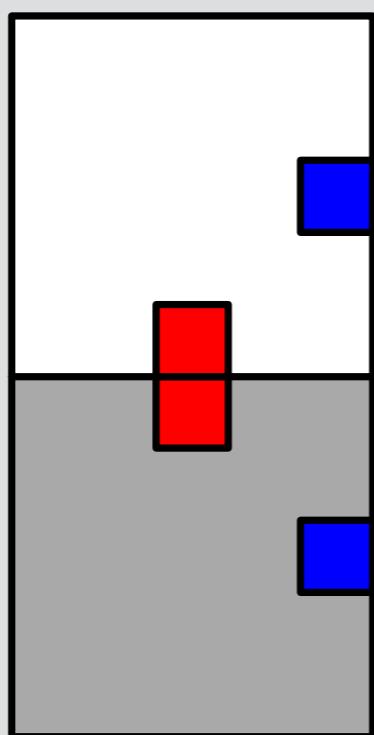
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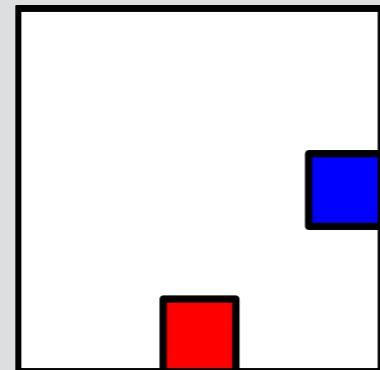


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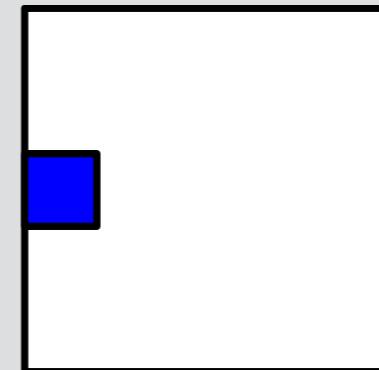


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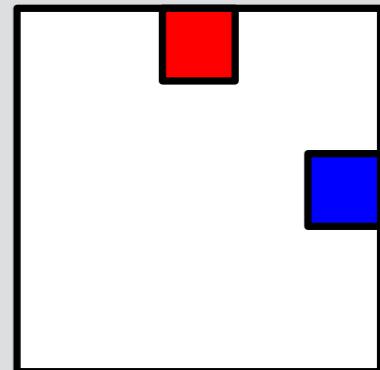
# Tile types



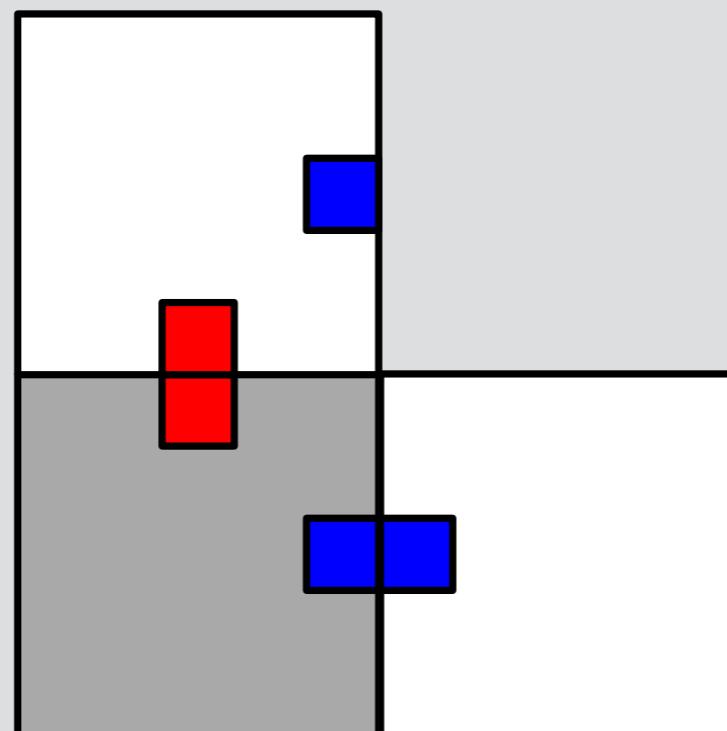
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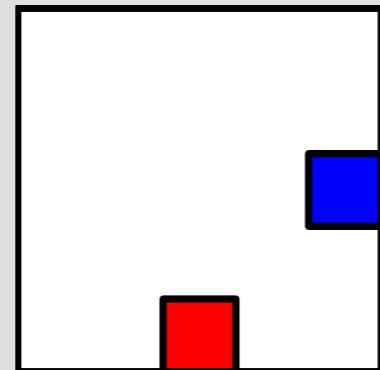


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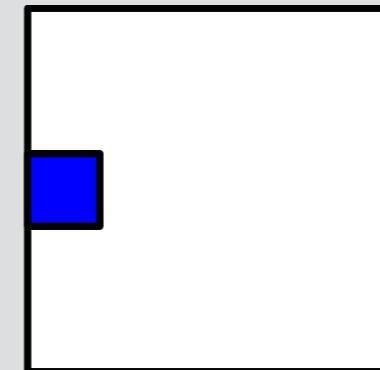


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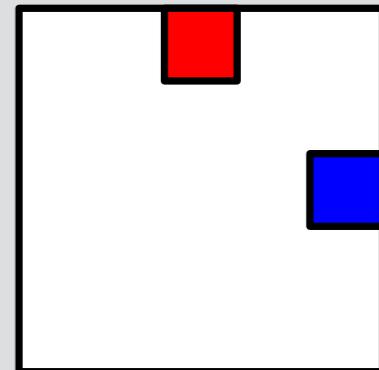
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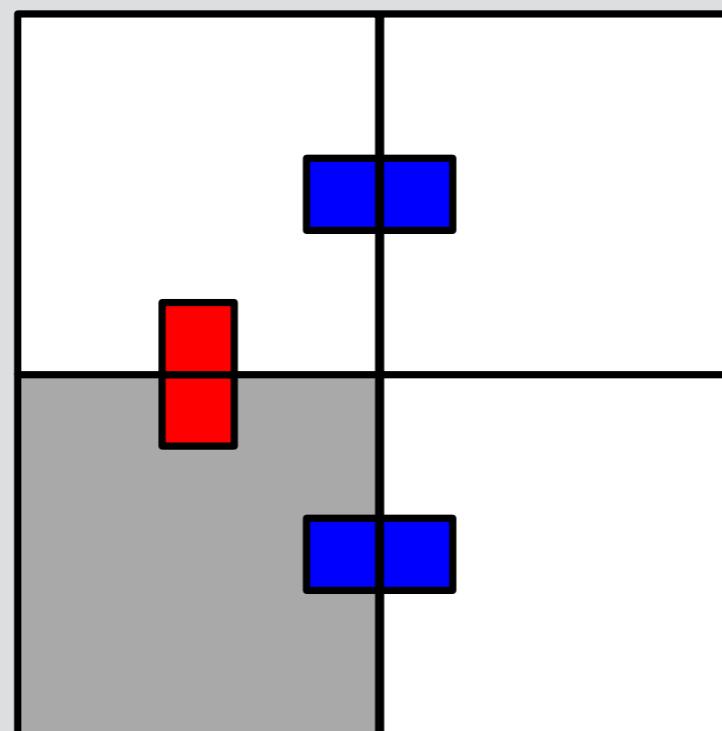
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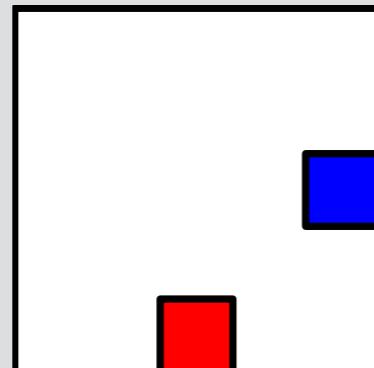


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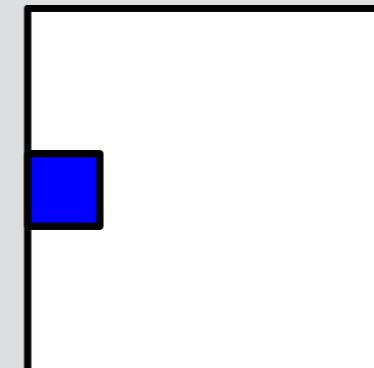


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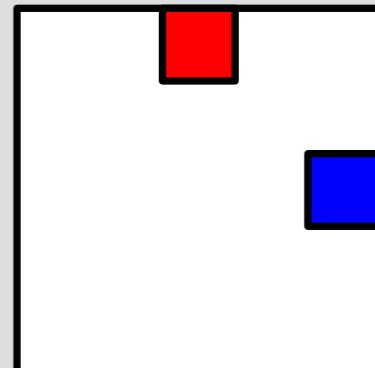
# Tile types



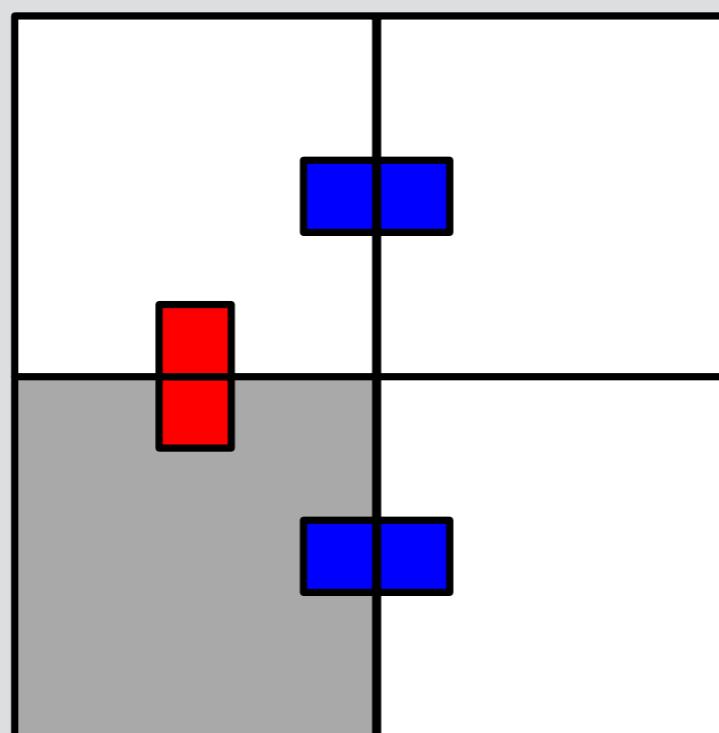
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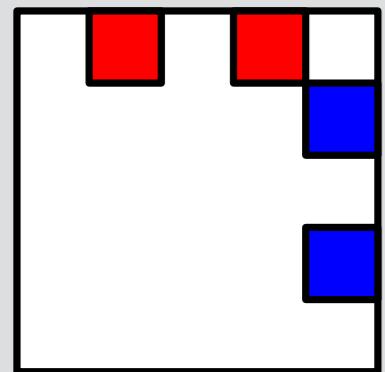


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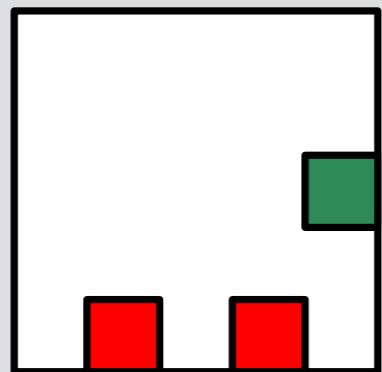


Seed tile

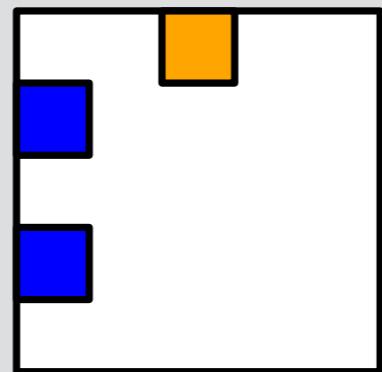
Temperature 1



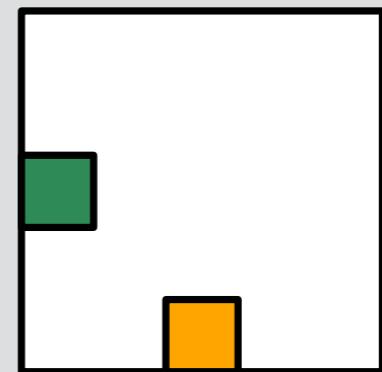
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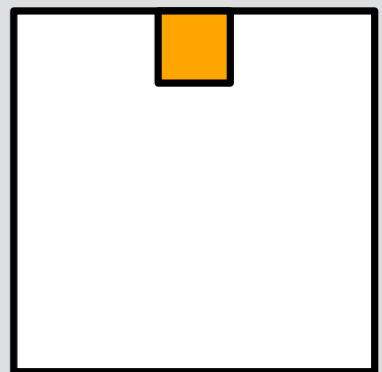
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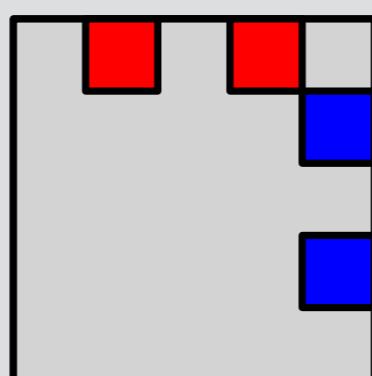
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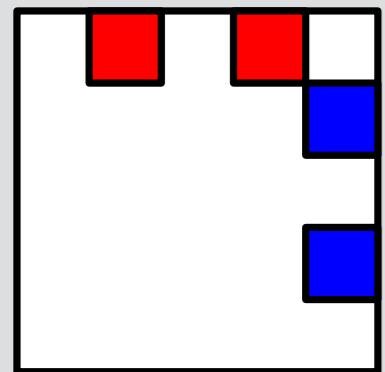
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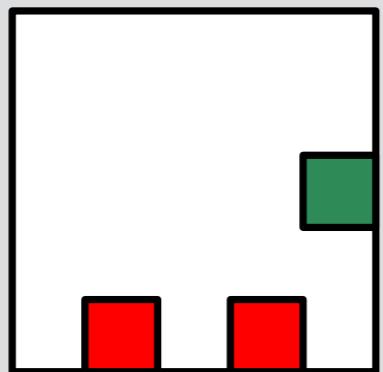
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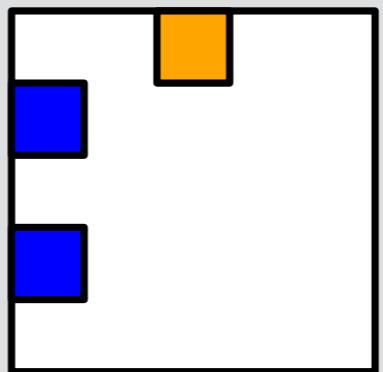
Temperature 2



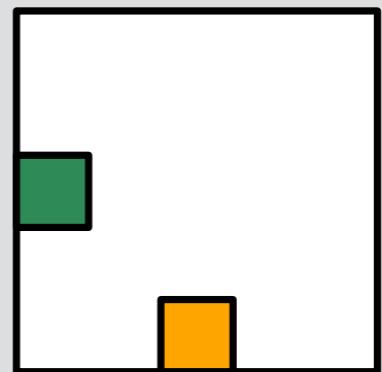
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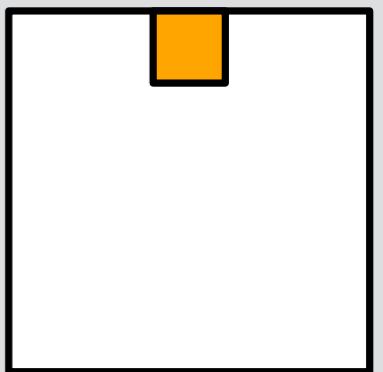
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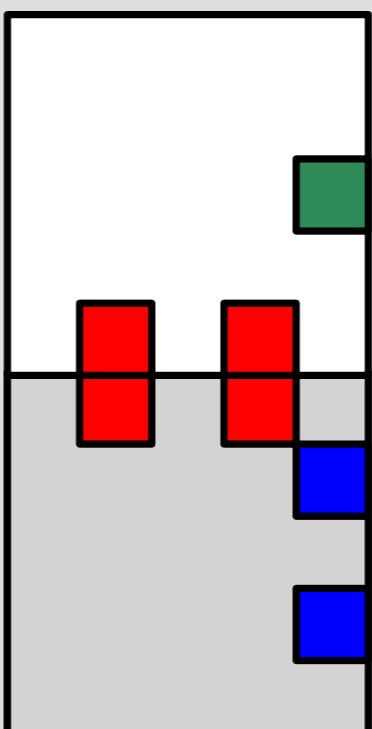
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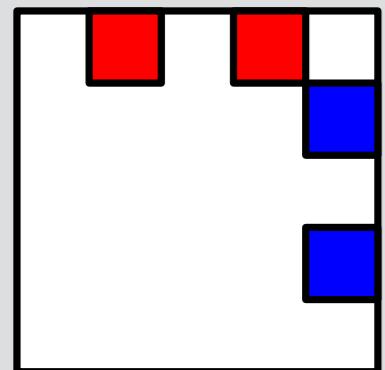
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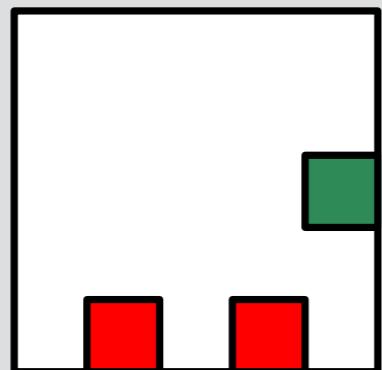
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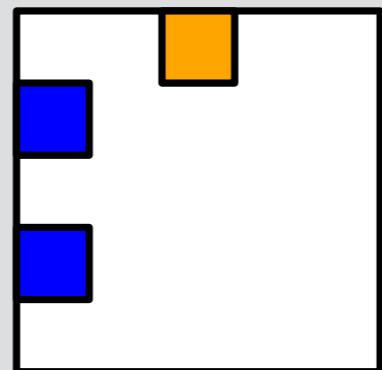
Temperature 2



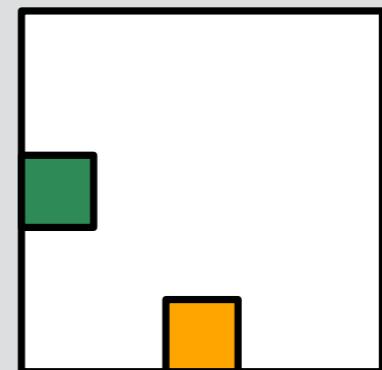
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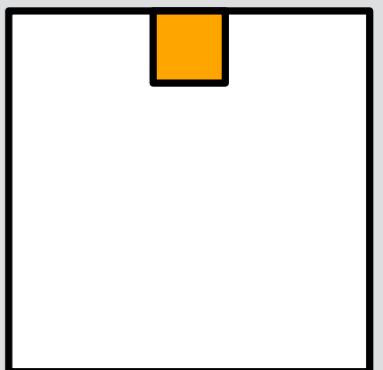
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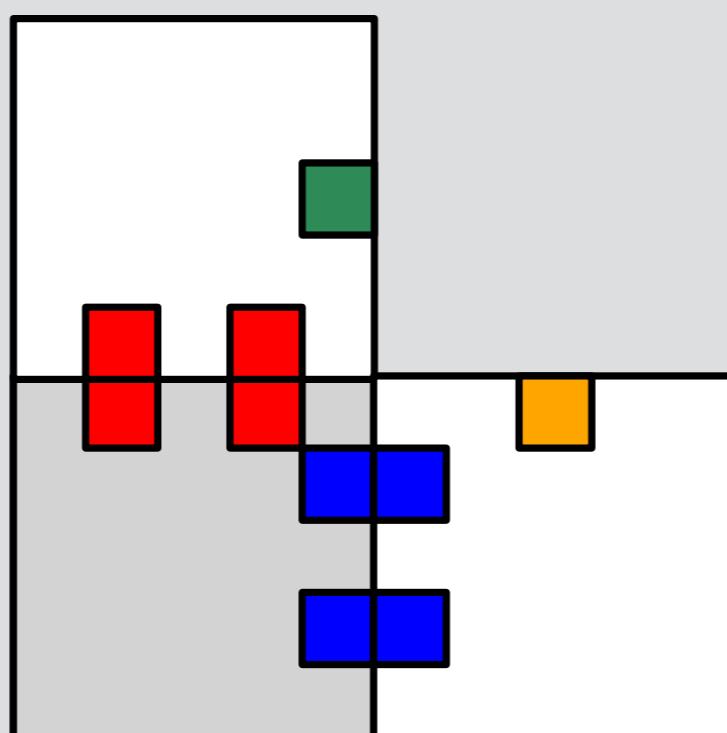
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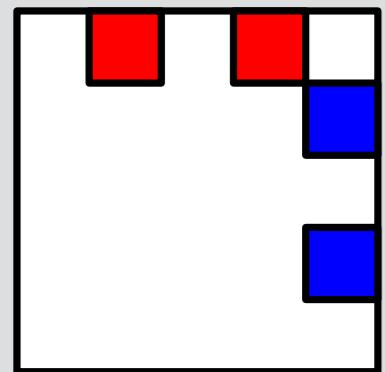
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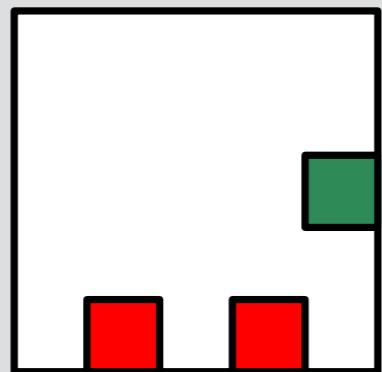
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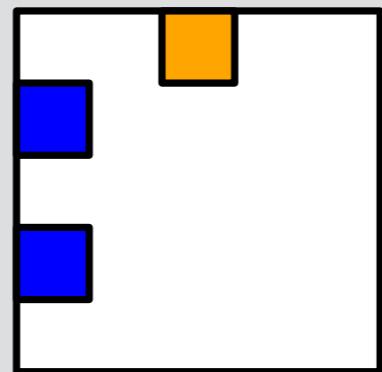
Temperature 2



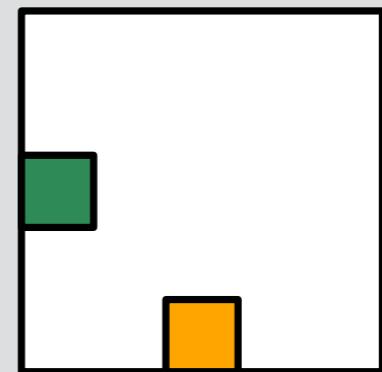
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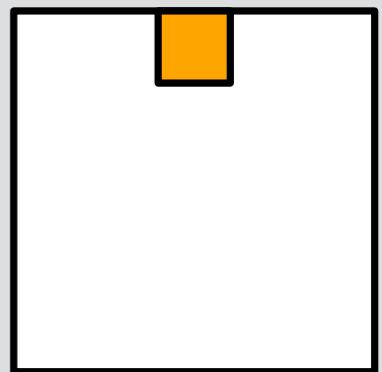
$\times\infty$



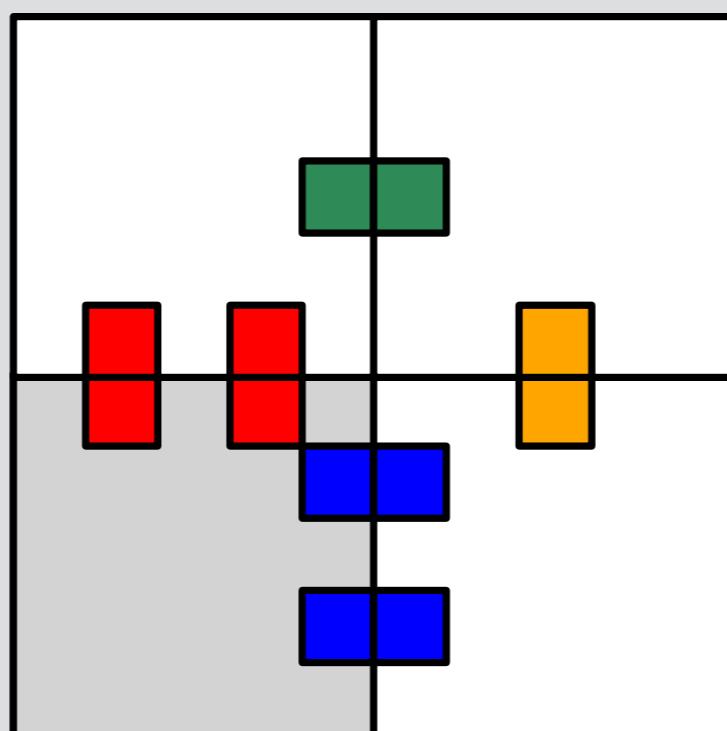
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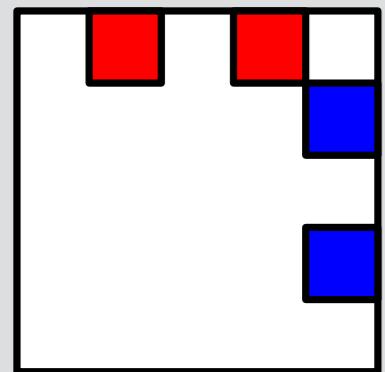
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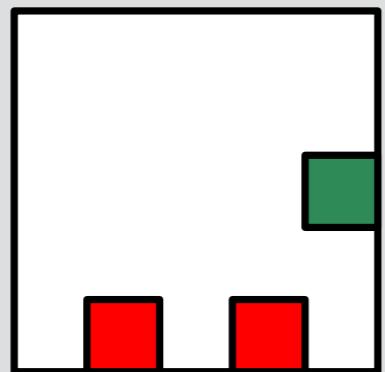
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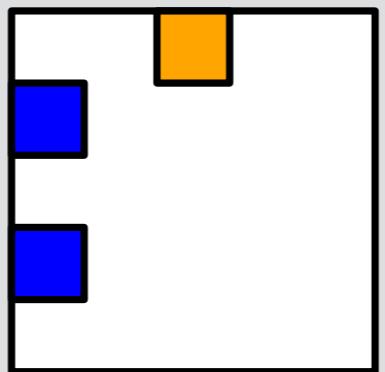
Temperature 2



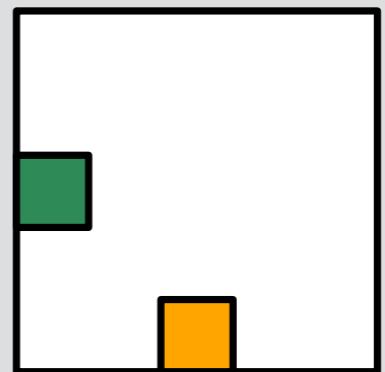
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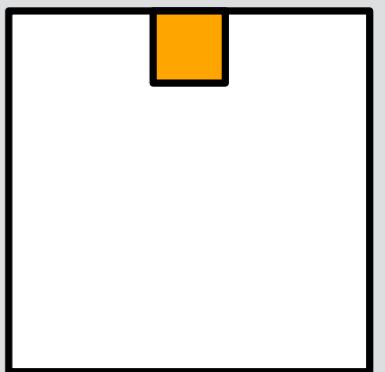
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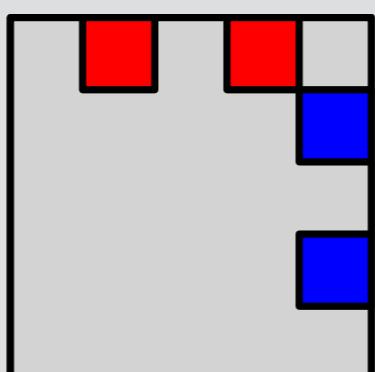
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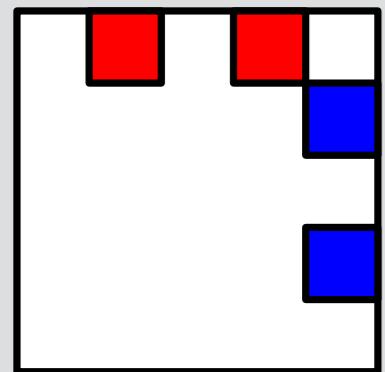
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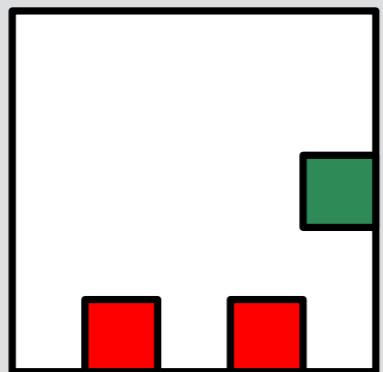
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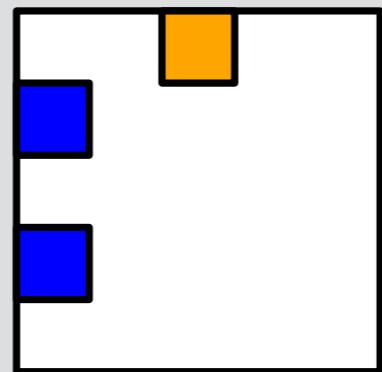
Temperature 1



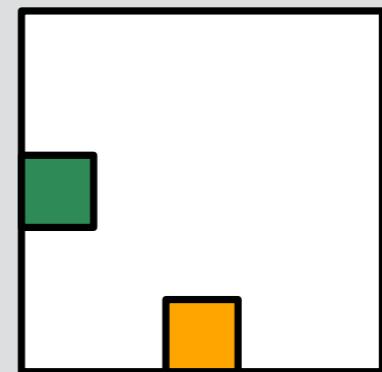
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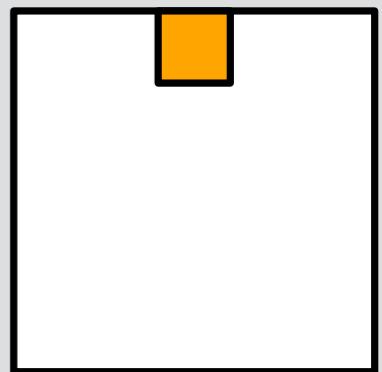
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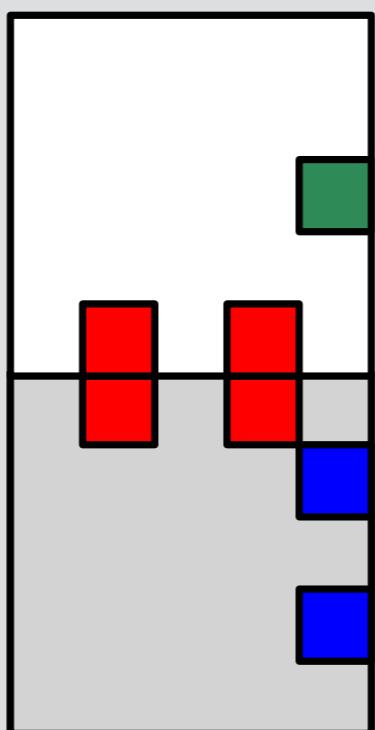
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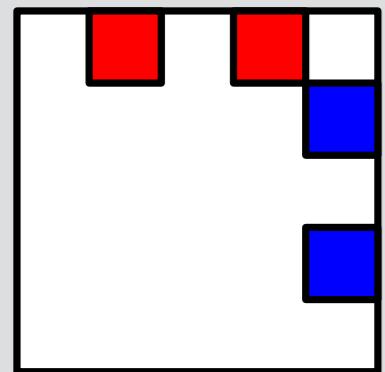
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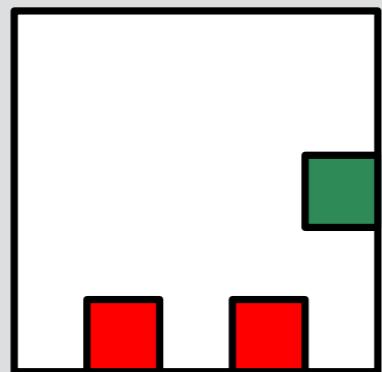
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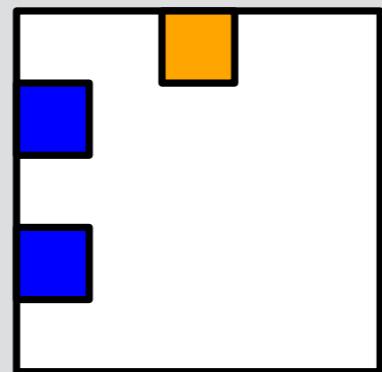
Temperature 1



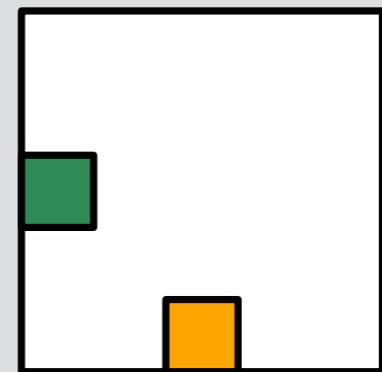
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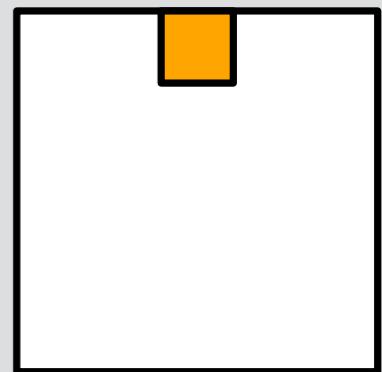
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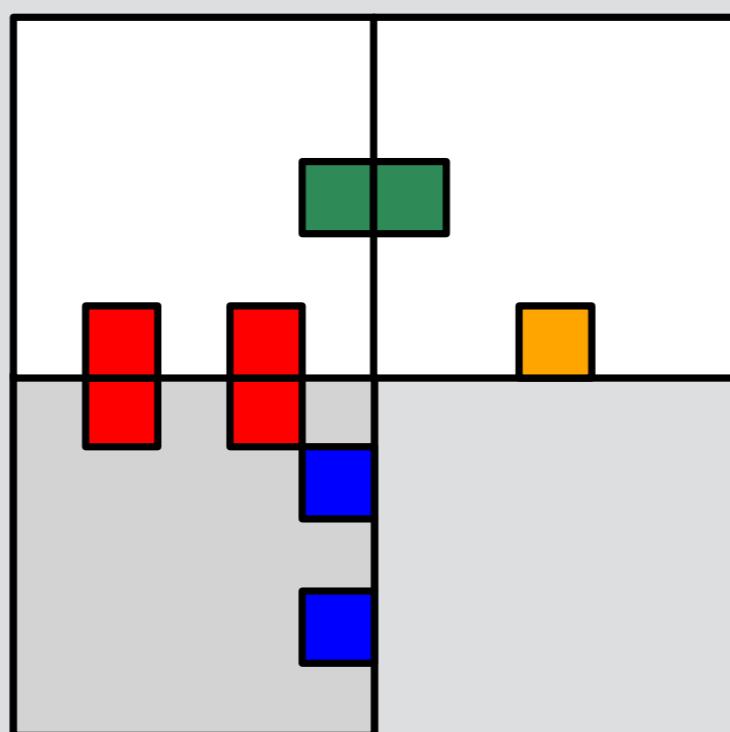
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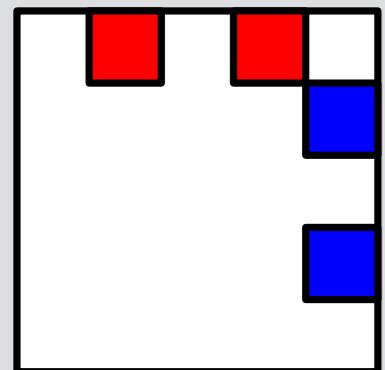
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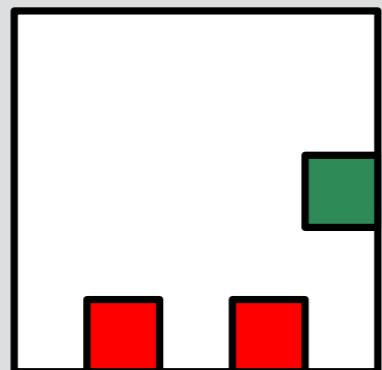
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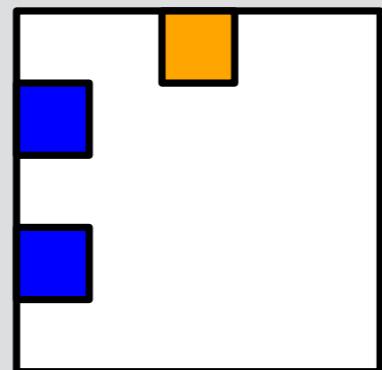
Temperature 1



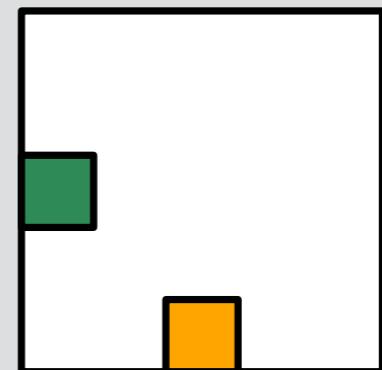
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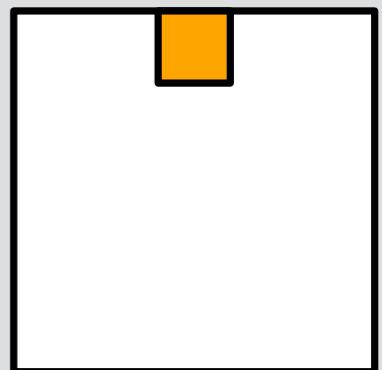
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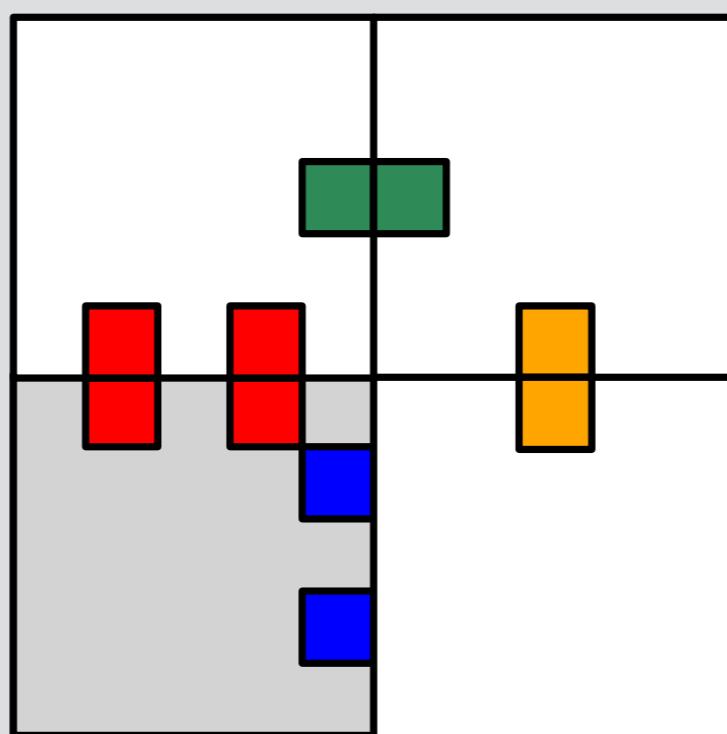
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$\times\infty$



$\times\infty$



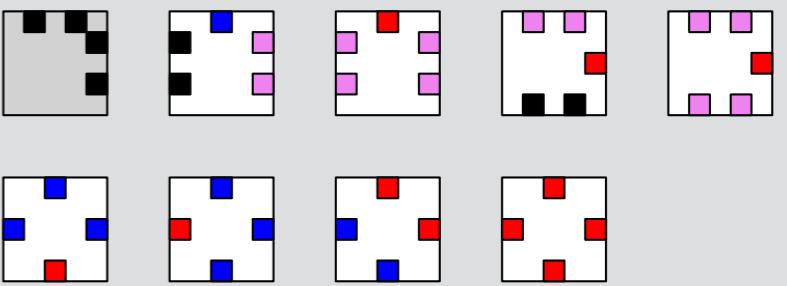
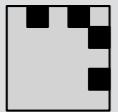
Temperature 1



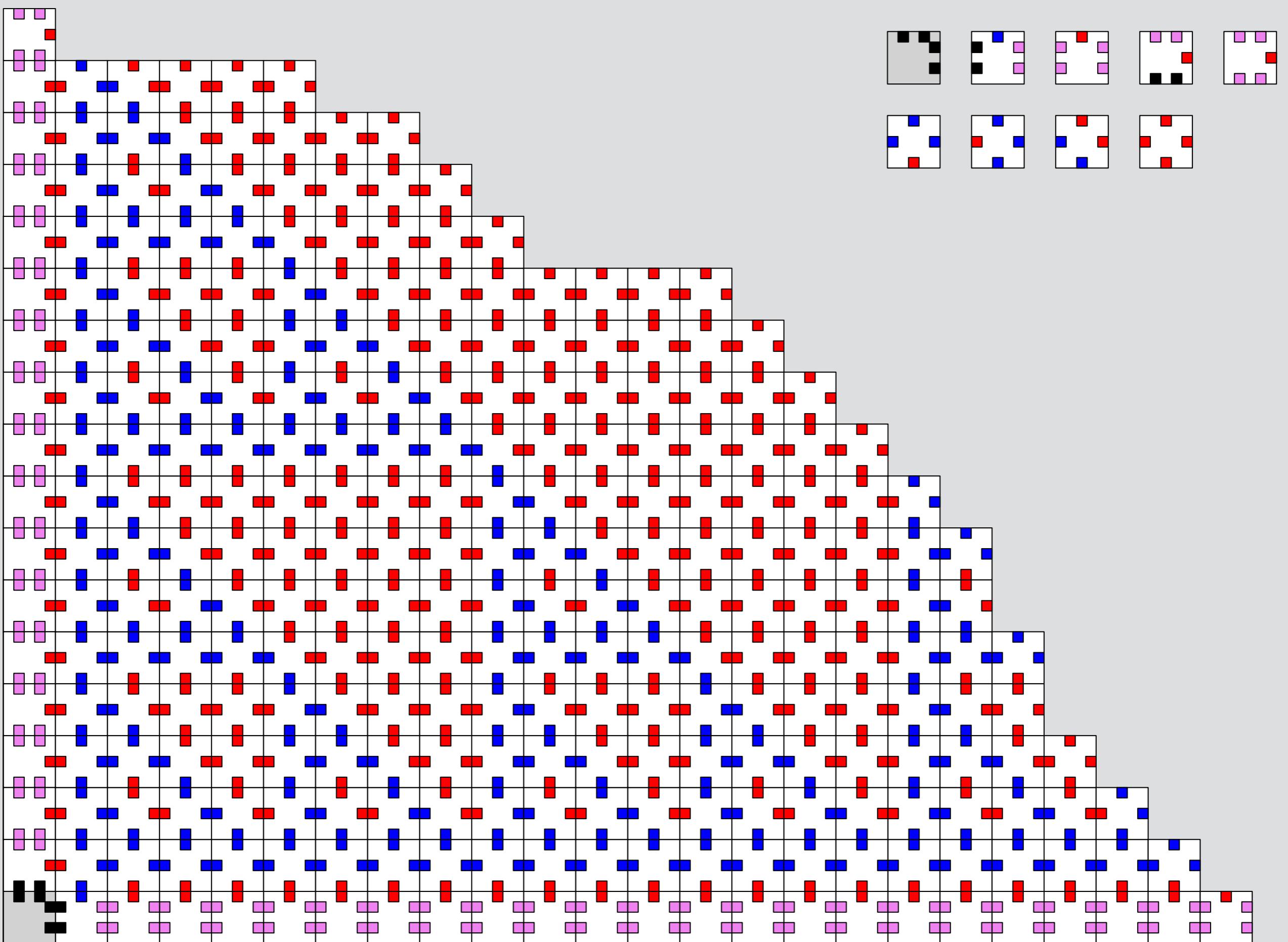
## abstract Tile Assembly Model (aTAM) [Winfree 1998]



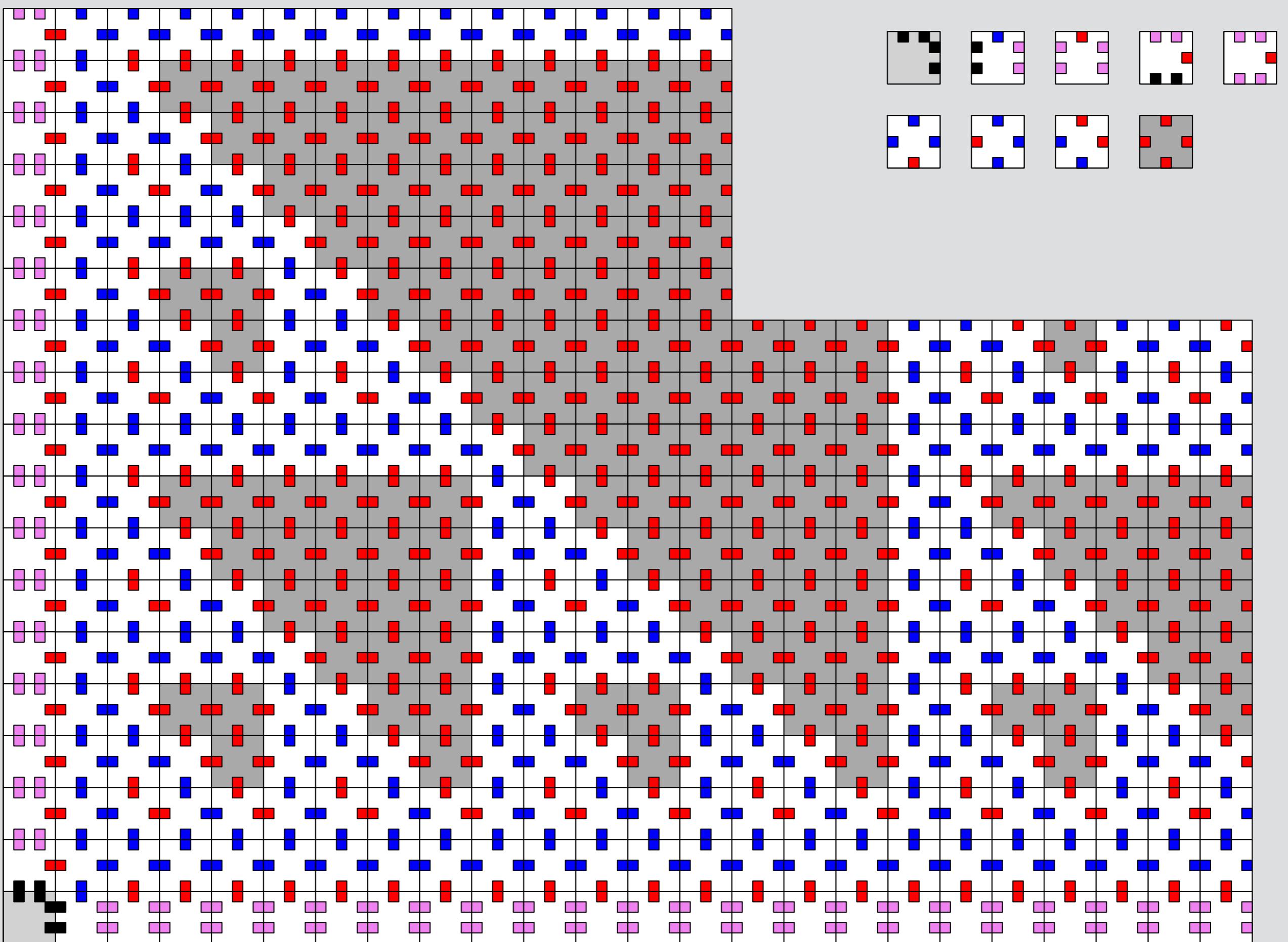
Temperature 1



Temperature 2



Temperature 2



Temperature 2

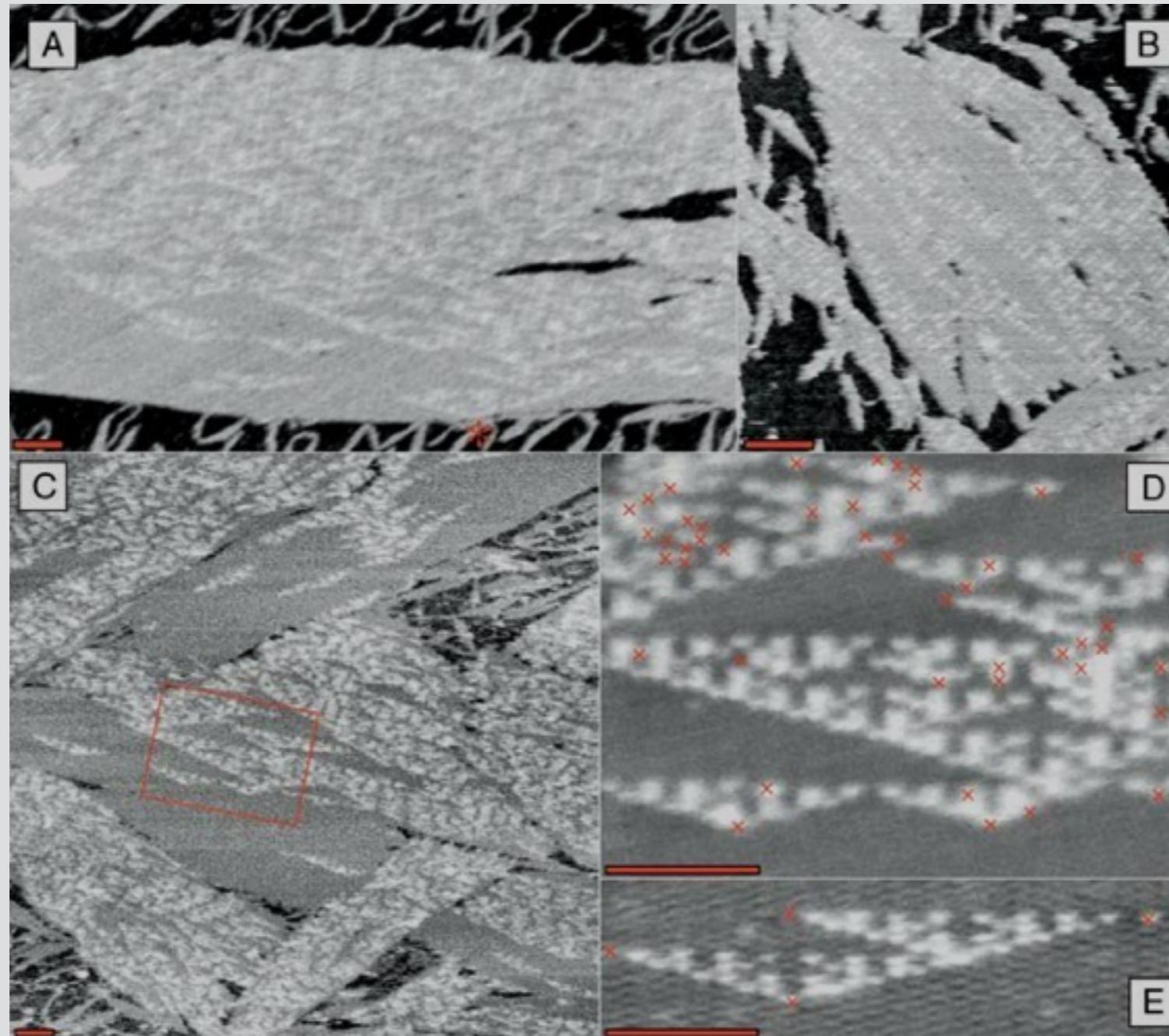


# Sierpinski triangle

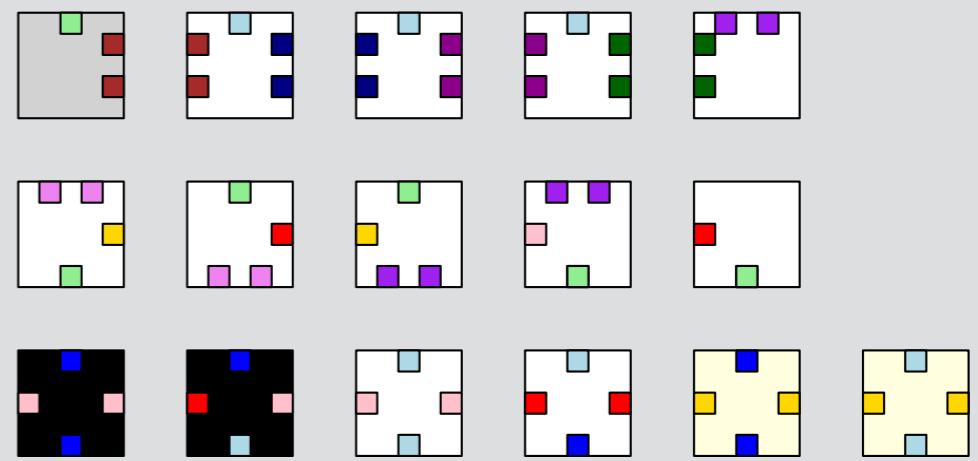
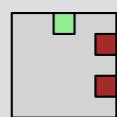
Temperature 2

# DNA Sierpinski triangles

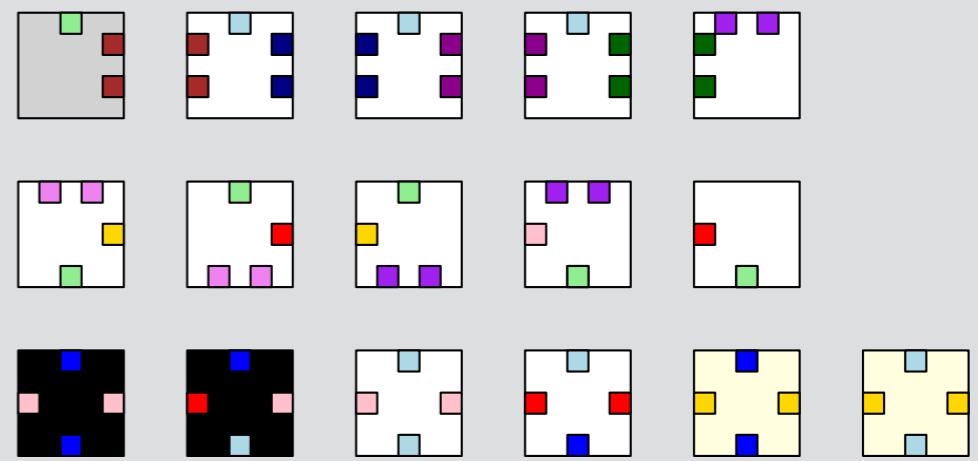
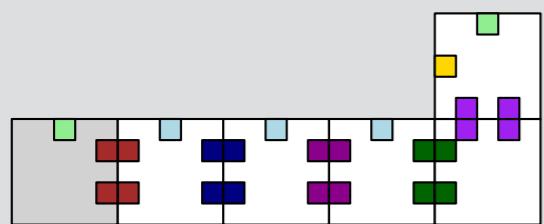
[Papadakis, Rothemund, Winfree 2004]:



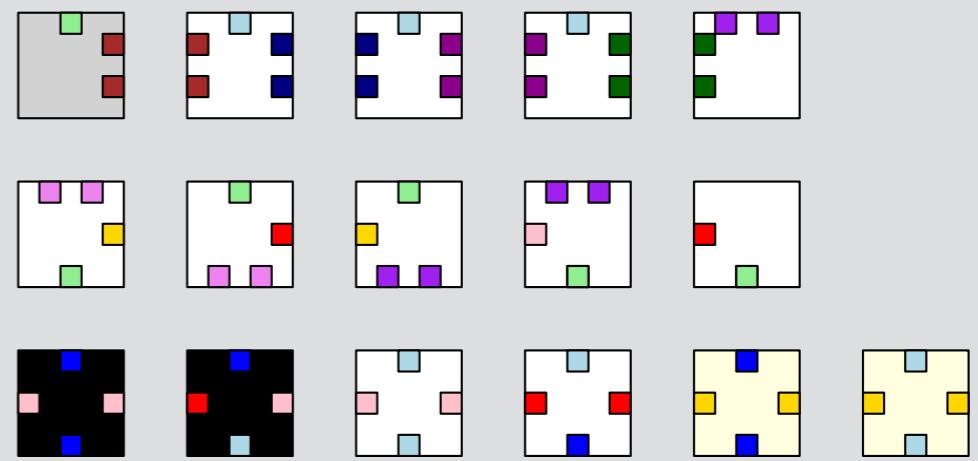
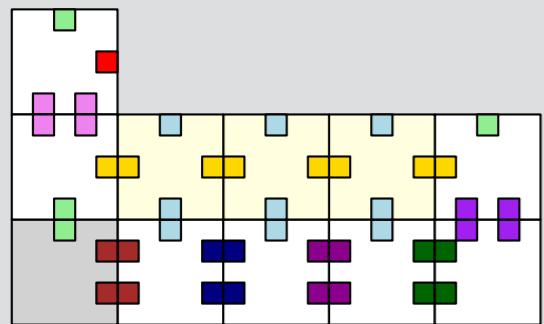
scale bars = 100 nm



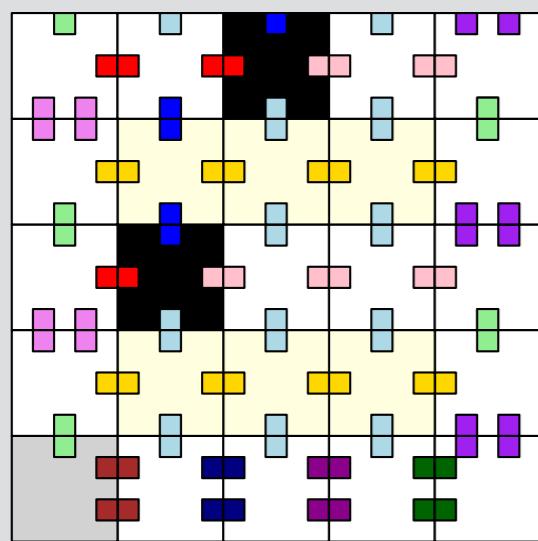
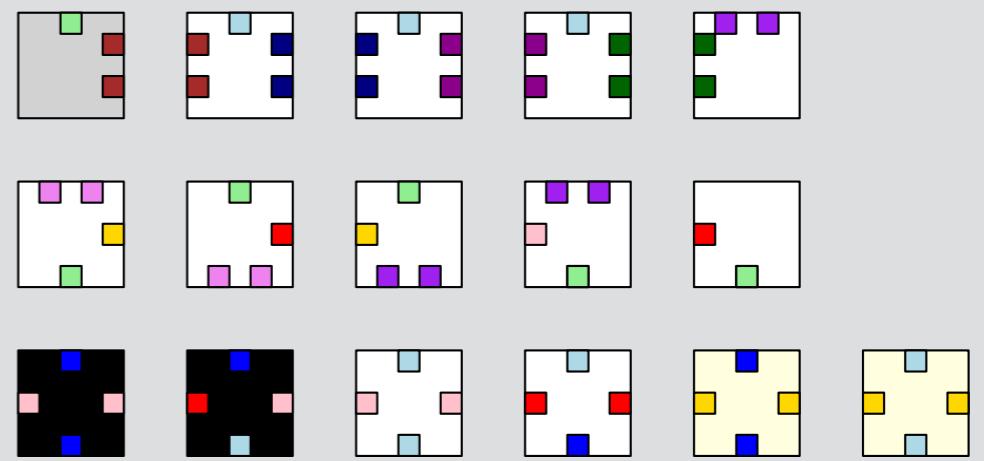
Temperature 2



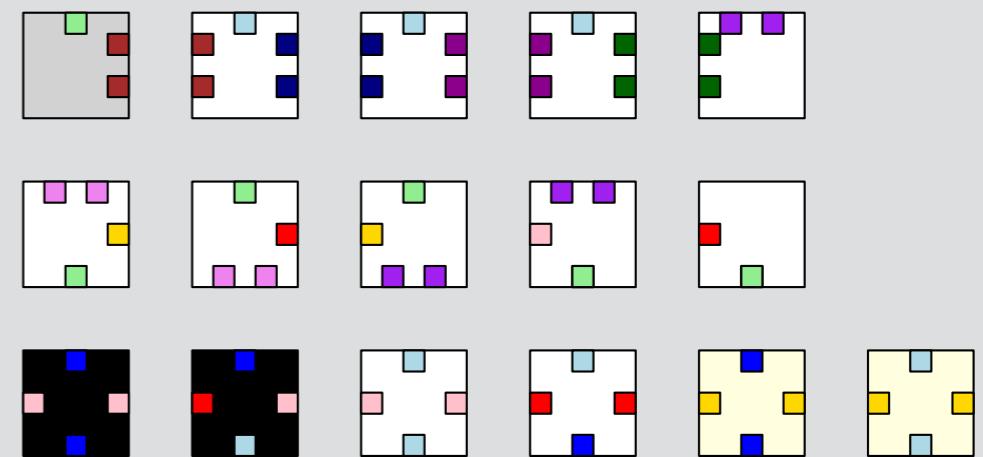
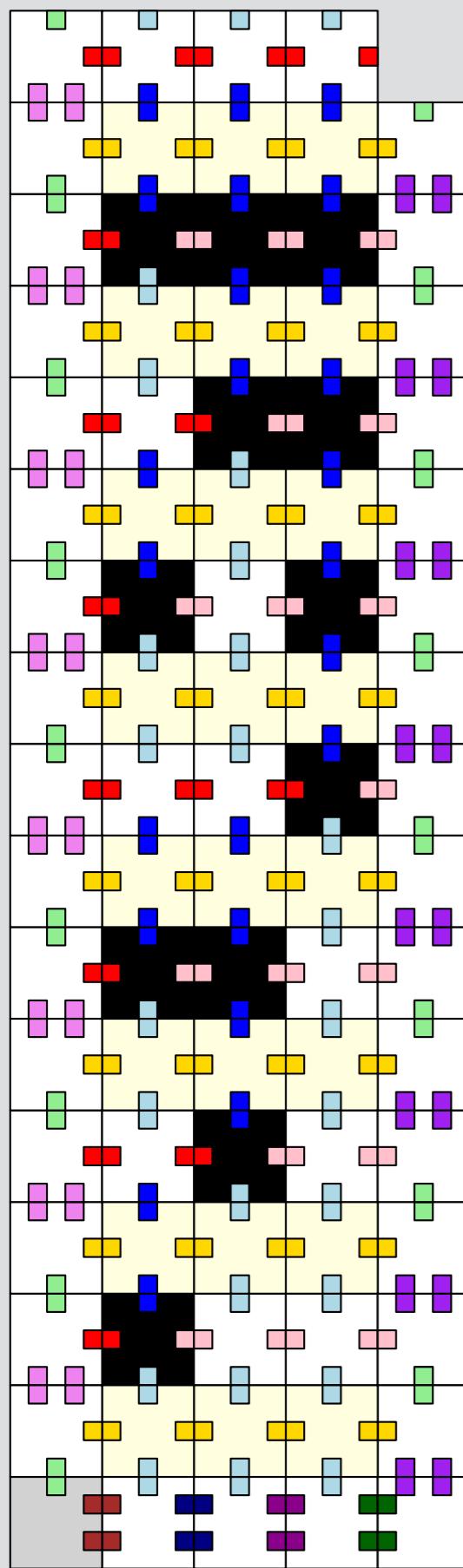
Temperature 2



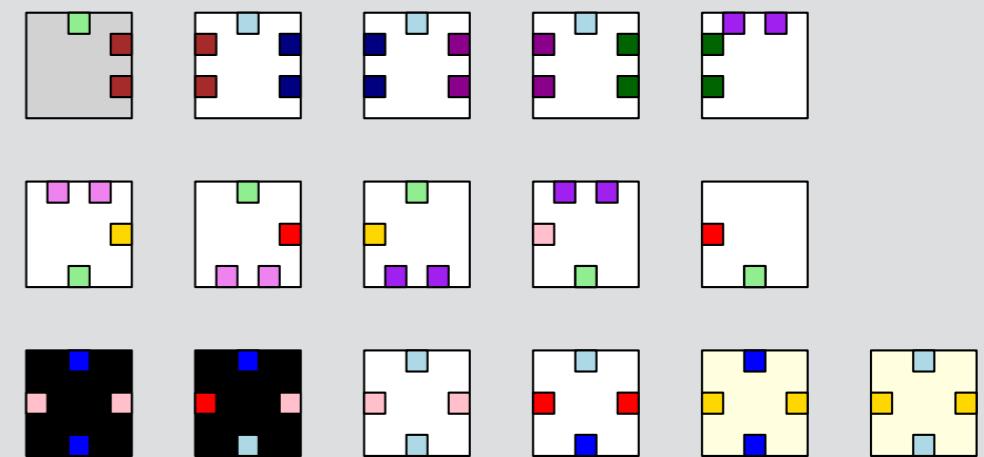
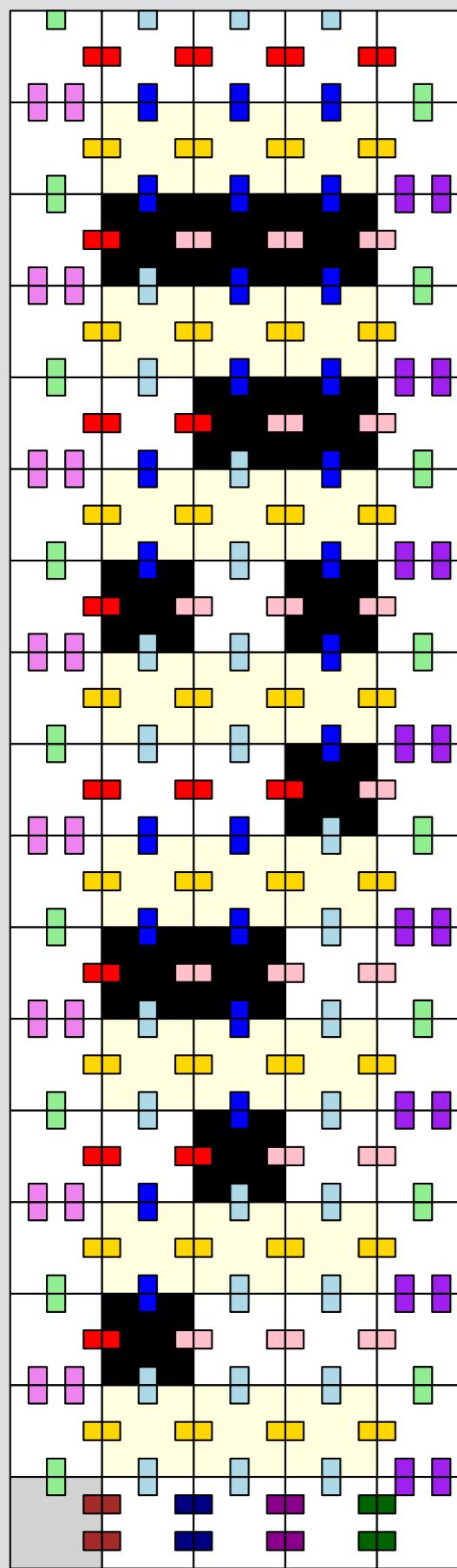
Temperature 2



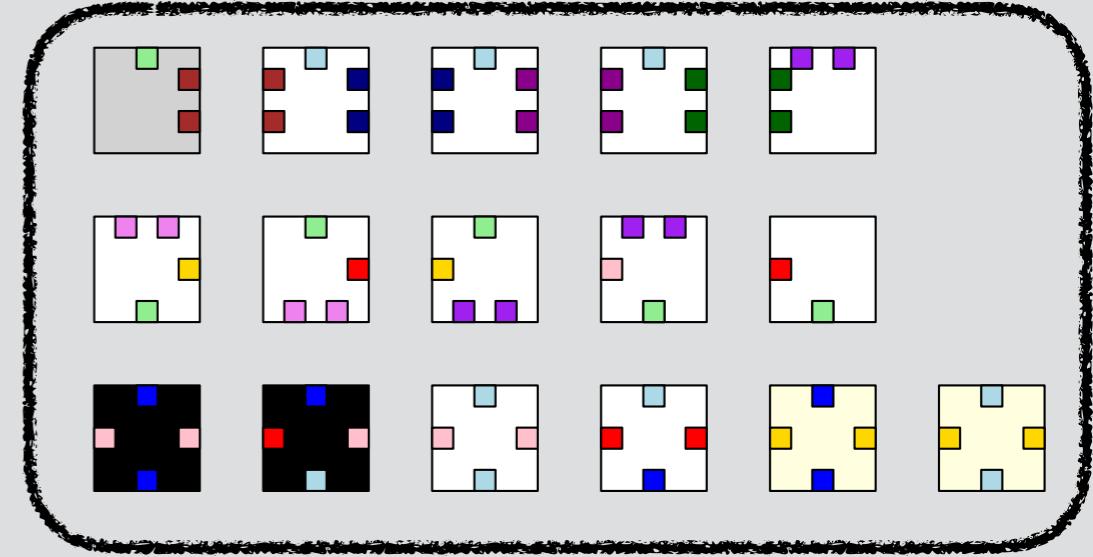
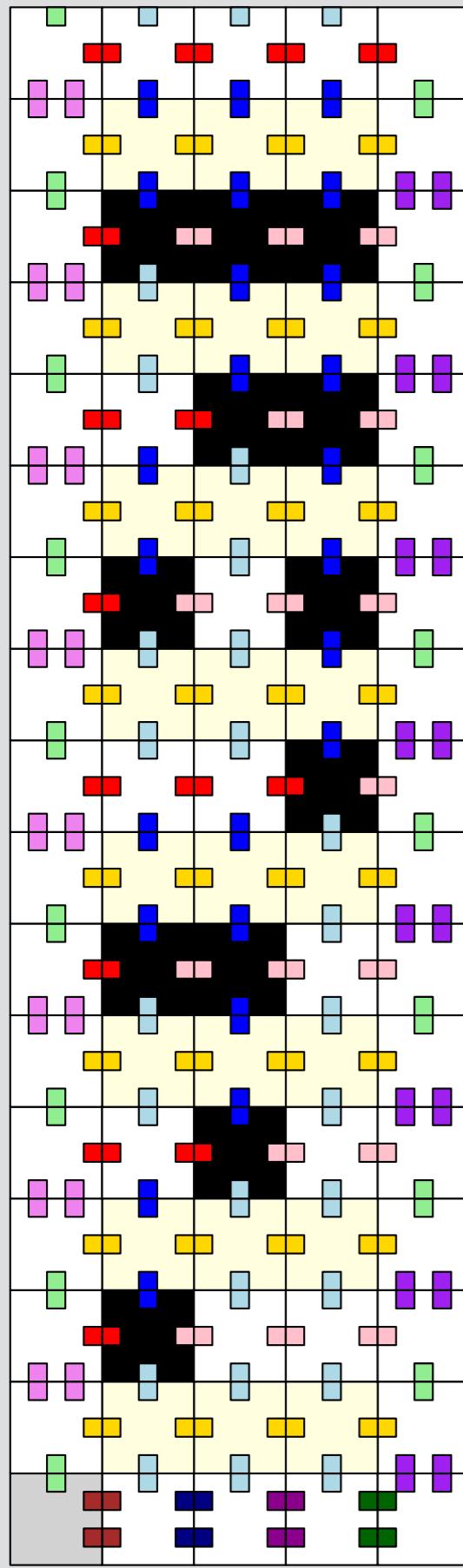
Temperature 2



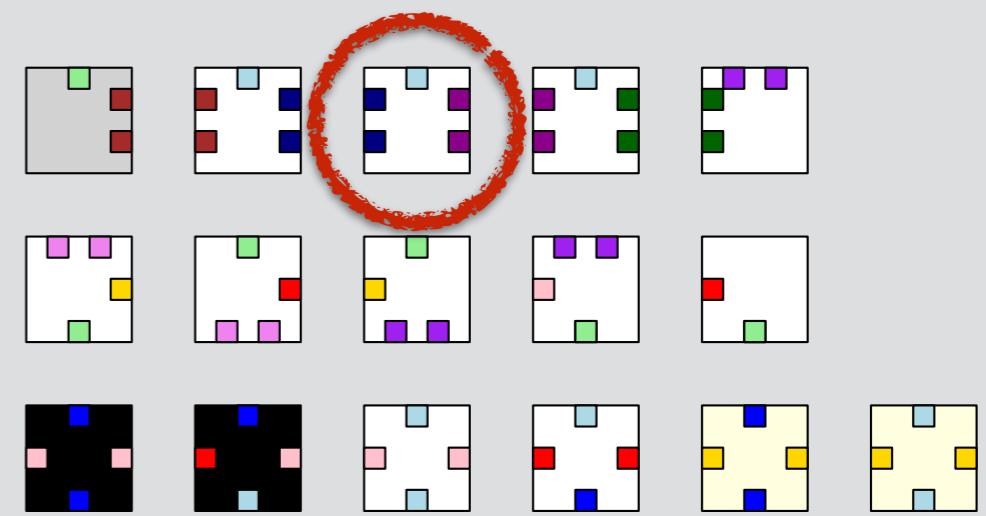
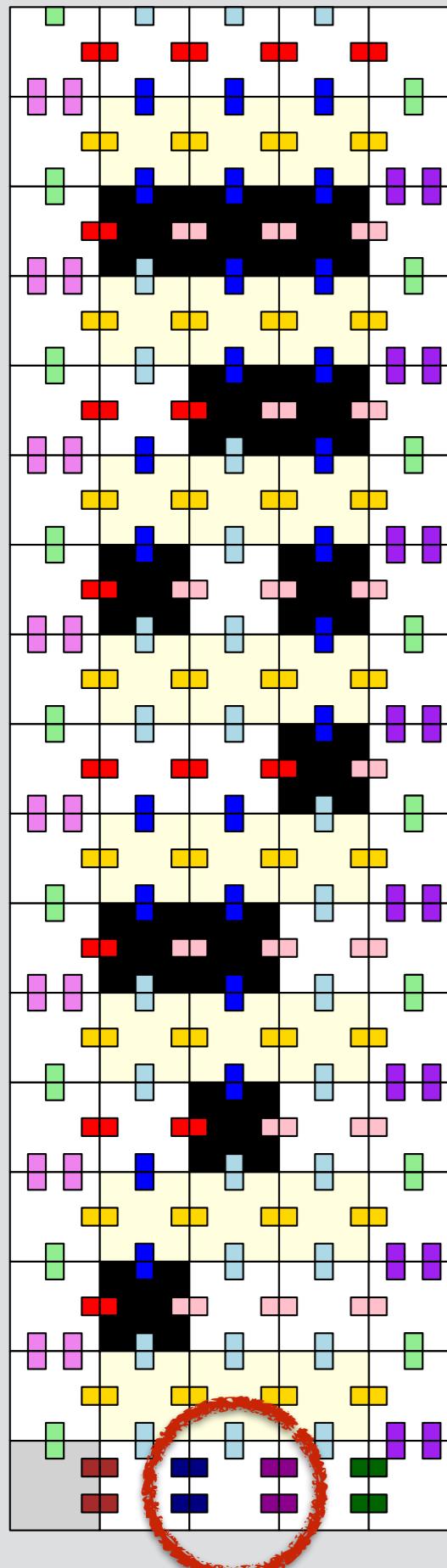
Temperature 2



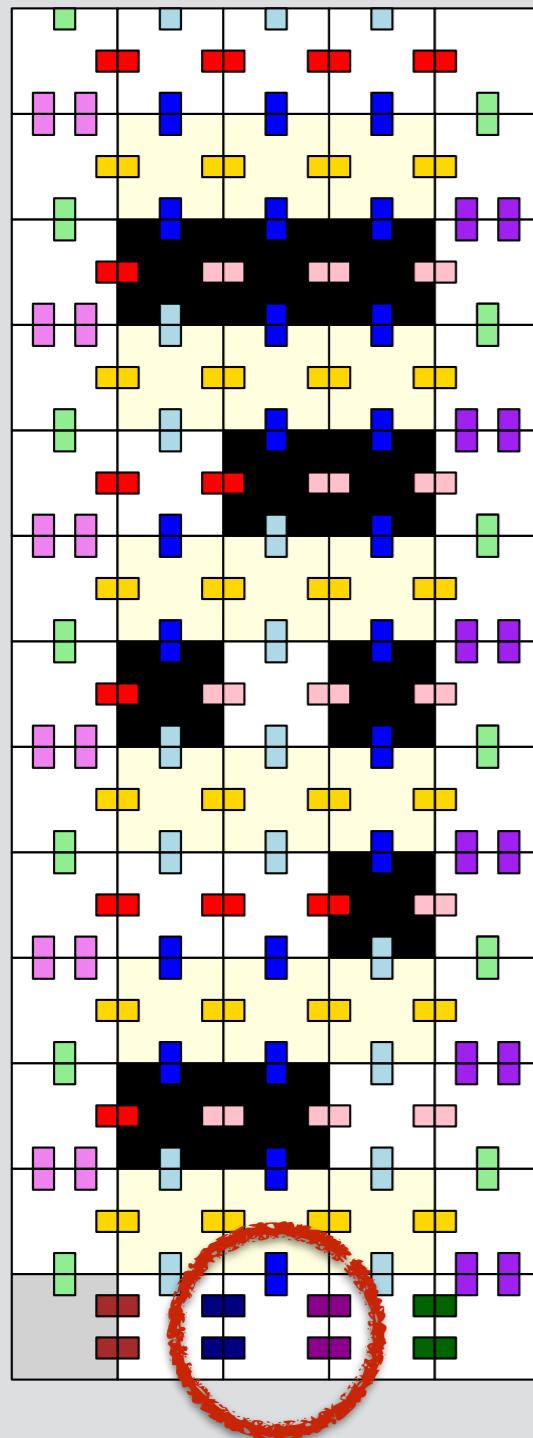
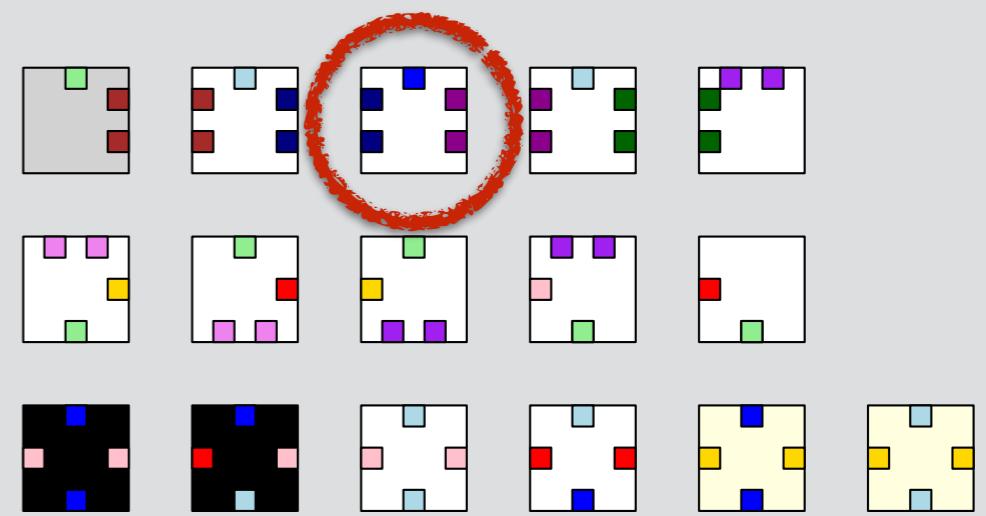
Temperature 2



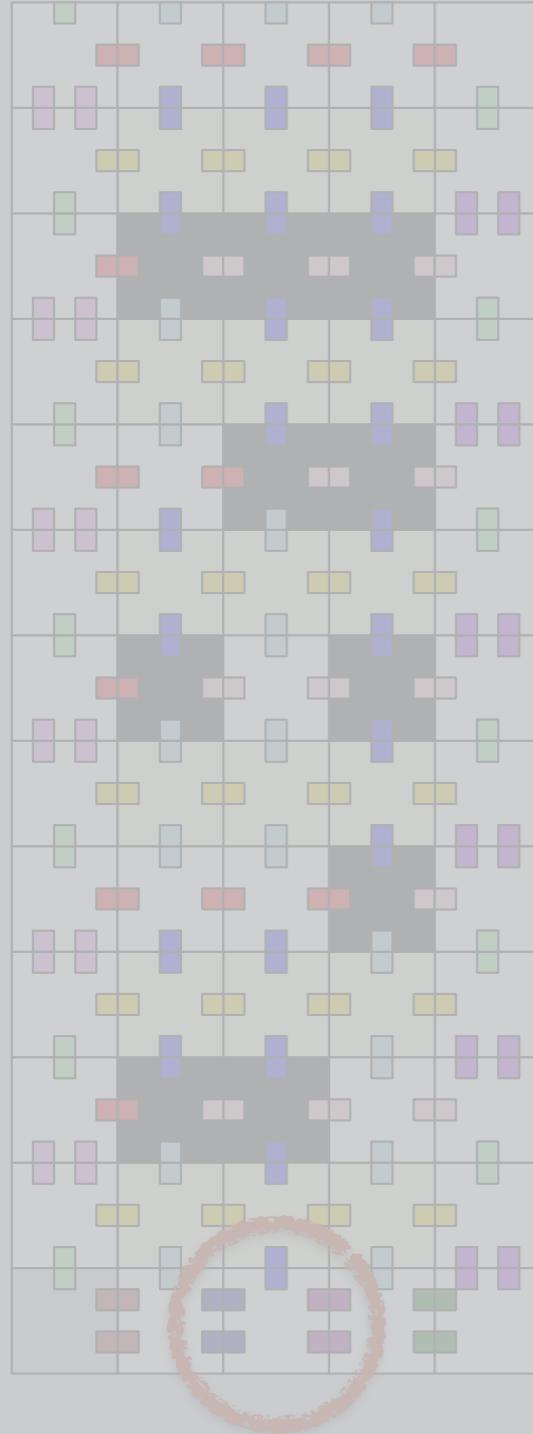
Temperature 2



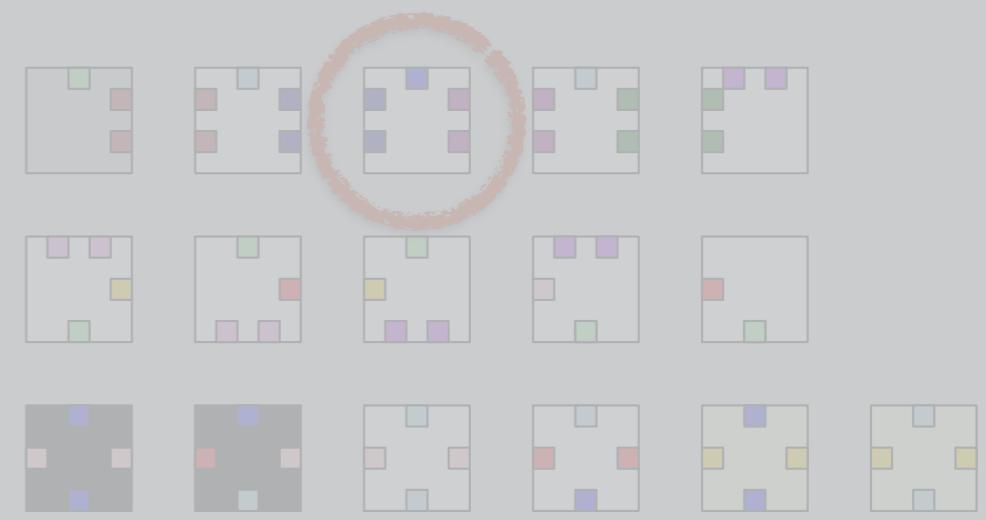
Temperature 2



Temperature 2



# Binary counter

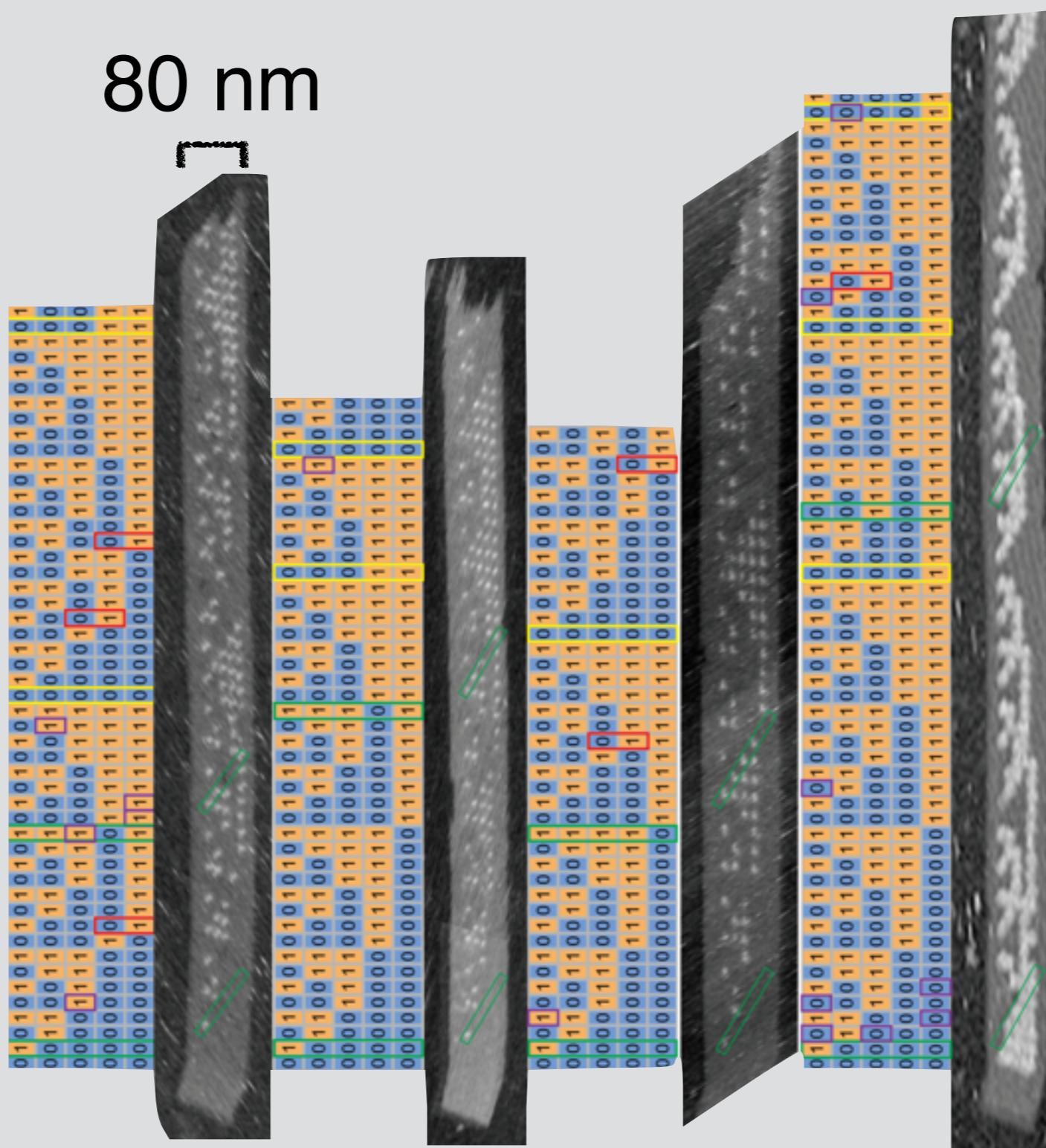


Temperature 2

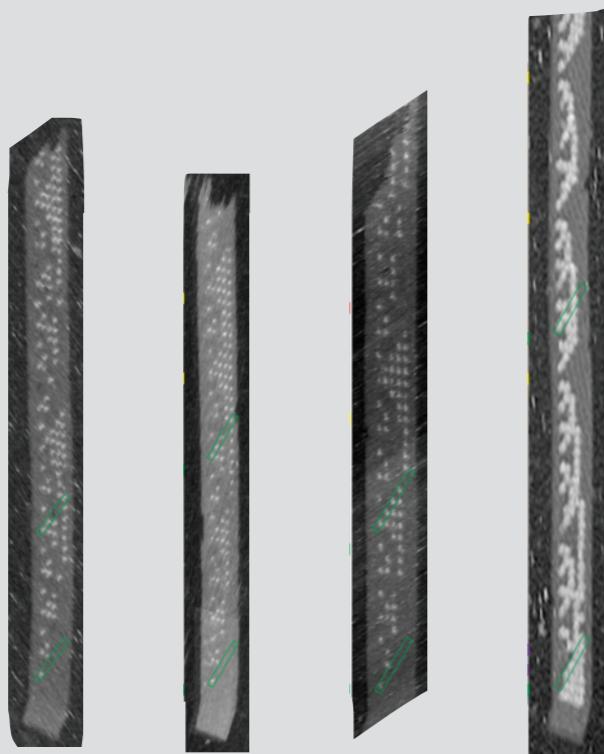
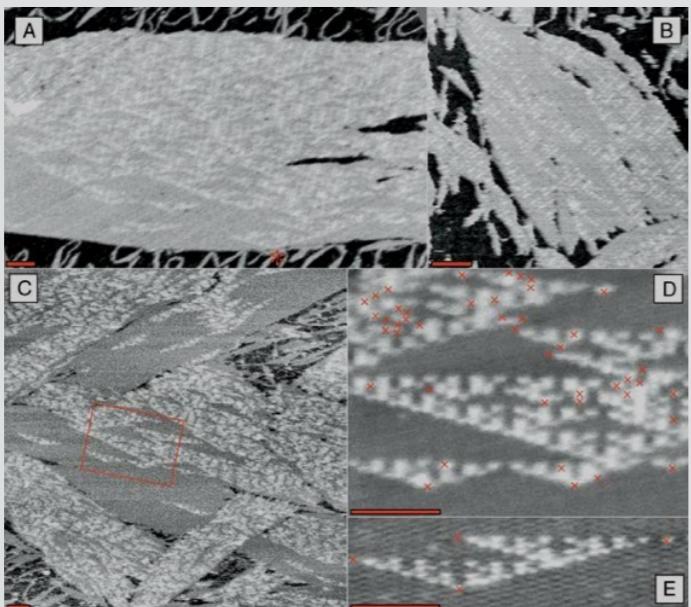
# DNA binary counters

[Evans, 2014]

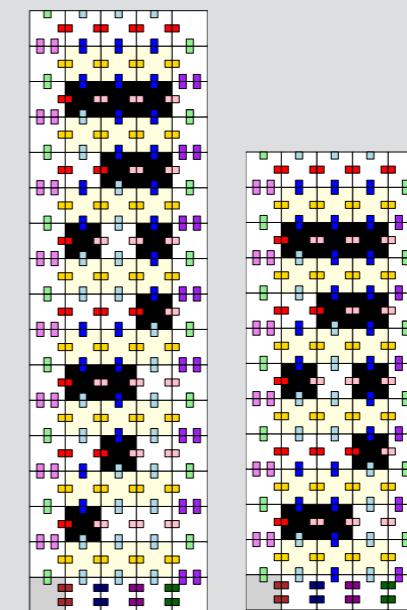
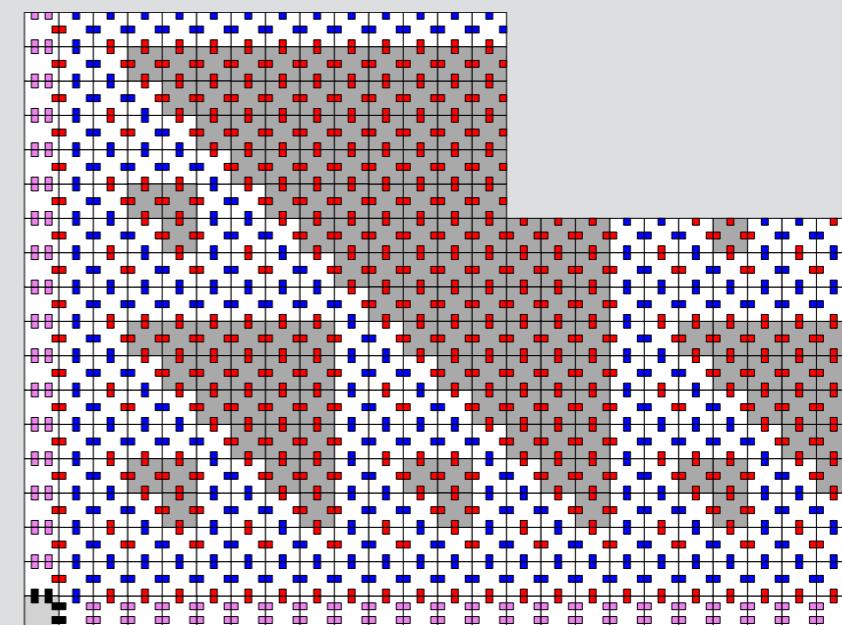
80 nm



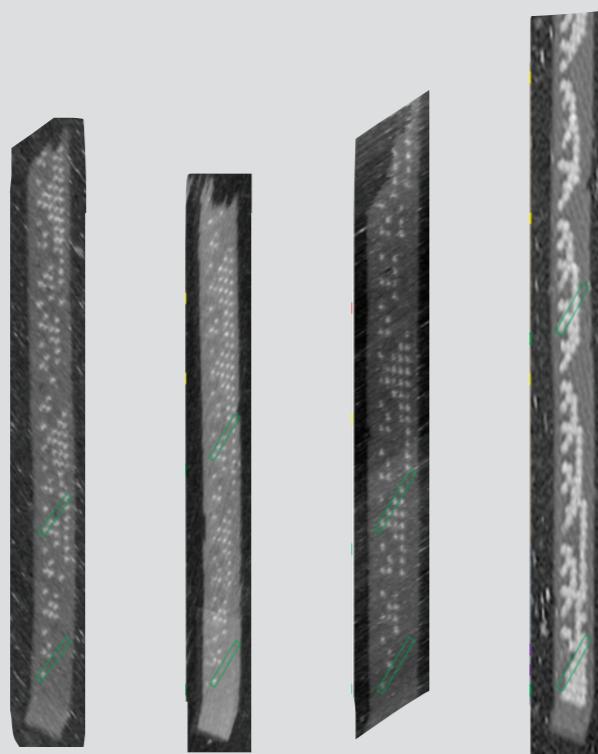
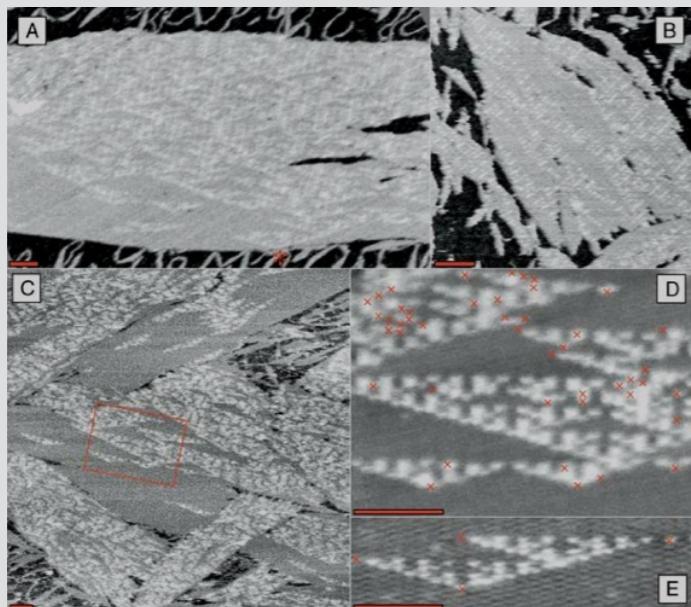
# DNA tile self-assembly:



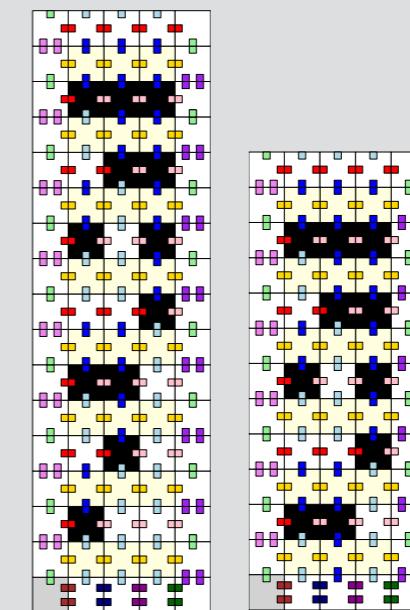
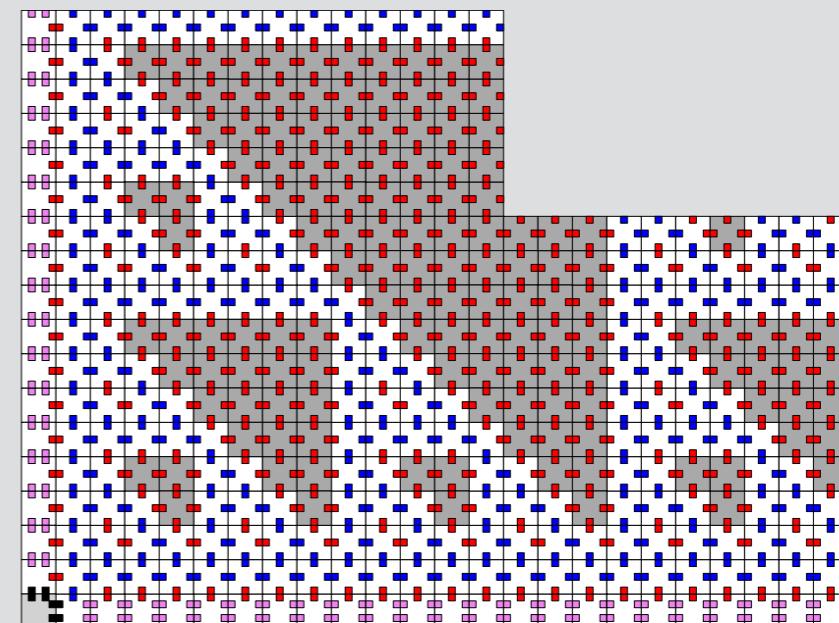
# Tile self-assembly:



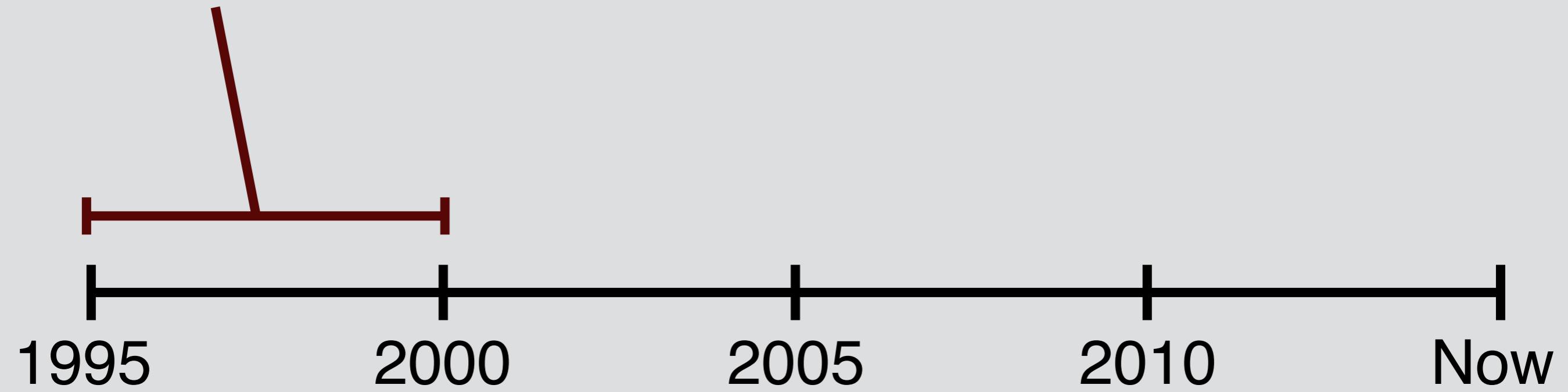
# DNA tile self-assembly:

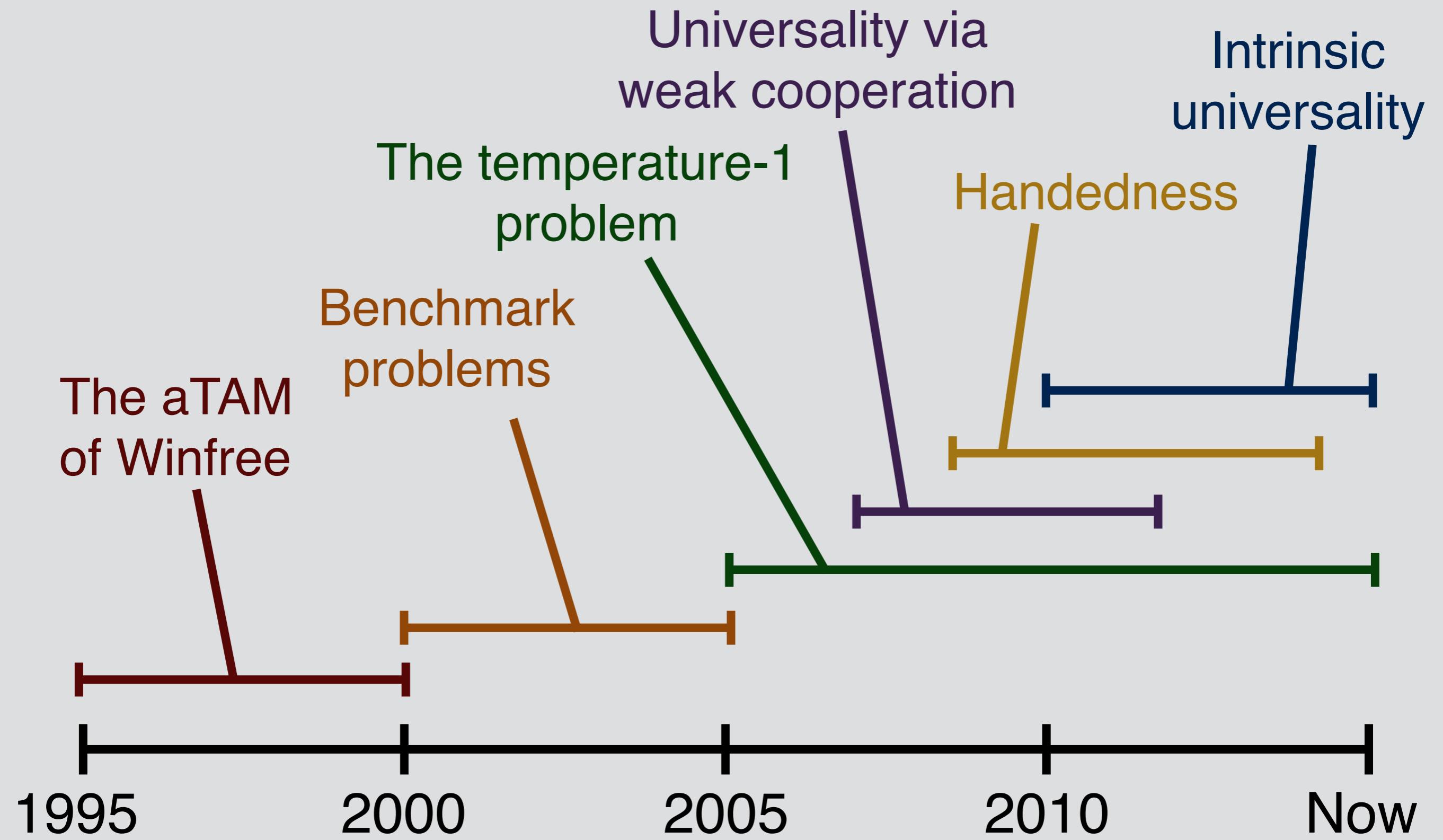


# Tile self-assembly:

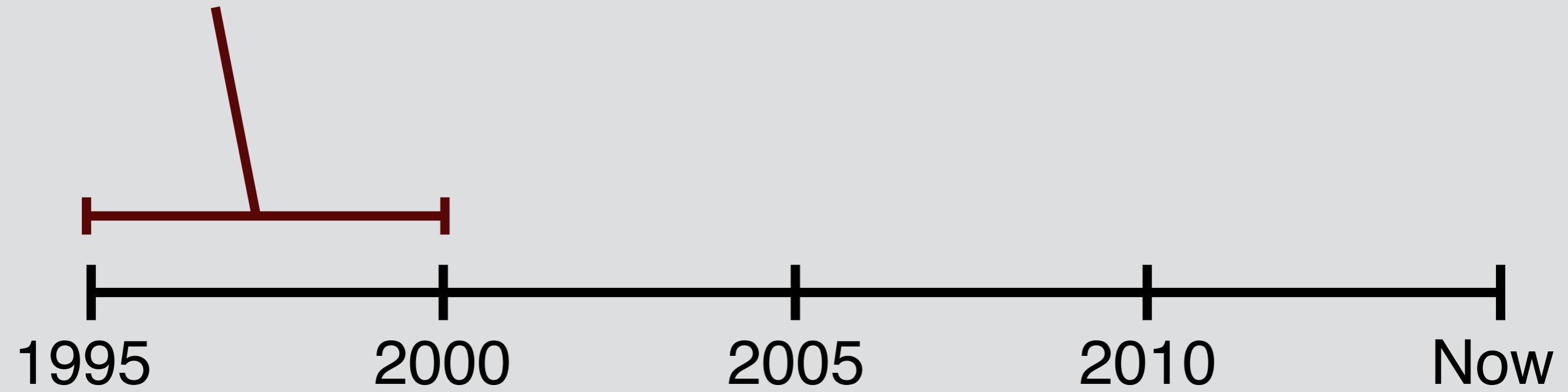


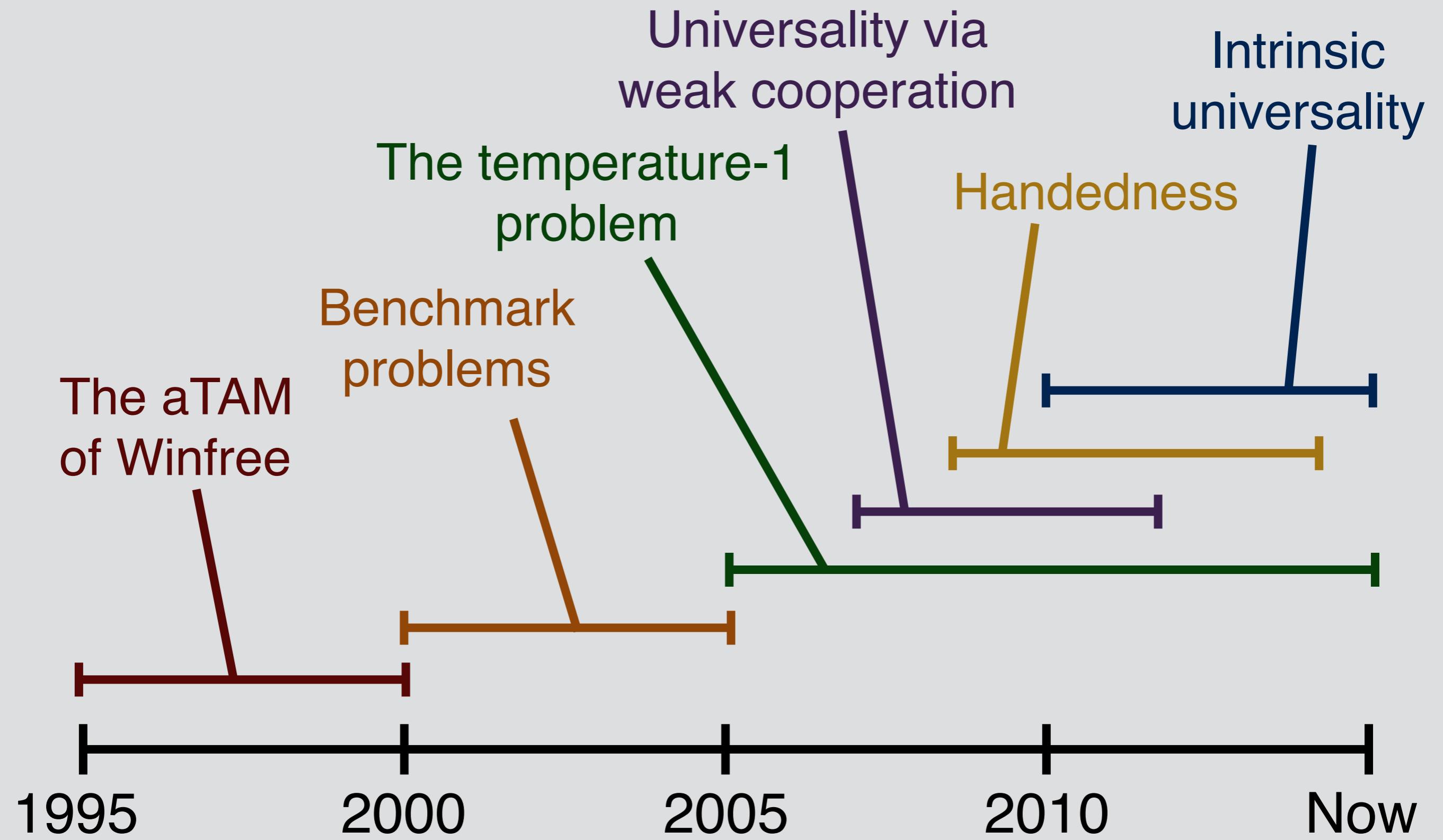
# The aTAM of Winfree





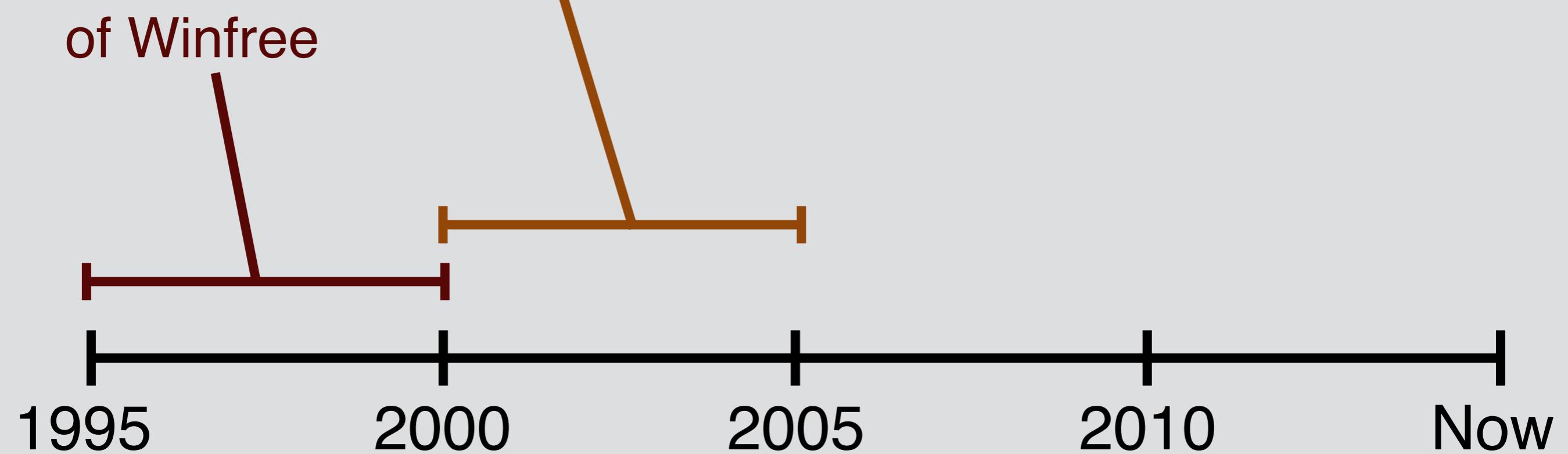
# The aTAM of Winfree

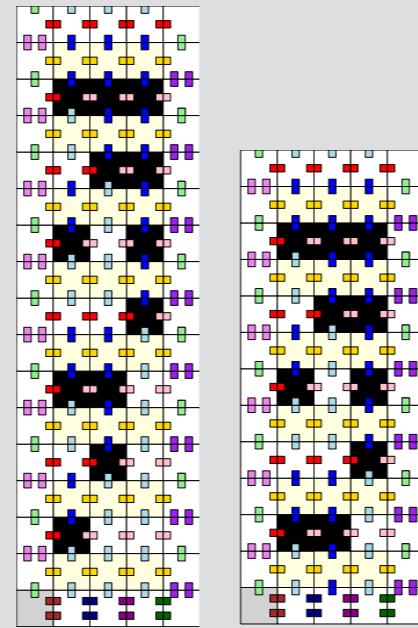
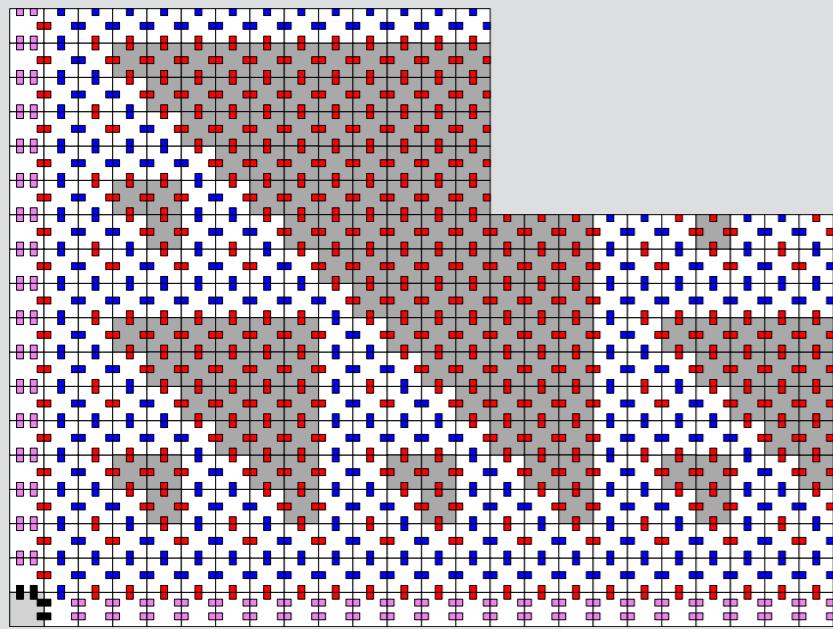




The aTAM  
of Winfree

Benchmark  
problems





aTAM is “capable”

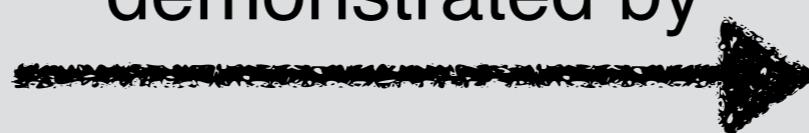
What is a “capable” model  
of self-assembly?

# The benchmarks of a capable self-assembly model

# The benchmarks of a capable self-assembly model

Algorithmic behavior

demonstrated by

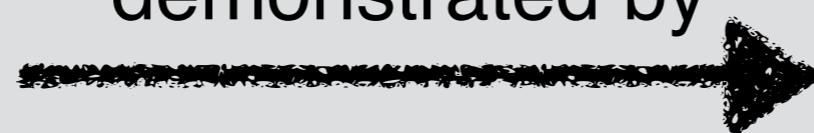


Universal computation

# The benchmarks of a capable self-assembly model

Usable  
Algorithmic  
behavior

demonstrated by



Efficient NxN  
square assembly  
Universal  
computation

# The benchmarks of a capable self-assembly model

Usable  
Algorithmic  
behavior

demonstrated by



Efficient NxN  
square assembly  
Universal  
computation

**Efficient = few tile types**  
(program size, time)

# The benchmarks of a capable self-assembly model

Algorithmic behavior

demonstrated by

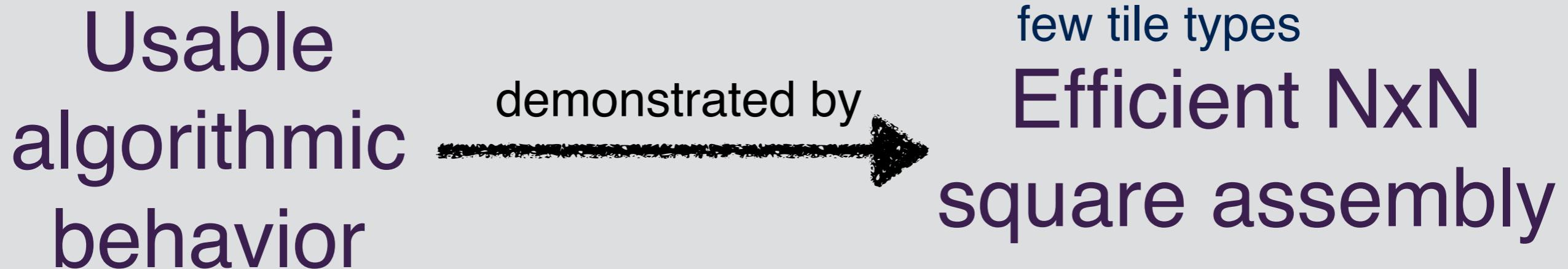
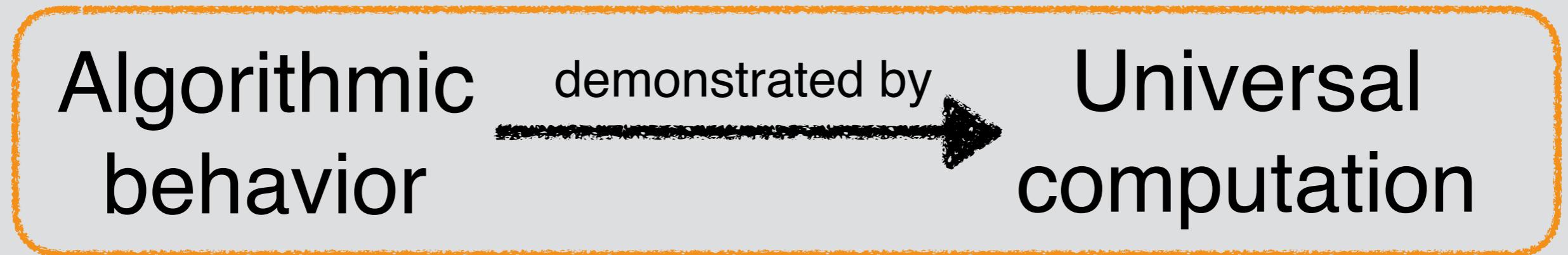
Universal computation

Usable  
algorithmic  
behavior

demonstrated by

few tile types  
**Efficient NxN**  
square assembly

# The benchmarks of a capable self-assembly model

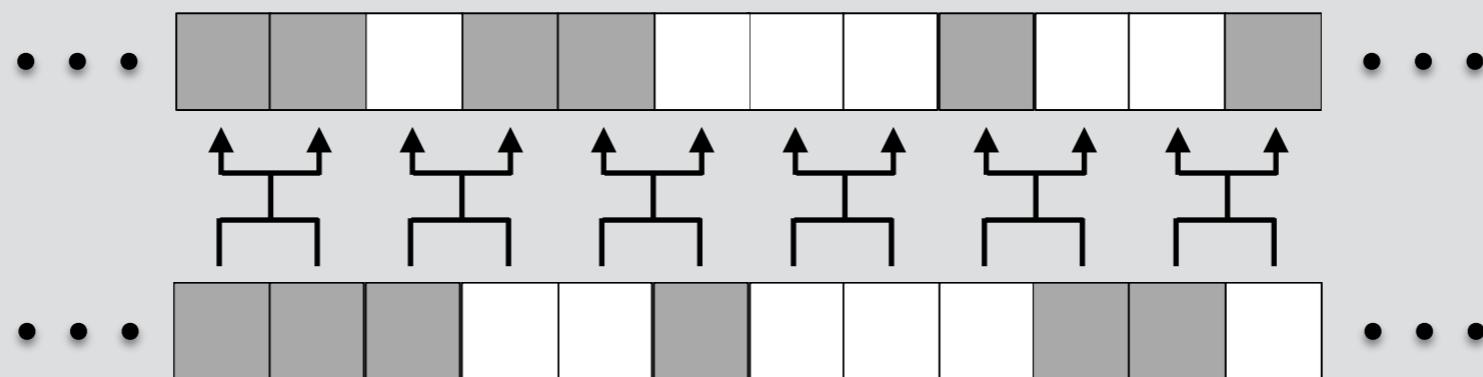


# Blocked cellular automata

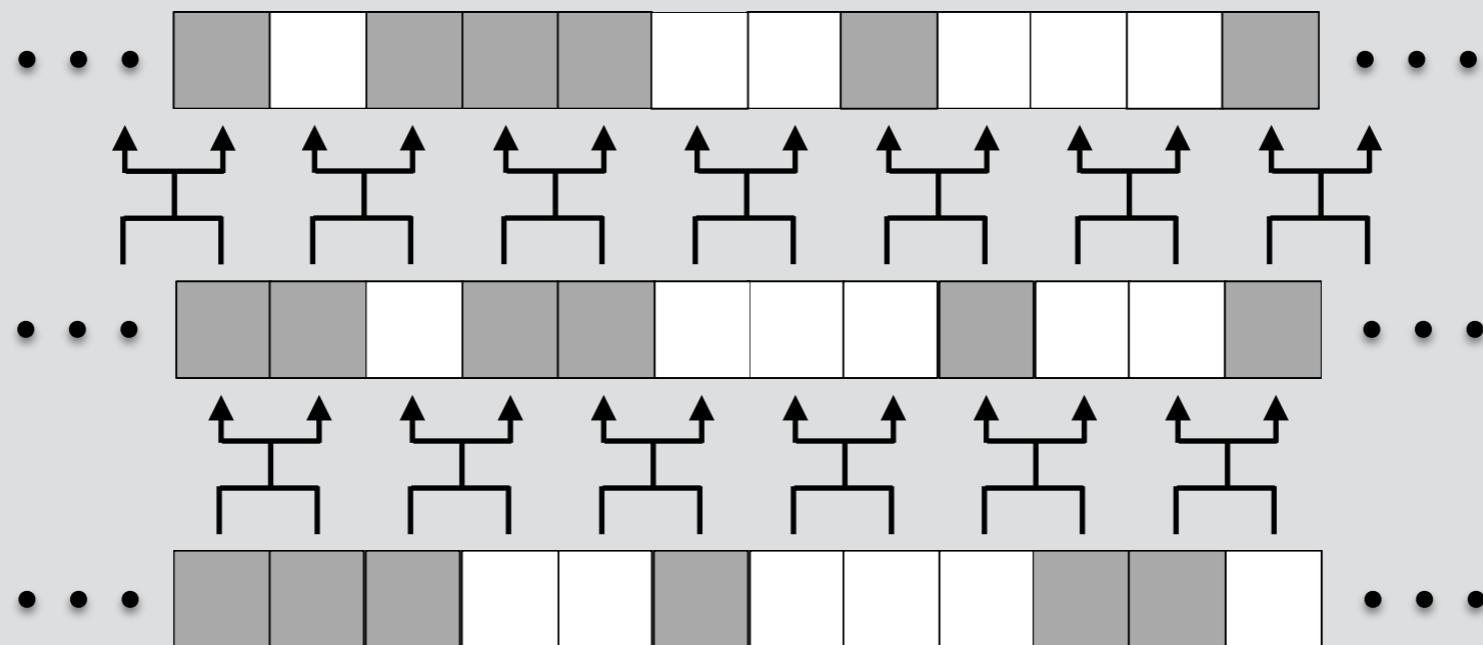
# Blocked cellular automata



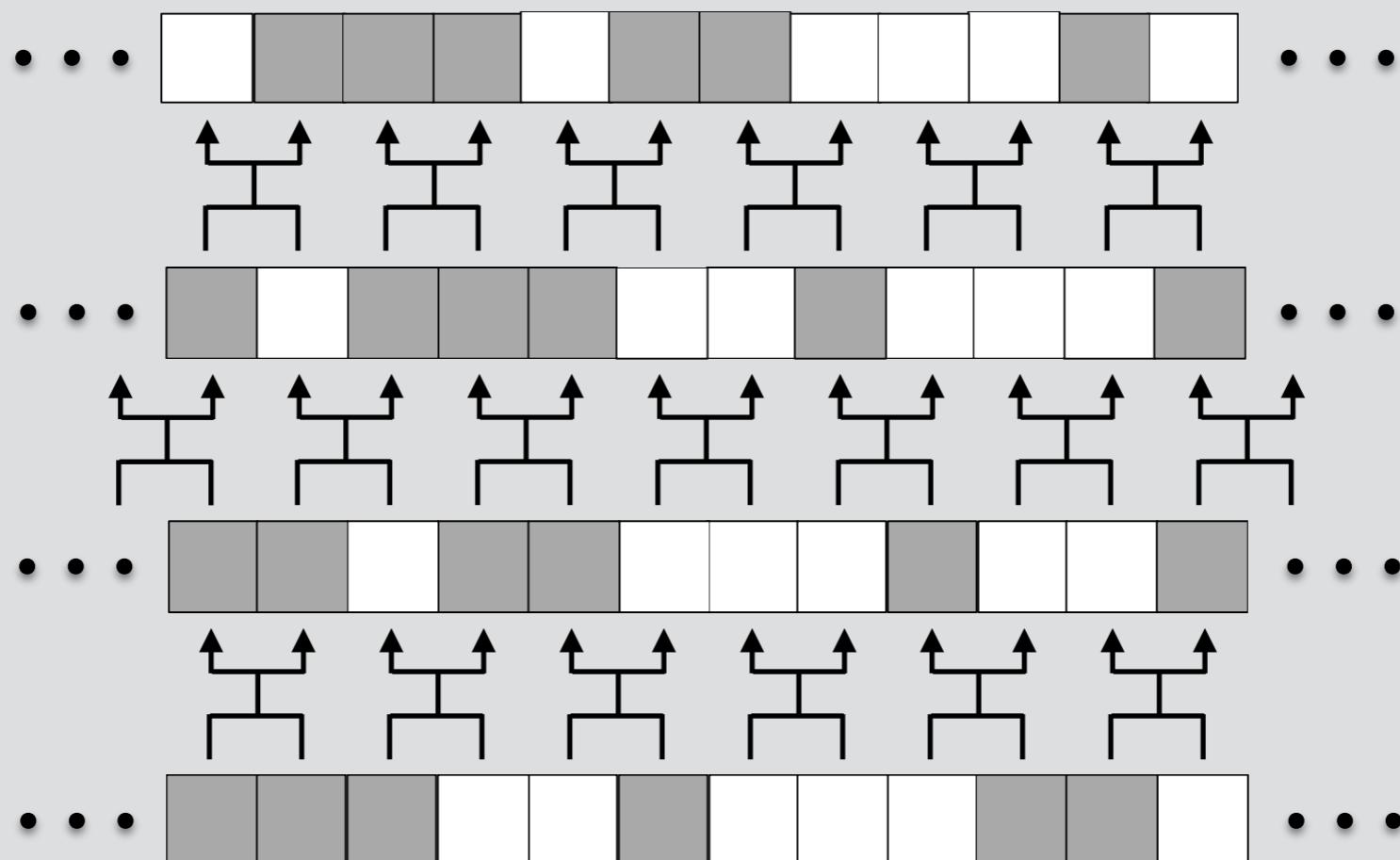
# Blocked cellular automata



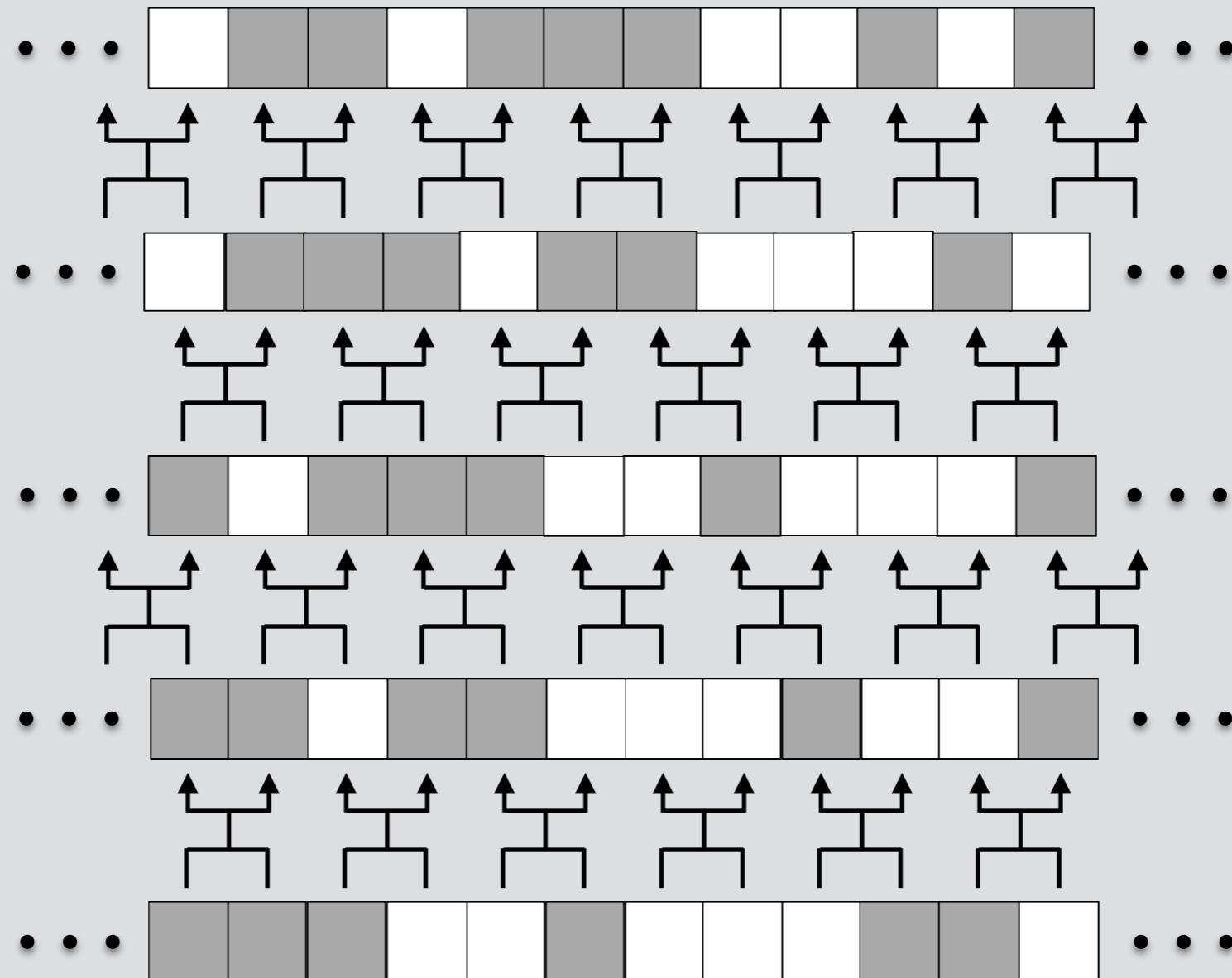
# Blocked cellular automata



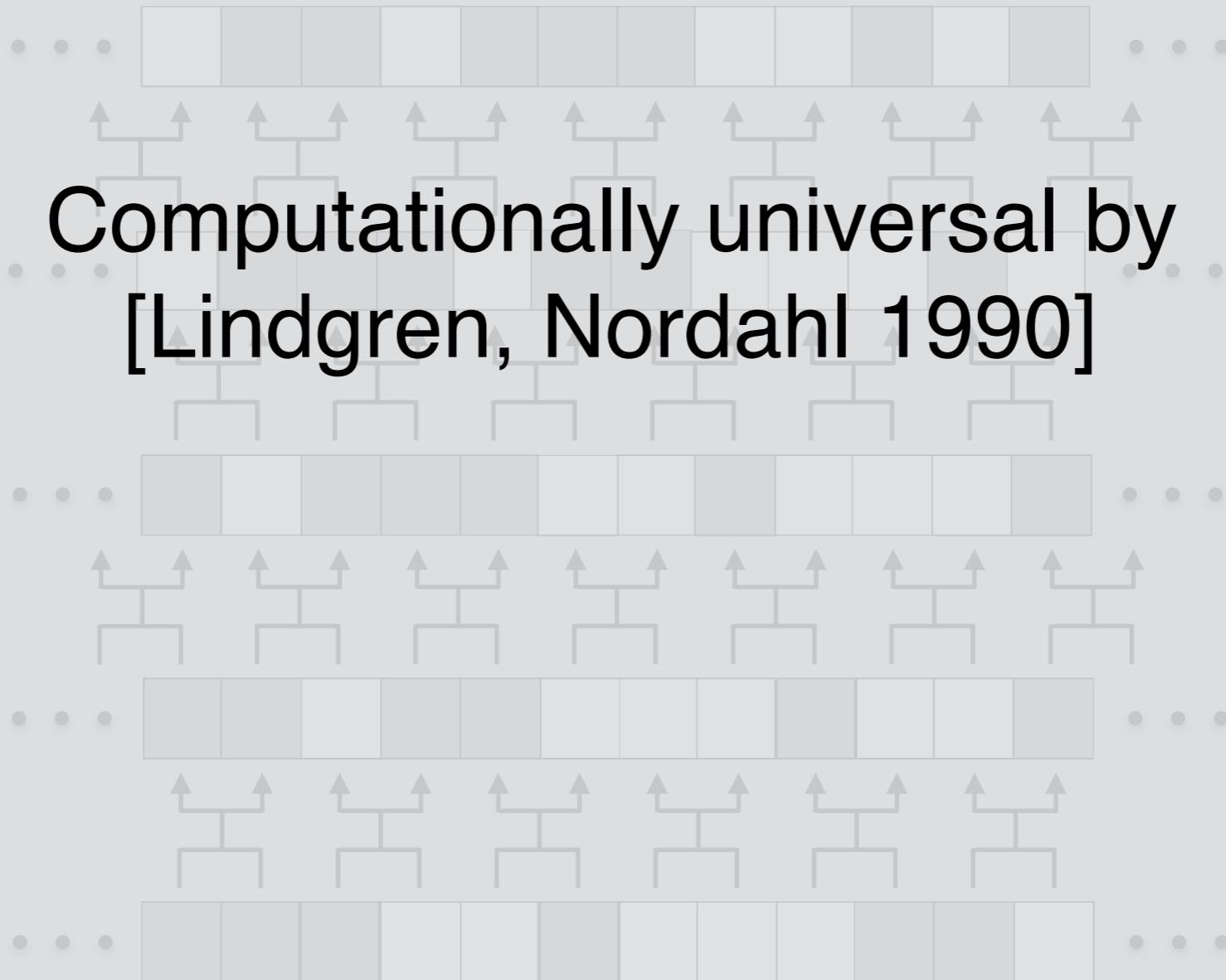
# Blocked cellular automata



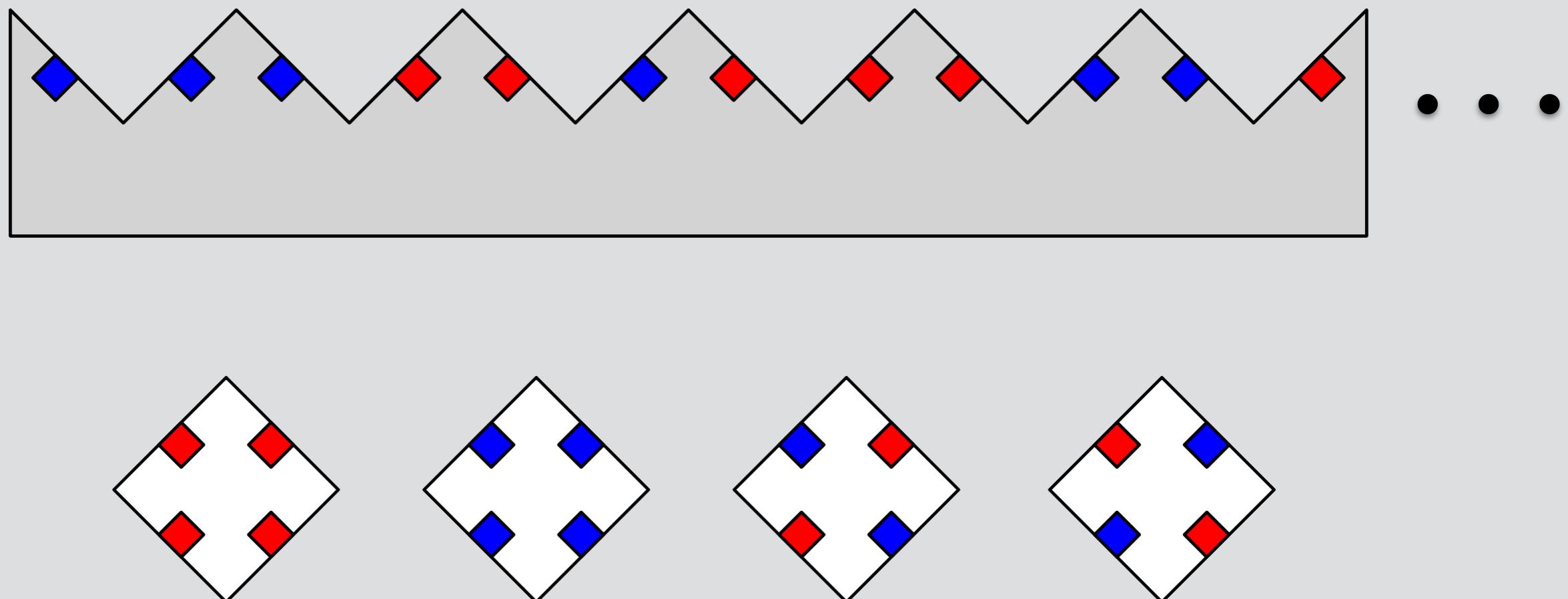
# Blocked cellular automata



# Blocked cellular automata



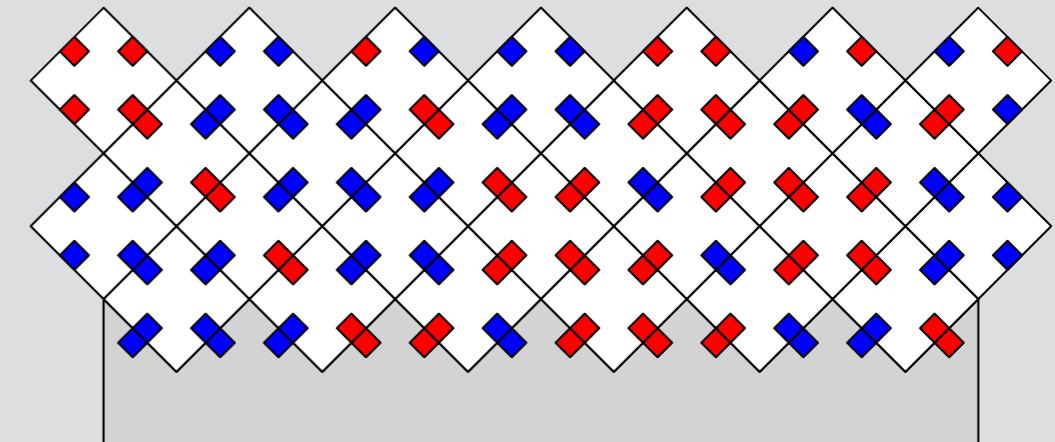
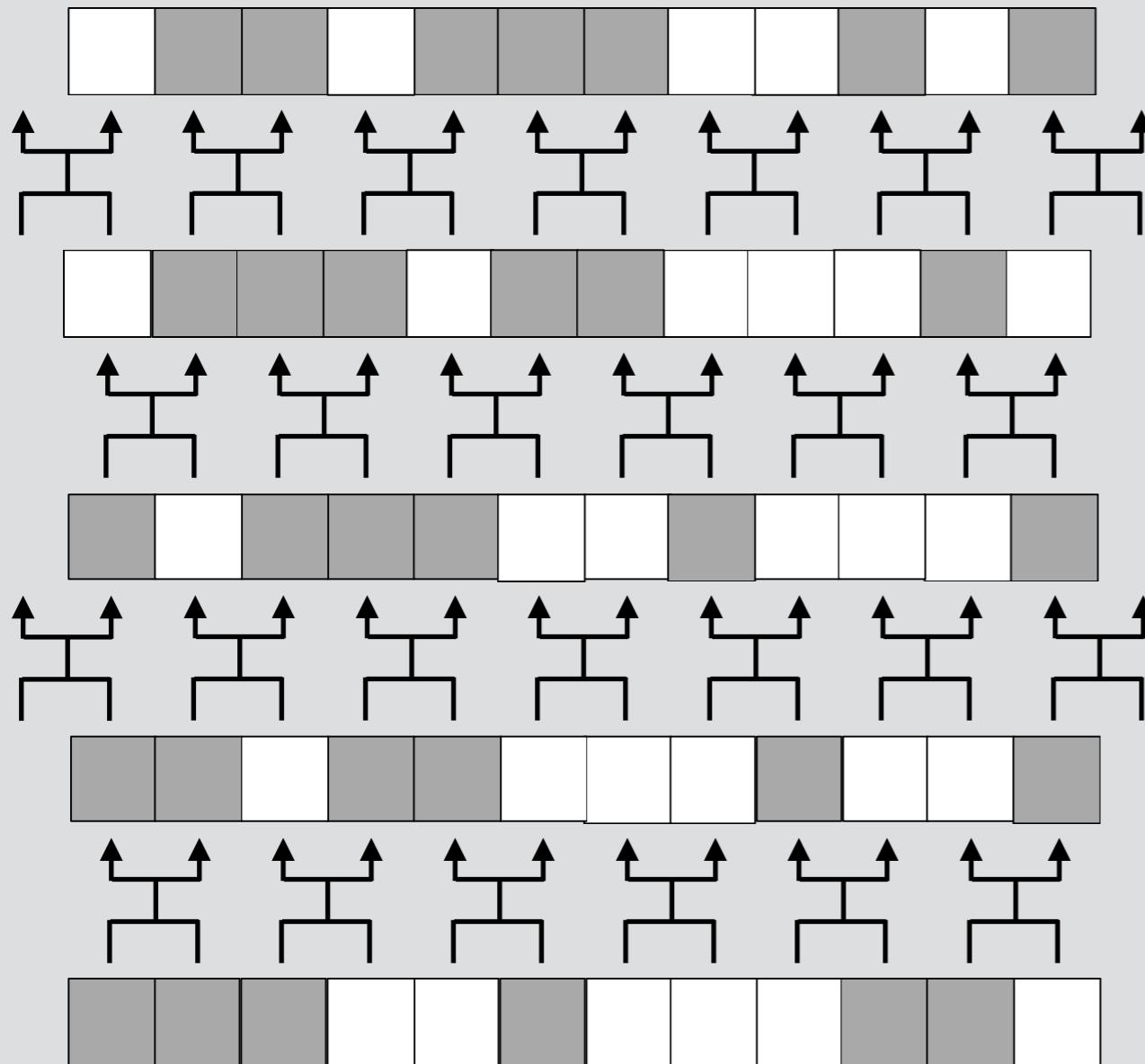
# Universal computation (via blocked CA simulation)



Temperature 2

# Universal computation (via blocked CA simulation)

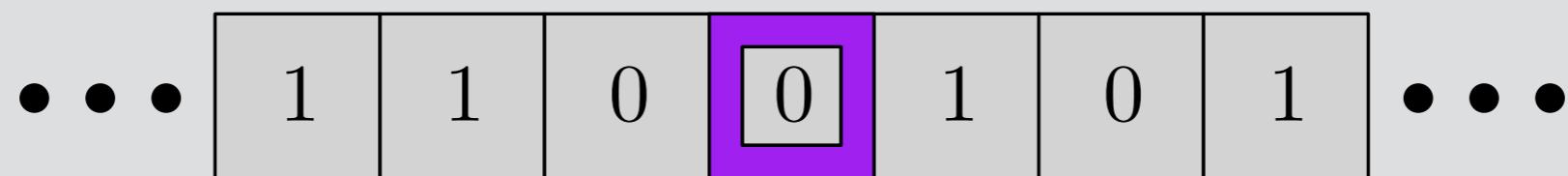
[Winfree 1998]



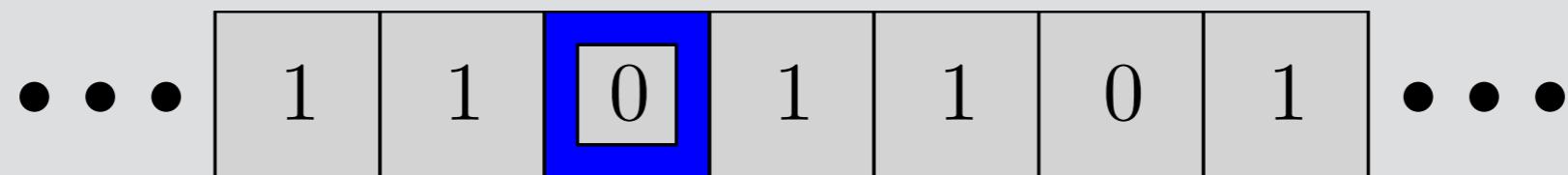
Temperature 2

# Universal computation (solution by “zig-zag” TM simulation)

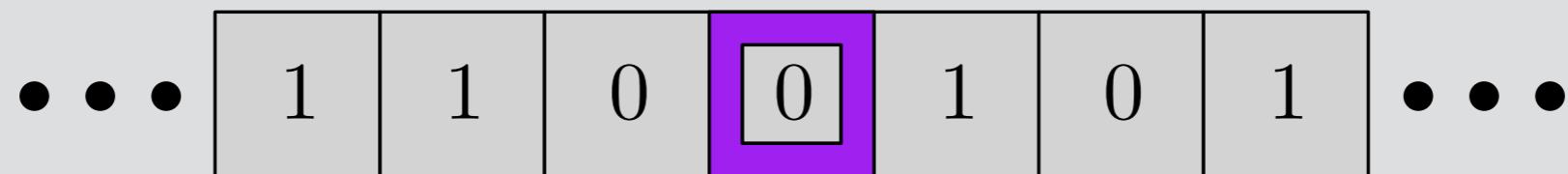
# Universal computation (solution by “zig-zag” TM simulation)



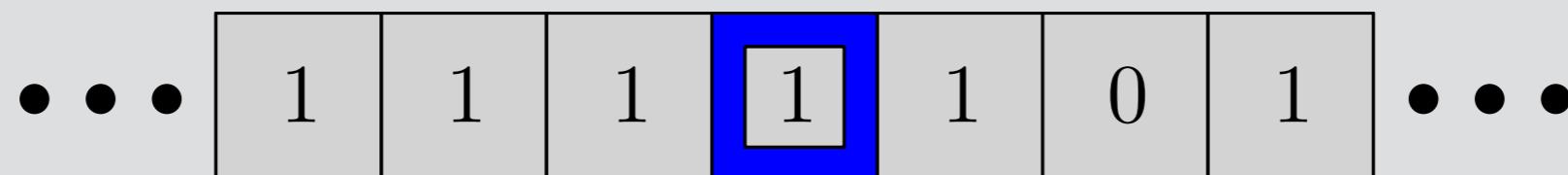
# Universal computation (solution by “zig-zag” TM simulation)



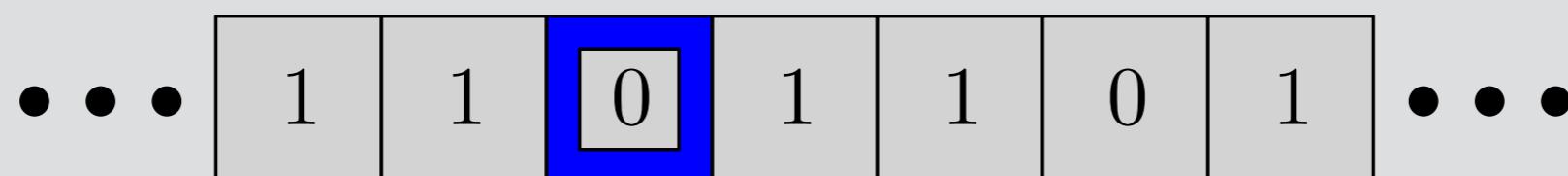
“write 1, move left, change to  $s_1$ ”



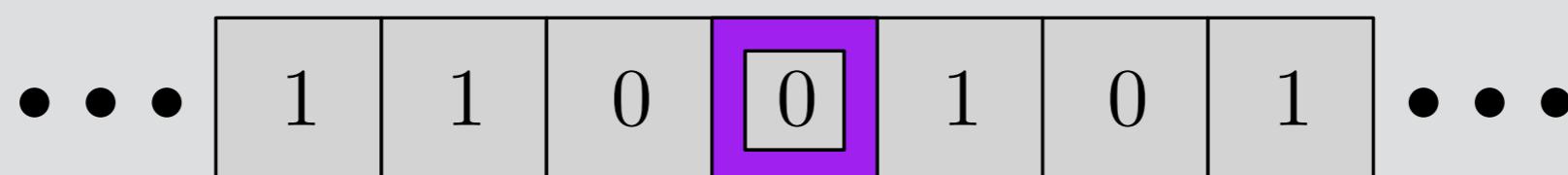
# Universal computation (solution by “zig-zag” TM simulation)



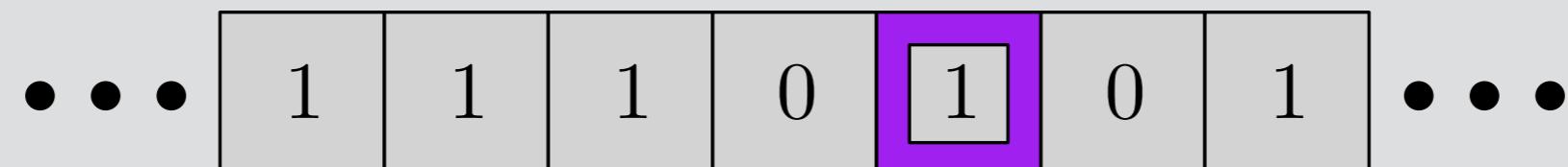
“write 1, move right, change to  $s_1$ ”



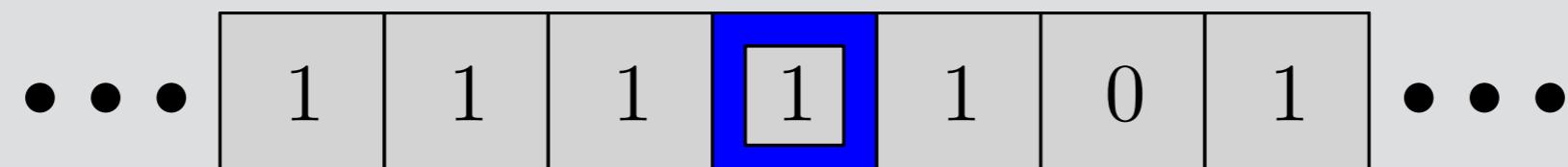
“write 1, move left, change to  $s_1$ ”



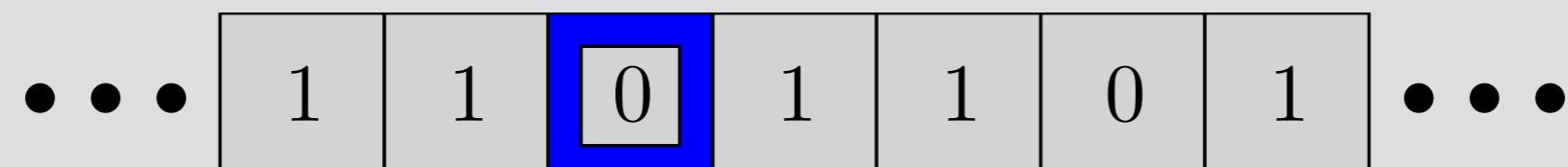
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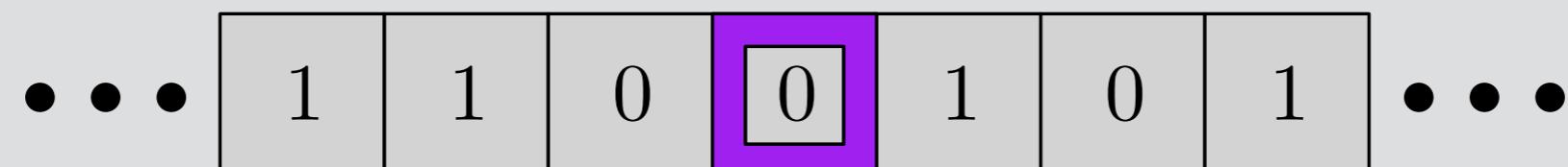
“write 0, move right, change to  $s_0$ ”



“write 1, move right, change to  $s_1$ ”



“write 1, move left, change to  $s_1$ ”



• • •

1	1	1	0	0	0	1
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• • •

“write 0, move right, change to  $s_0$ ”

• • •

1	1	1	0	1	0	1
---	---	---	---	---	---	---

• • •

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• • •

1	1	1	1	1	0	1
---	---	---	---	---	---	---

• • •

“write 1, move right, change to  $s_1$ ”

• • •

1	1	0	1	1	0	1
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• • •

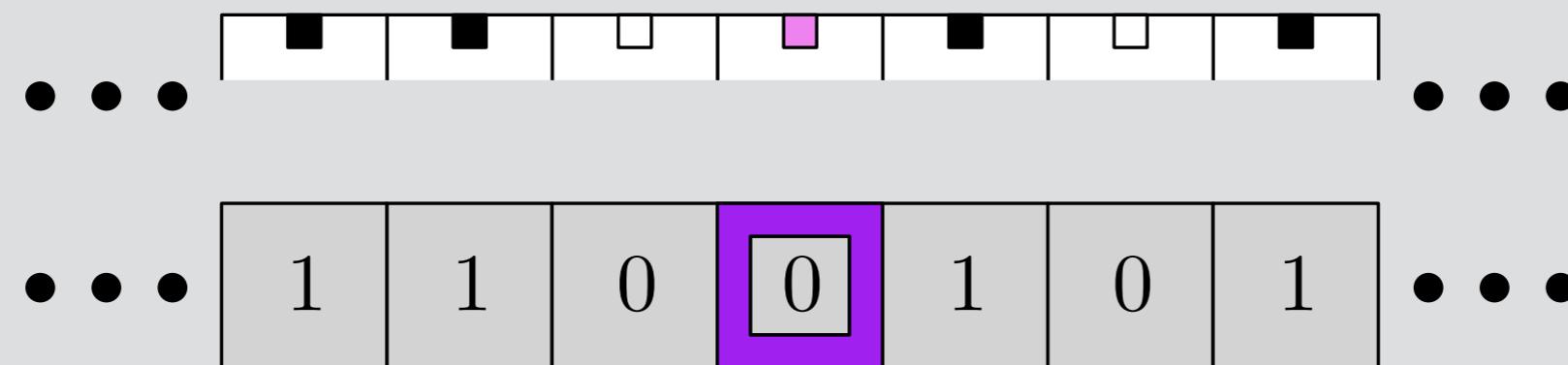
“write 1, move left, change to  $s_1$ ”

• • •

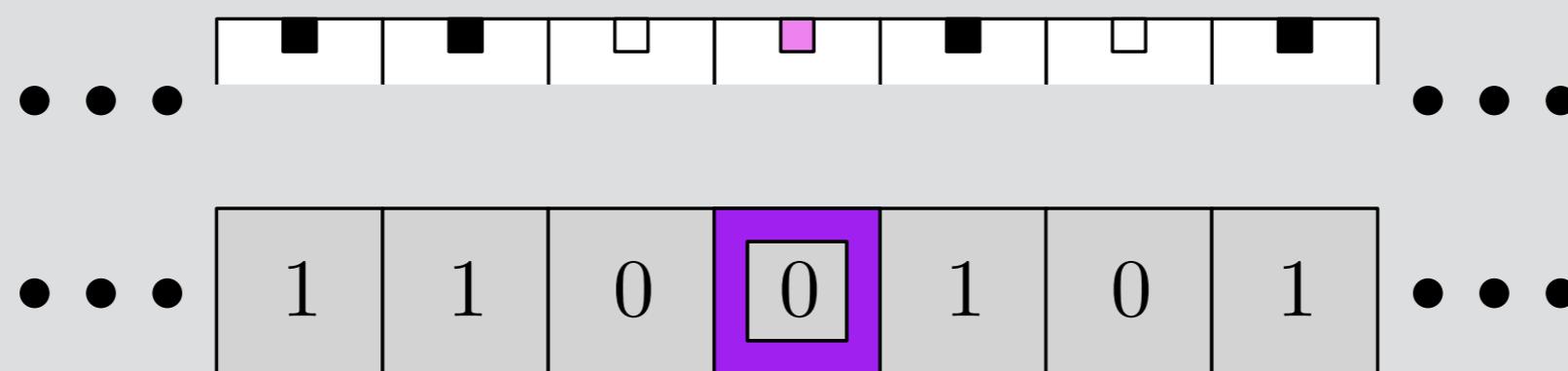
1	1	0	0	1	0	1
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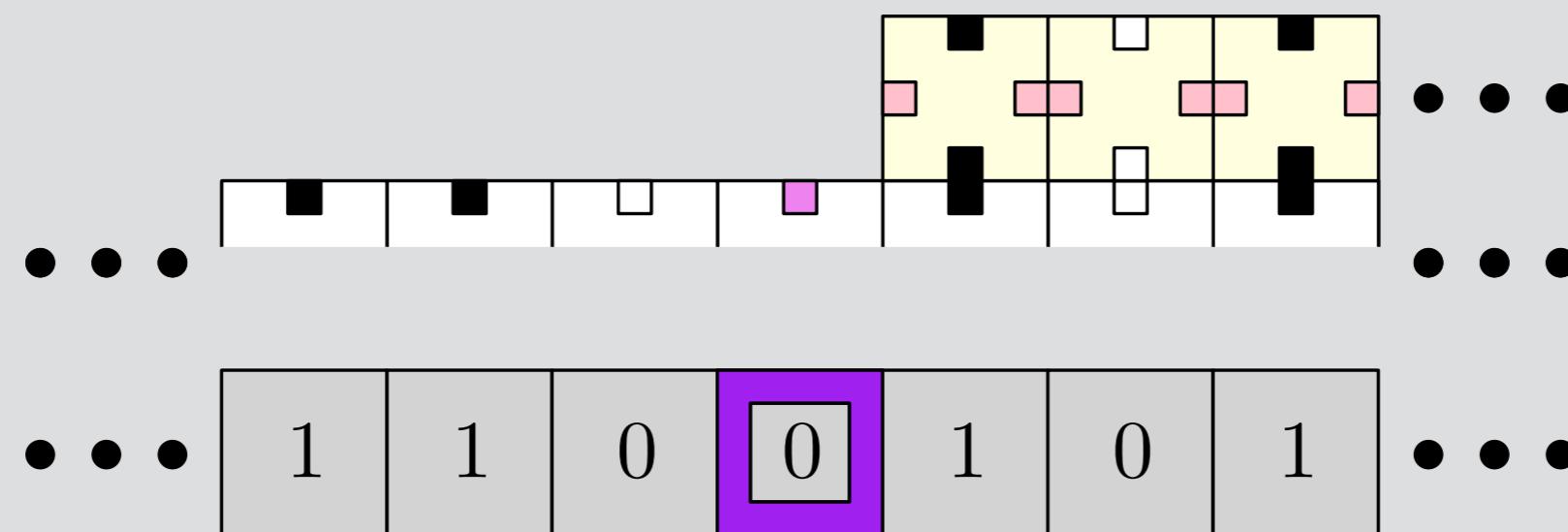


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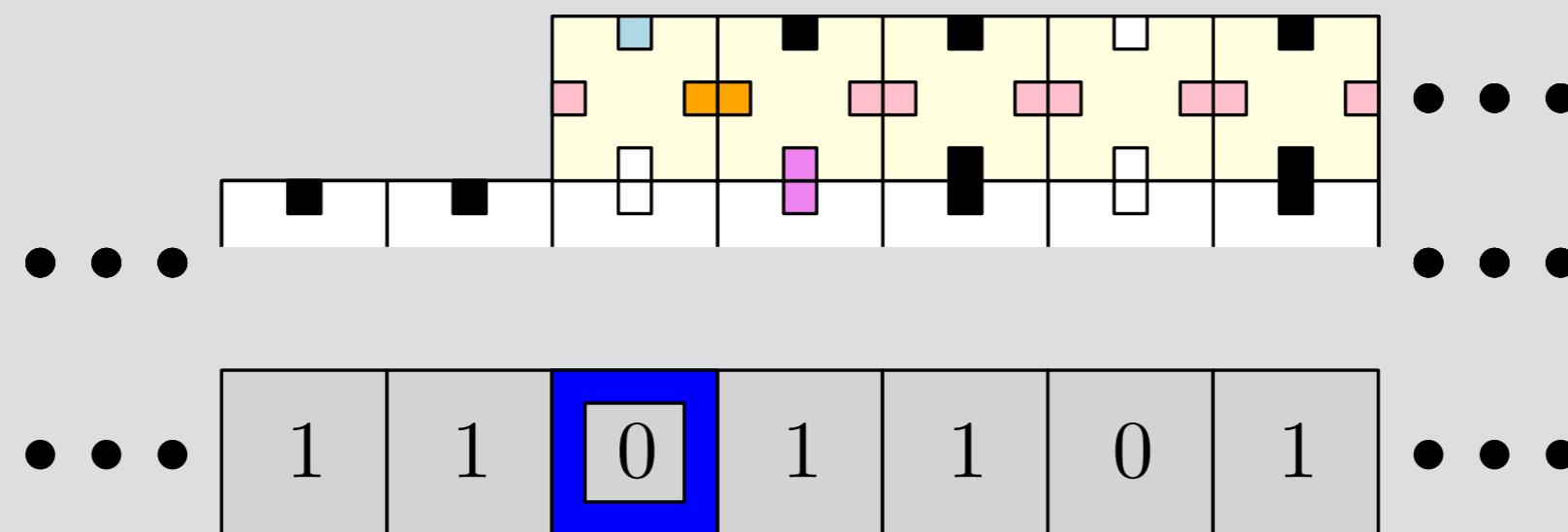
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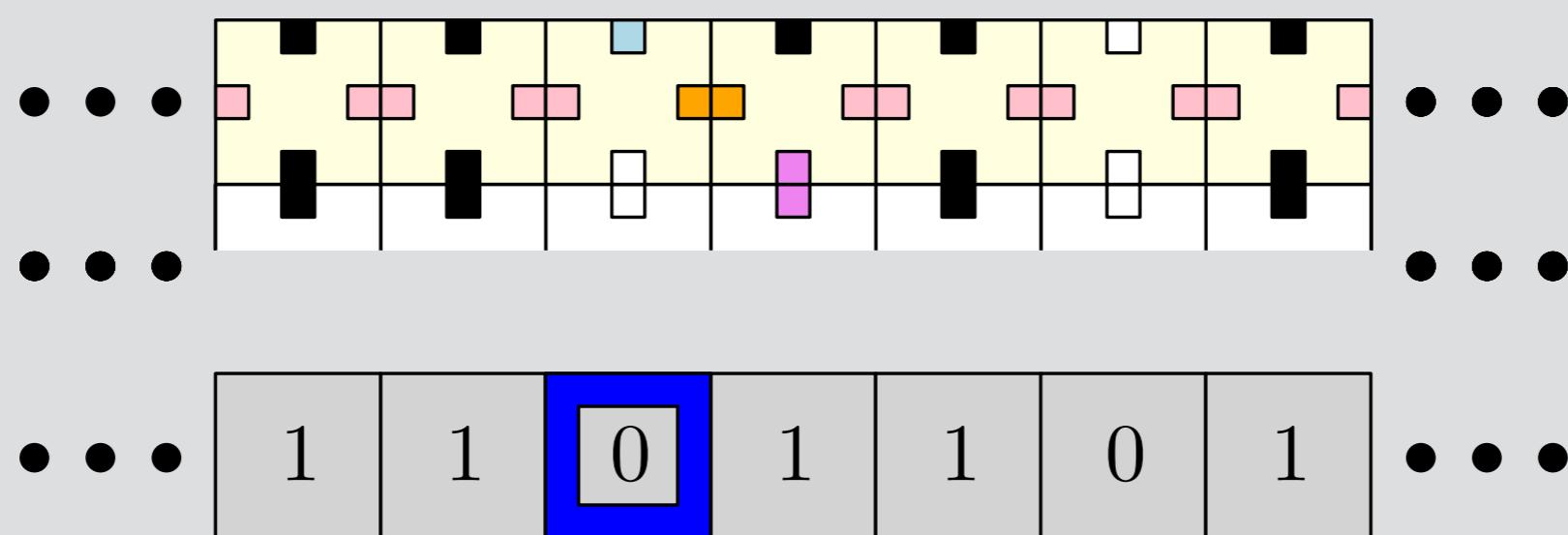
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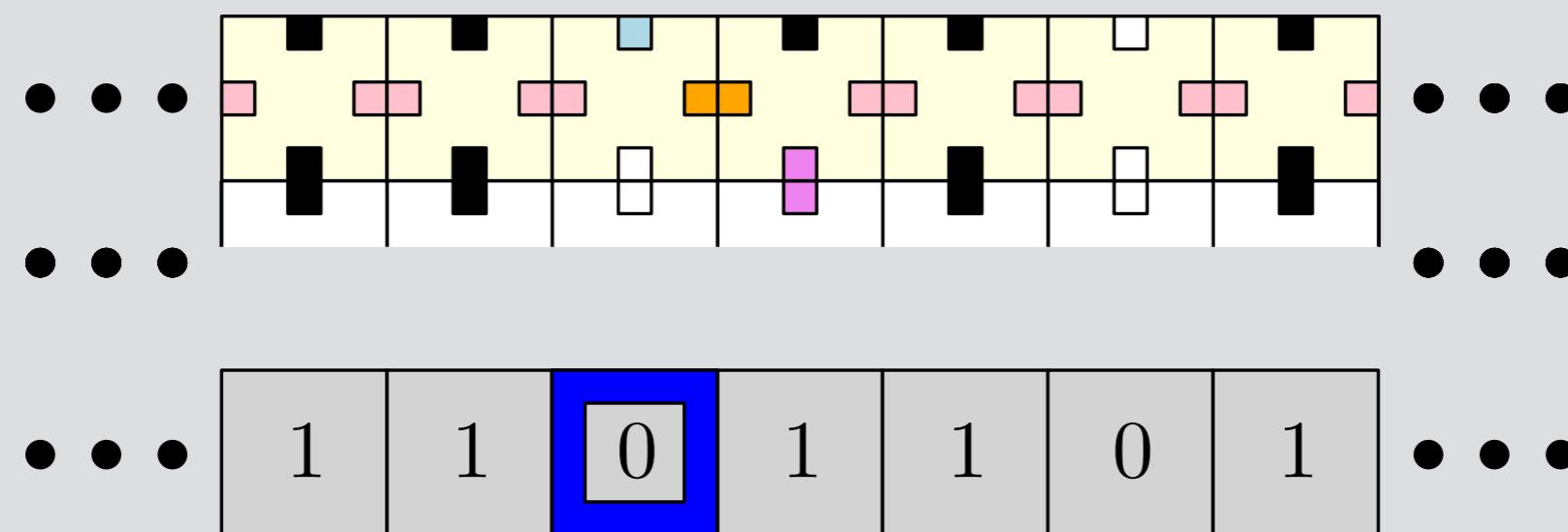
“write 1, move left, change to  $s_1$ ”

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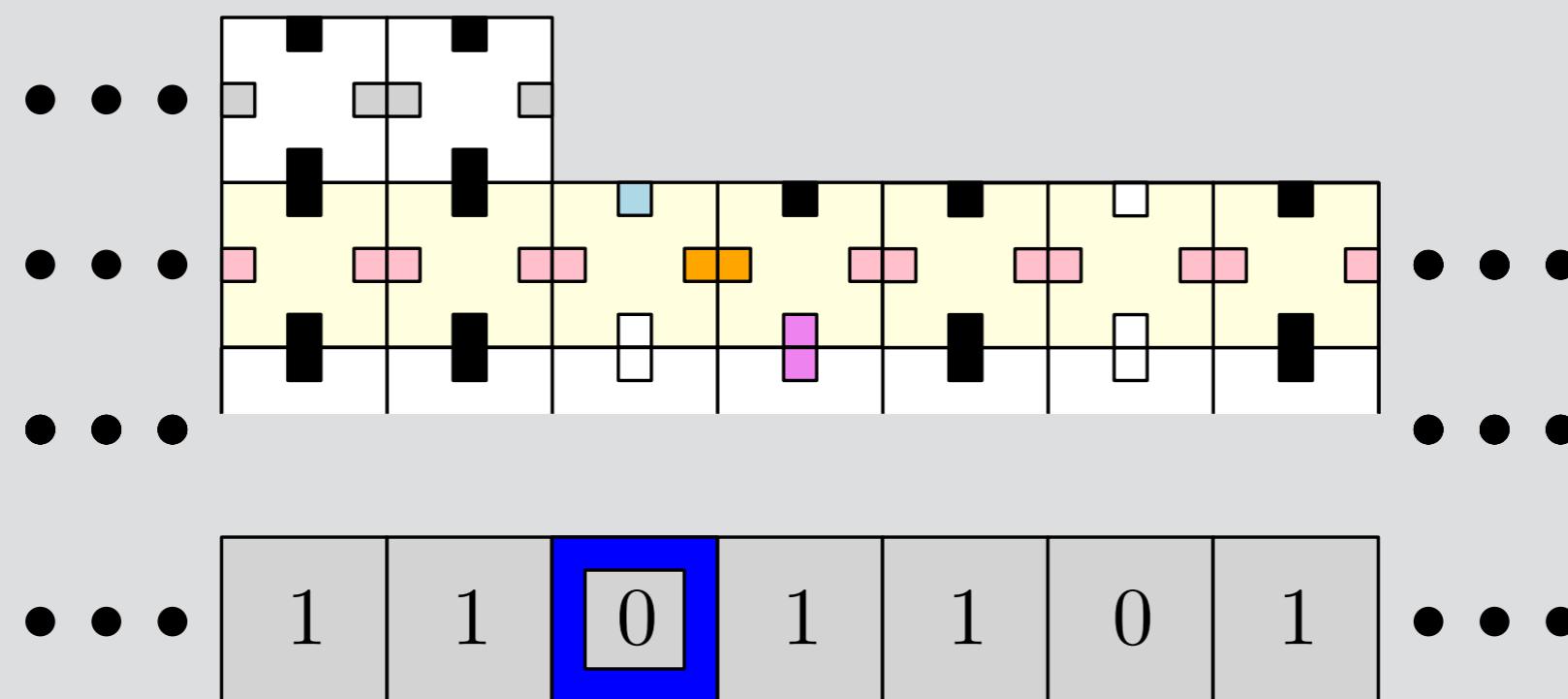
## Temperature 2

# Universal computation (solution by “zig-zag” TM simulation)



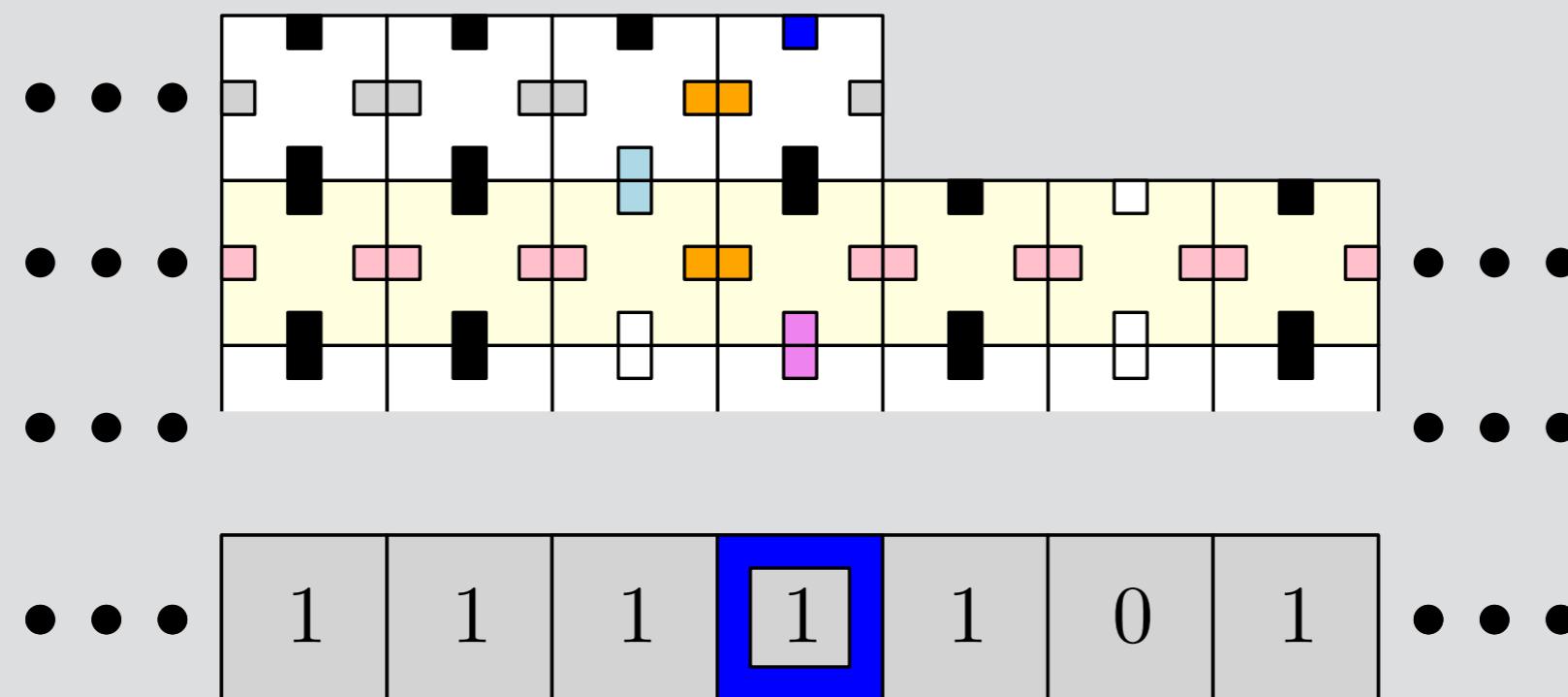
“write 1, move right, change to  $s_1$ ”

# Universal computation (solution by “zig-zag” TM simulation)



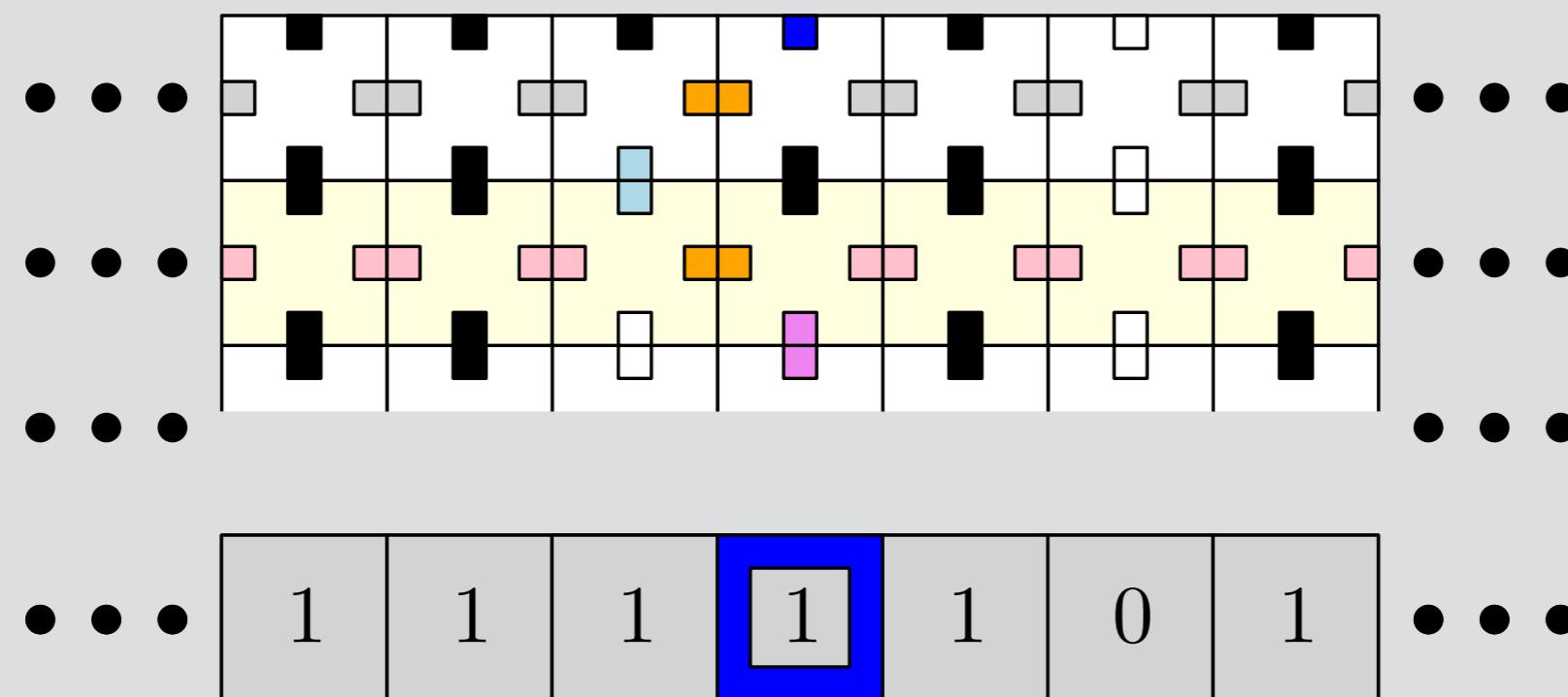
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# Universal computation (solution by “zig-zag” TM simulation)



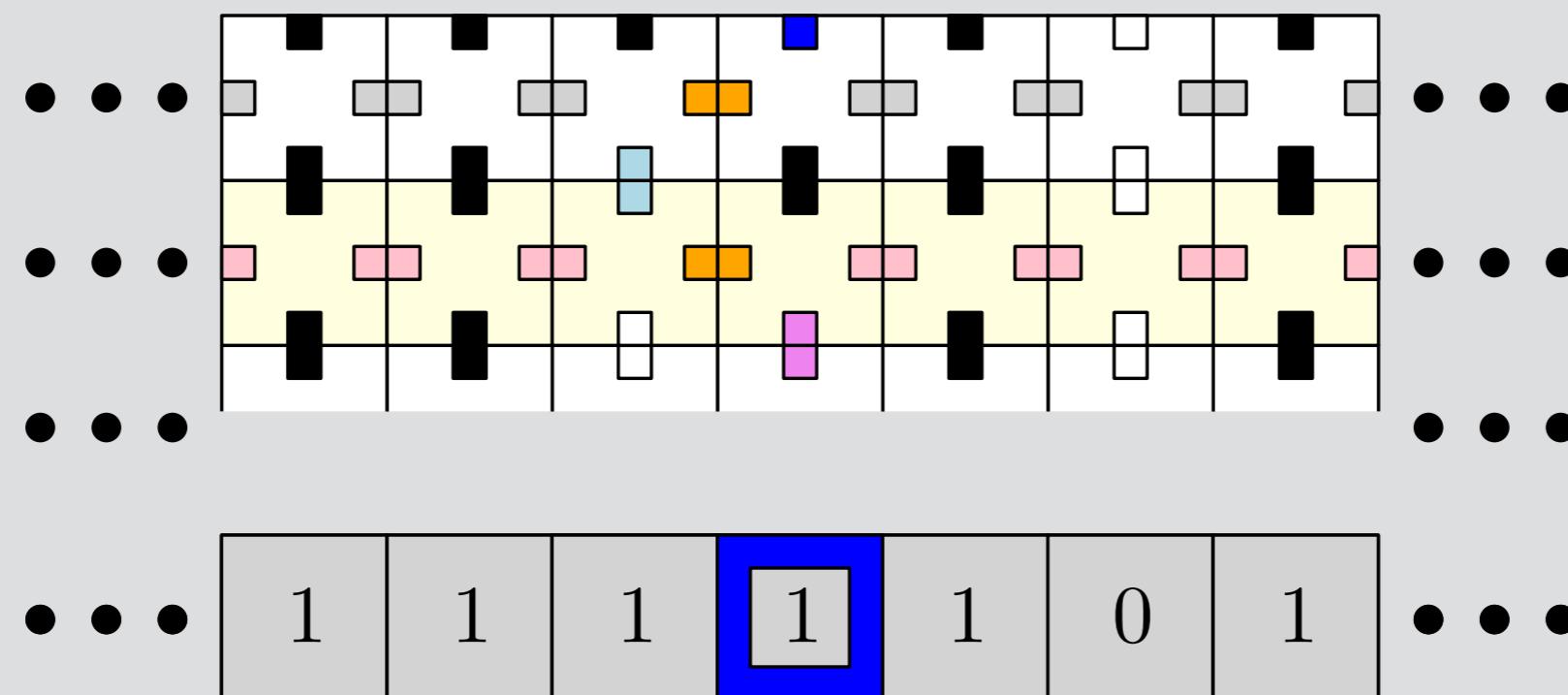
“write 1, move right, change to  $s_1$ ”

# Universal computation (solution by “zig-zag” TM simulation)



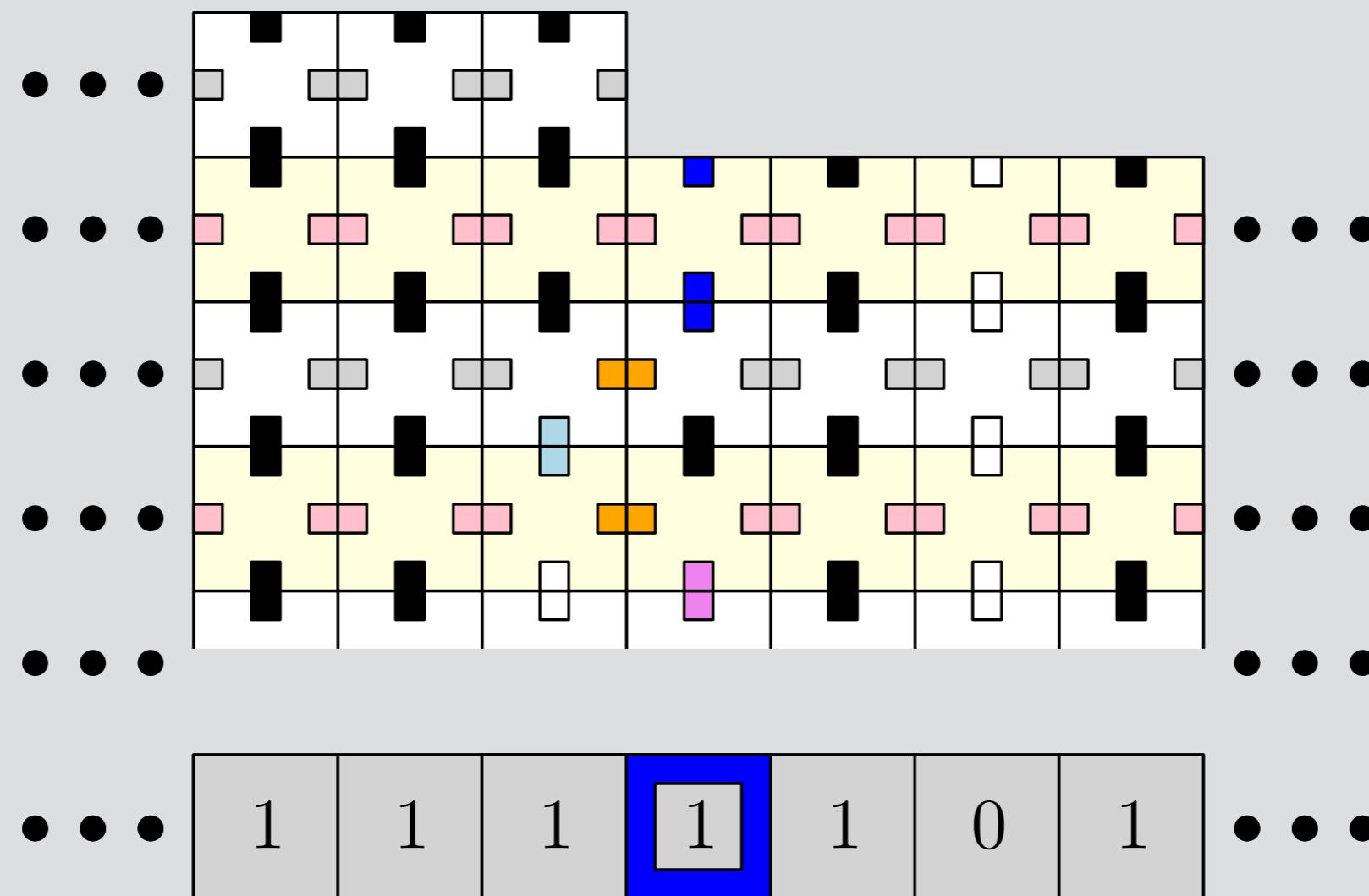
Temperature 2

# Universal computation (solution by “zig-zag” TM simulation)



“write 0, move right, change to  $s_0$ ”

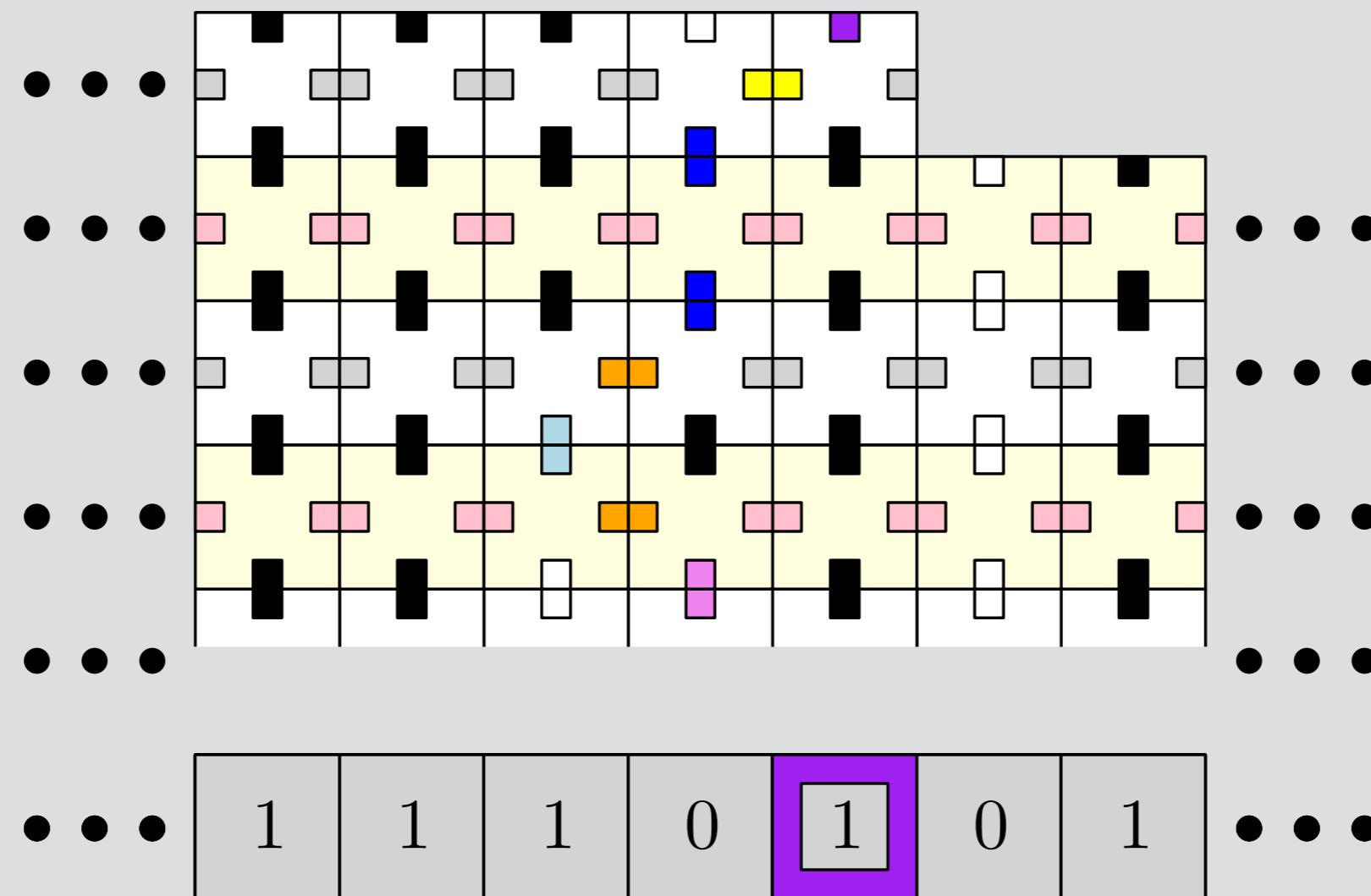
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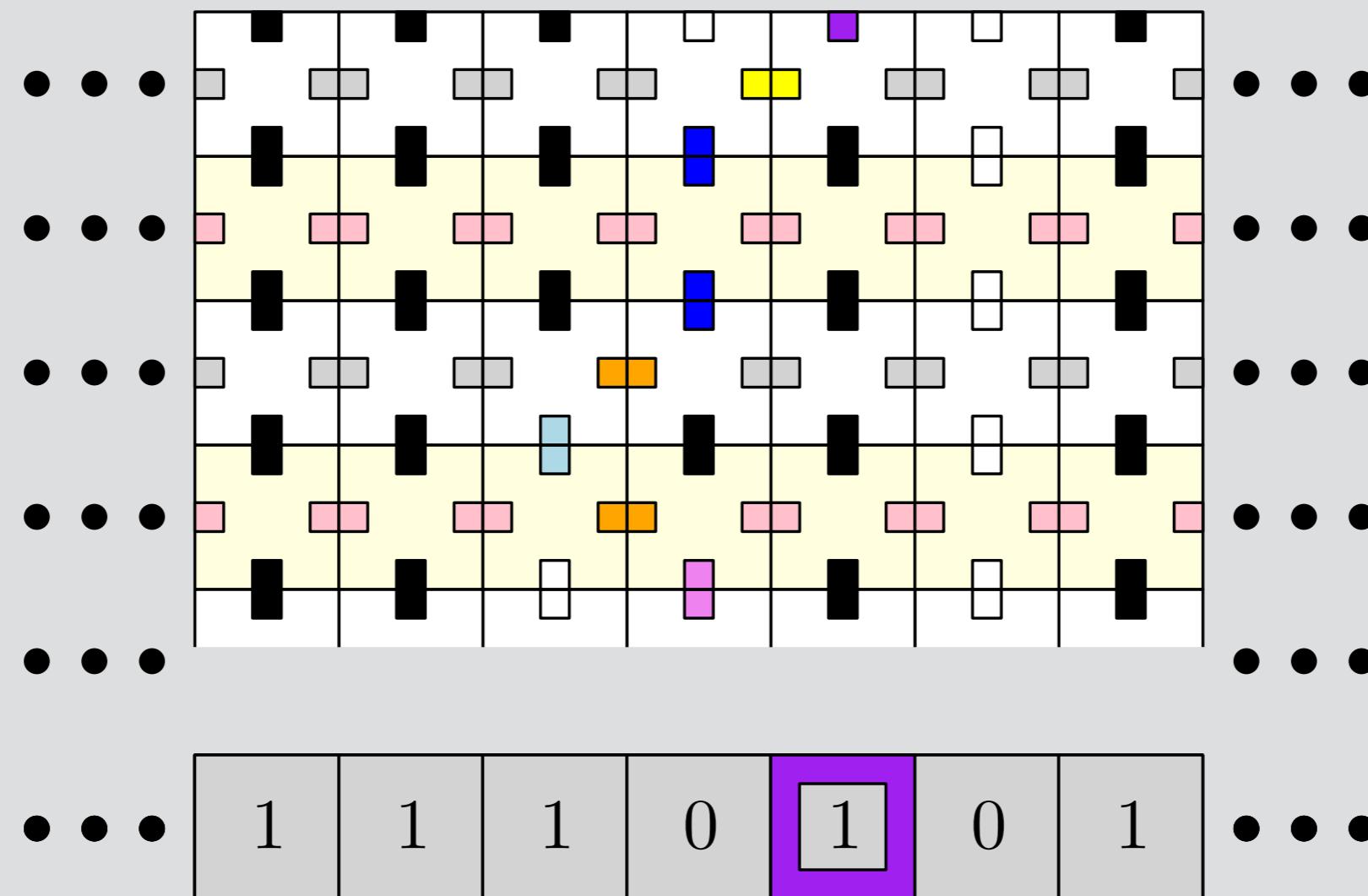
Temperature 2

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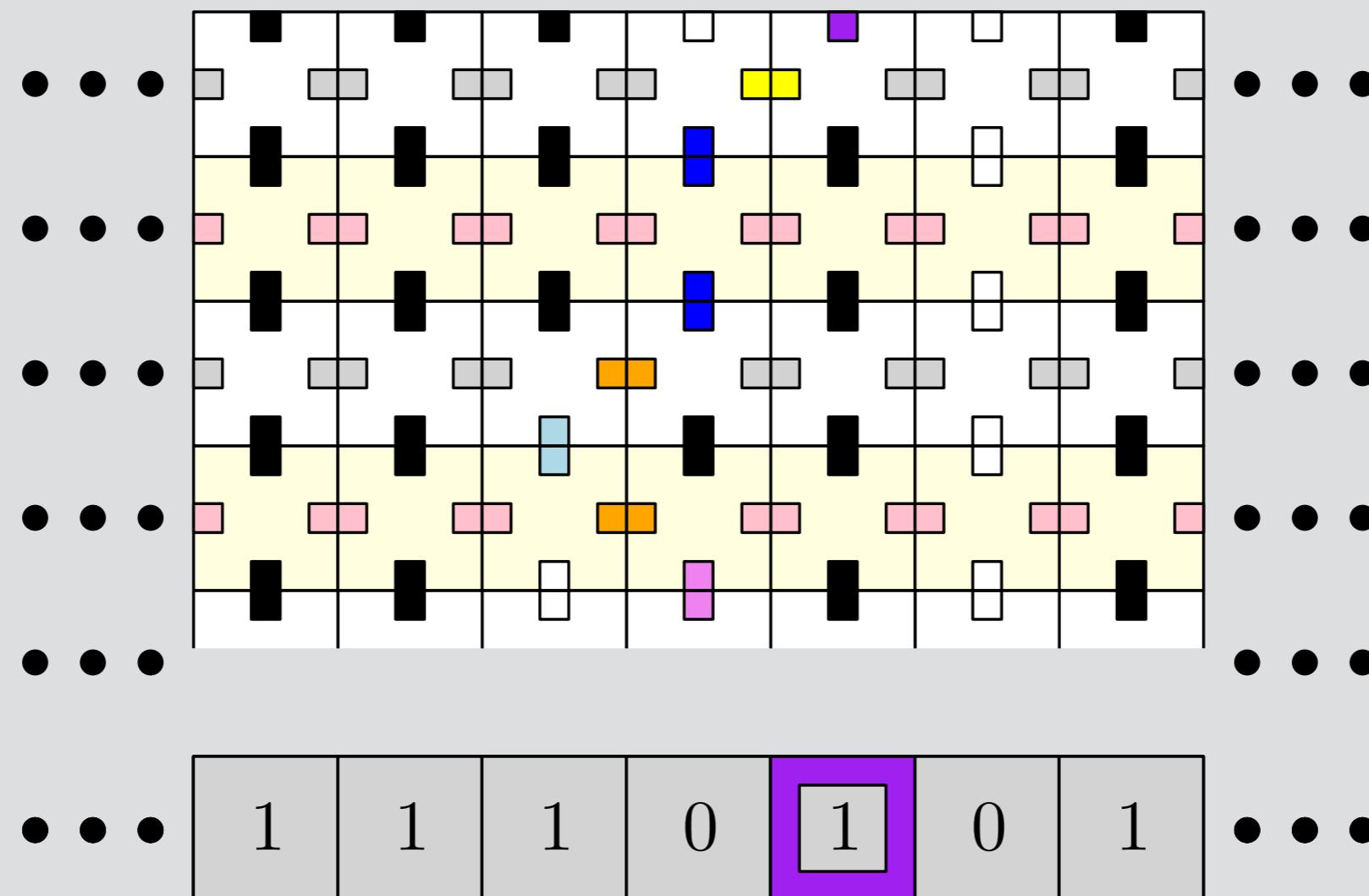
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Temperature 2

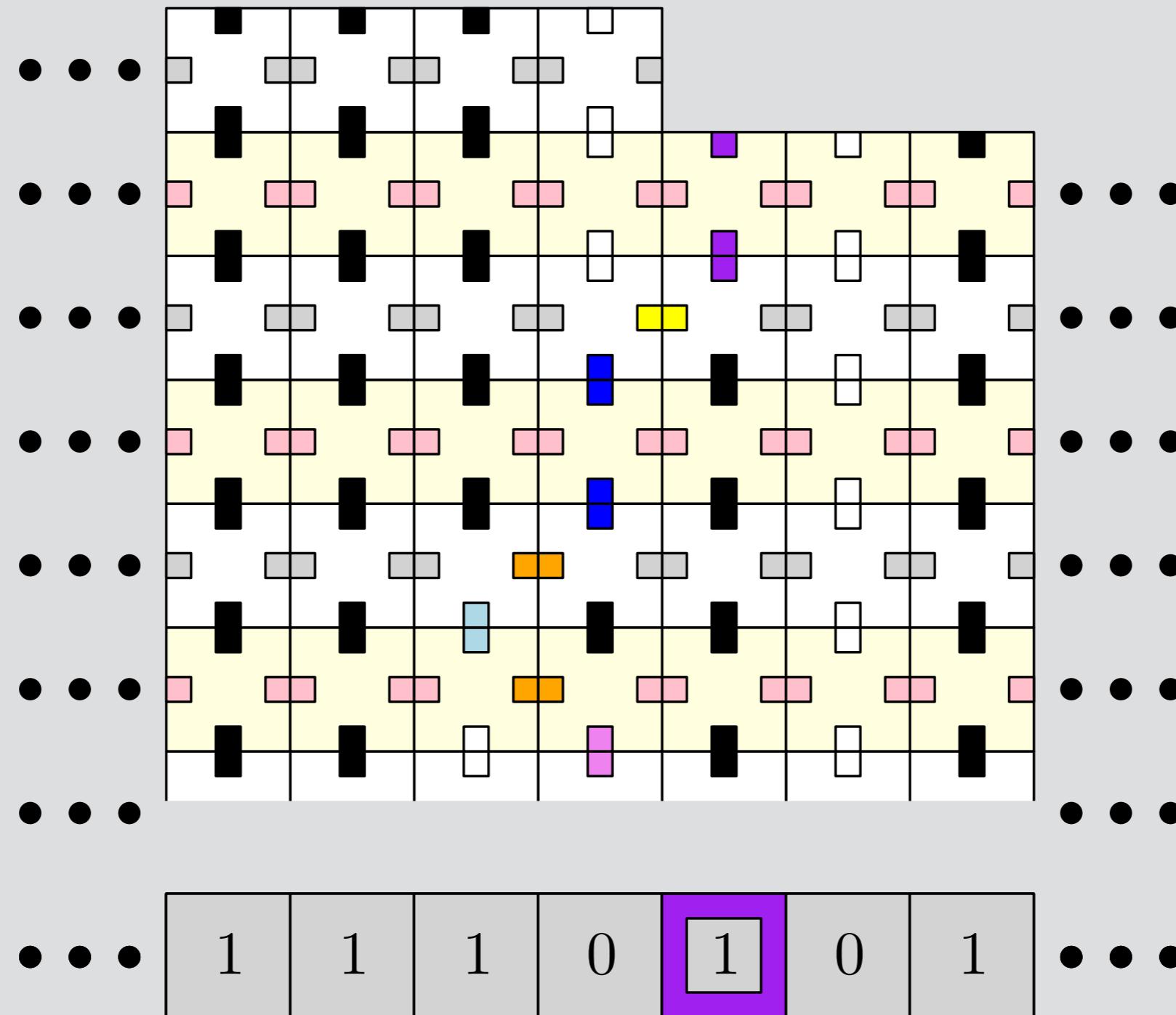
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Temperature 2

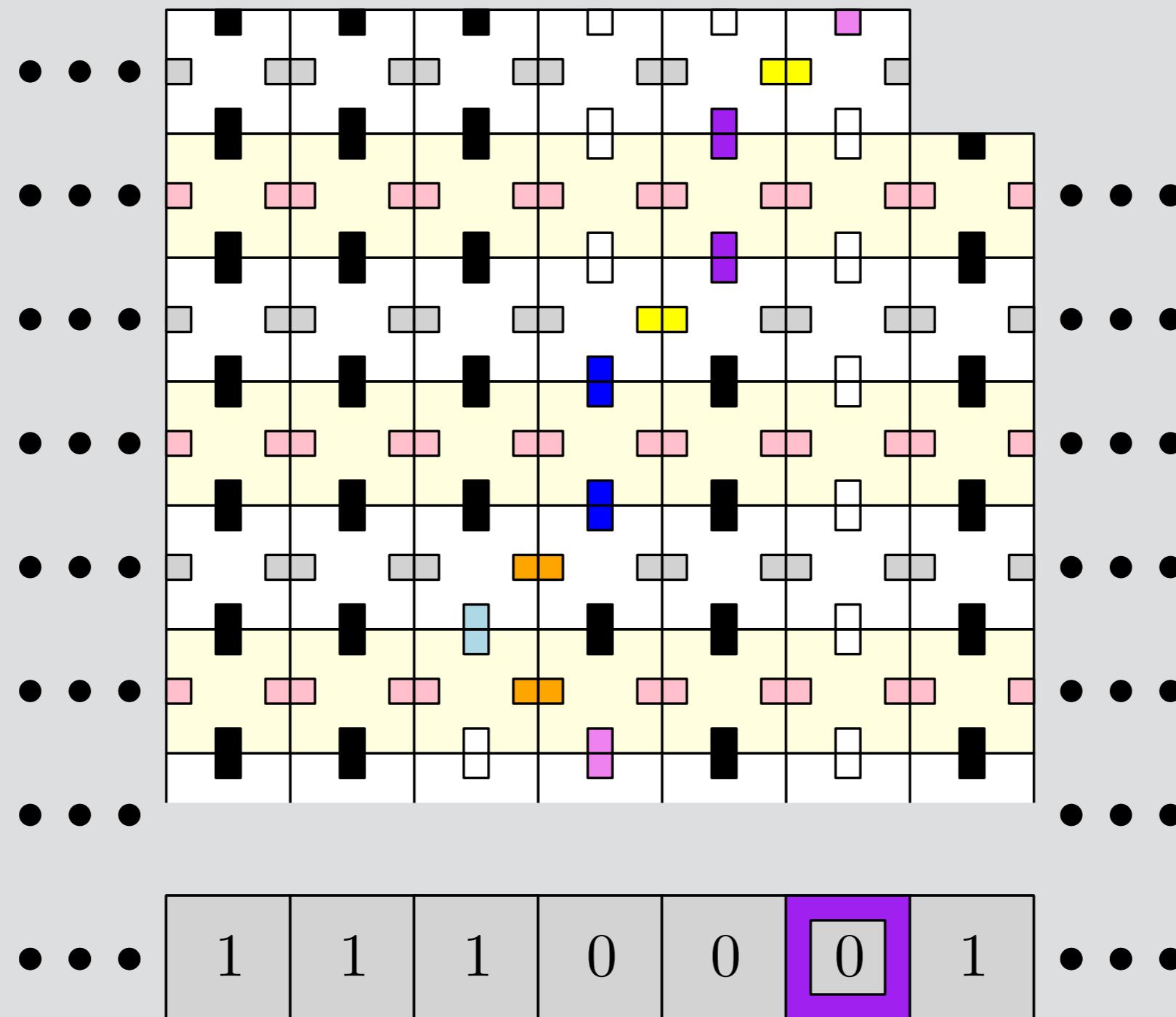
# Universal computation



“write 0, move right, change to  $s_0$ ”

Temperature 2

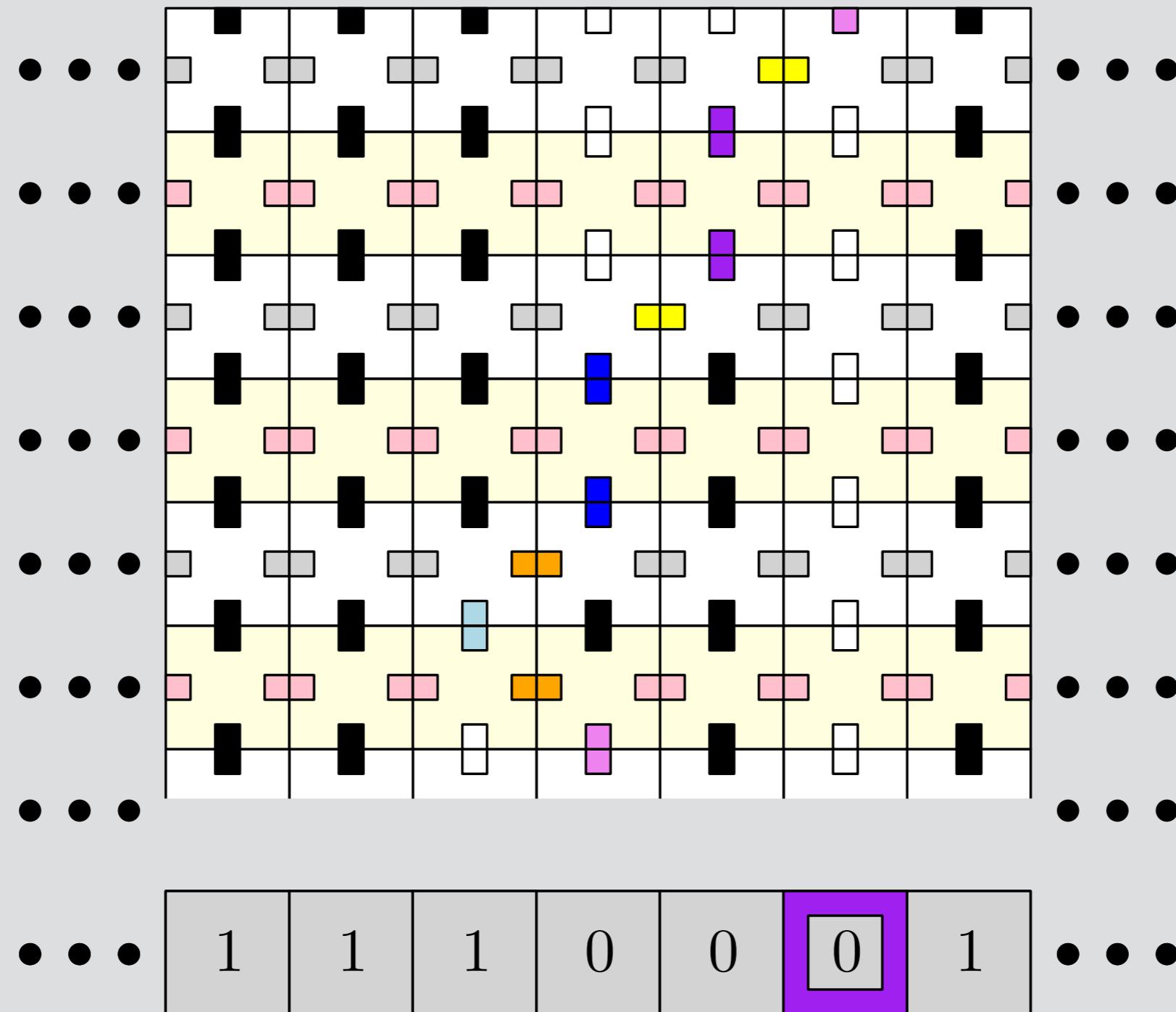
# Universal computation



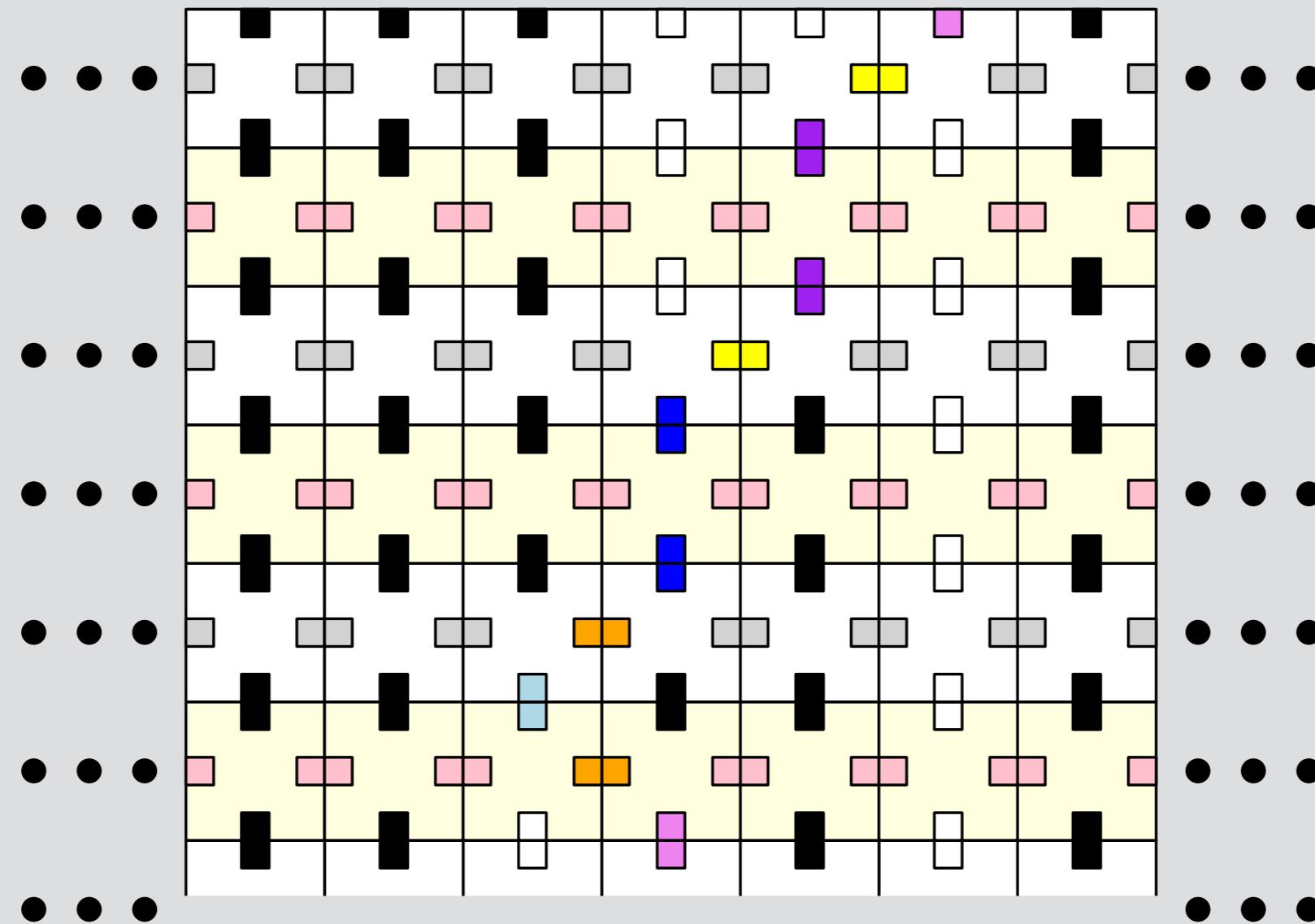
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Temperature 2

# Universal computation



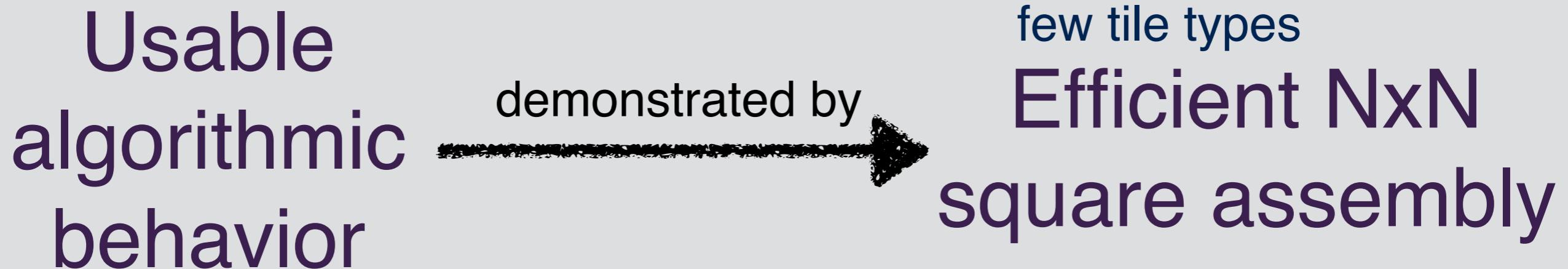
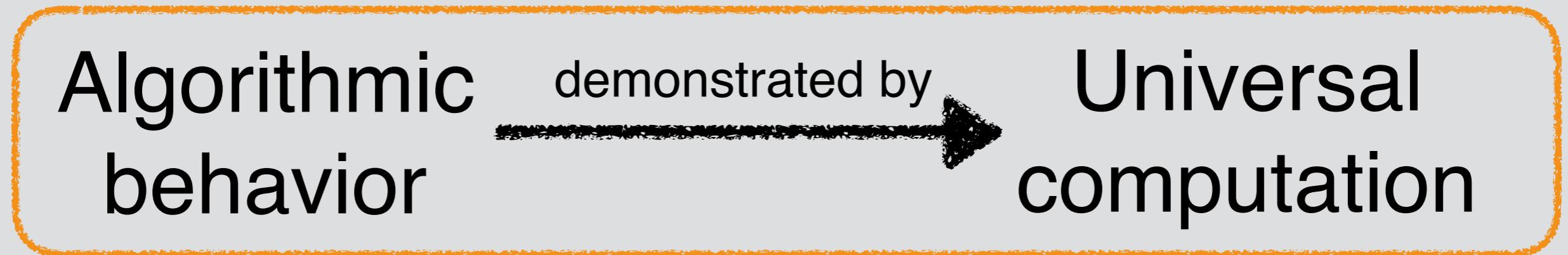
# Universal computation



[Rothenmund, Winfree 2000]

Temperature 2

# The benchmarks of a capable self-assembly model



# The benchmarks of a capable self-assembly model

Algorithmic behavior

demonstrated by

Universal computation

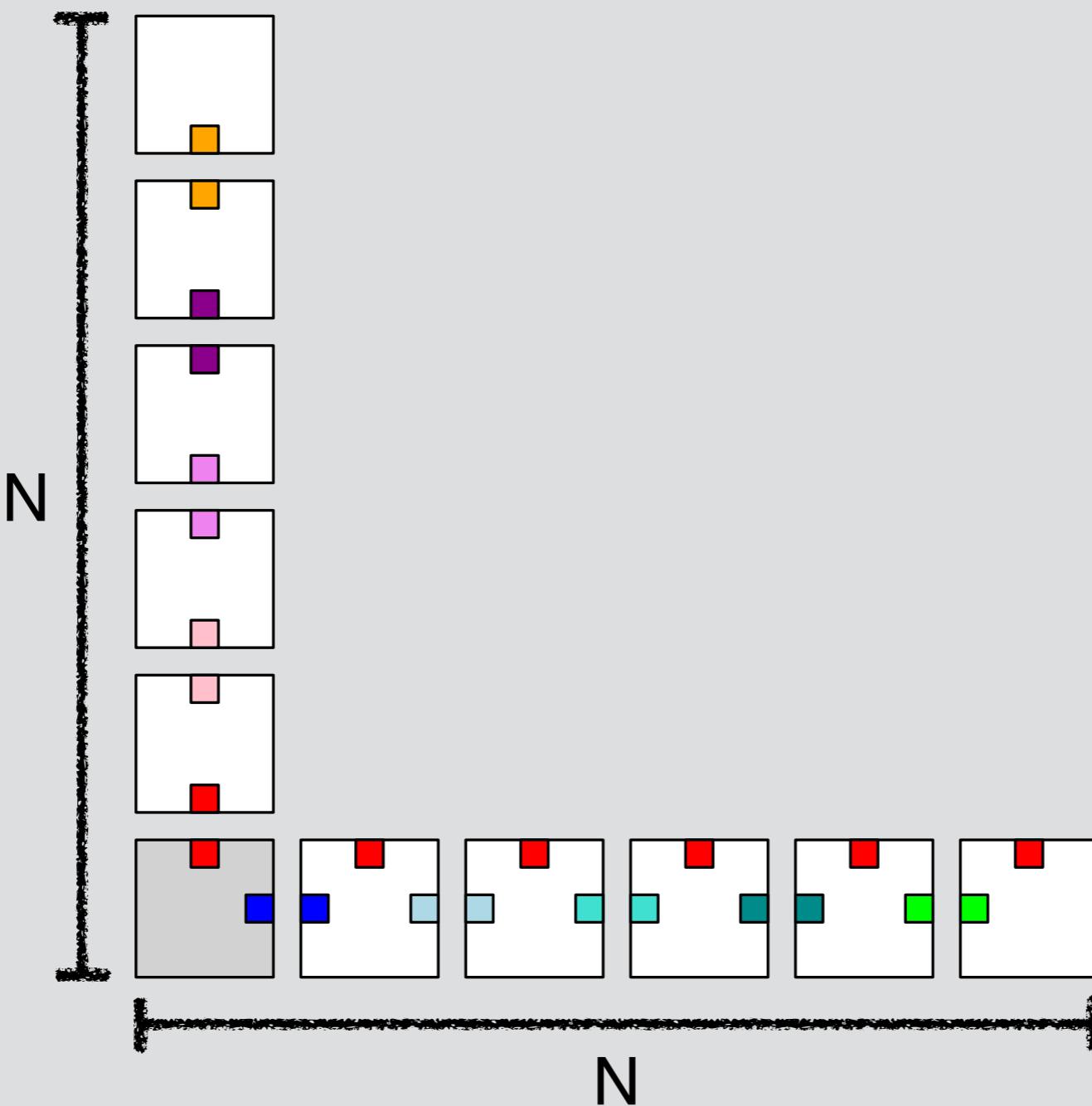
Usable algorithmic behavior

demonstrated by

few tile types  
**Efficient NxN square assembly**

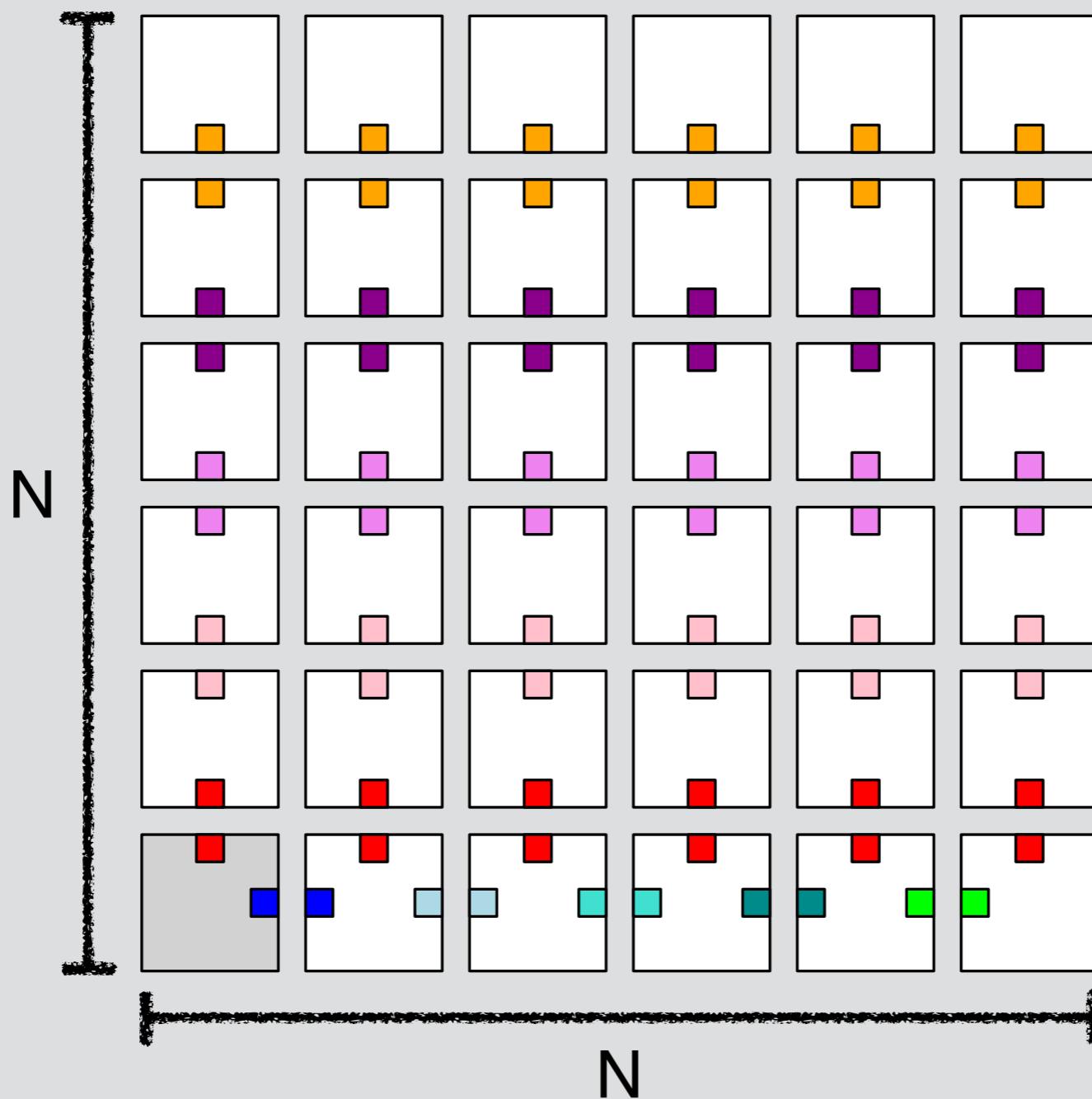
# Efficient assembly of NxN square

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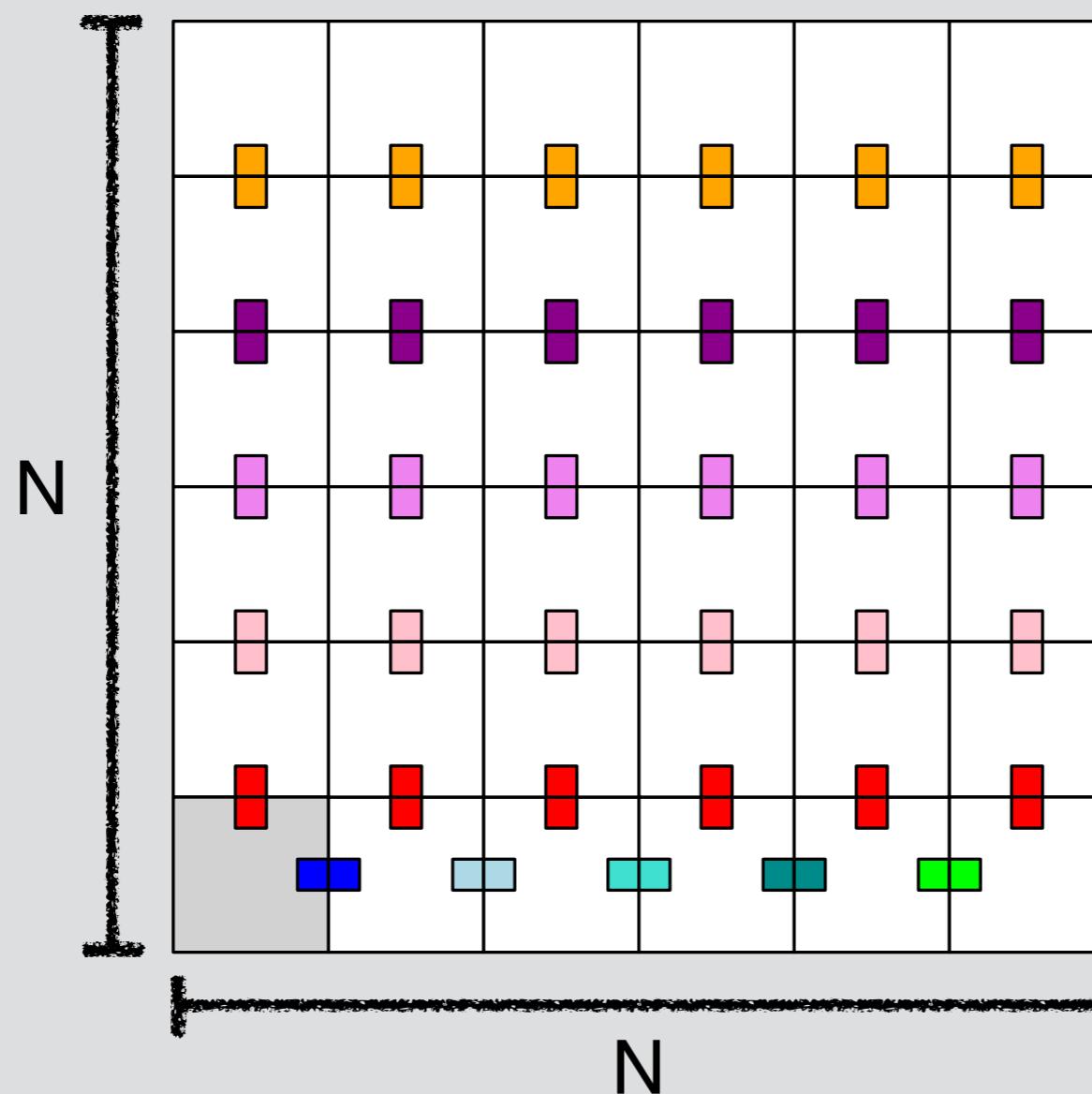
Temperature 1

# Efficient assembly of NxN square



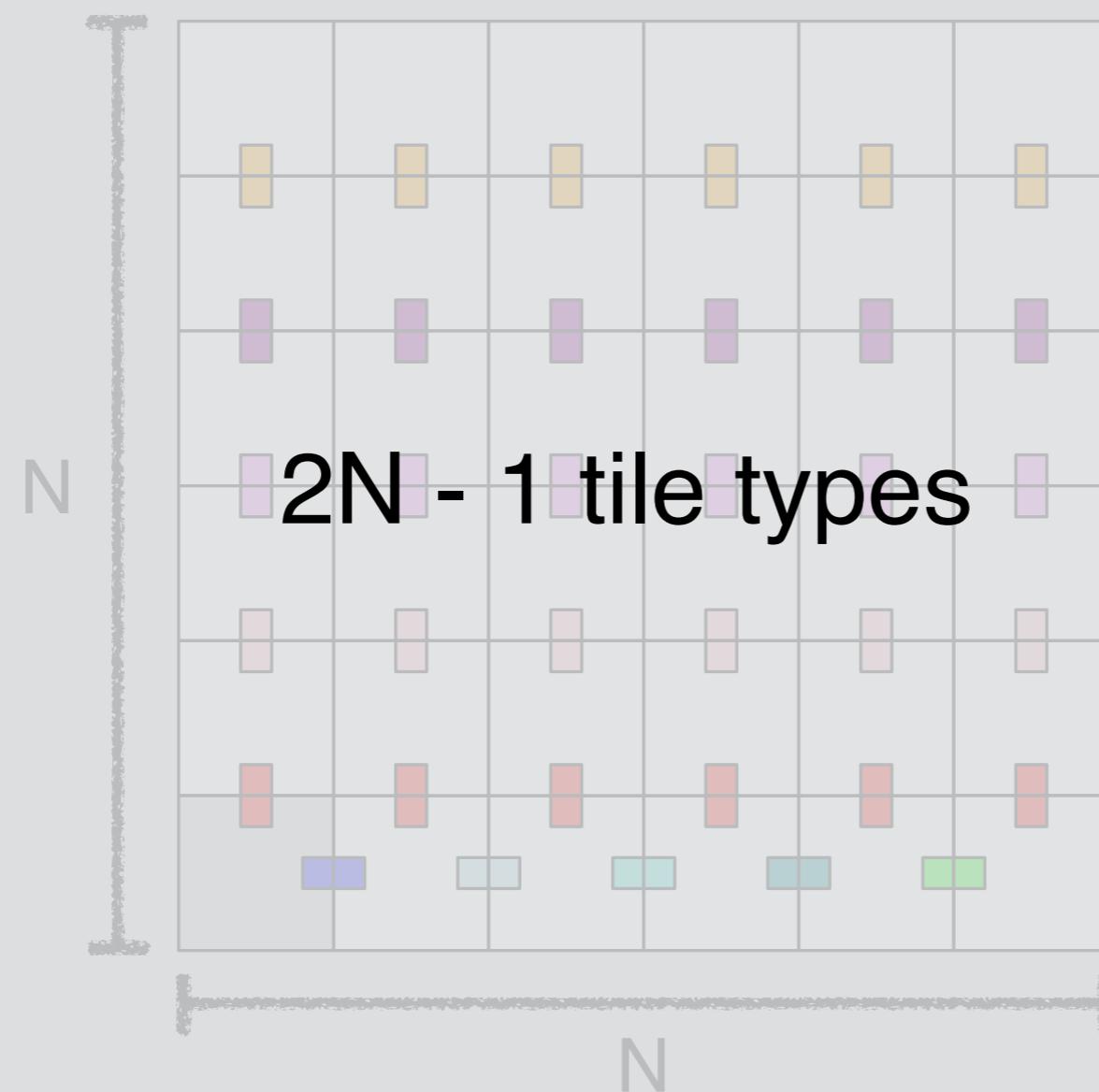
Temperature 1

# Efficient assembly of NxN square



Temperature 1

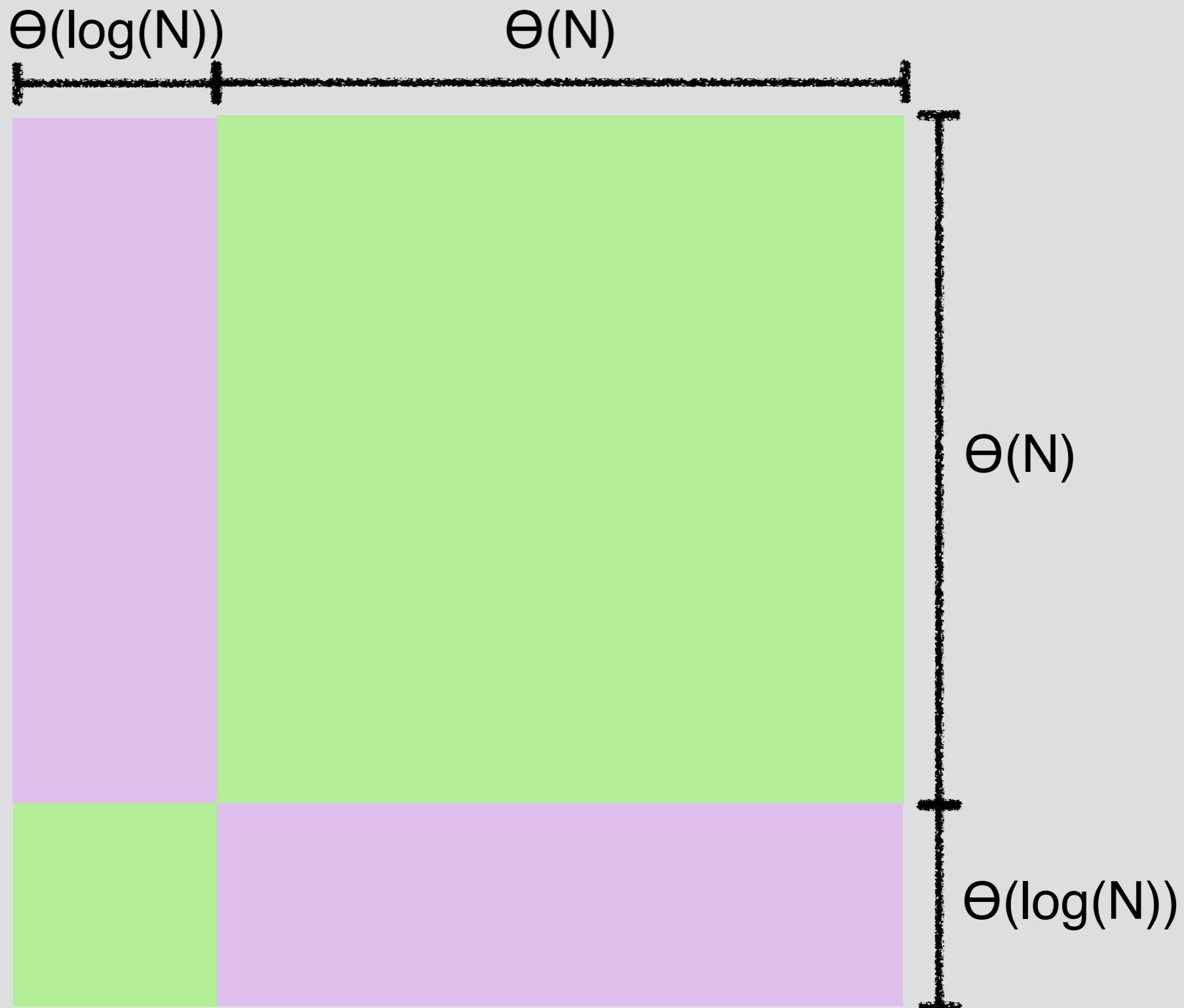
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Temperature 1

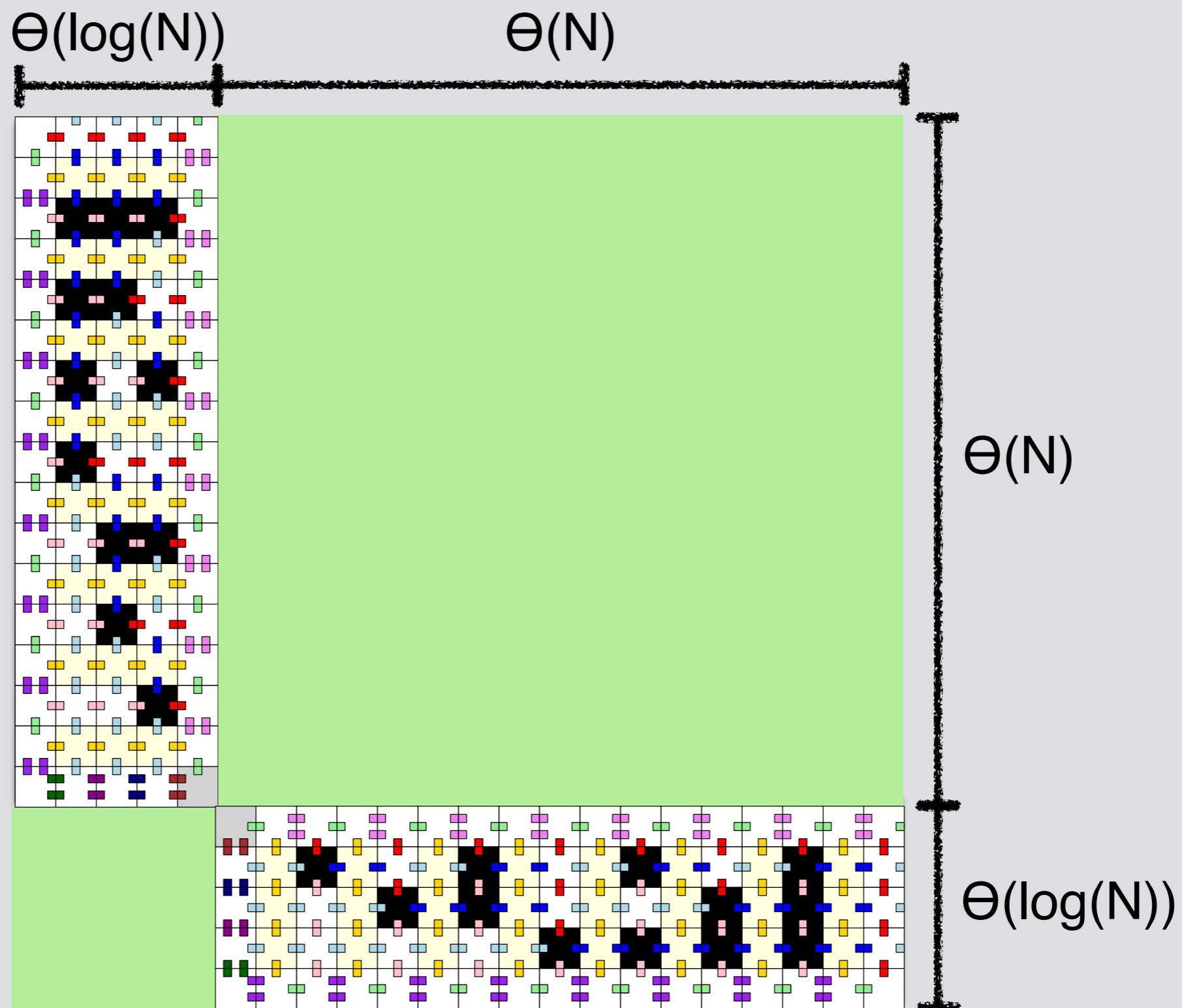
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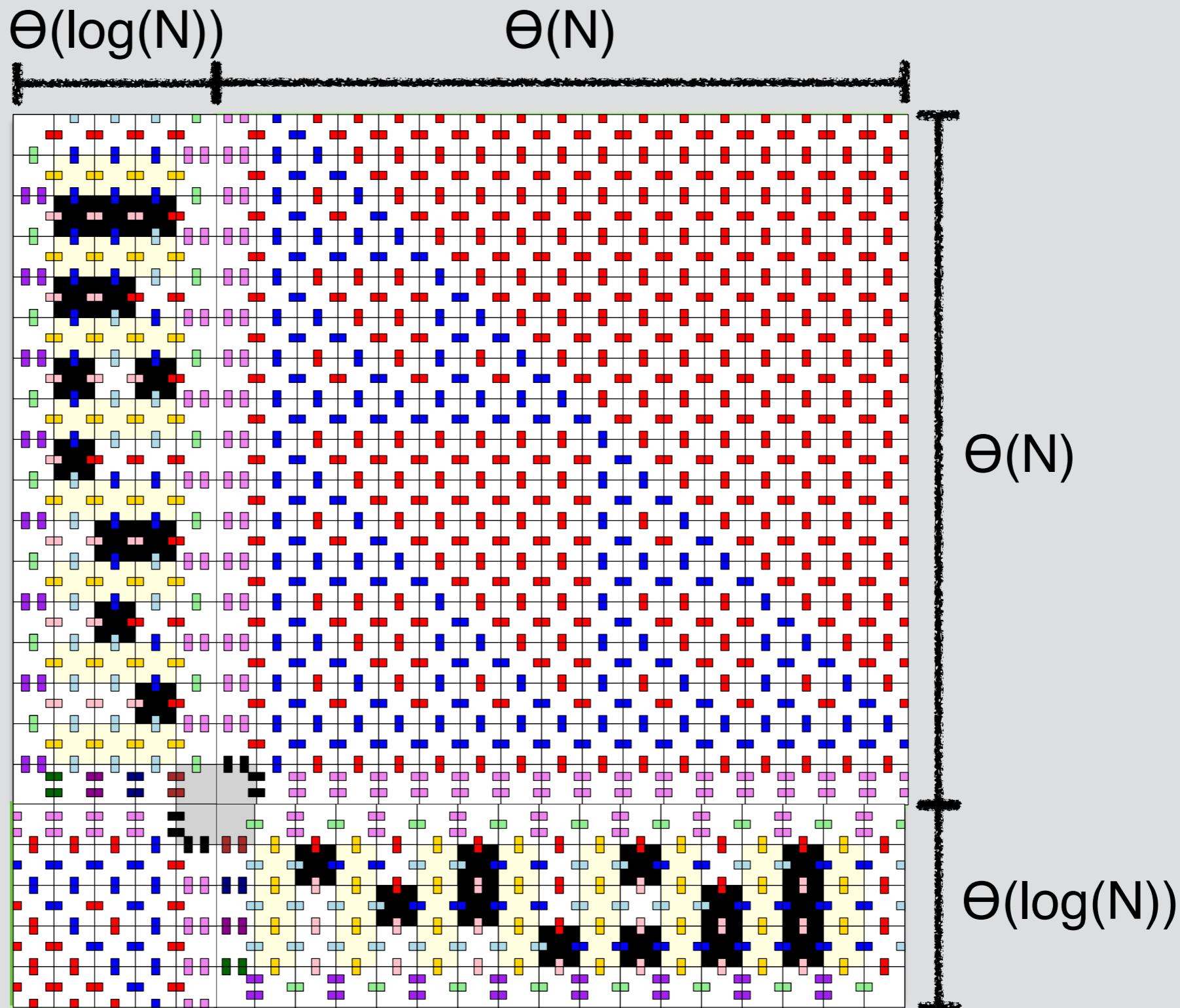
Temperature 2

# Efficient assembly of NxN square



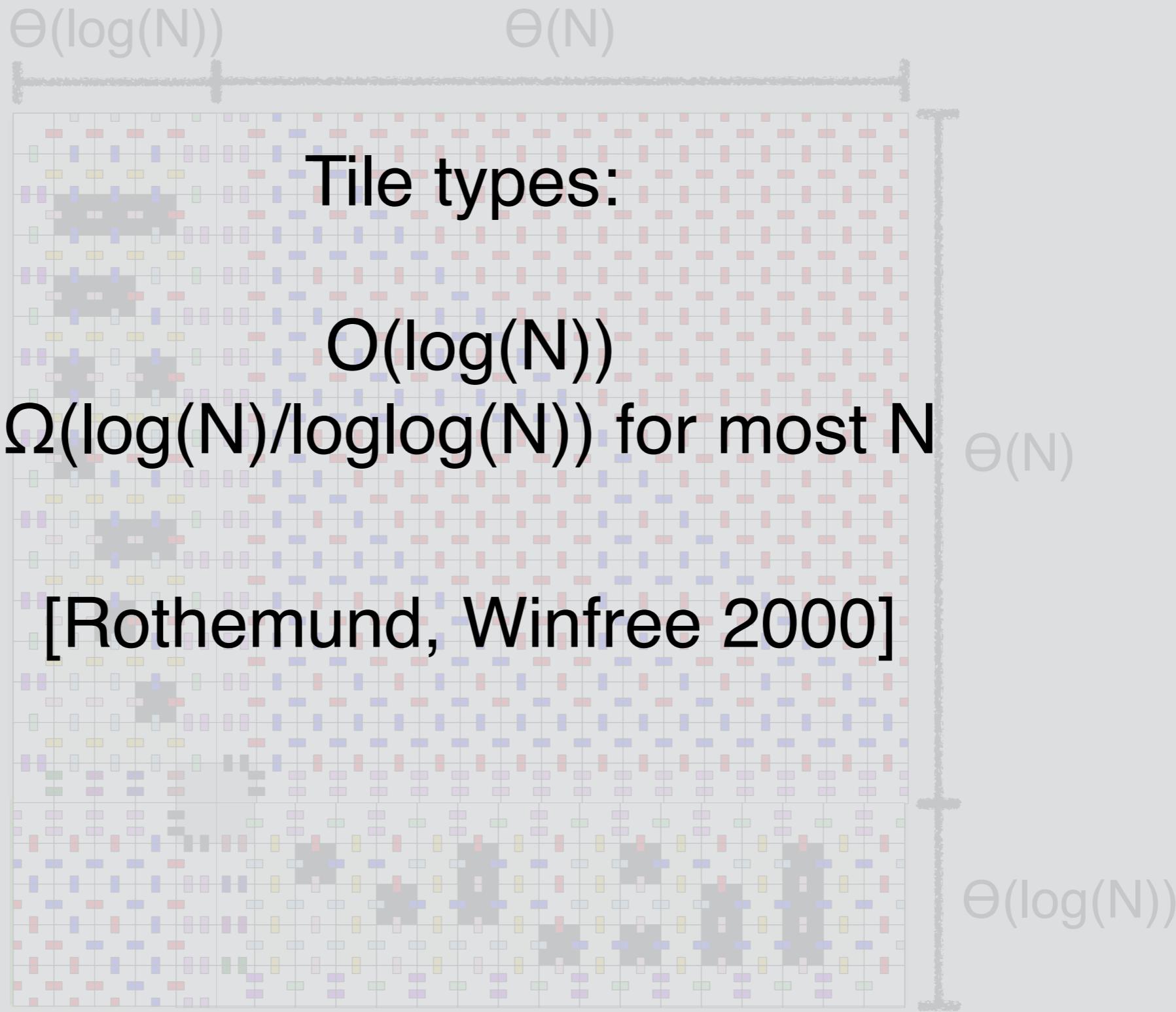
Temperature 2

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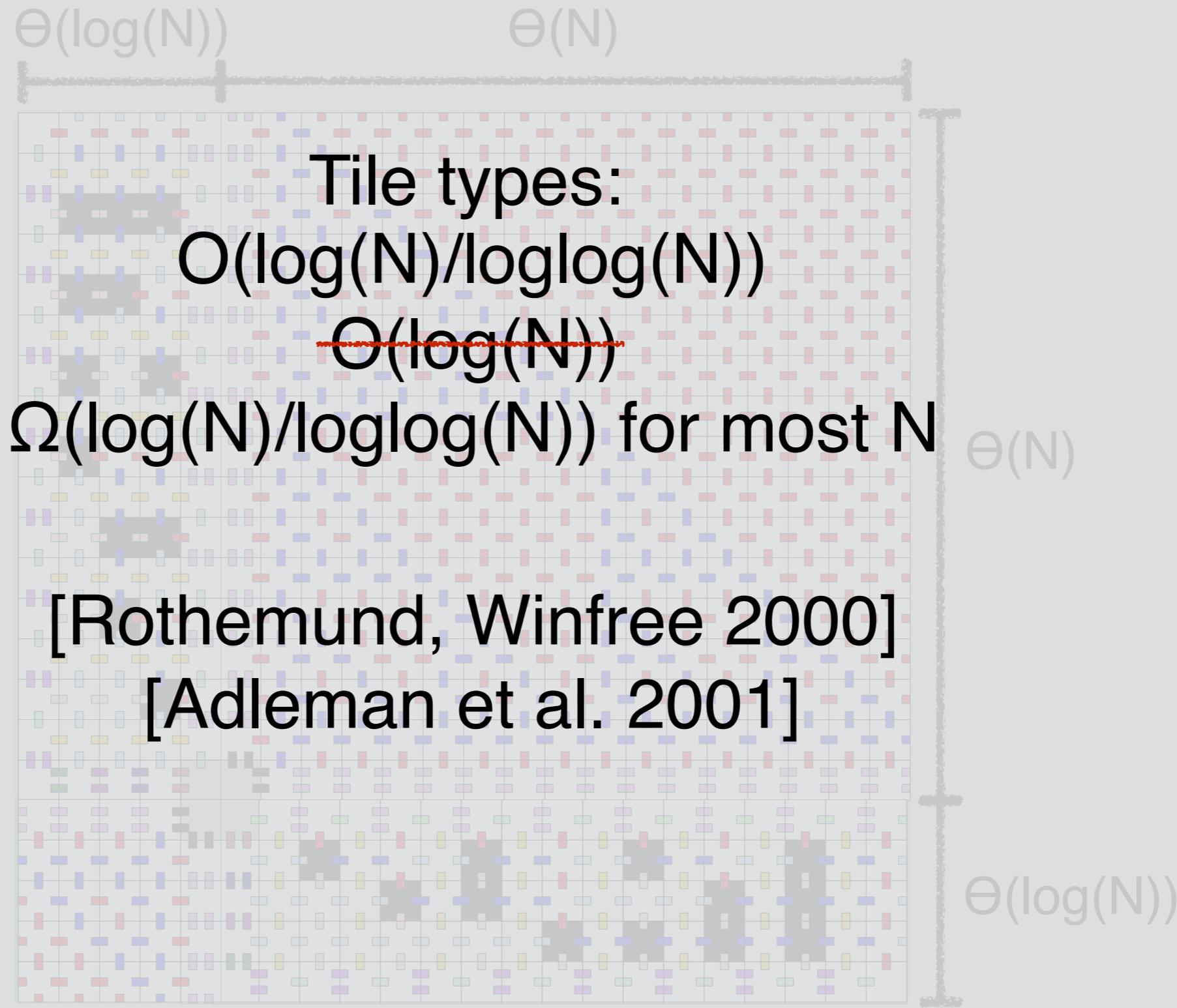
Temperature 2

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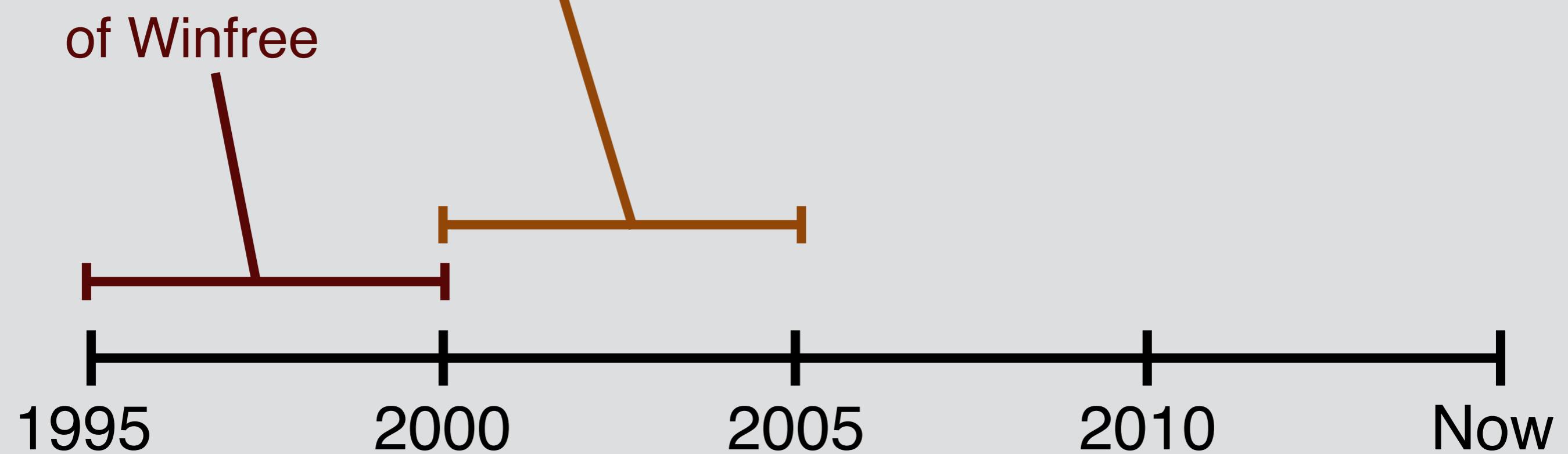
Temperature 2

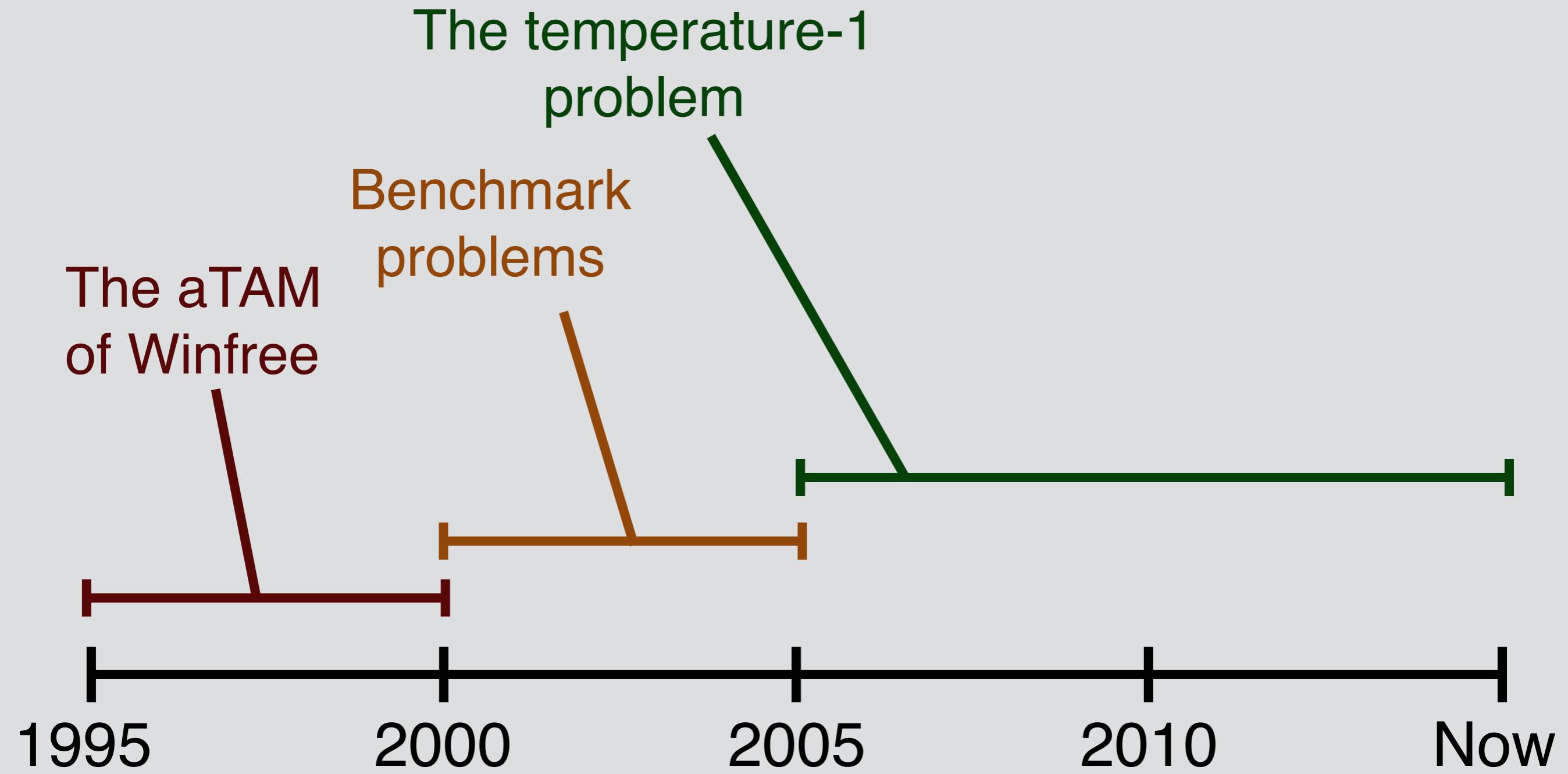
# Efficient assembly of NxN square



The aTAM  
of Winfree

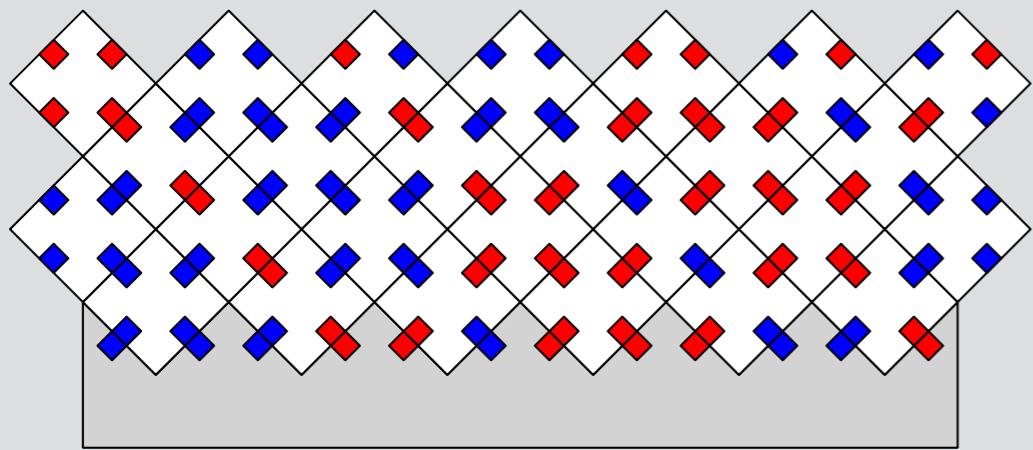
Benchmark  
problems





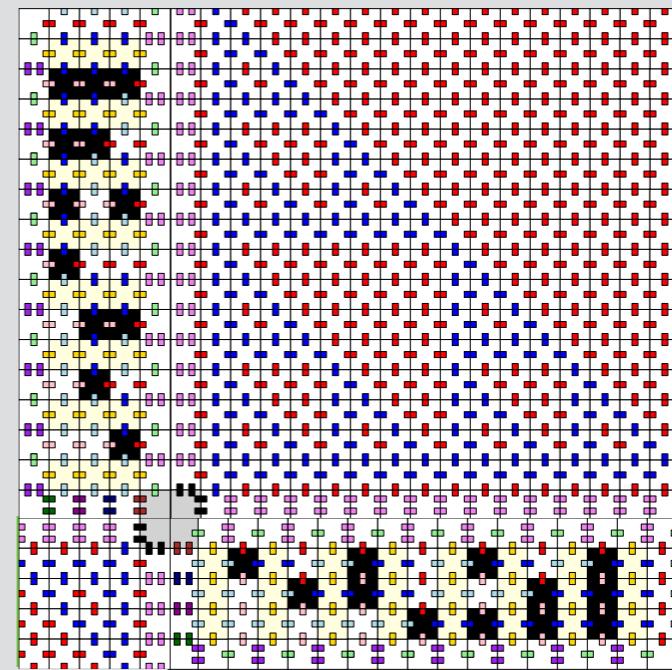
# The benchmarks of a capable self-assembly model

Universal computation



Temperature 2

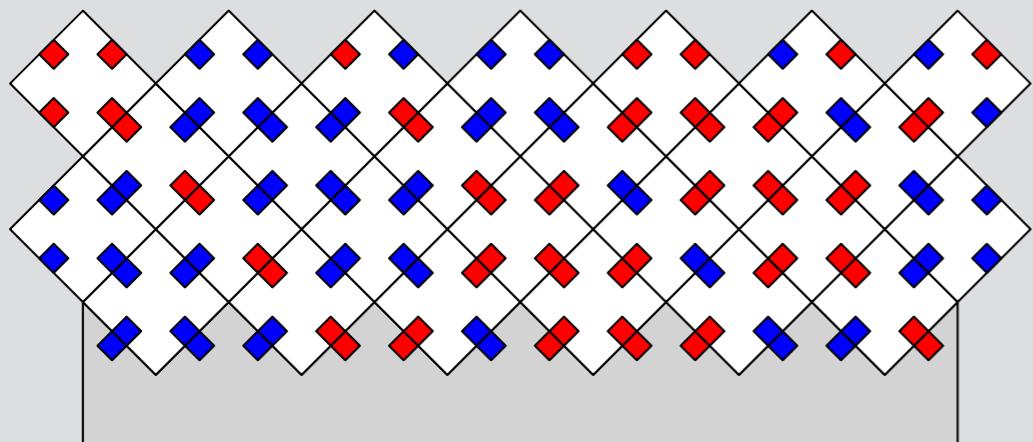
NxN square assembly  
w/O( $\log(N)$ ) tile types



Temperature 2

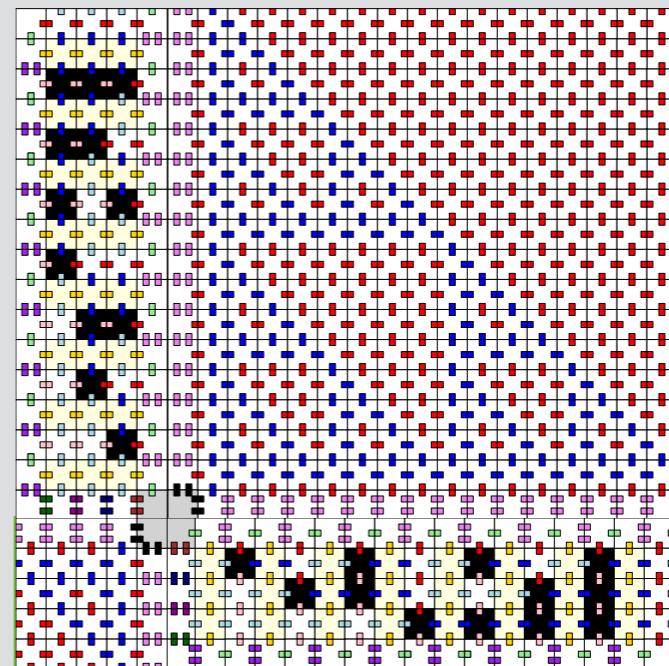
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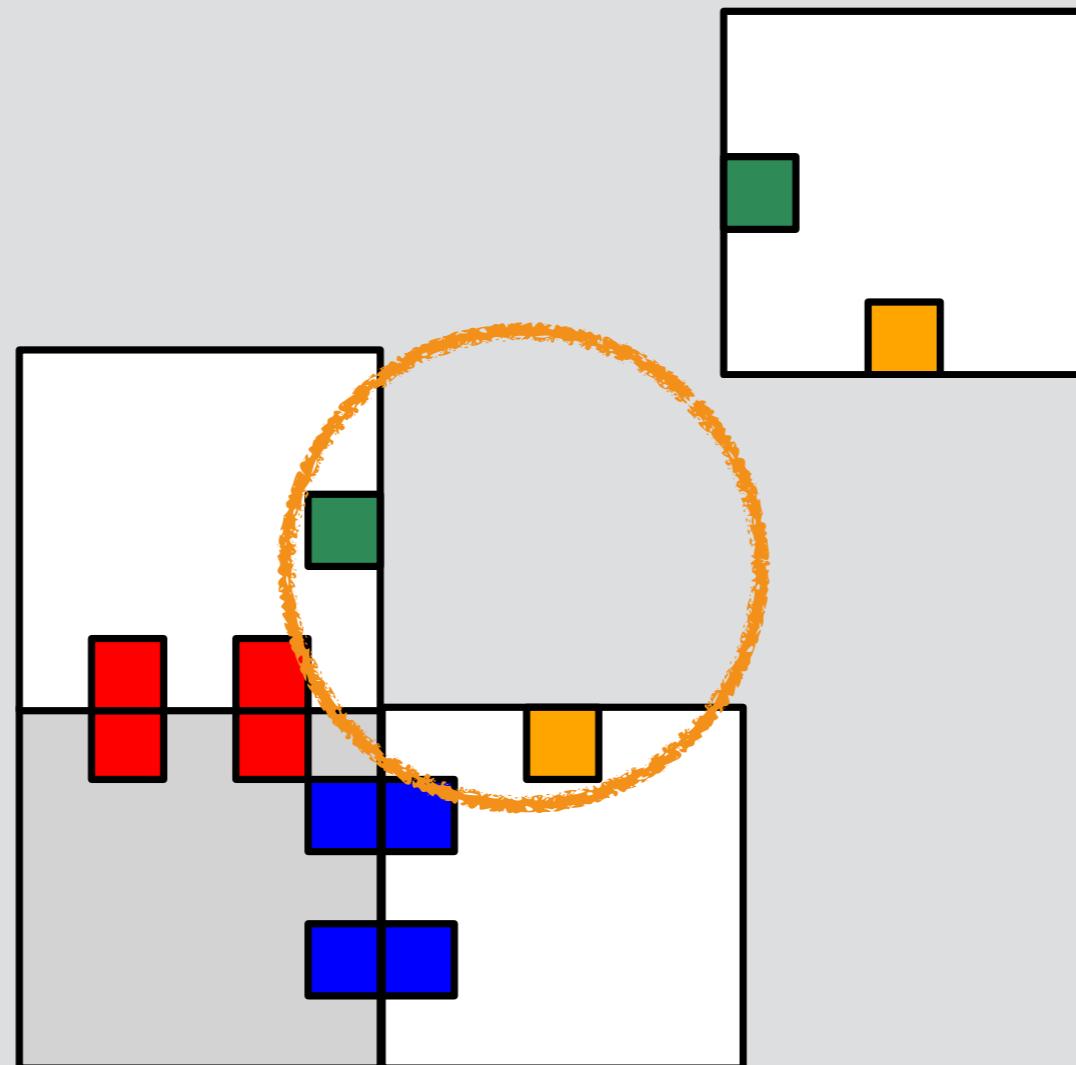
Temperature 2

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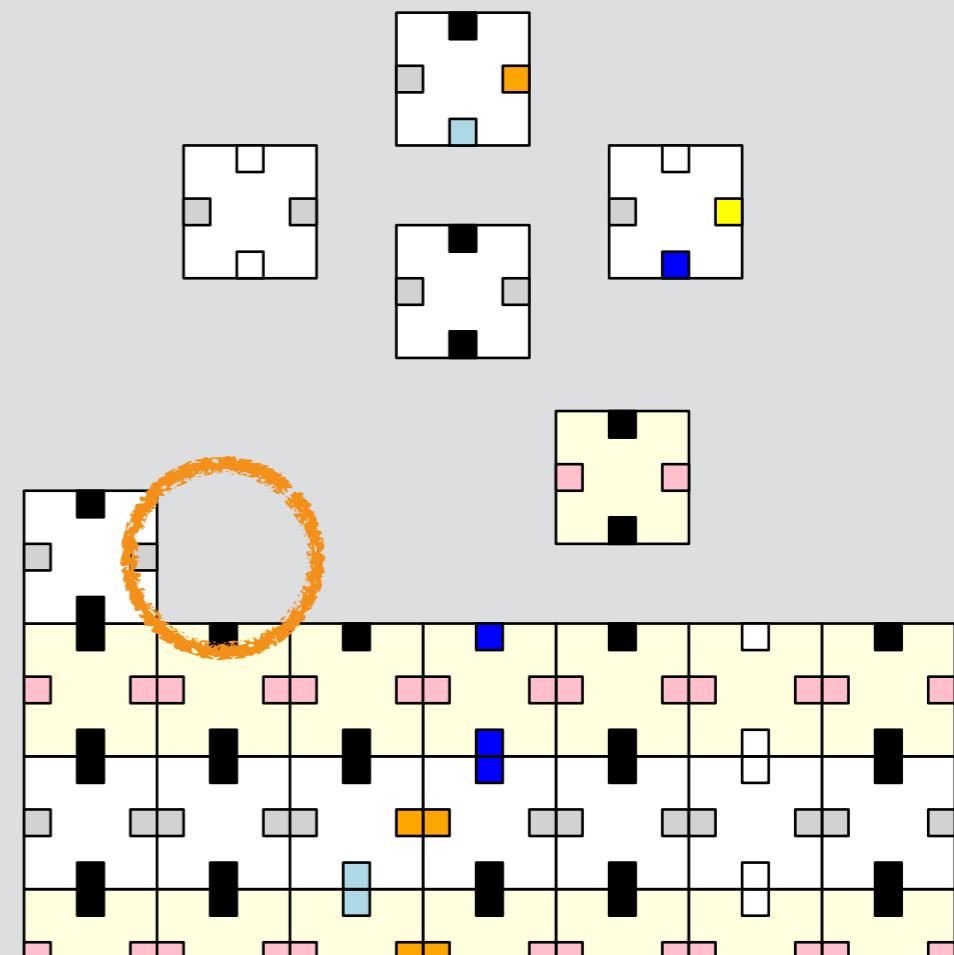
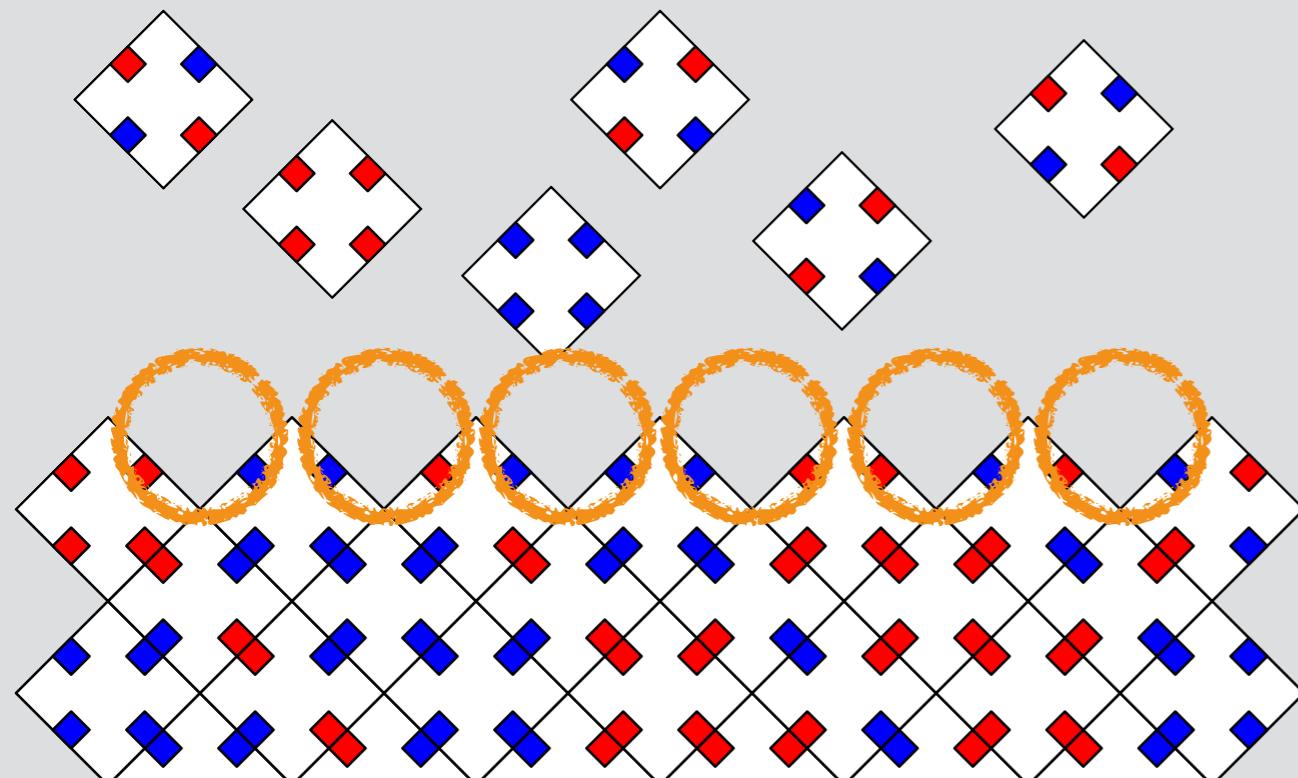


Temperature 2

Temperature 2 can do  
*cooperative bonding.*

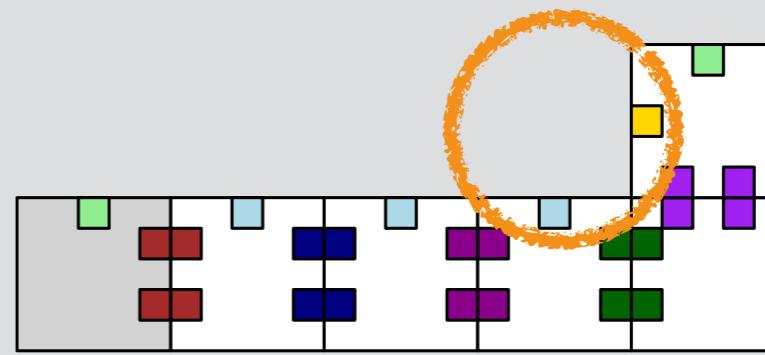
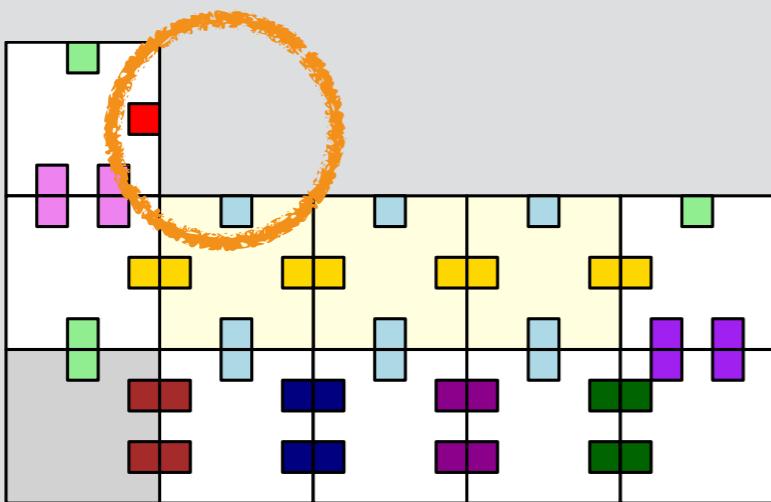


# Computation uses cooperative bonding:

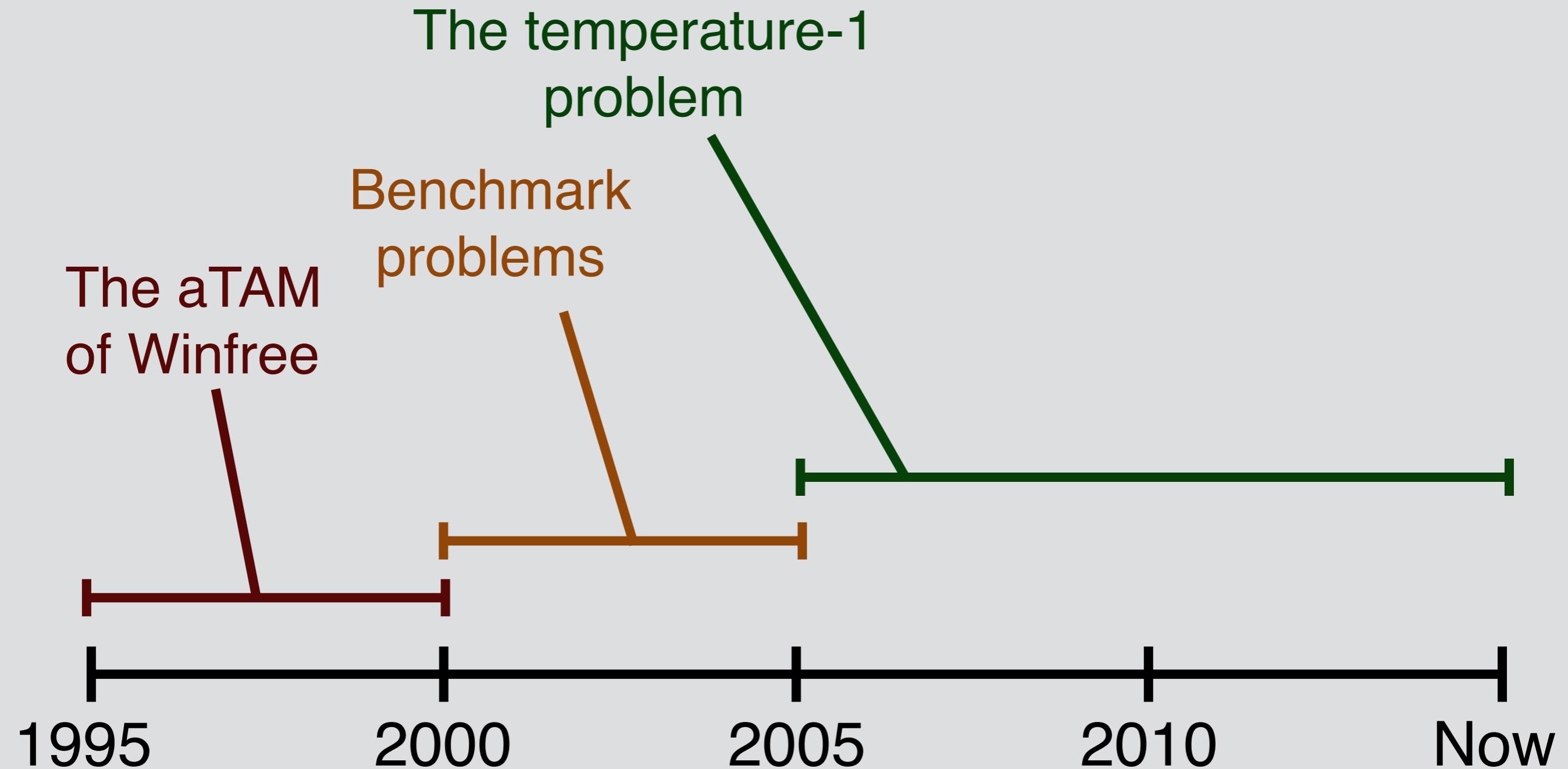


Is this required?

# Efficient square assembly uses cooperative bonding:



Is this required?

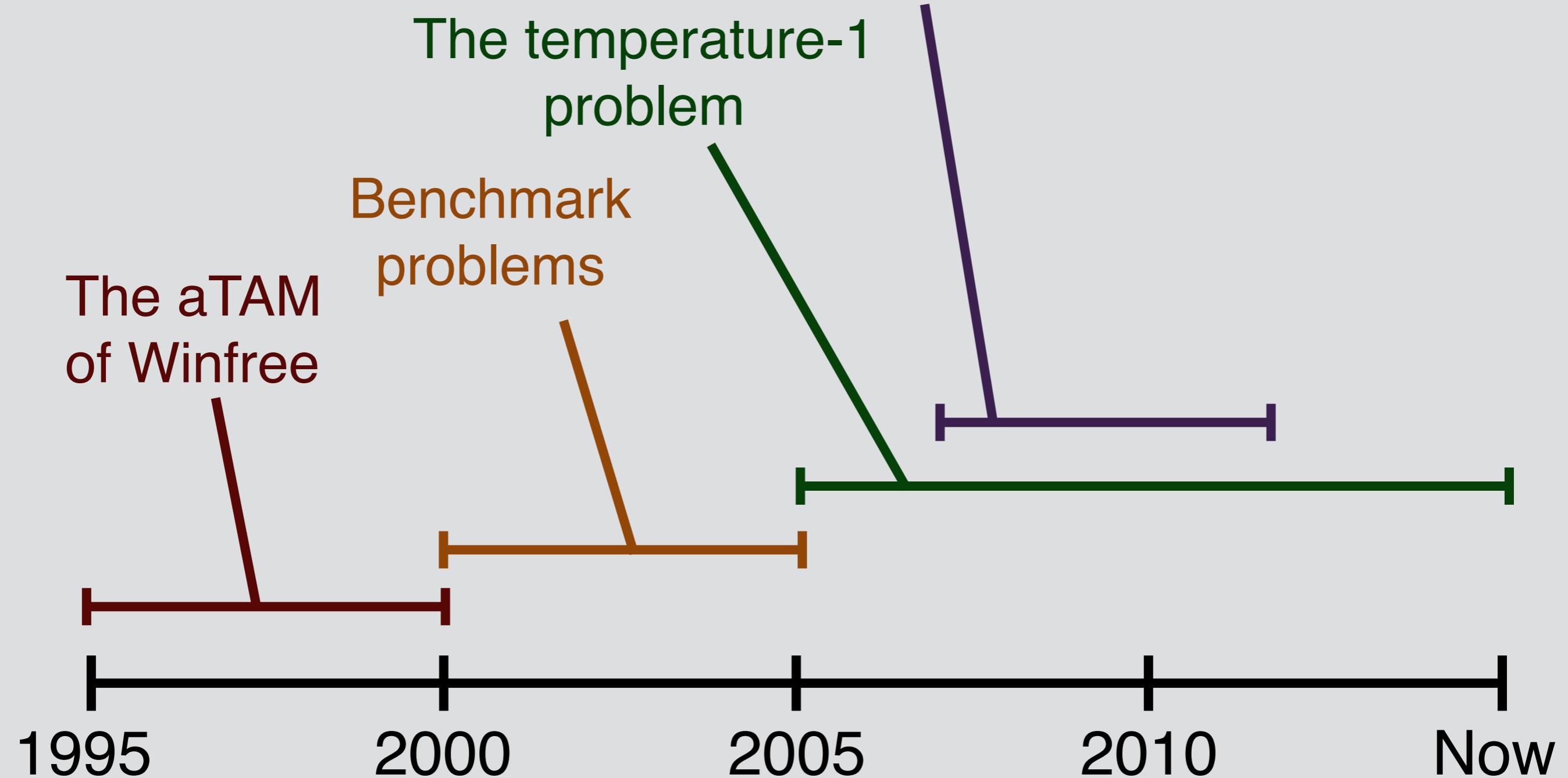


Universality via  
weak cooperation

The temperature-1  
problem

Benchmark  
problems

The aTAM  
of Winfree



# The temperature-1 problem:

Is the aTAM at temperature 1  
computationally universal?

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Open since 2000, conjecture: no.

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Can an augmented temperature-1 aTAM  
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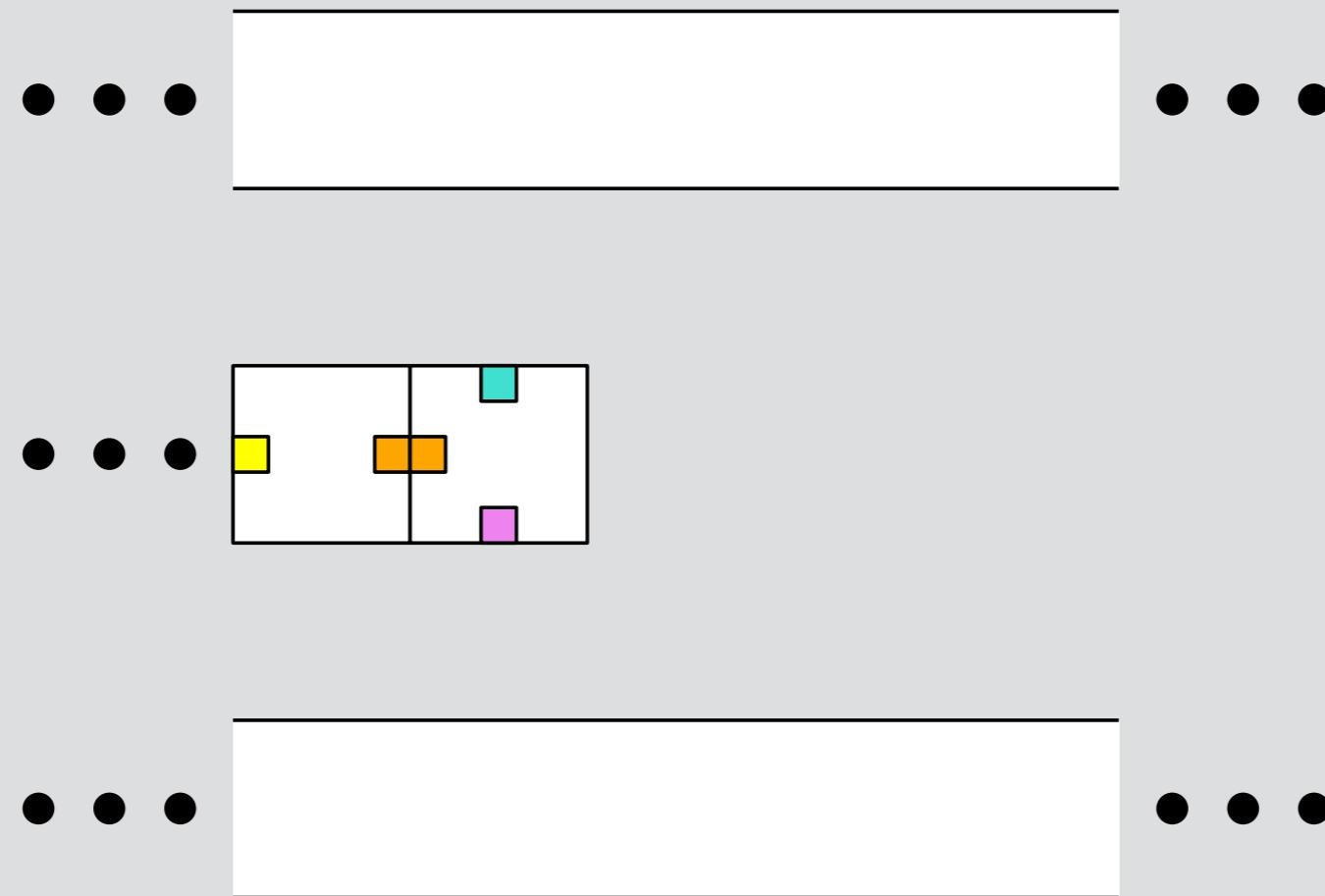
Open since 2000, conjecture: no.

“weak cooperation”

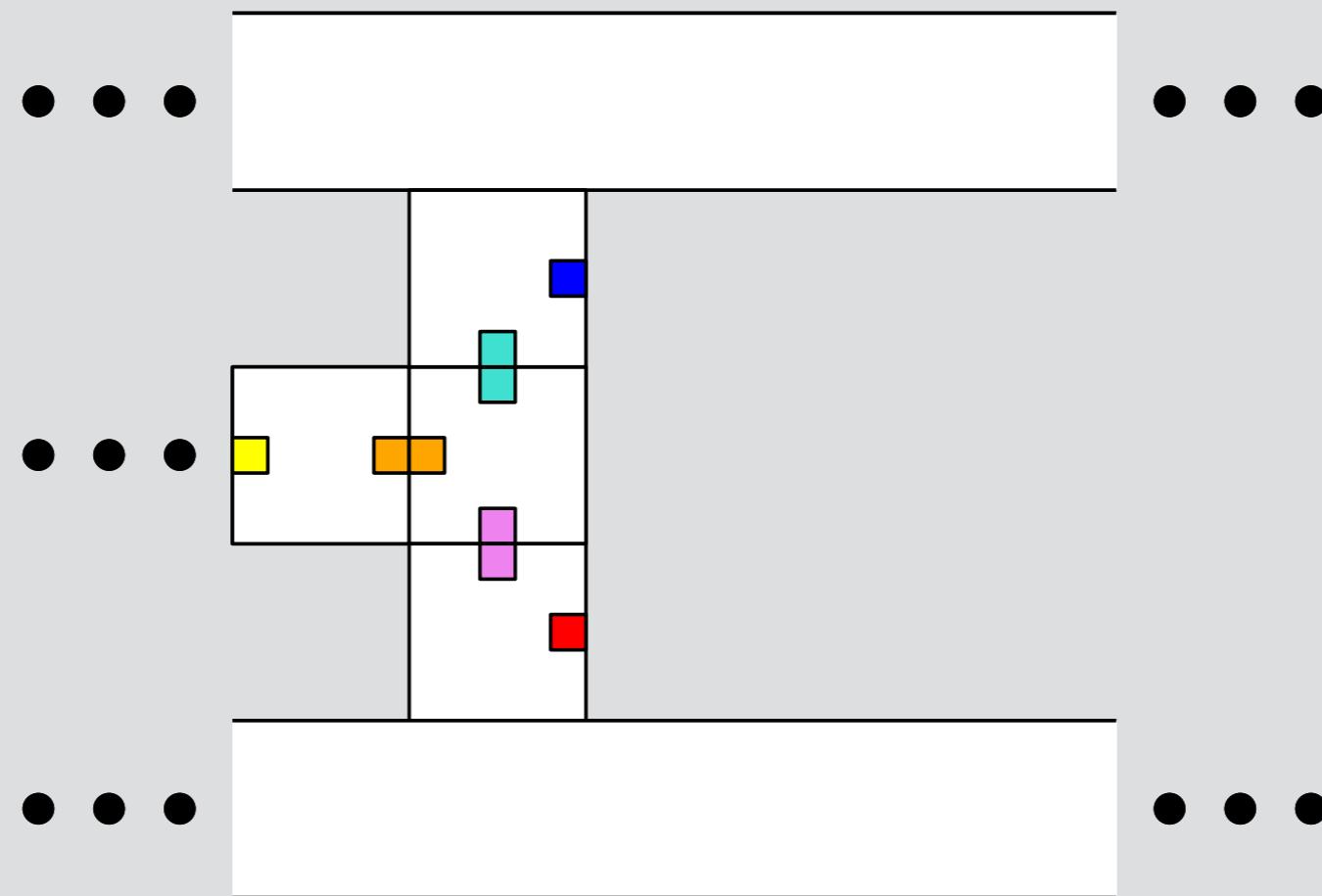
Can an augmented temperature-1 aTAM  
be computationally universal?

Yes, several ways.

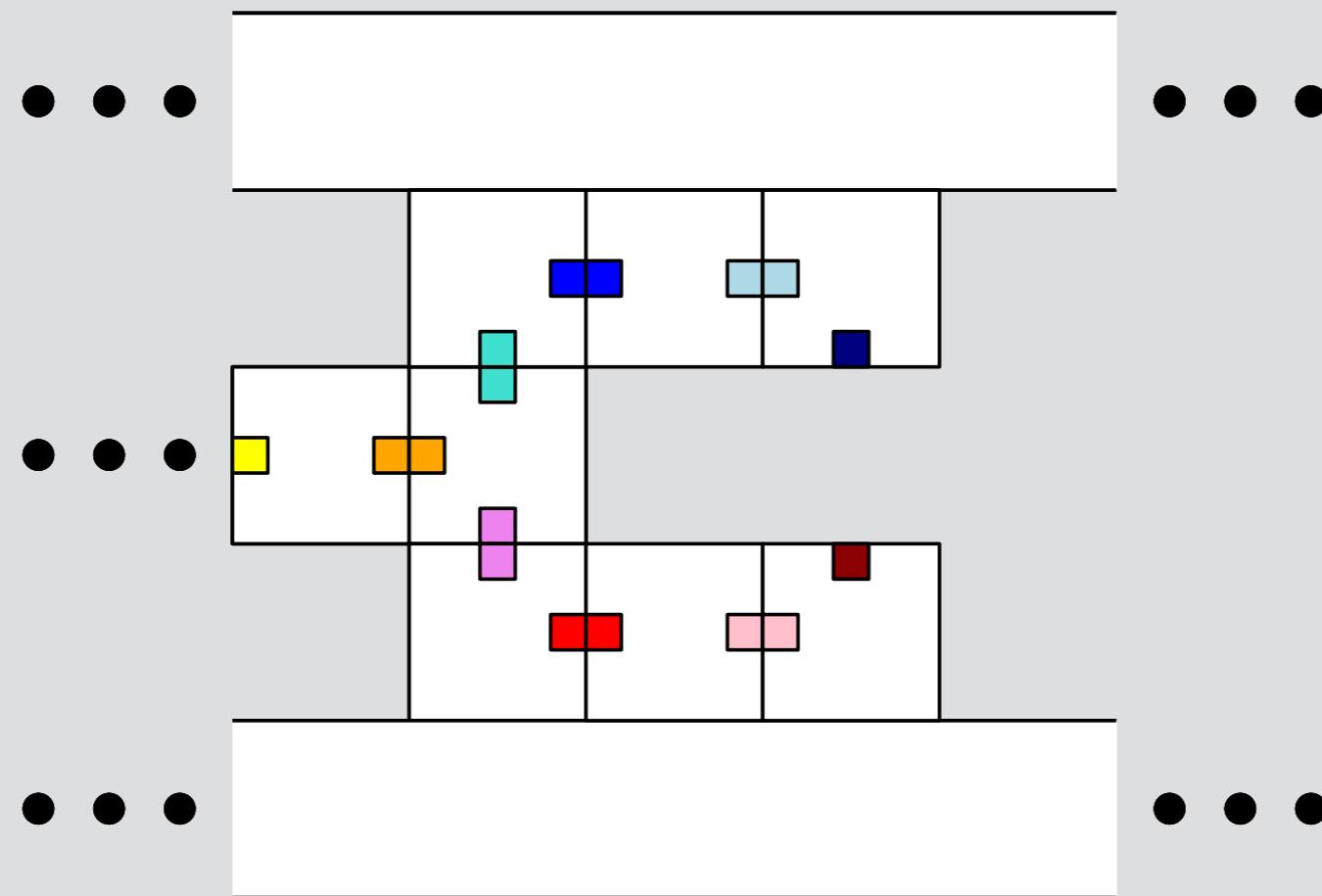
# Reading a bit w/o cooperation



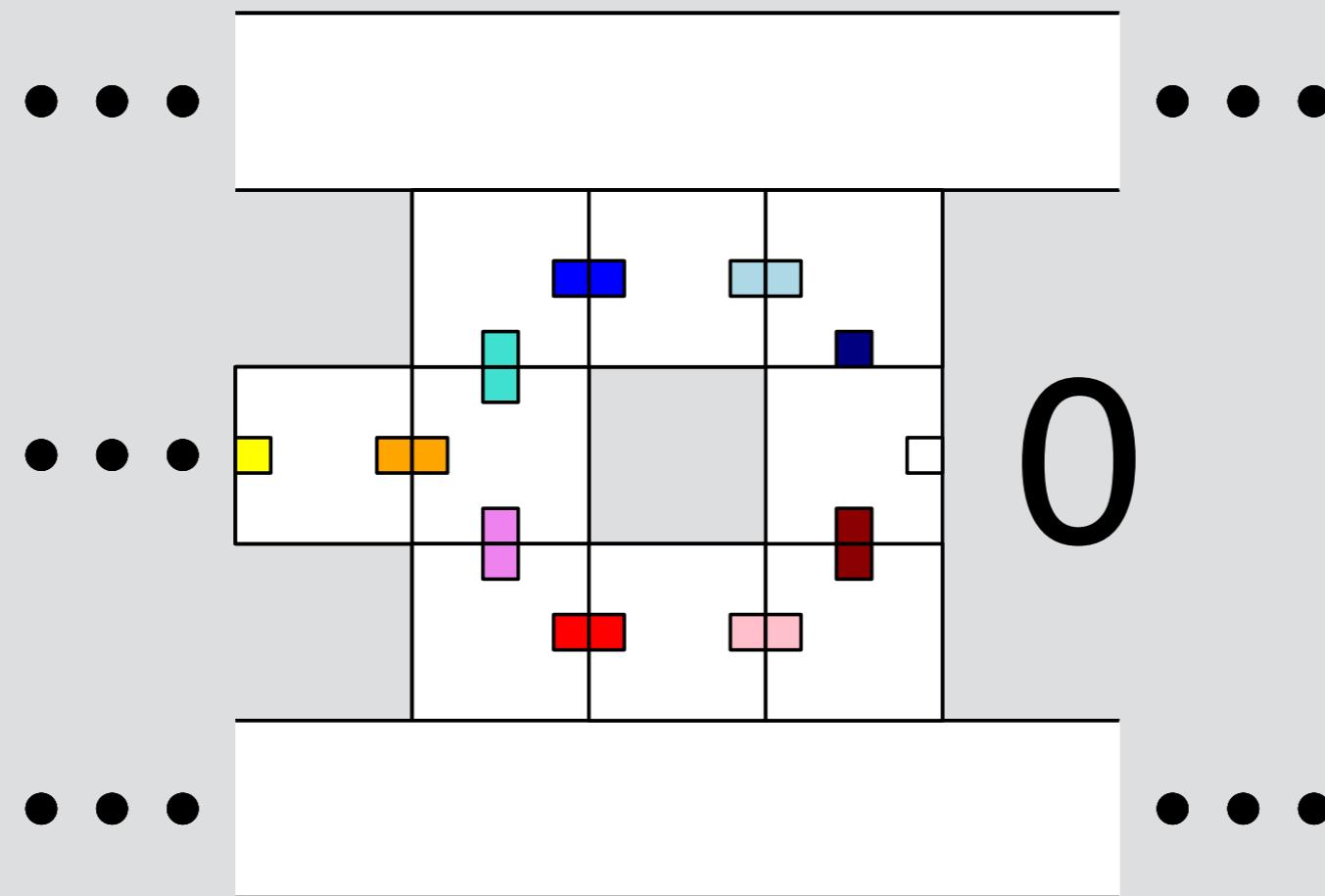
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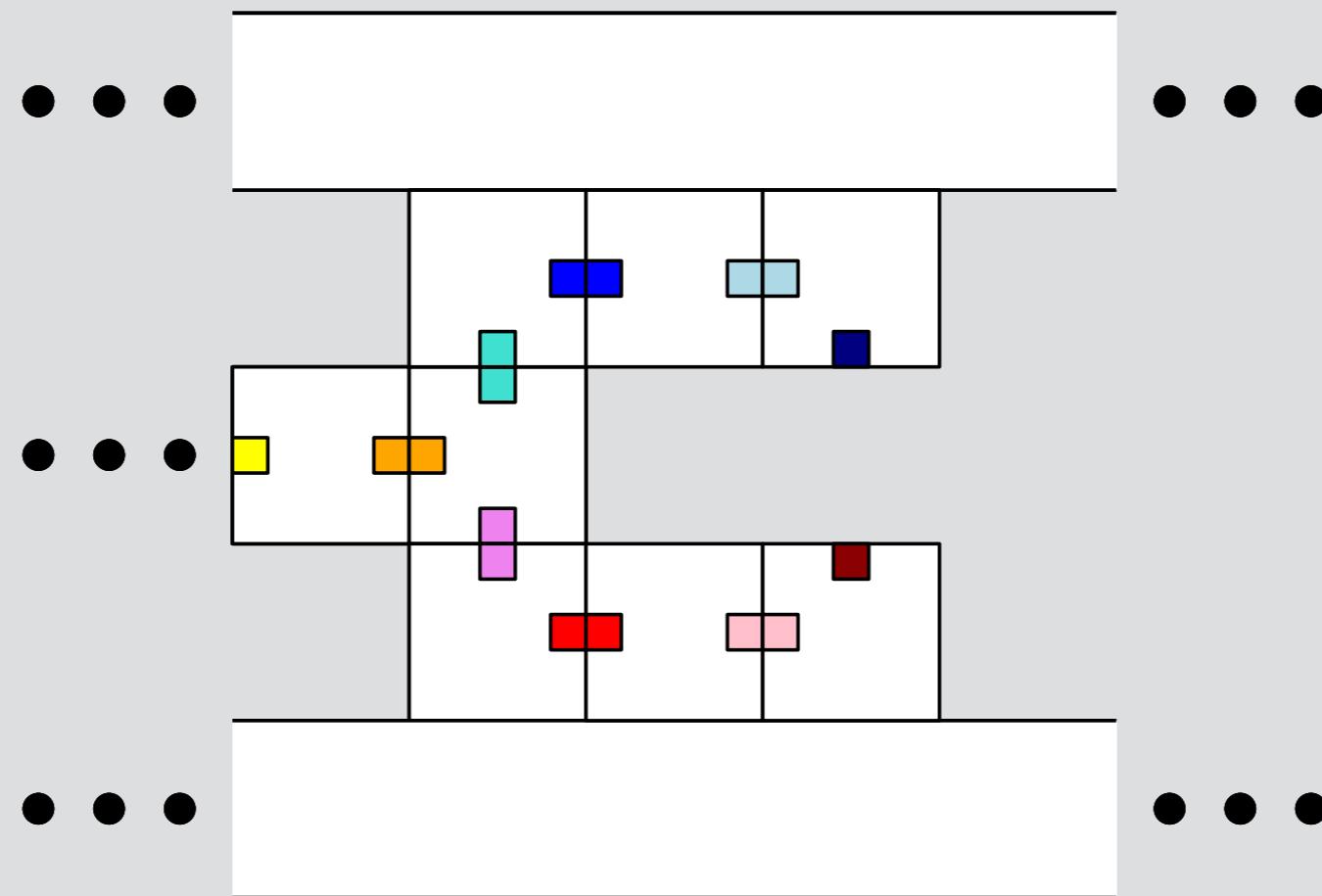
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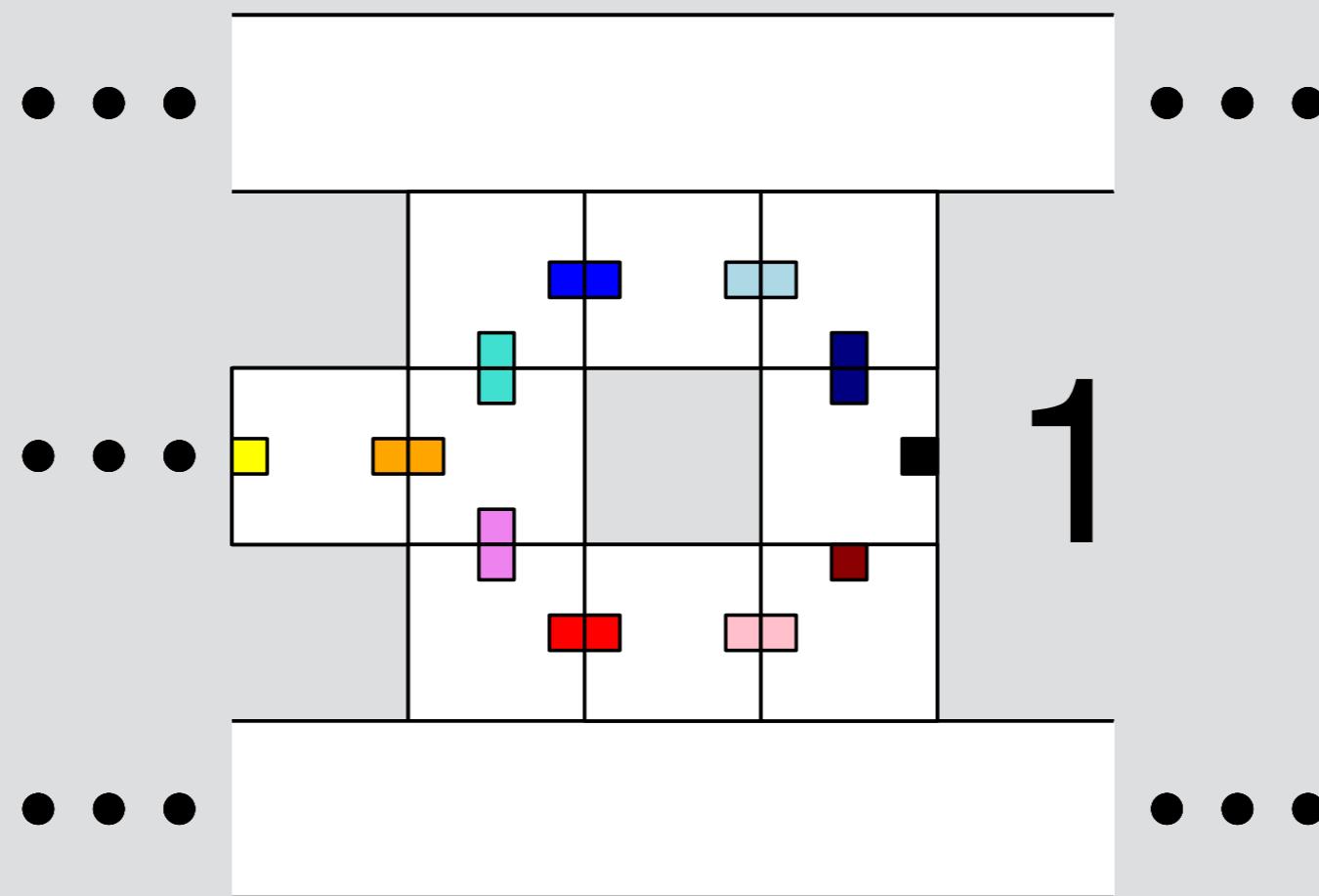
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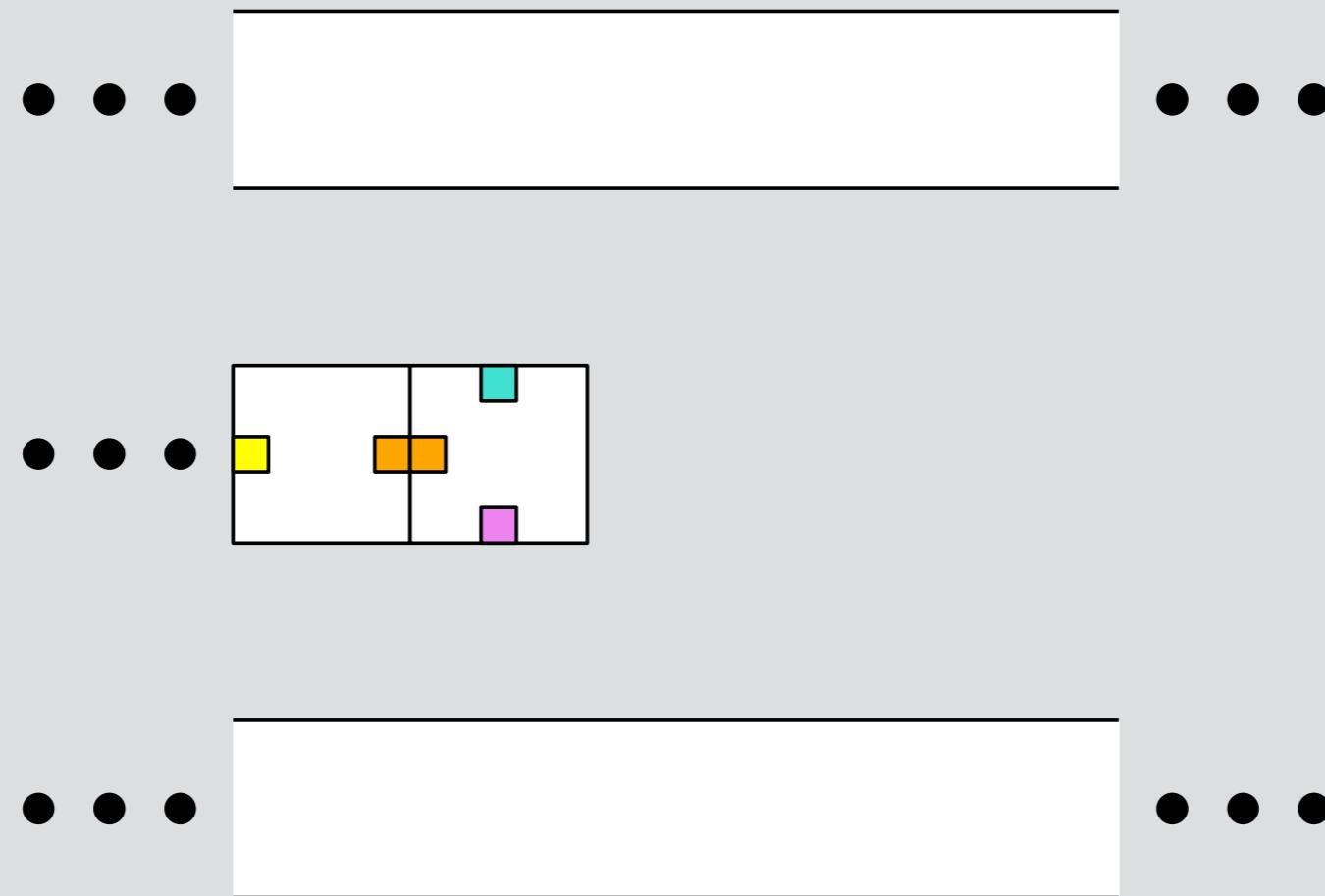
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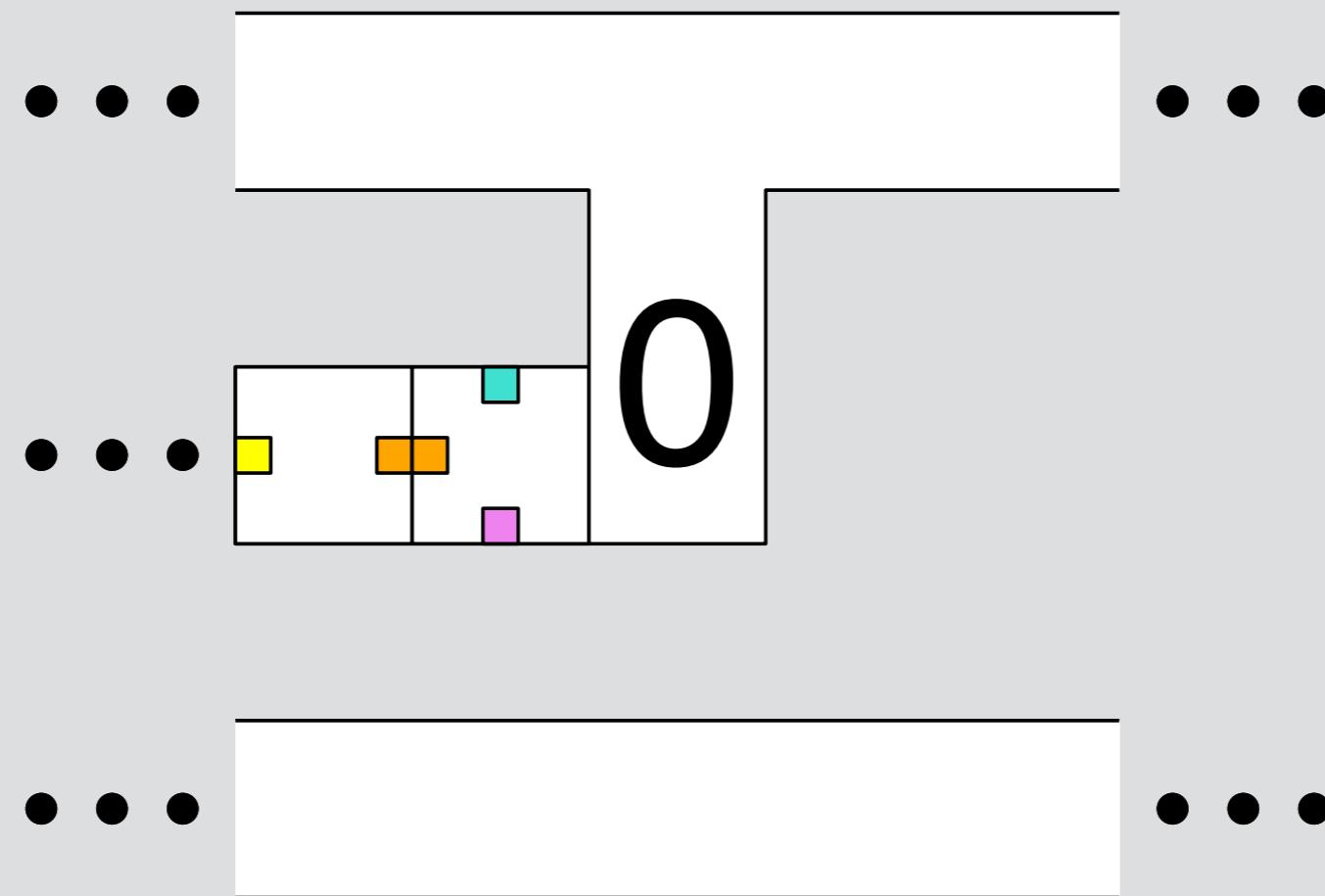
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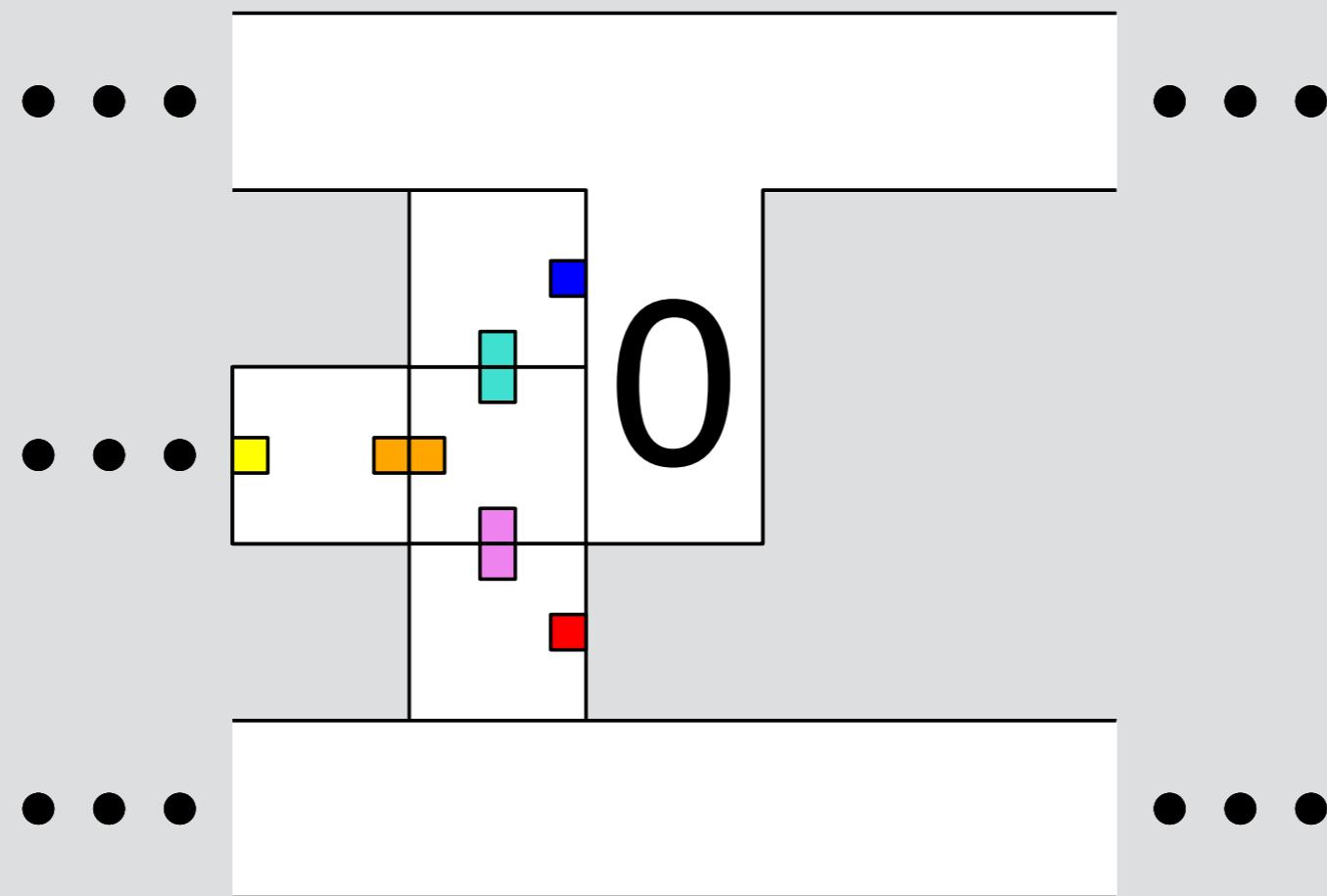
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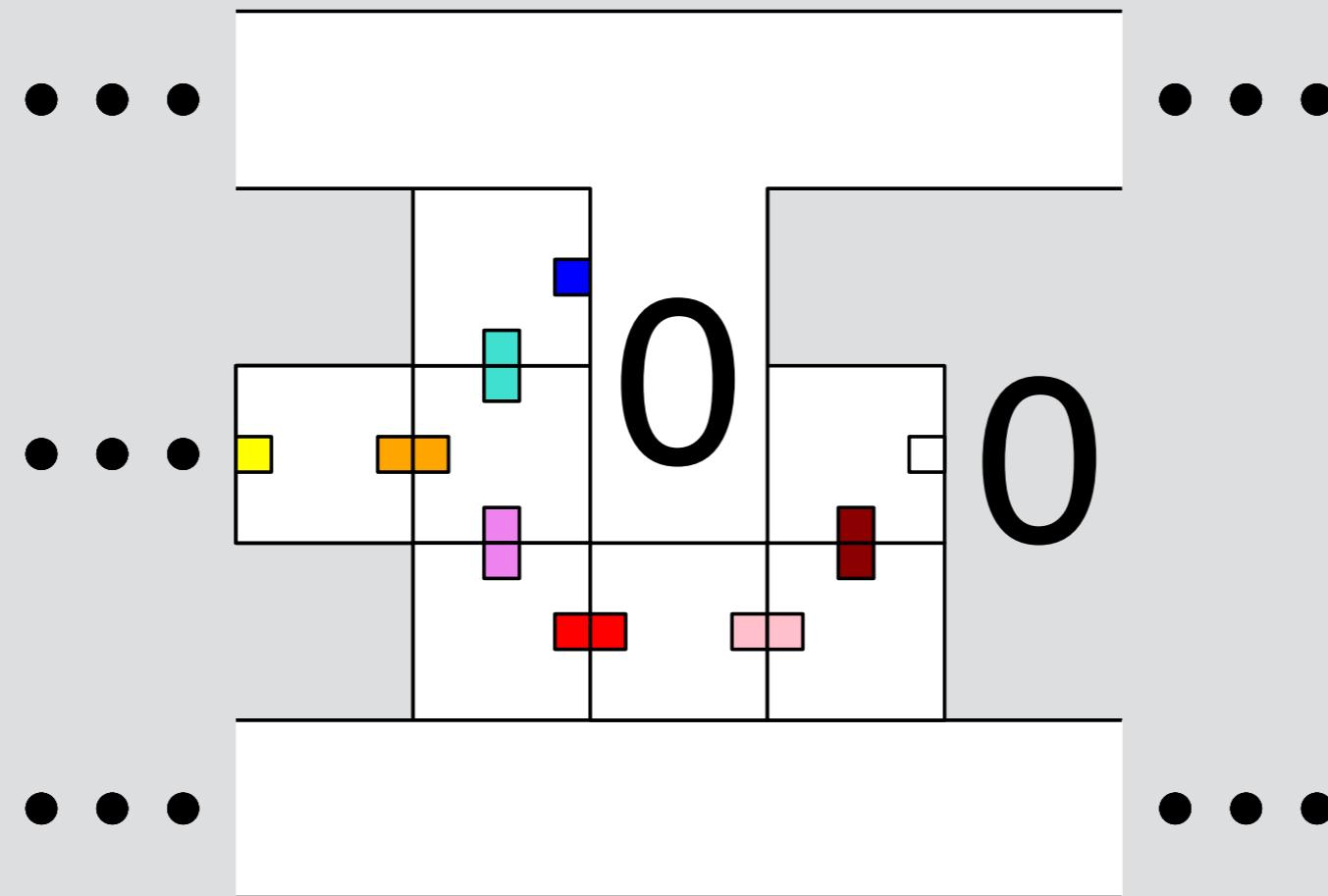
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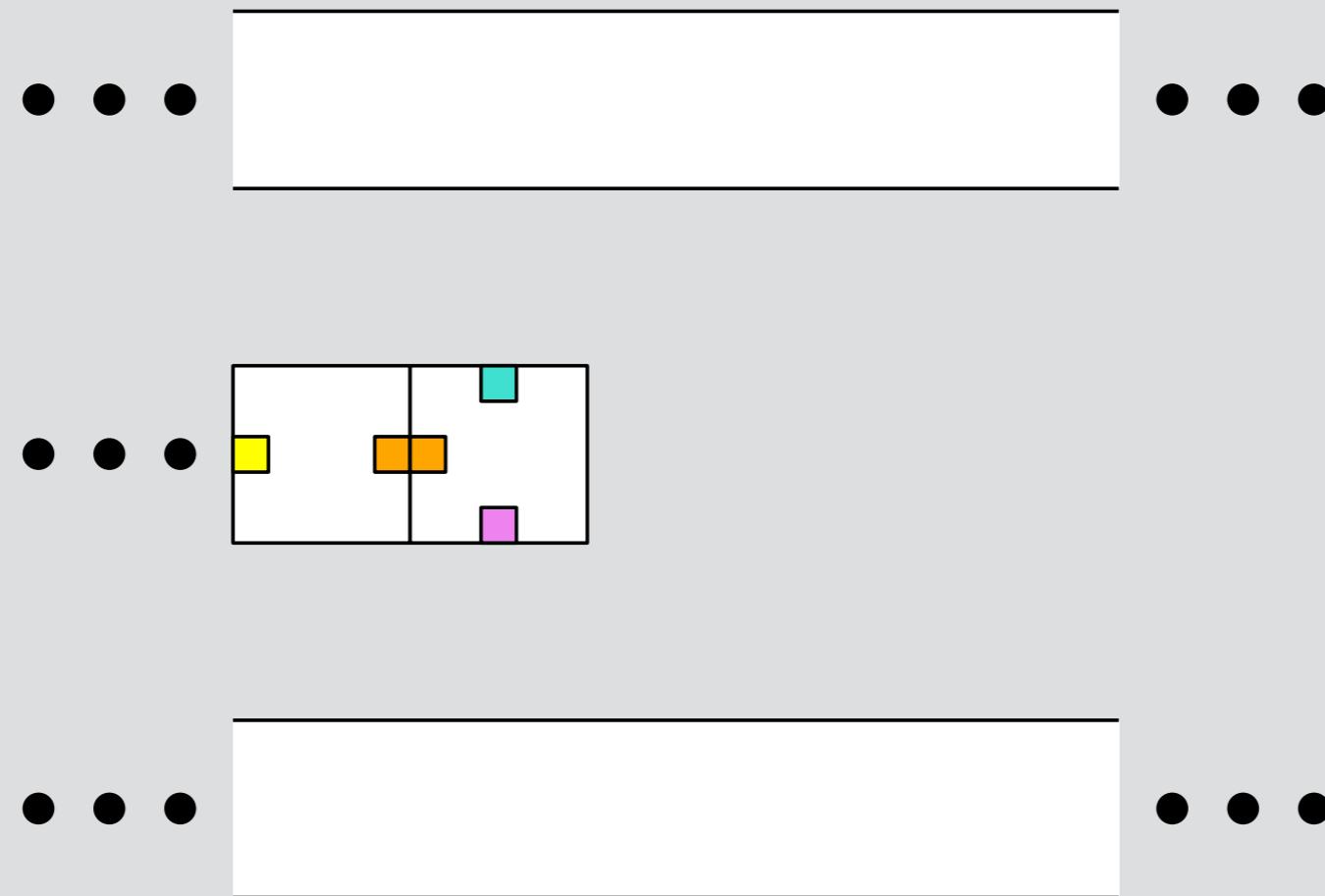
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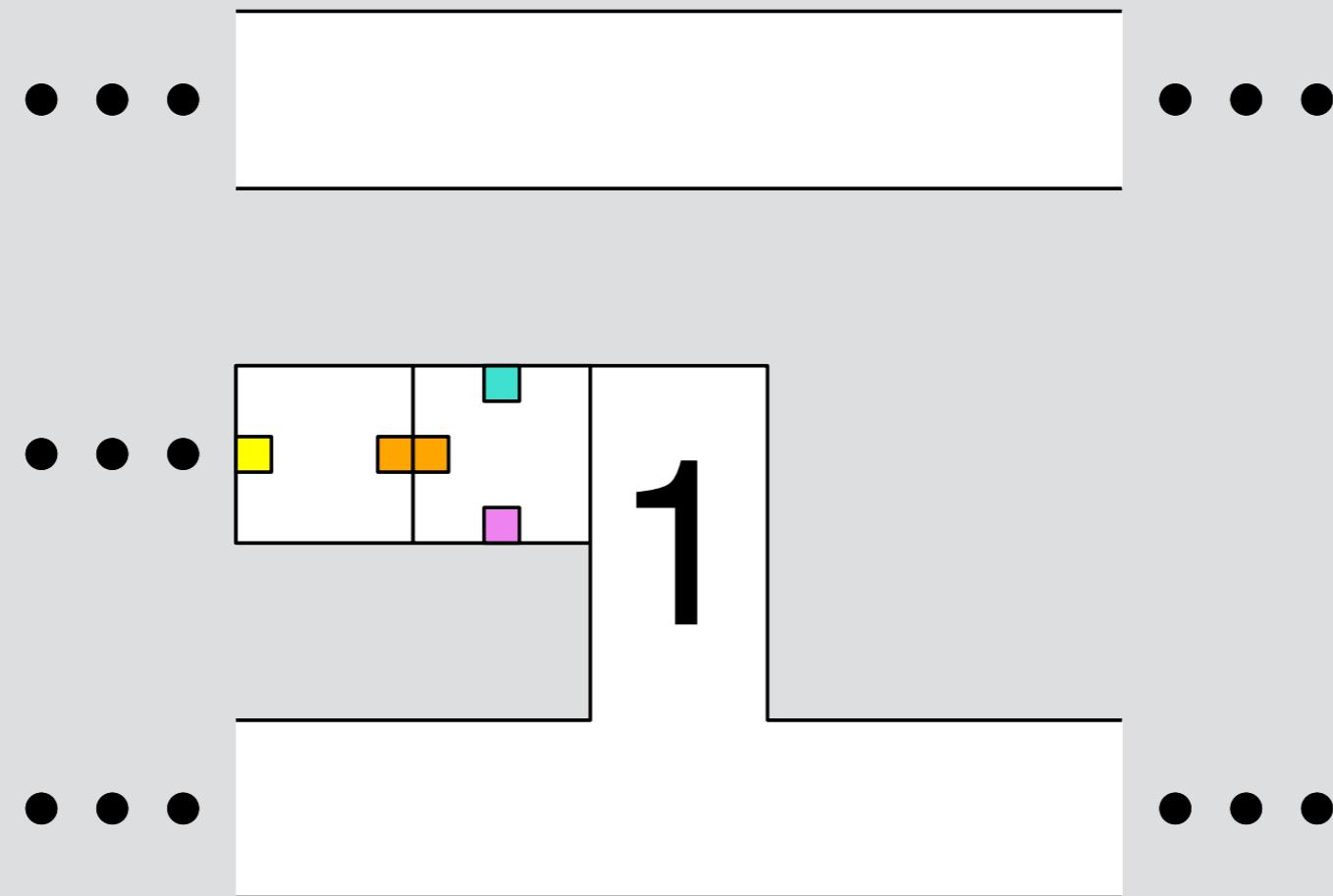
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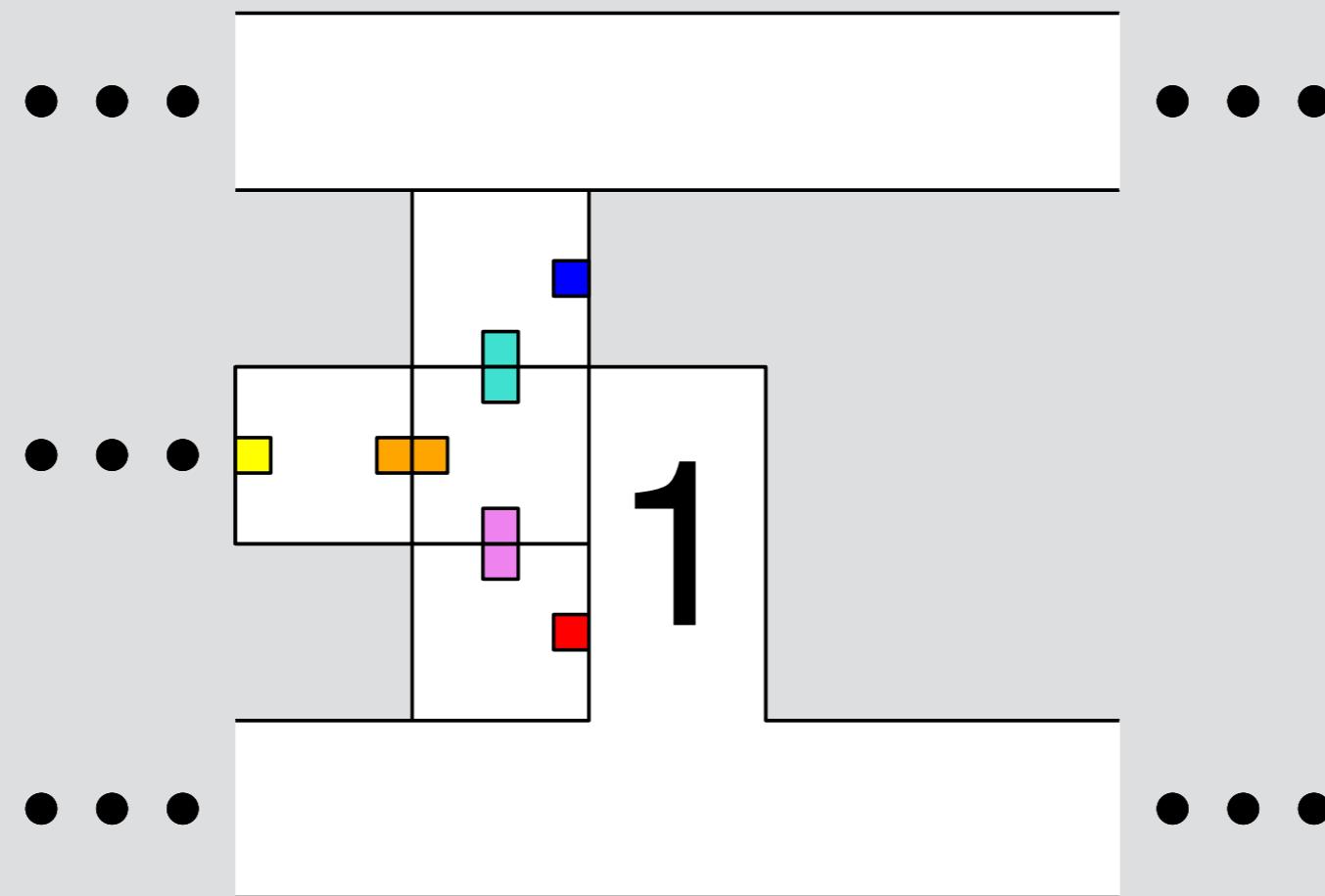
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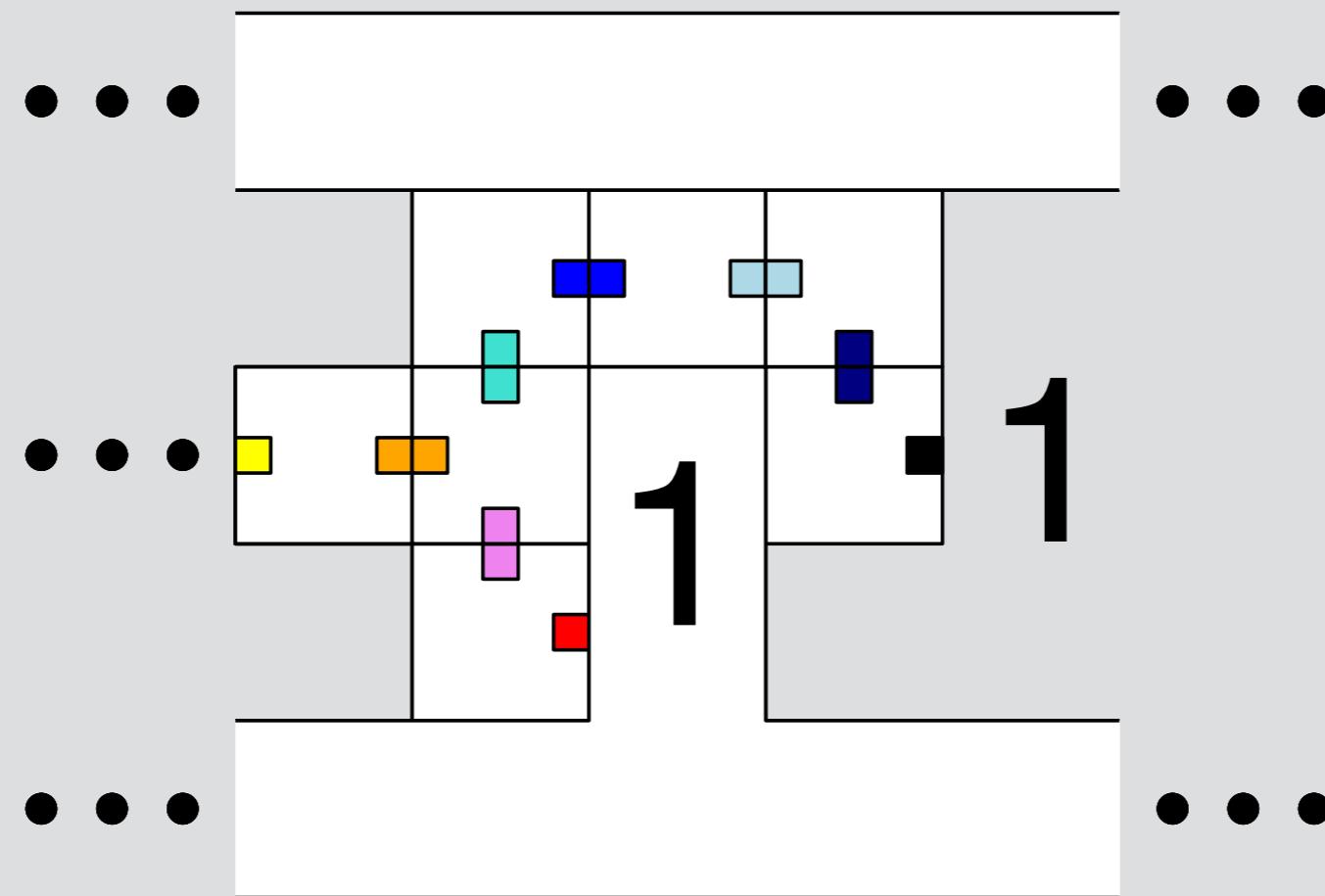
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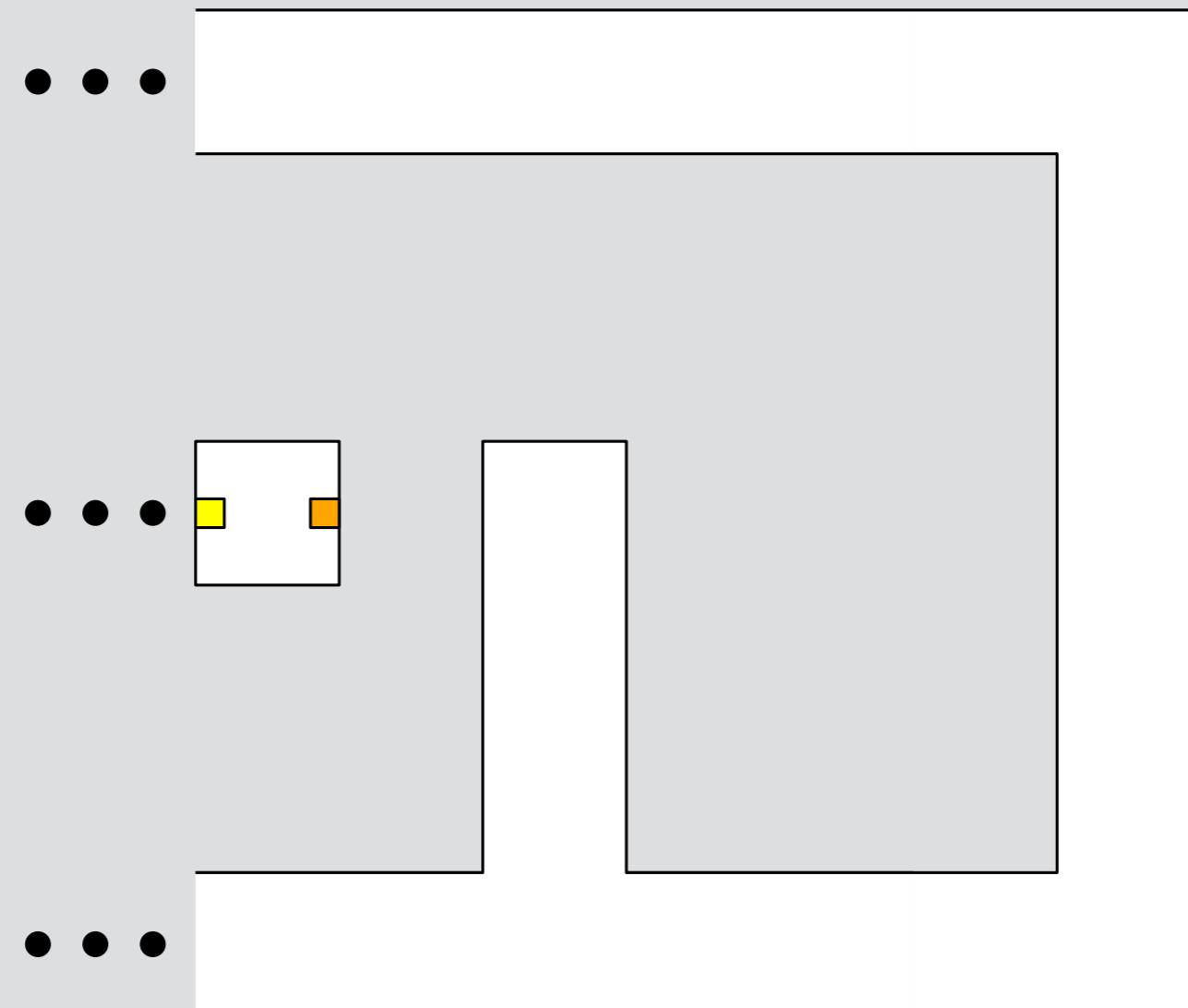
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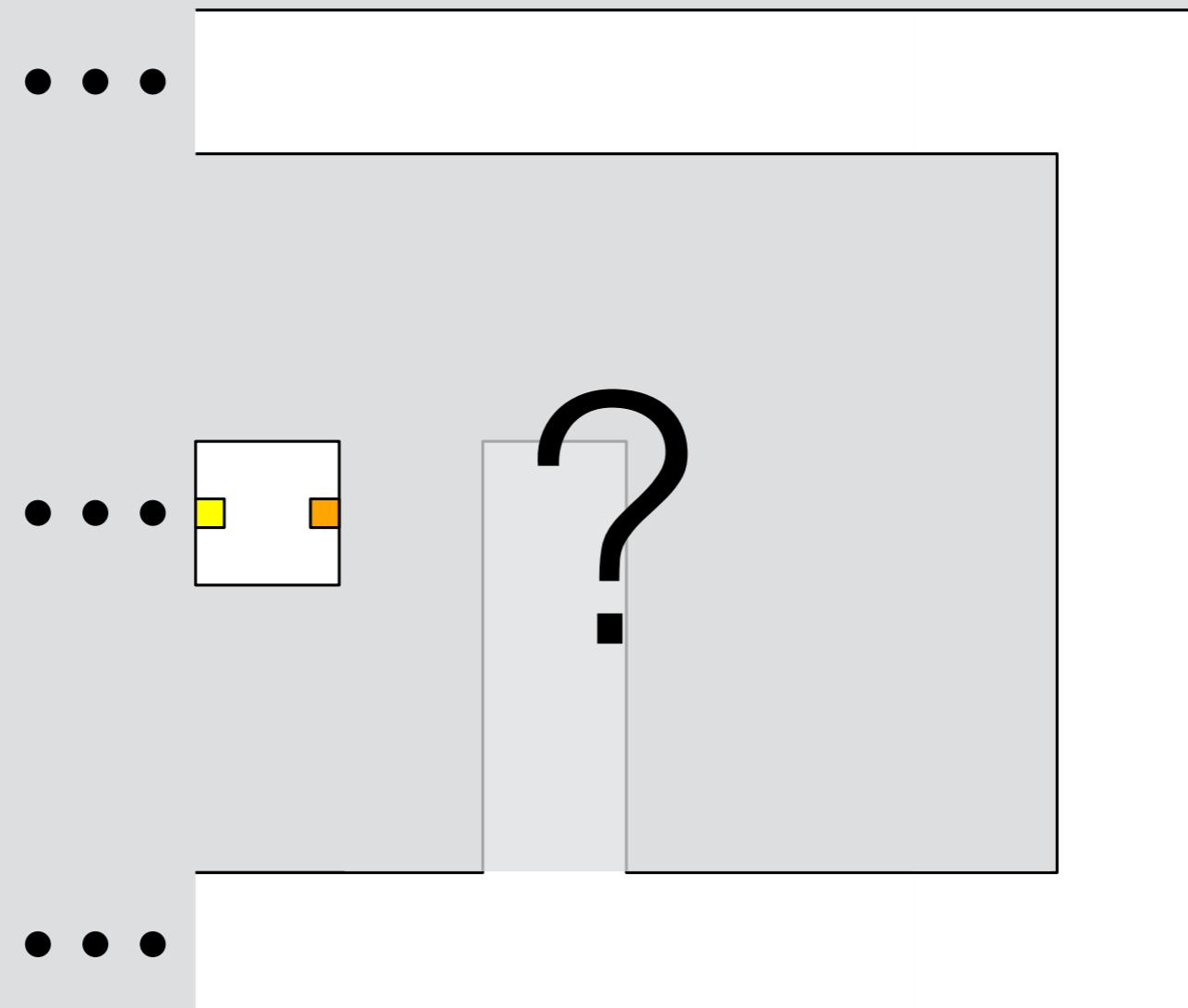
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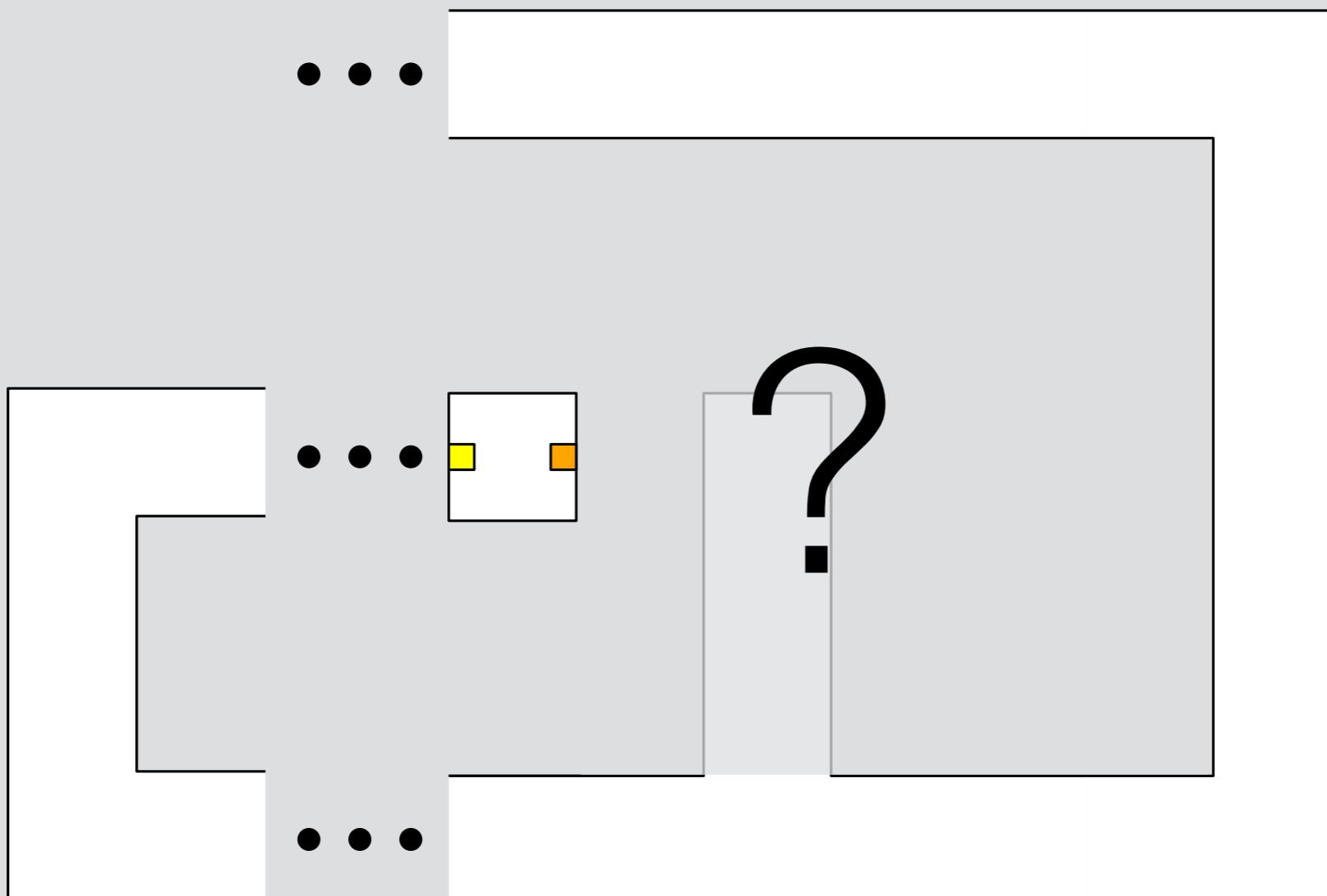
# A planarity problem



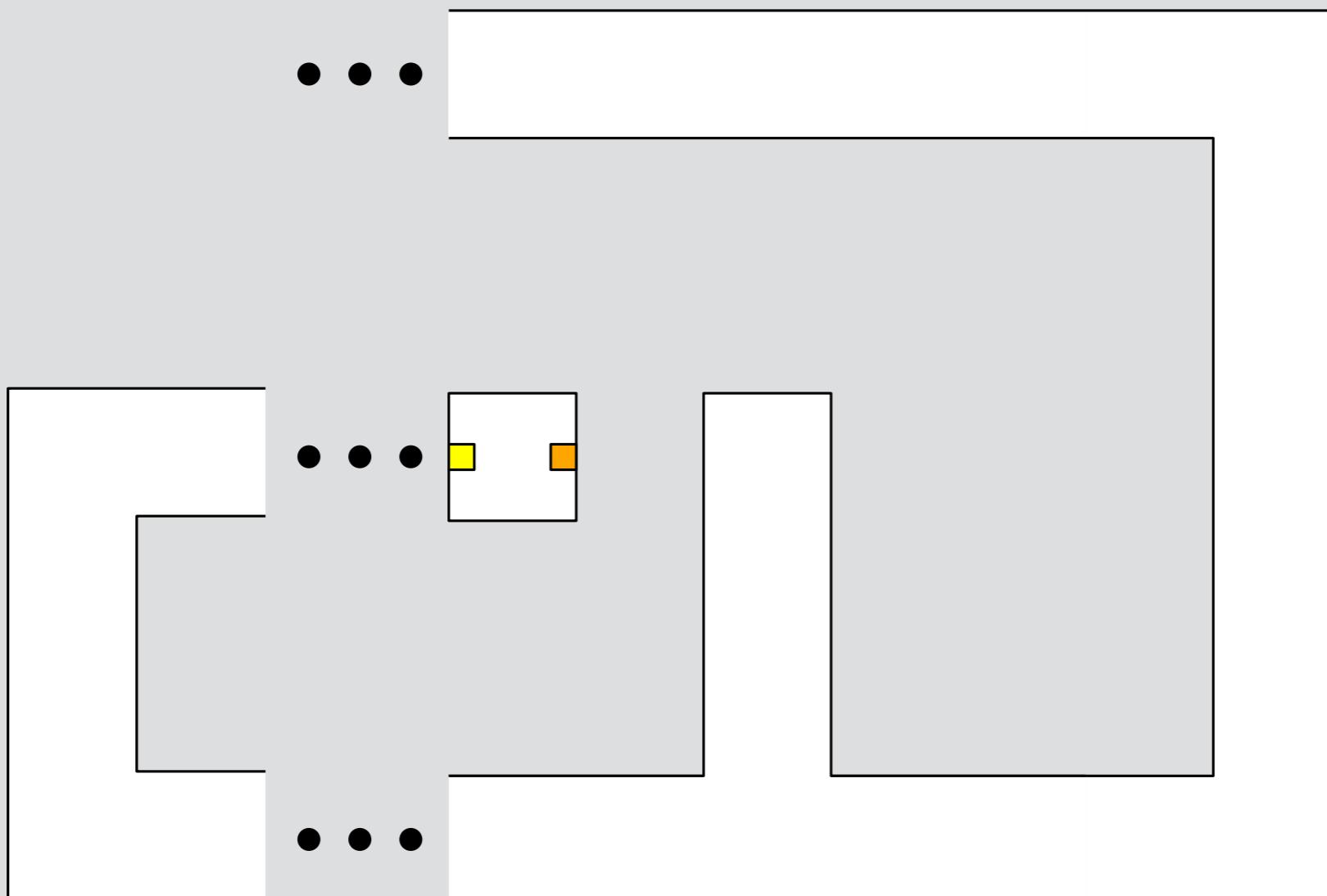
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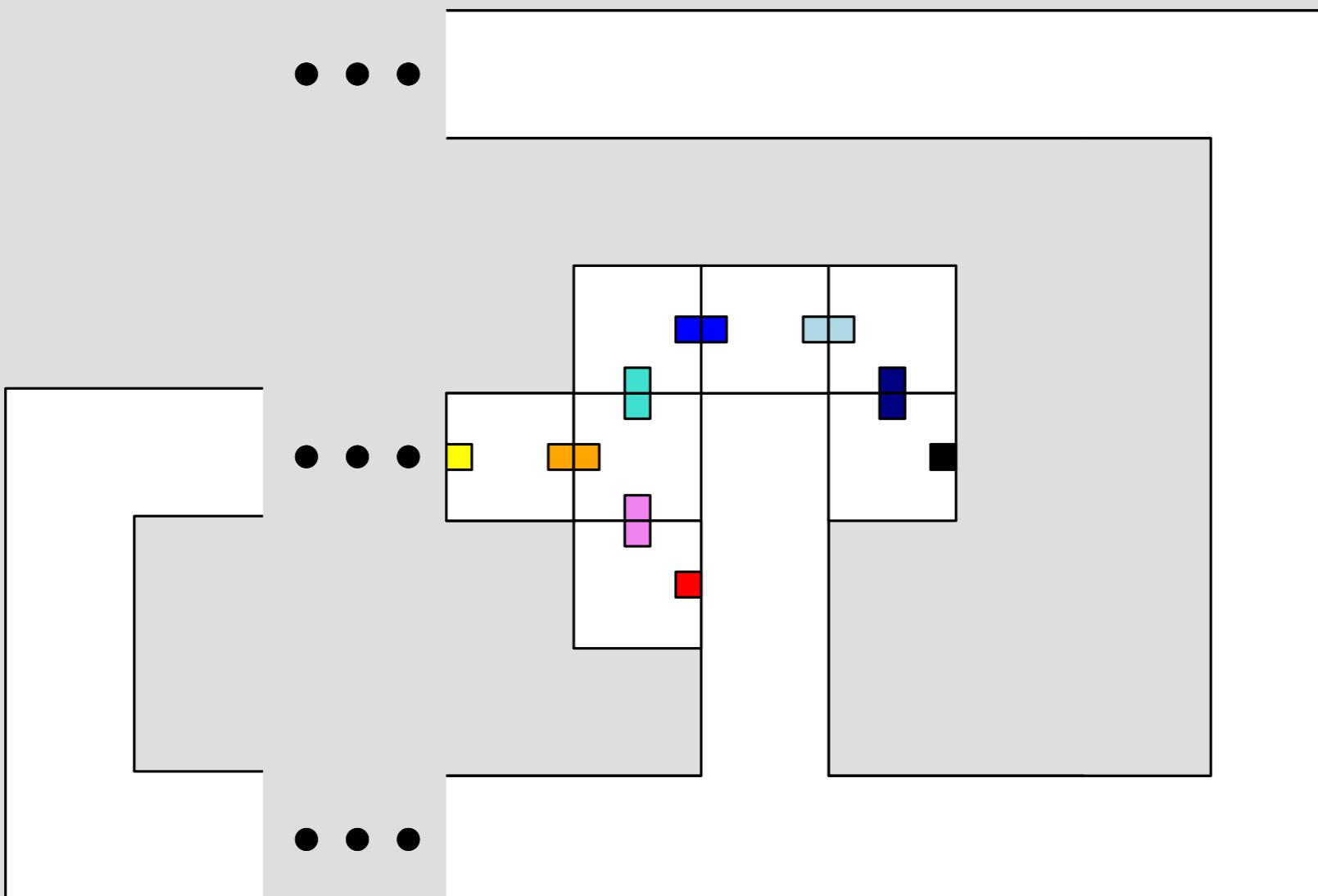
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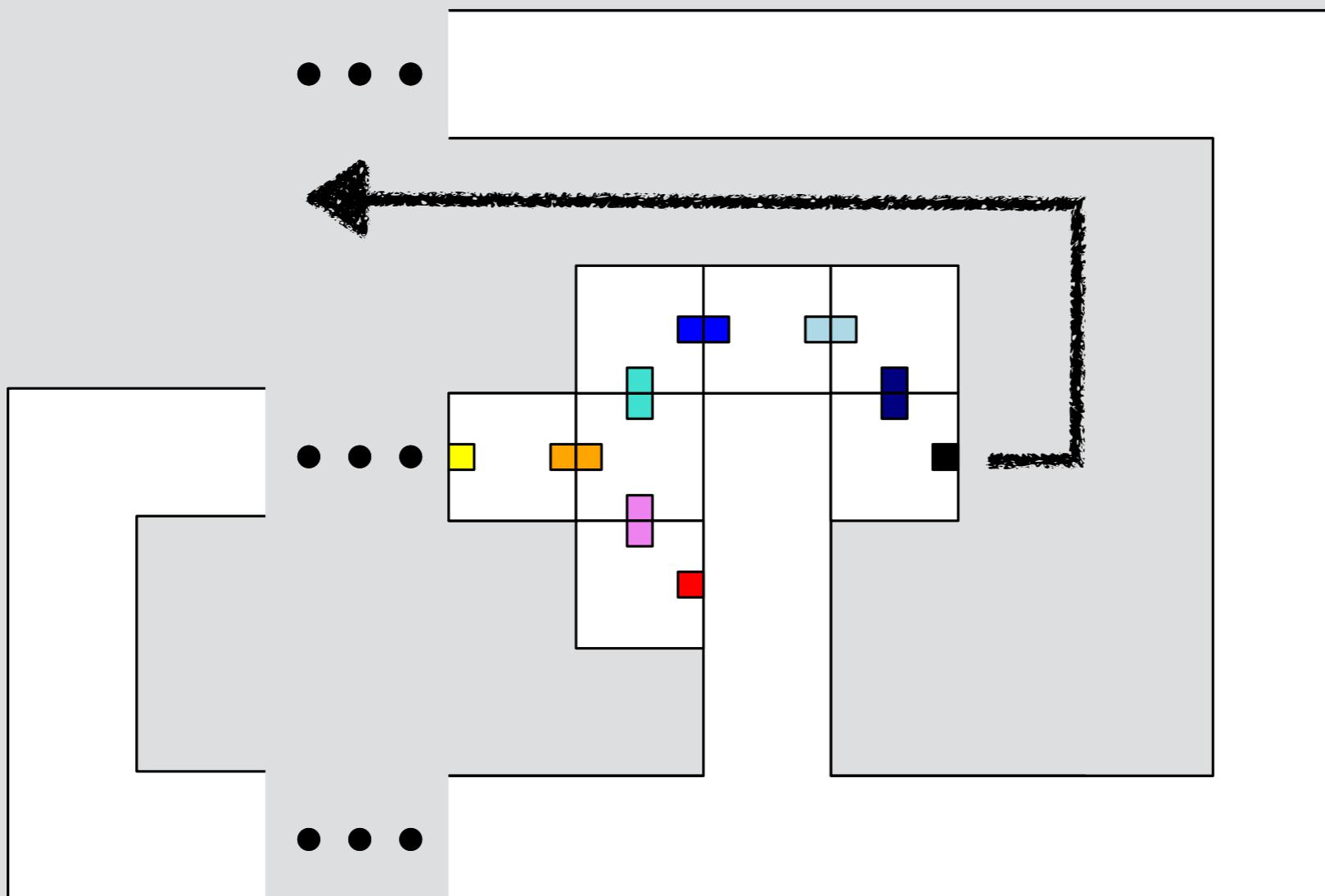
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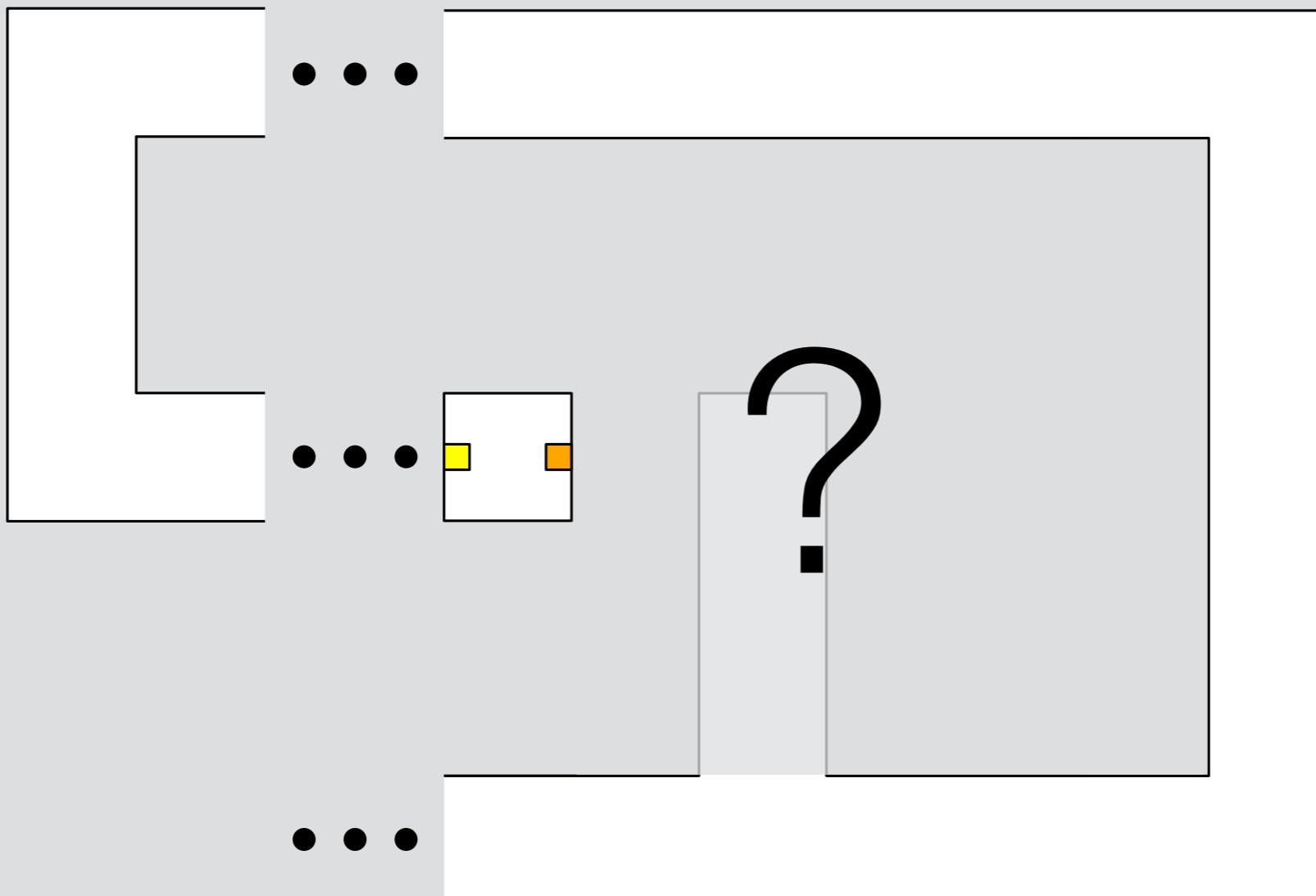
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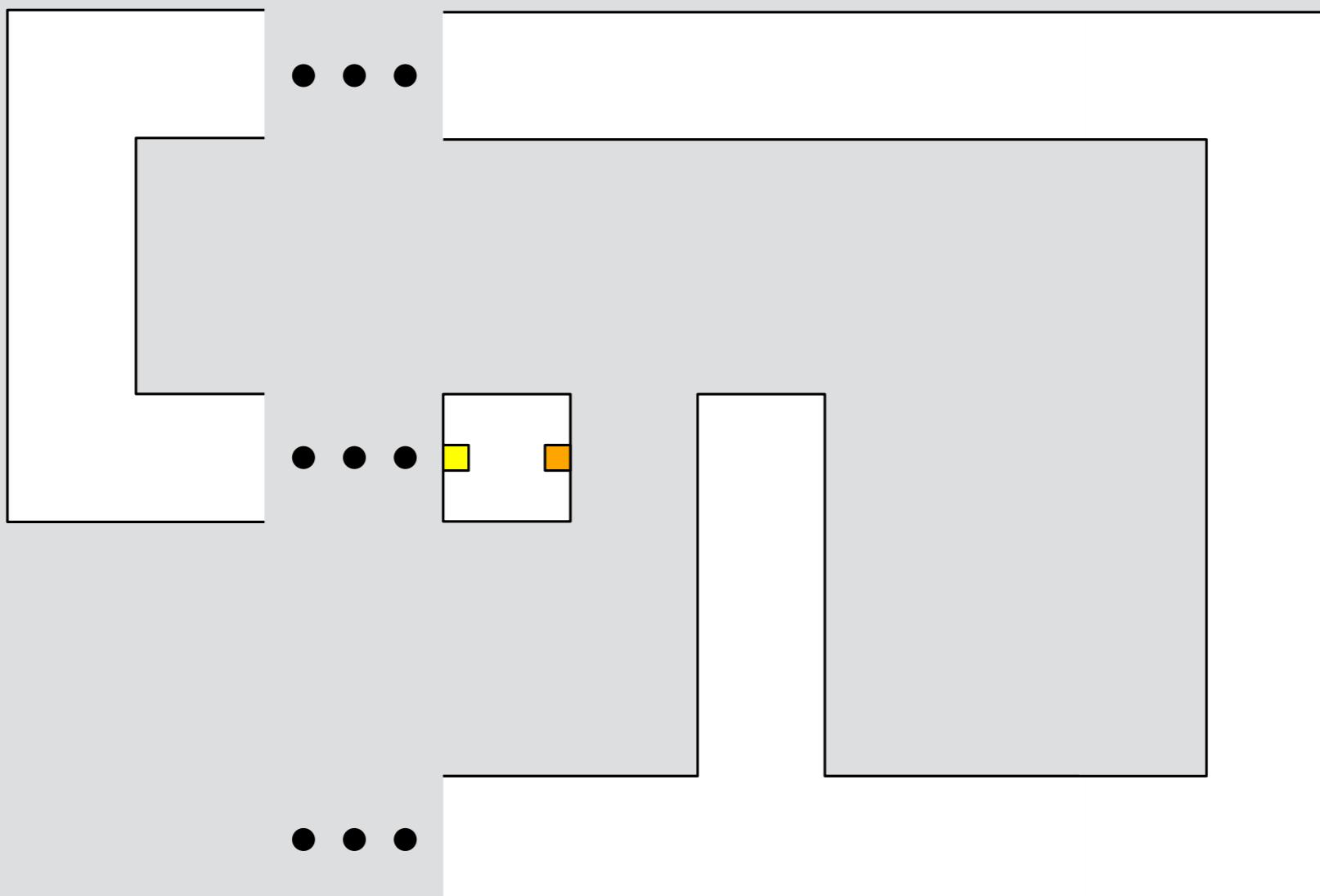
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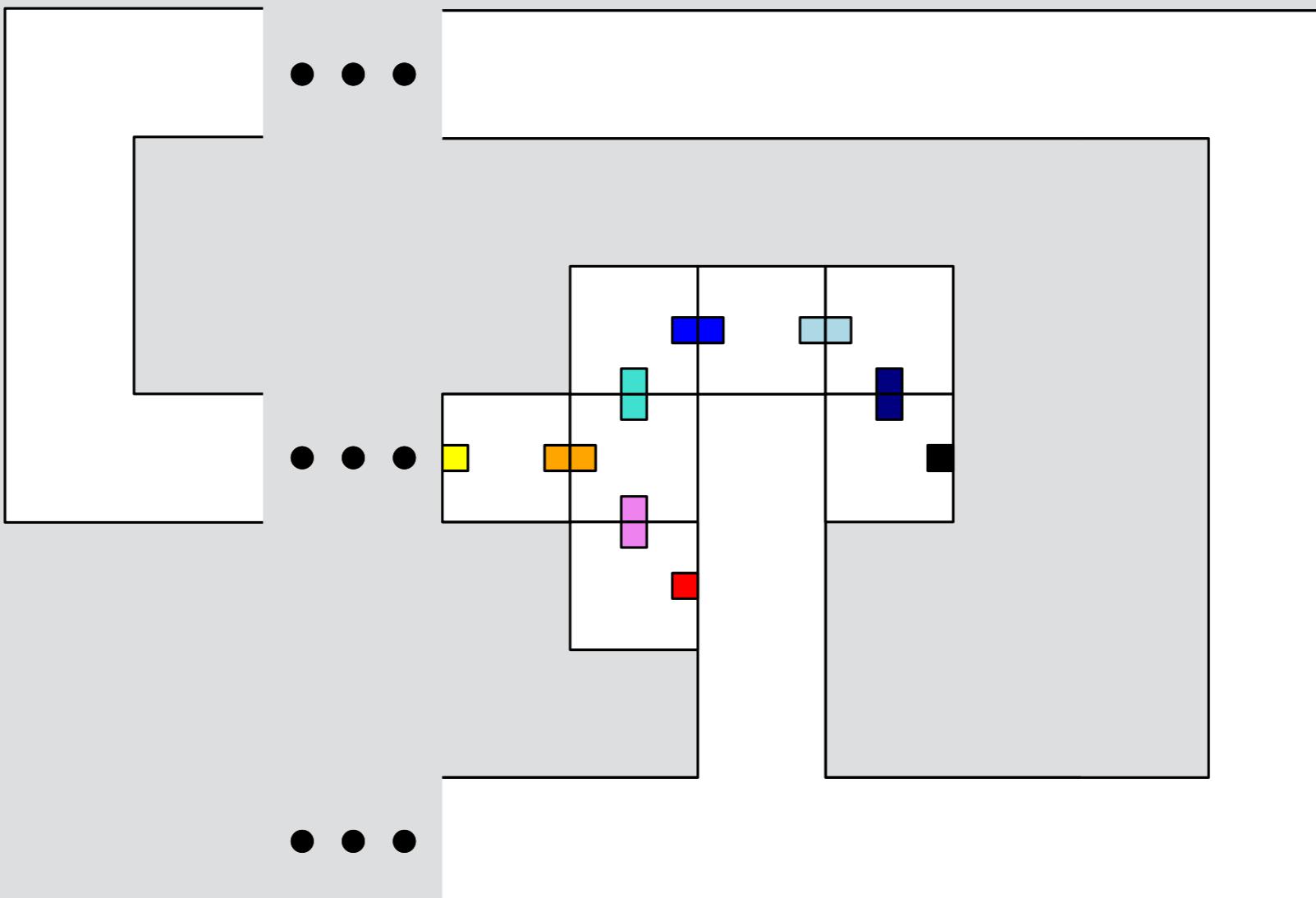
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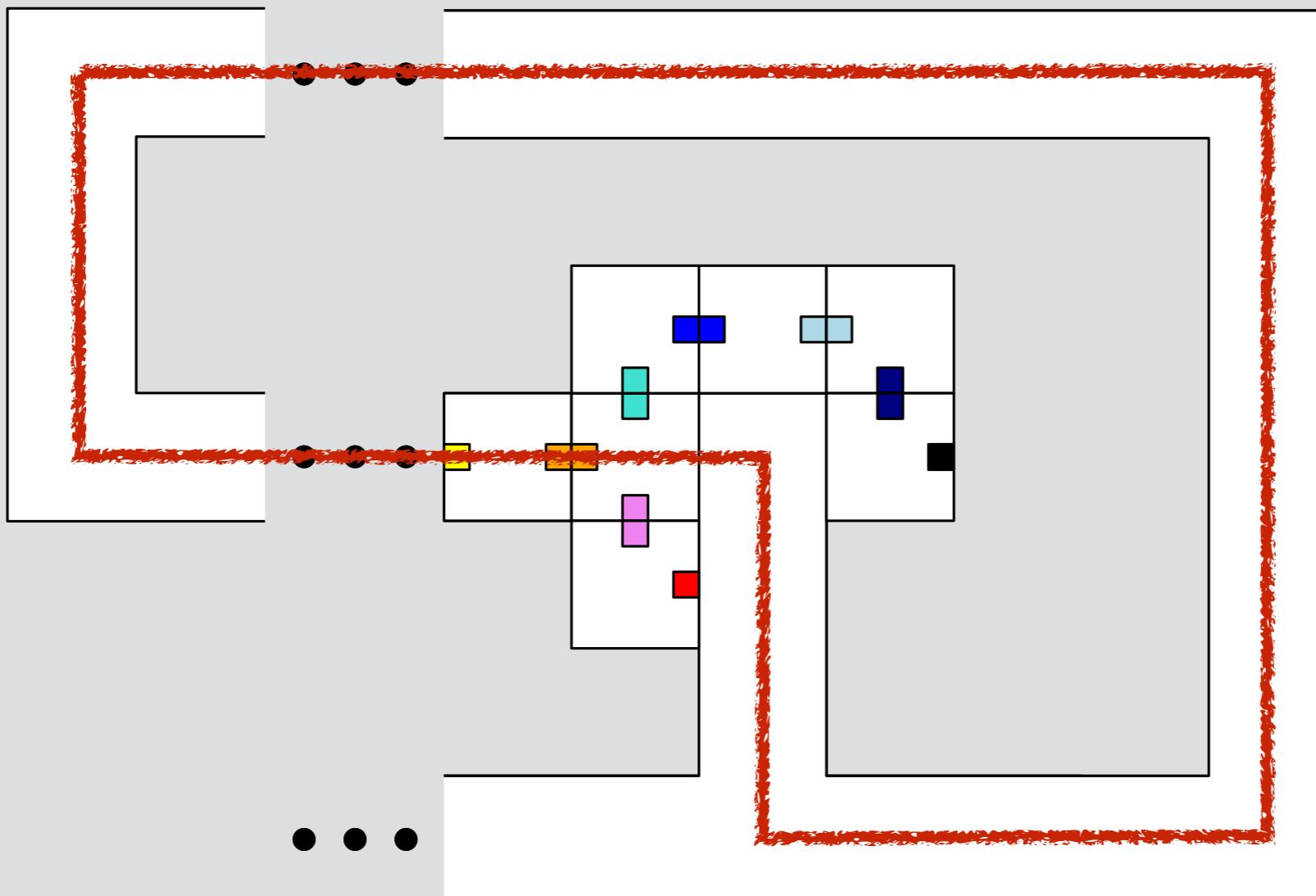
# A planarity problem



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# A planarity problem



Planarity is the barrier to reading without cooperative bonding.



Can read, but might get trapped.

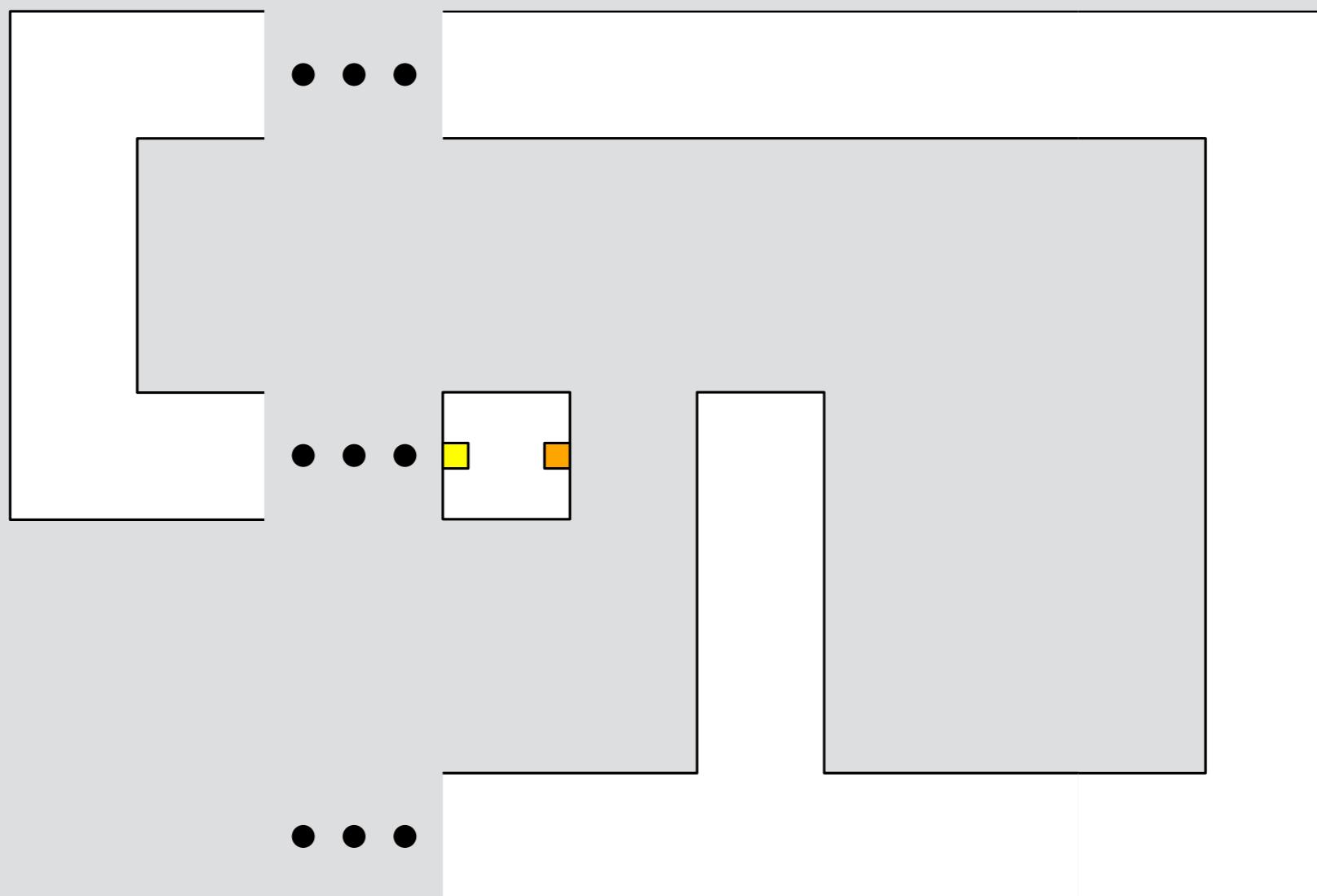
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# Approach 1: 3D

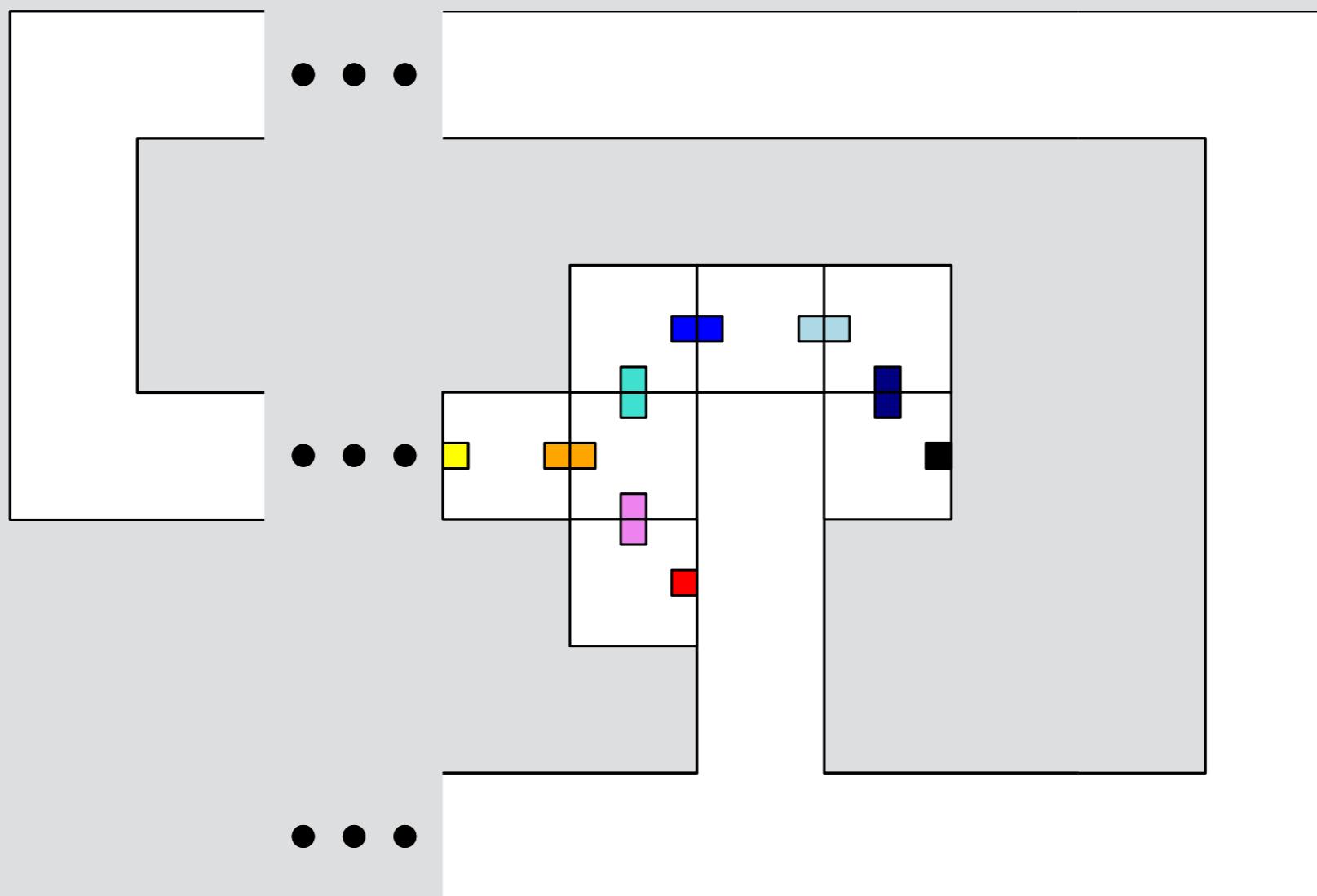
[Cook, Fu, Schweller 2012]



Temperature 1

# Approach 1: 3D

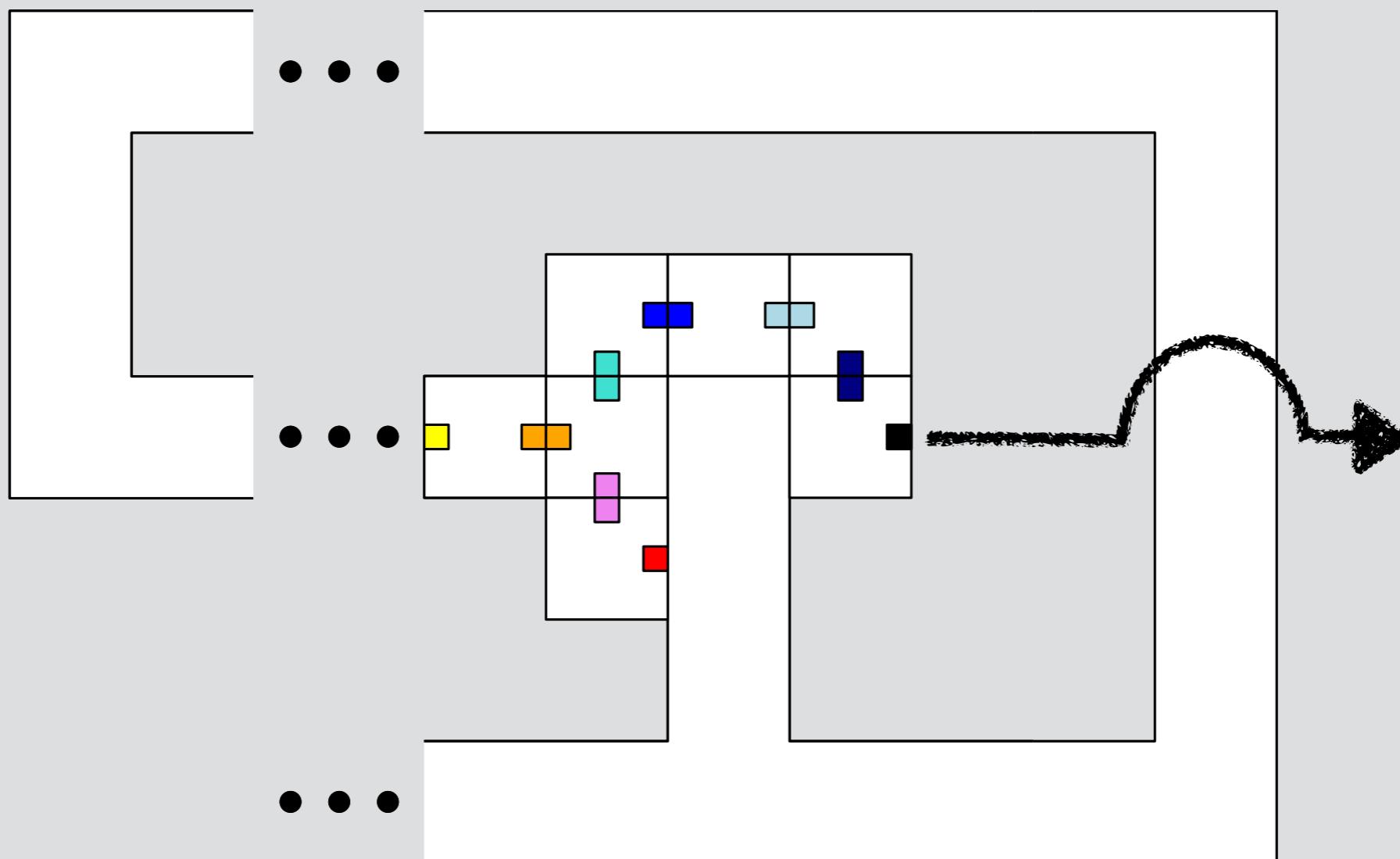
[Cook, Fu, Schweller 2012]



Temperature 1

# Approach 1: 3D

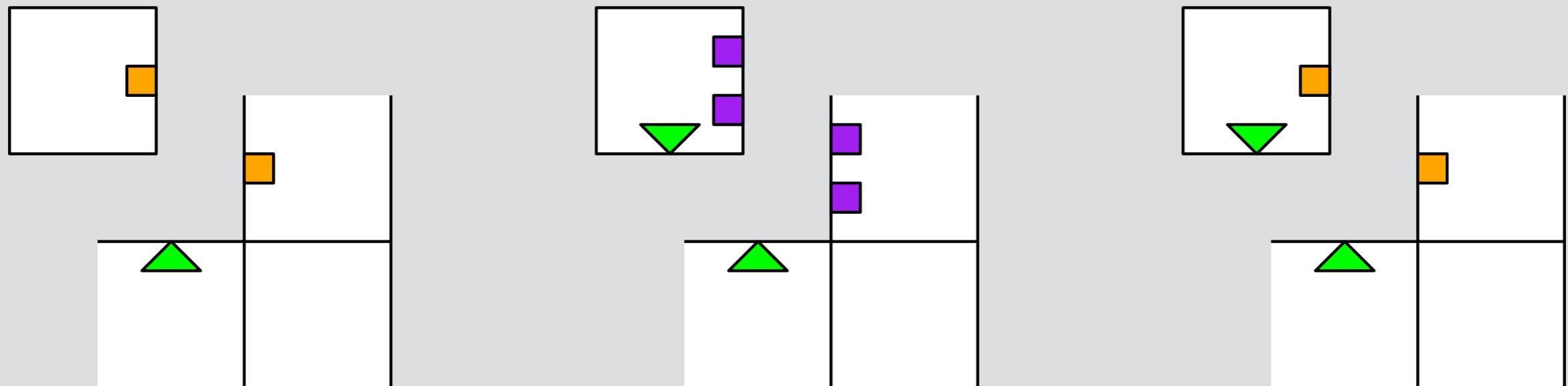
[Cook, Fu, Schweller 2012]



Temperature 1

# Approach 2: Negative Glue

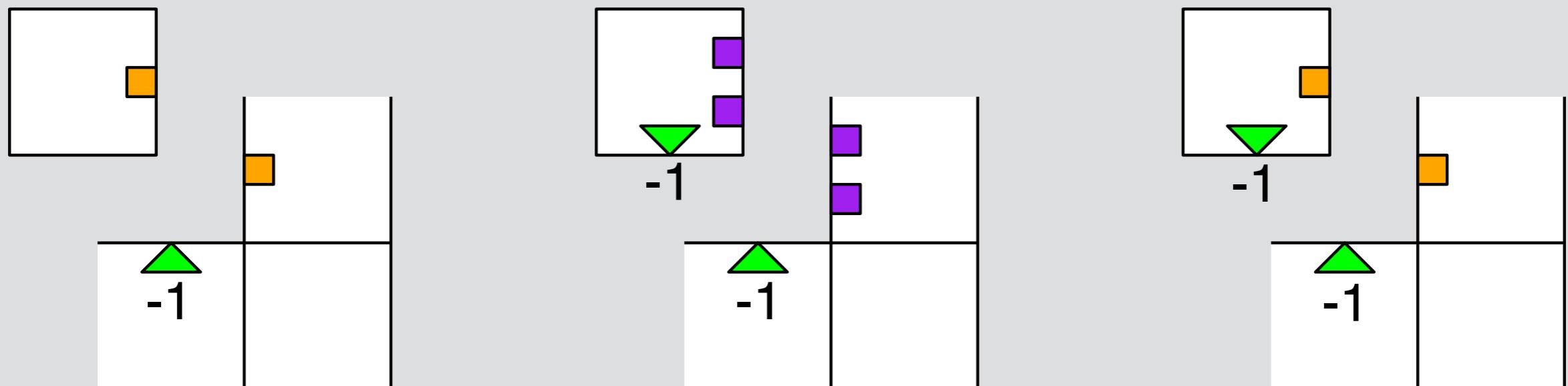
[Patitz, Schweller, Summers 2011]



Temperature 1

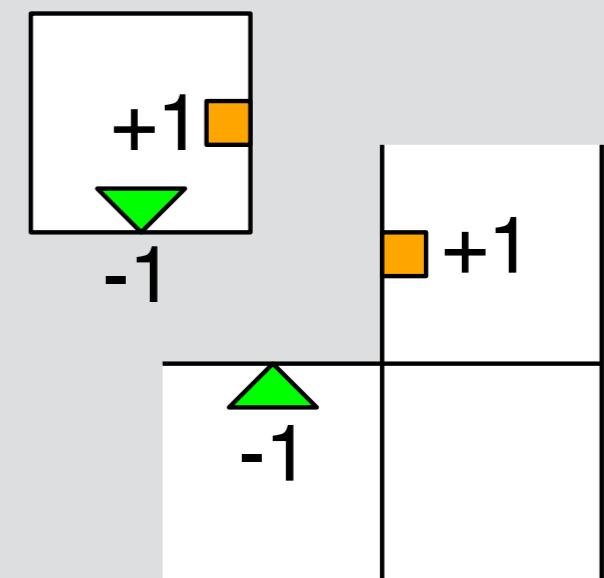
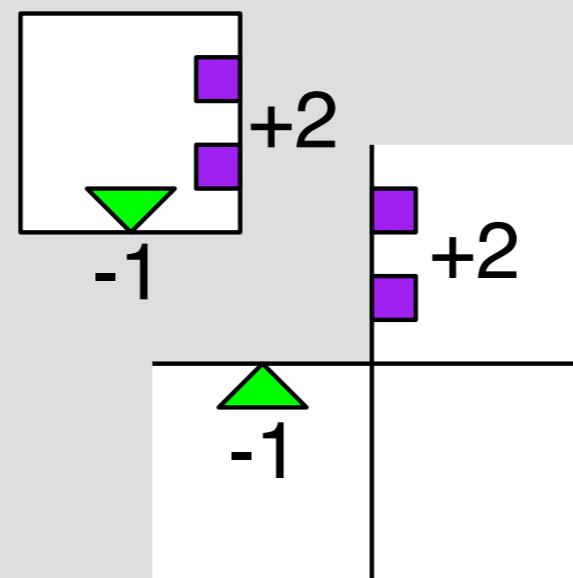
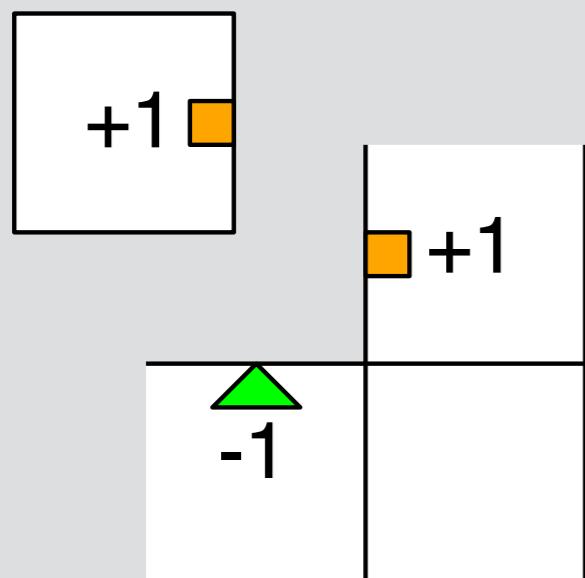
# Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]



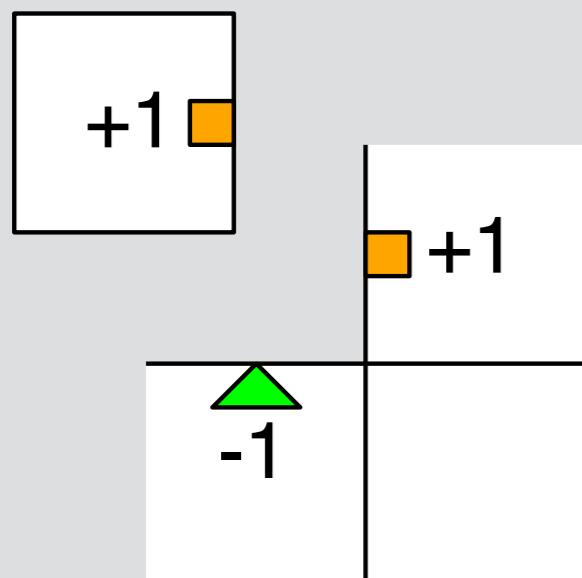
# Approach 2: Negative Glue

[Patitz, Schweller, Summers 2011]

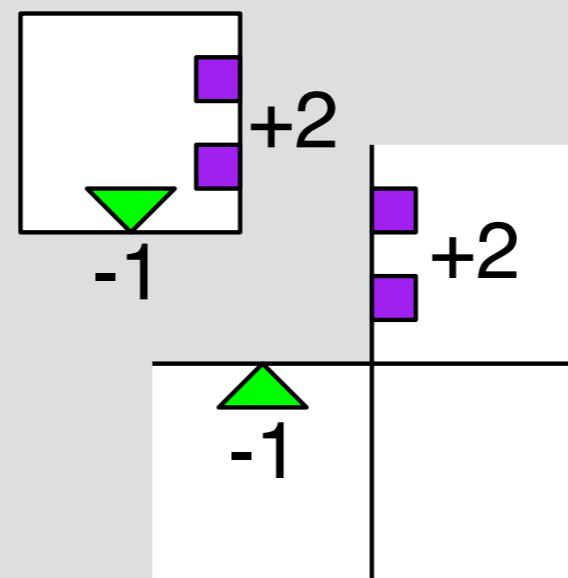


# Approach 2: Negative Glue

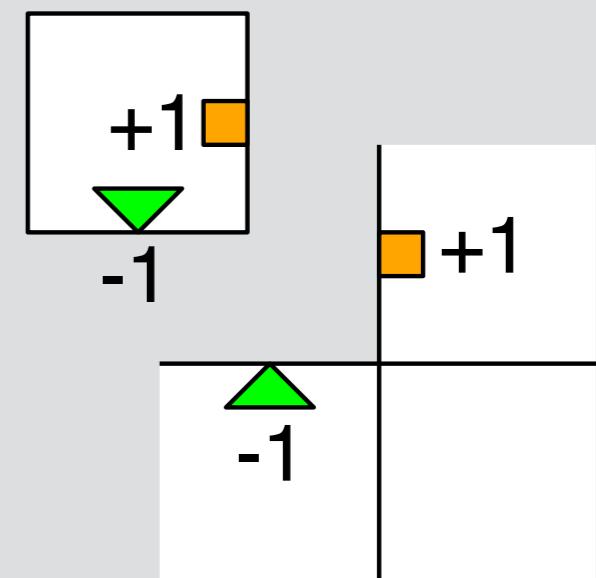
[Patitz, Schweller, Summers 2011]



YES



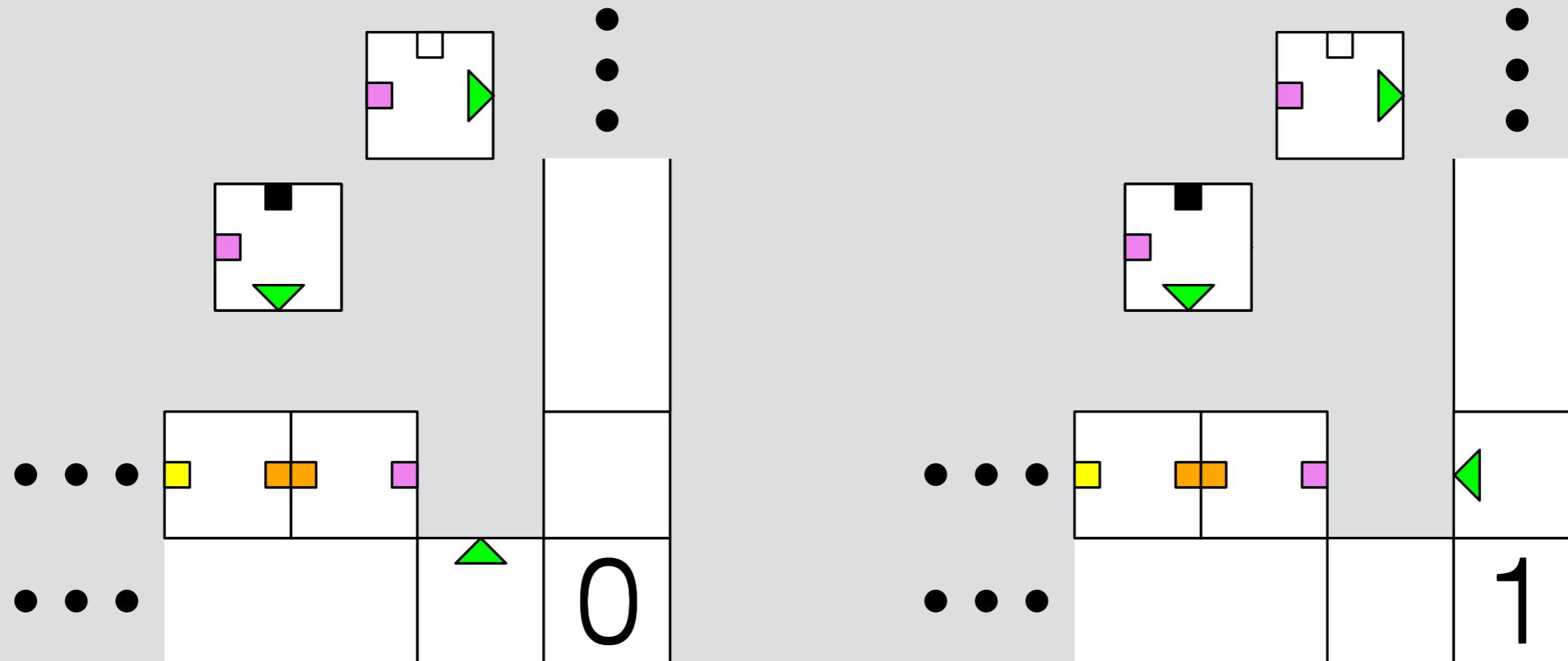
YES



NO

# Approach 2: Negative Glue

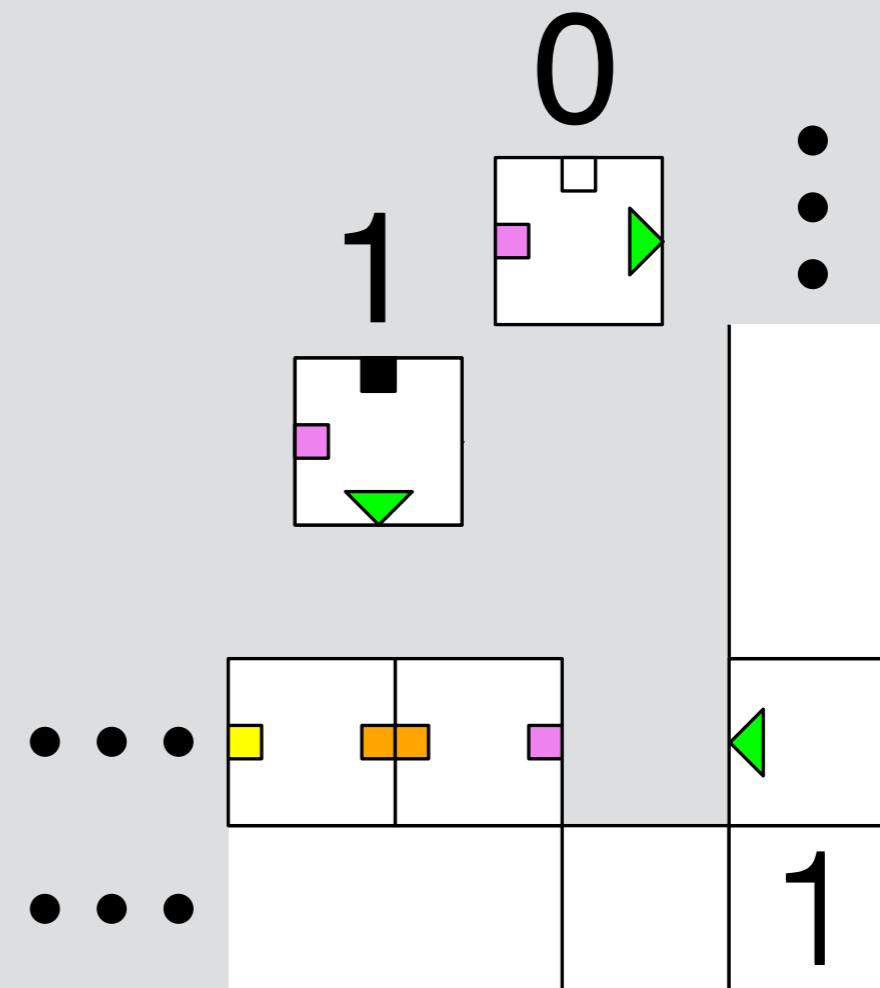
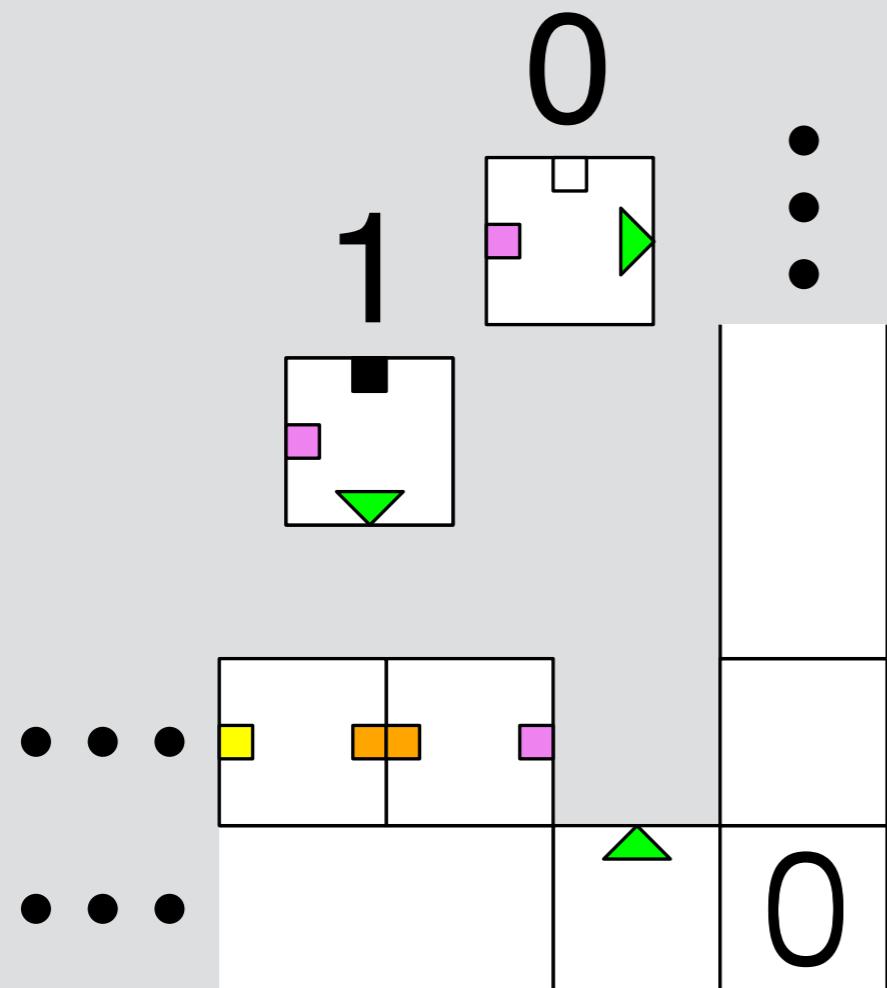
[Patitz, Schweller, Summers 2011]



Temperature 1

# Approach 2: Negative Glue

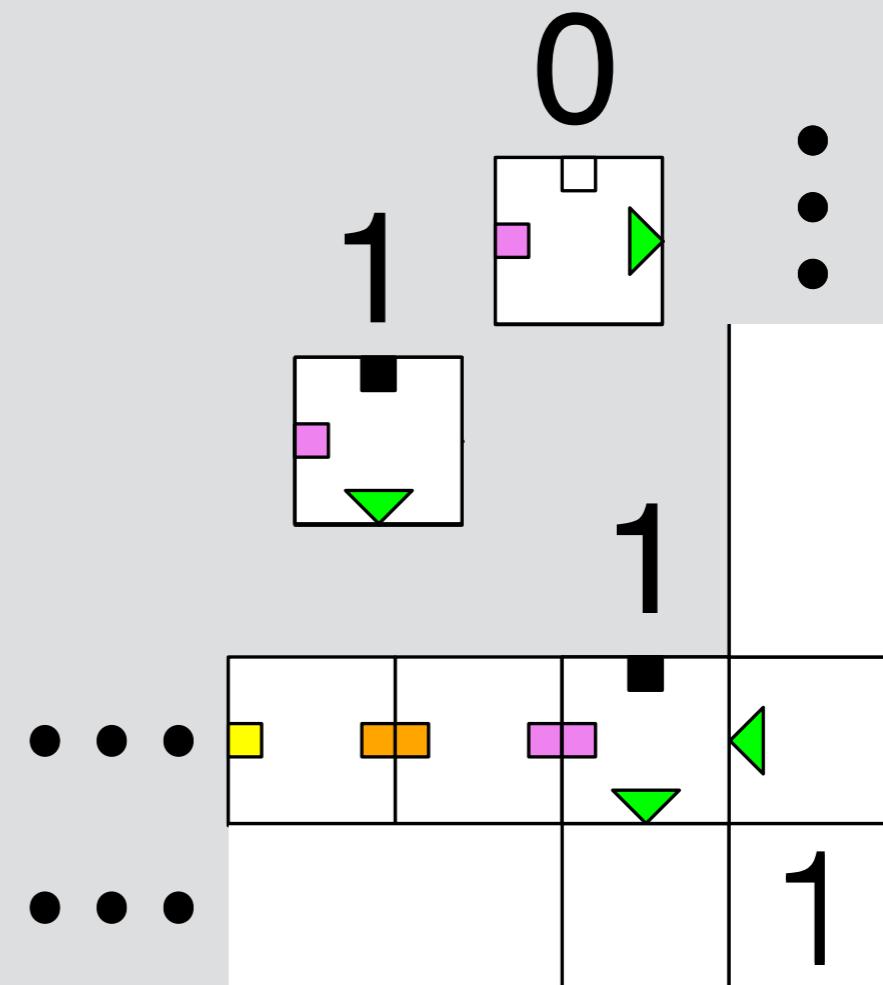
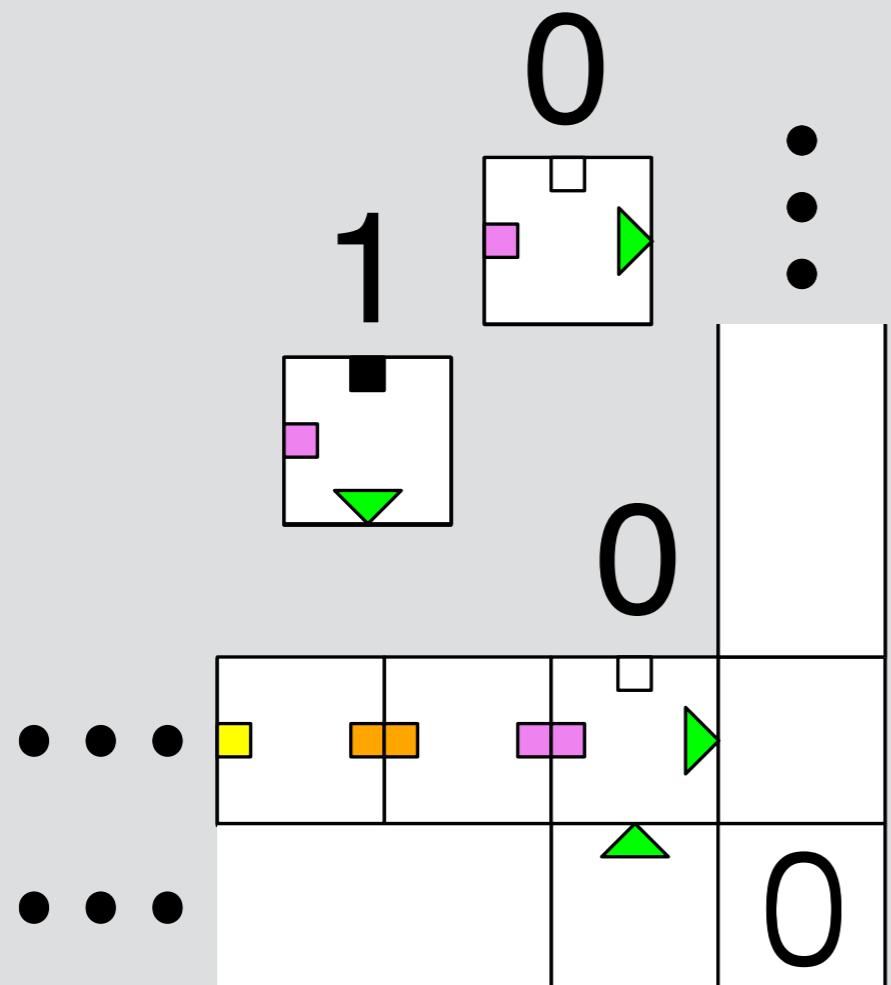
[Patitz, Schweller, Summers 2011]



Temperature 1

# Approach 2: Negative Glue

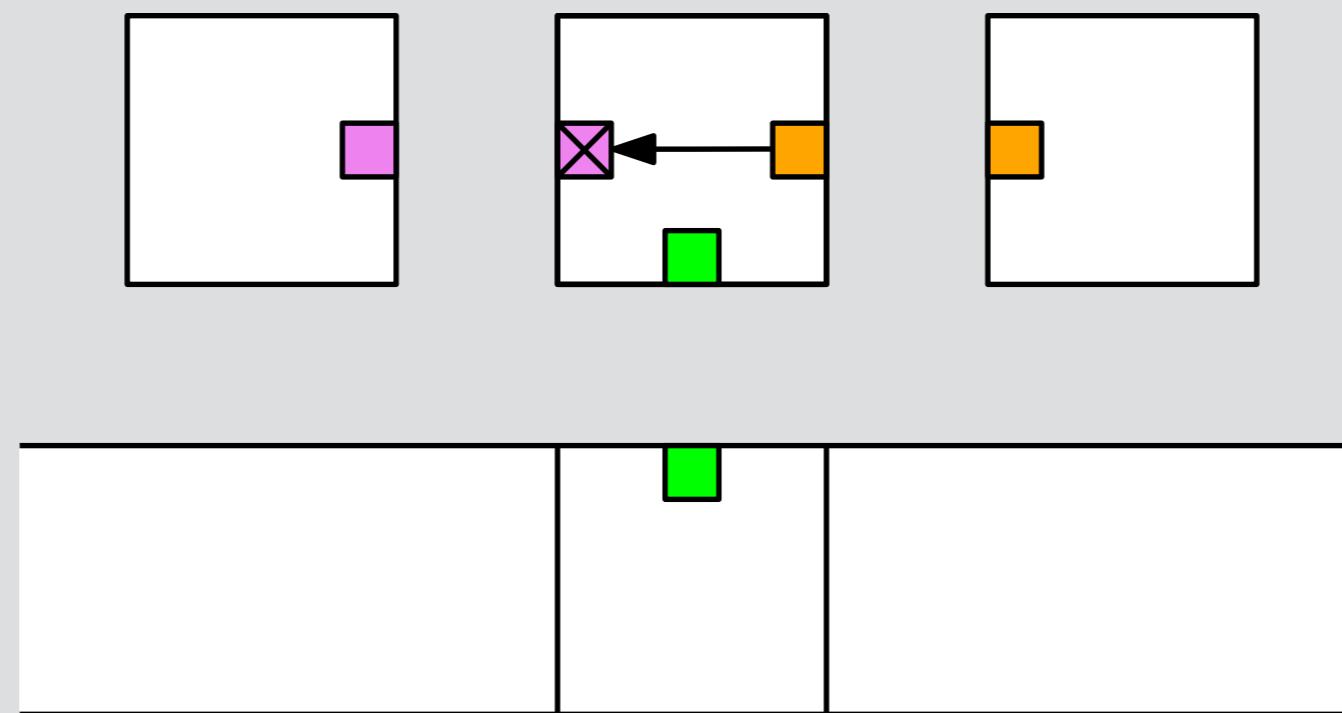
[Patitz, Schweller, Summers 2011]



Temperature 1

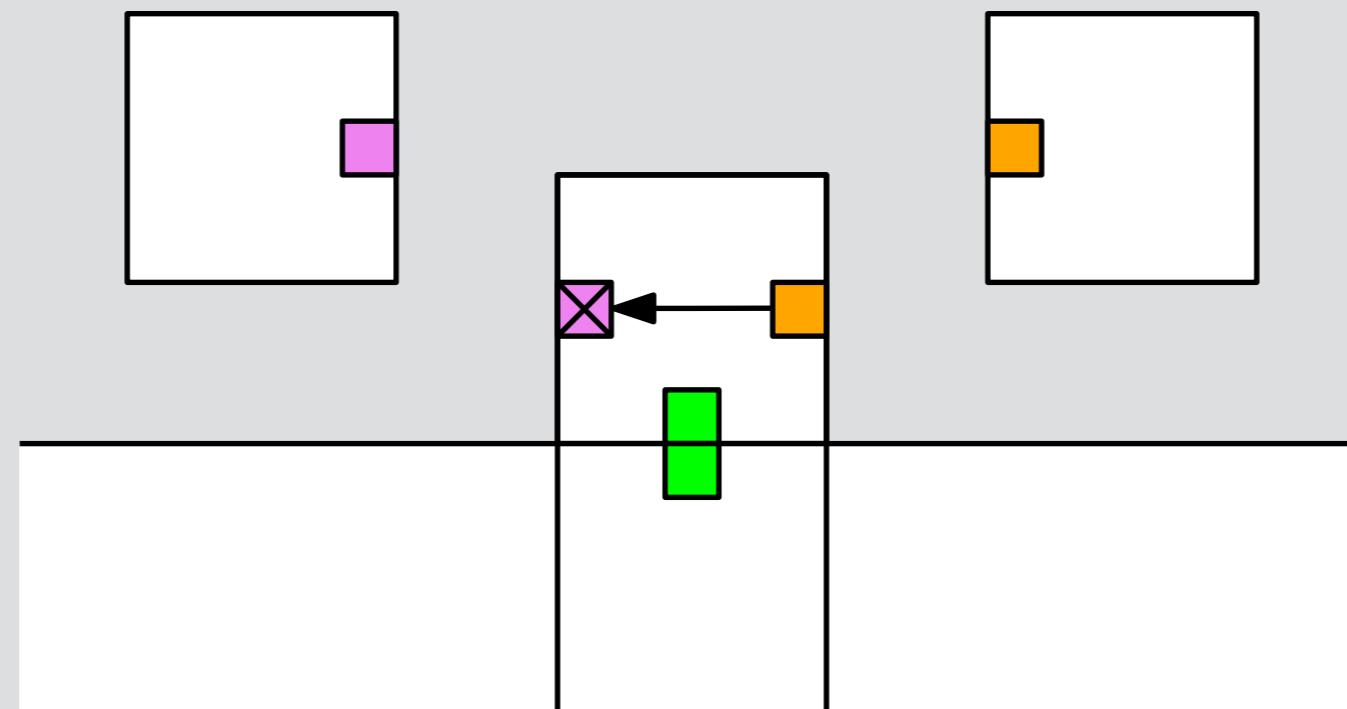
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



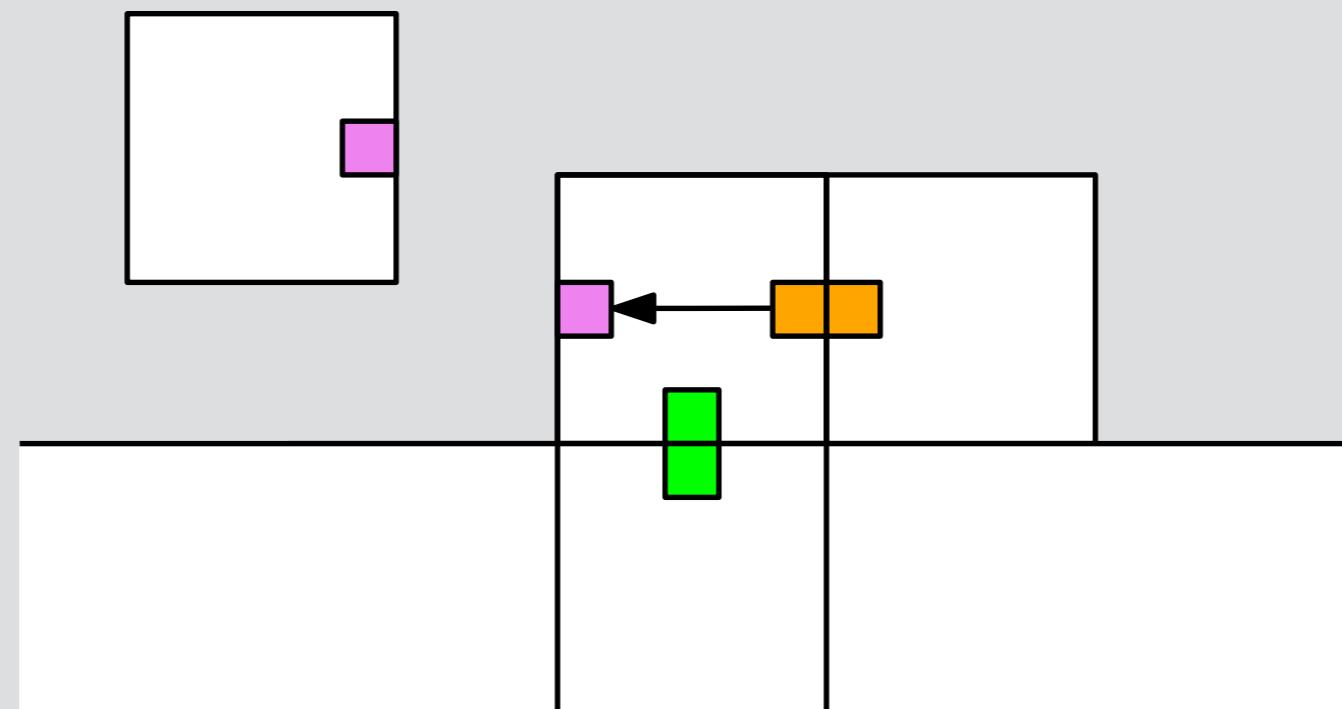
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



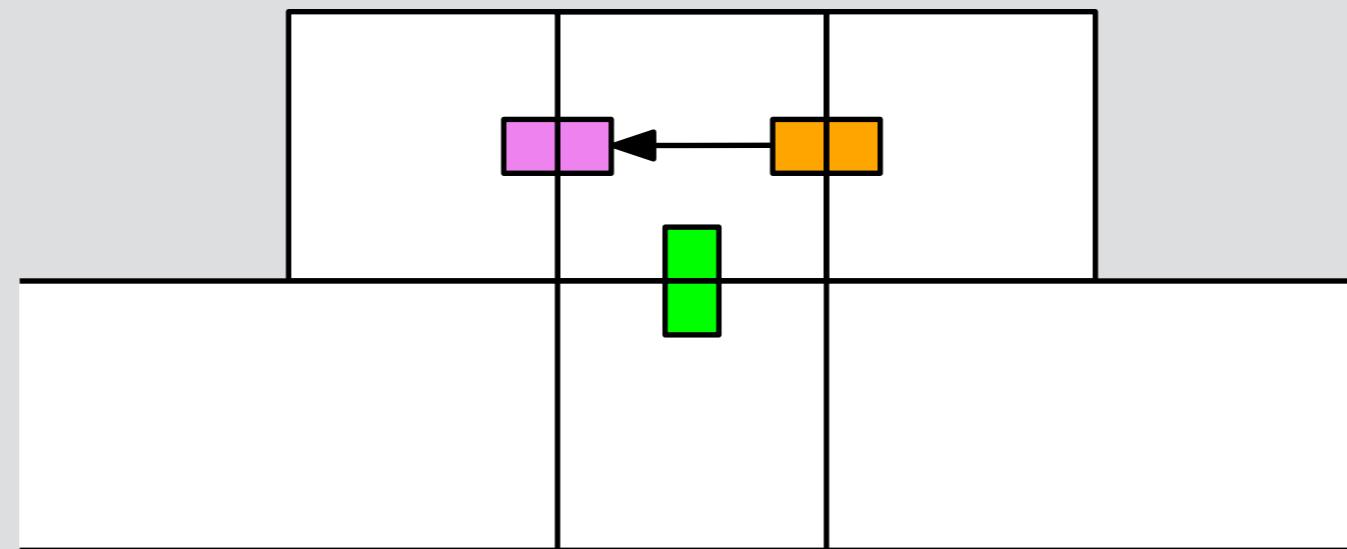
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



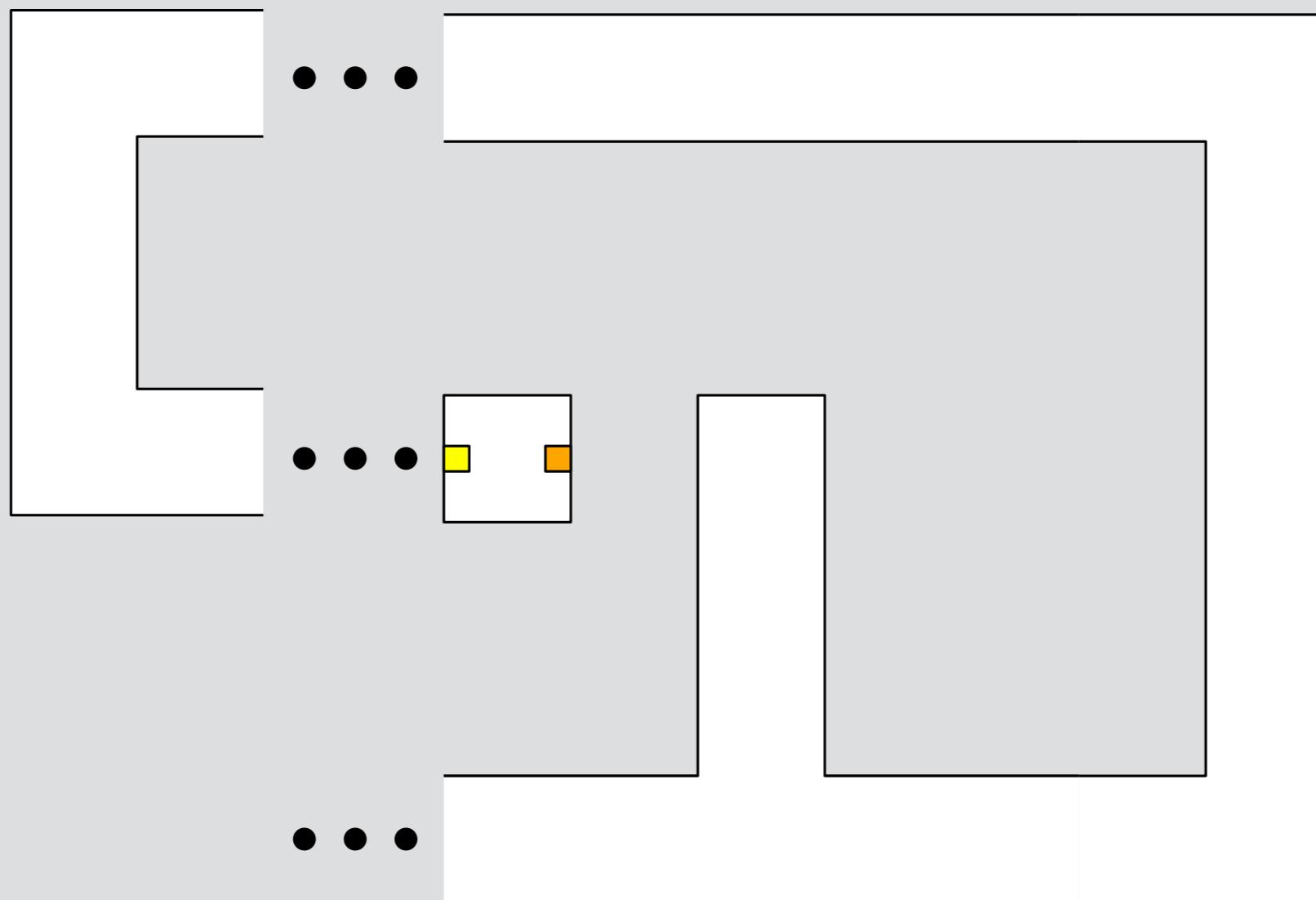
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



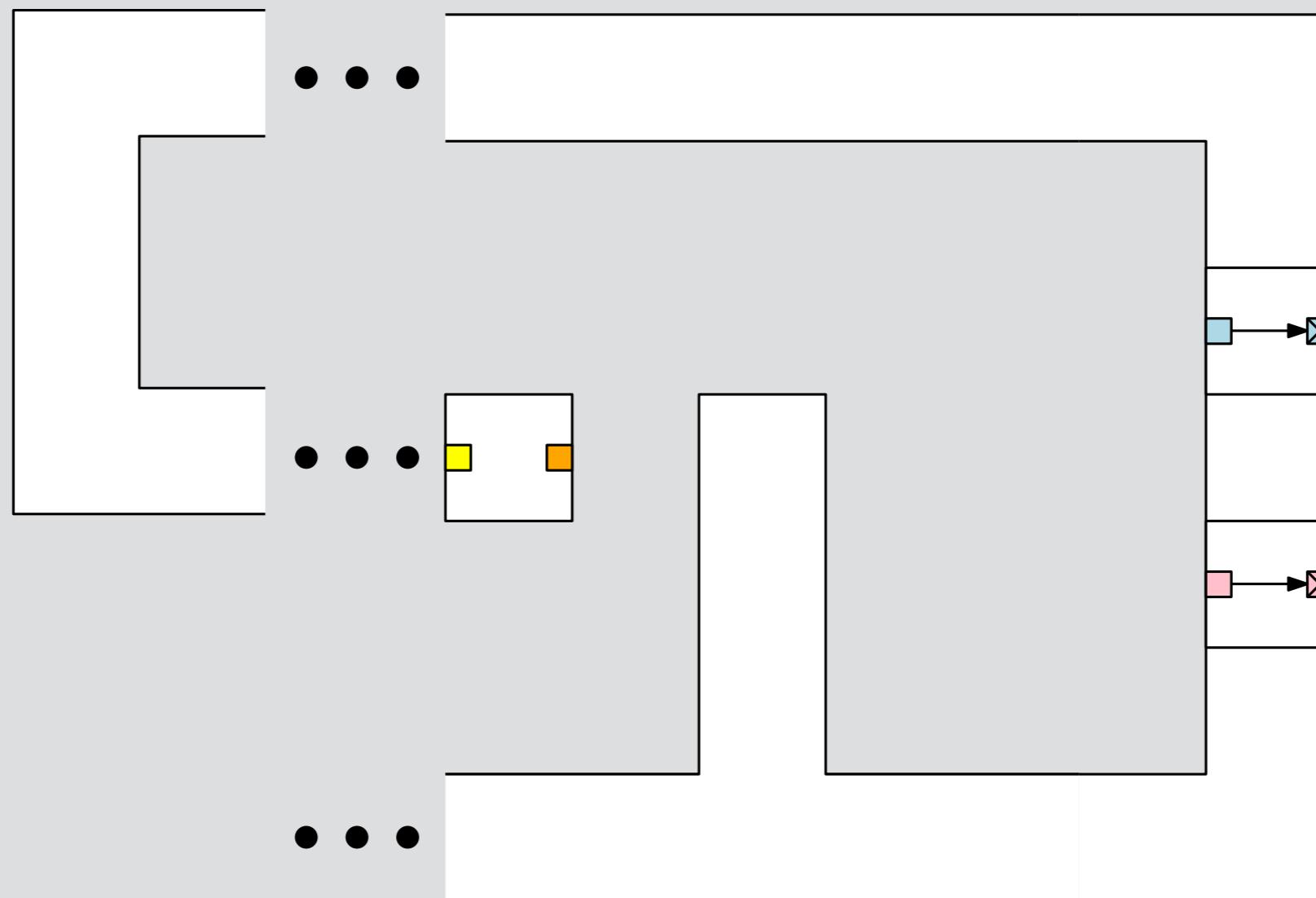
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



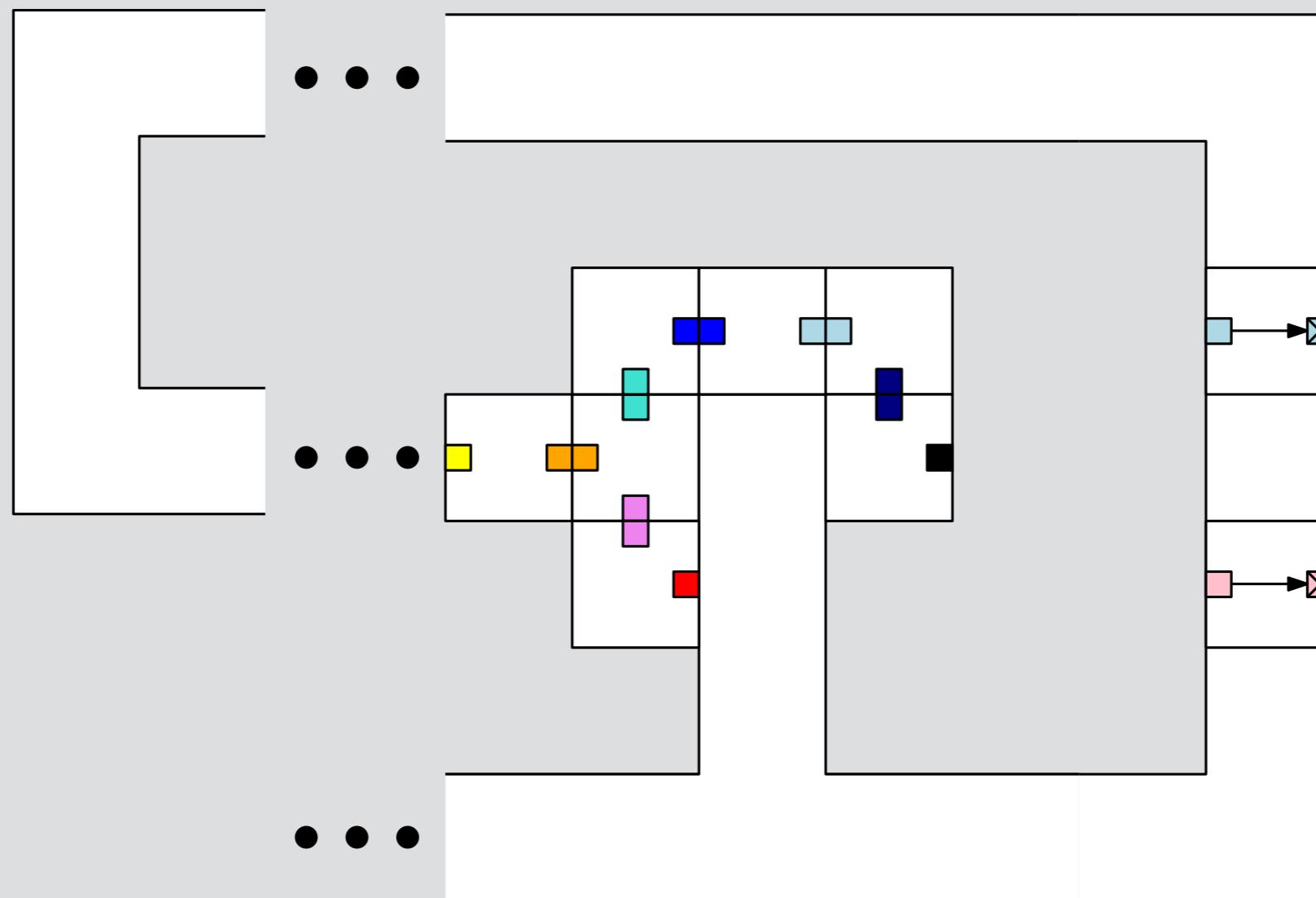
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



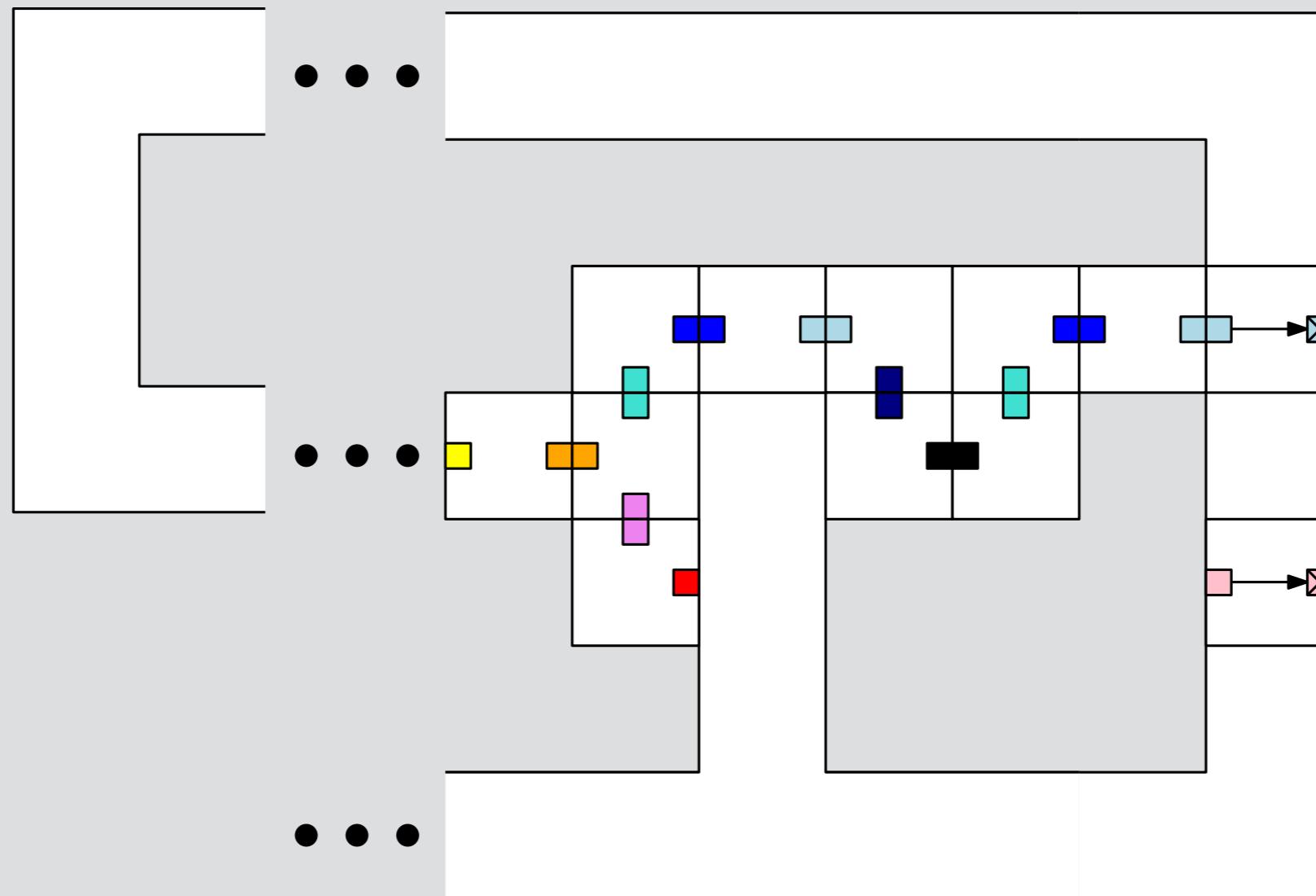
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



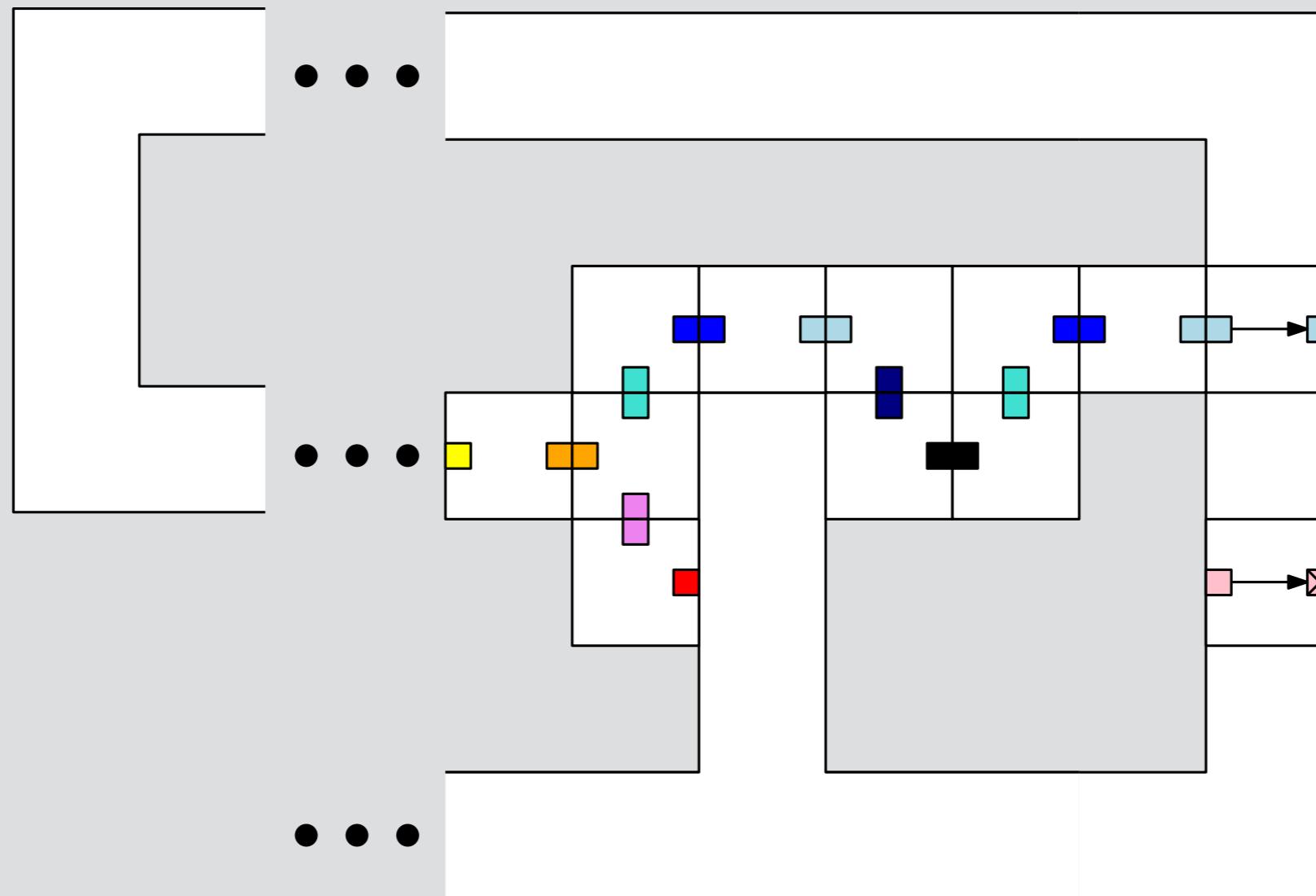
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



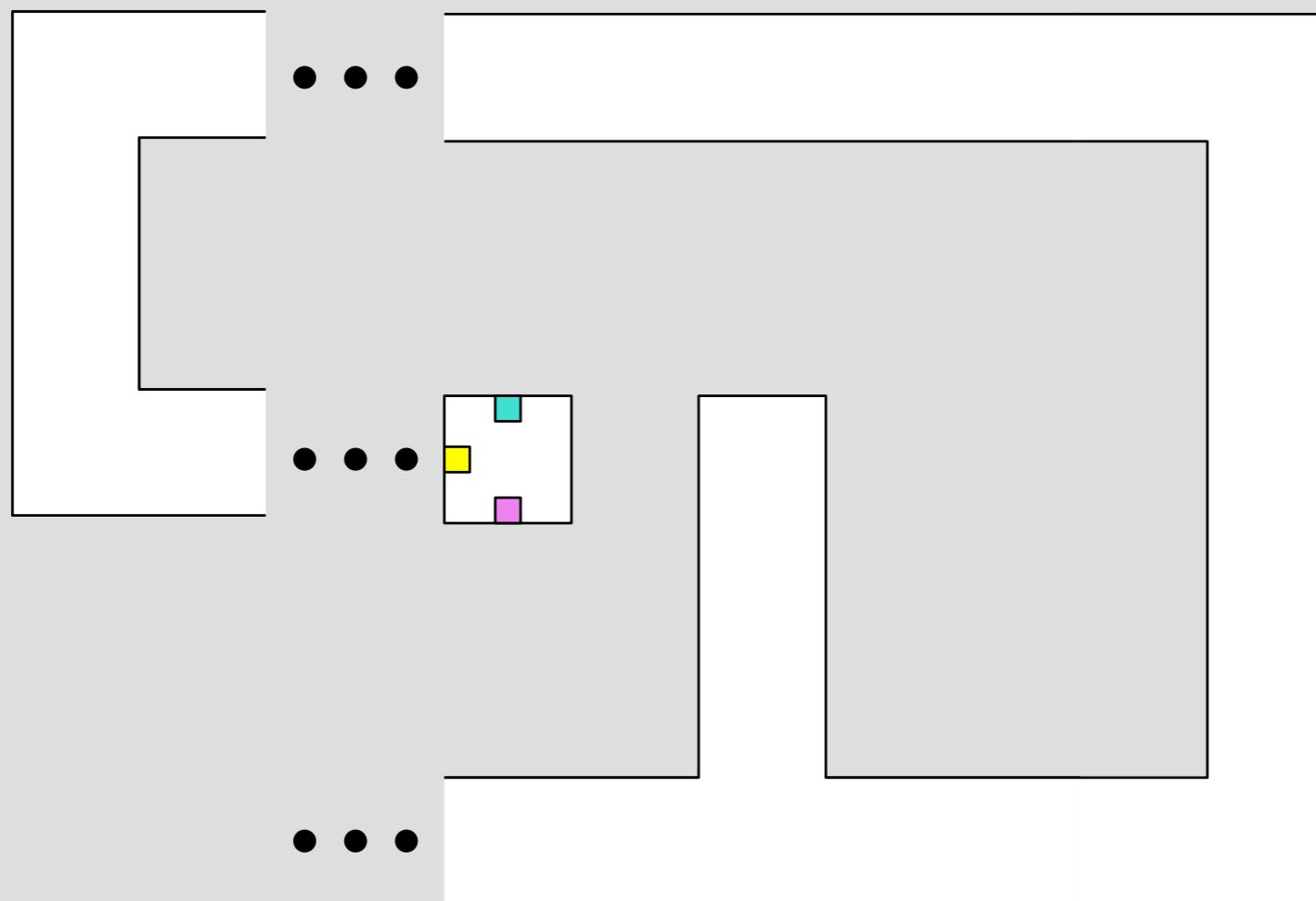
# Approach 3: Signal Tiles

[Padilla et al. 2014], [Jonoska, Karpenko 2014]



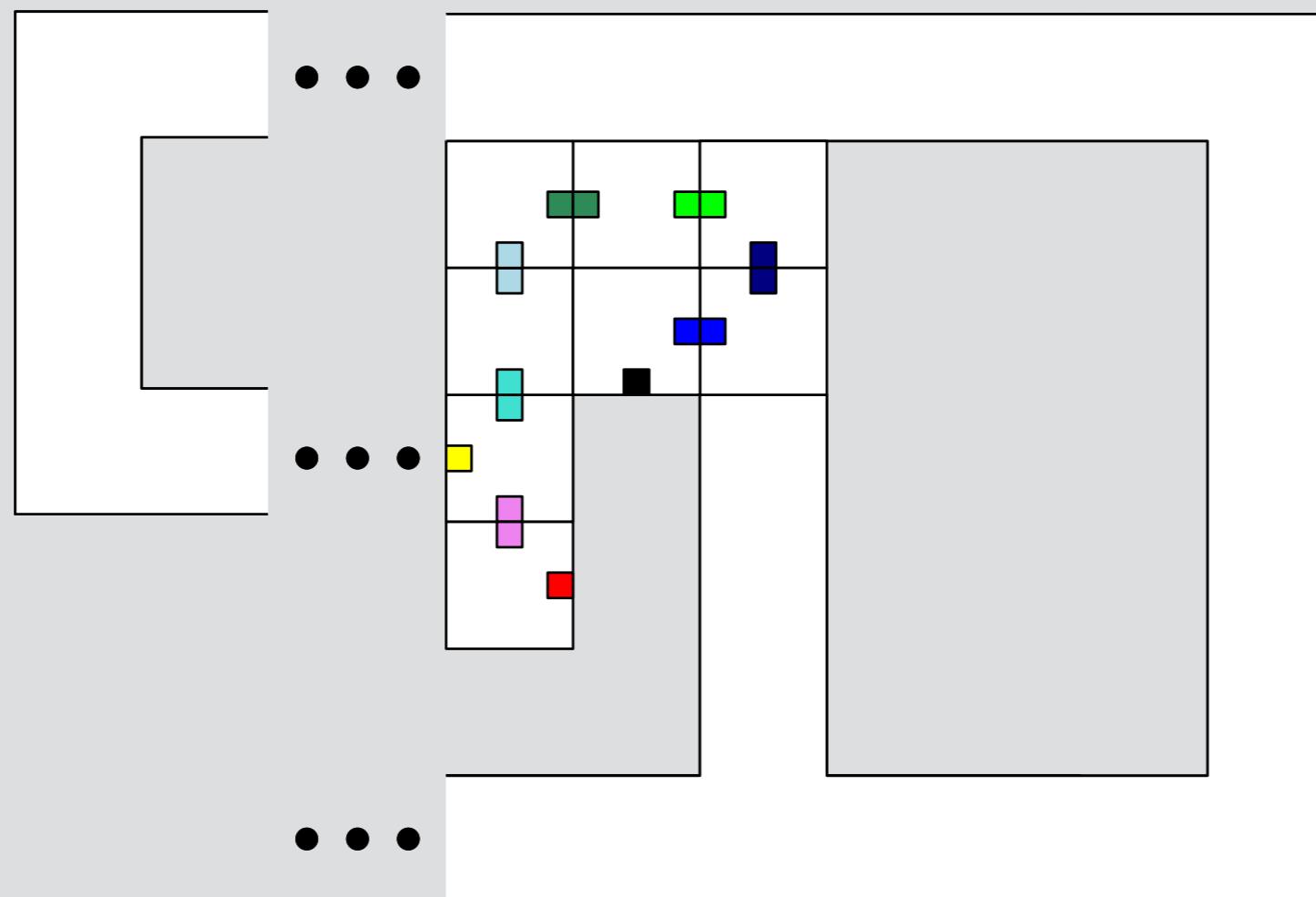
# Approach 4: Polyomino Tiles

[Hendricks et al. 2014], [Fekete et al. 2015]



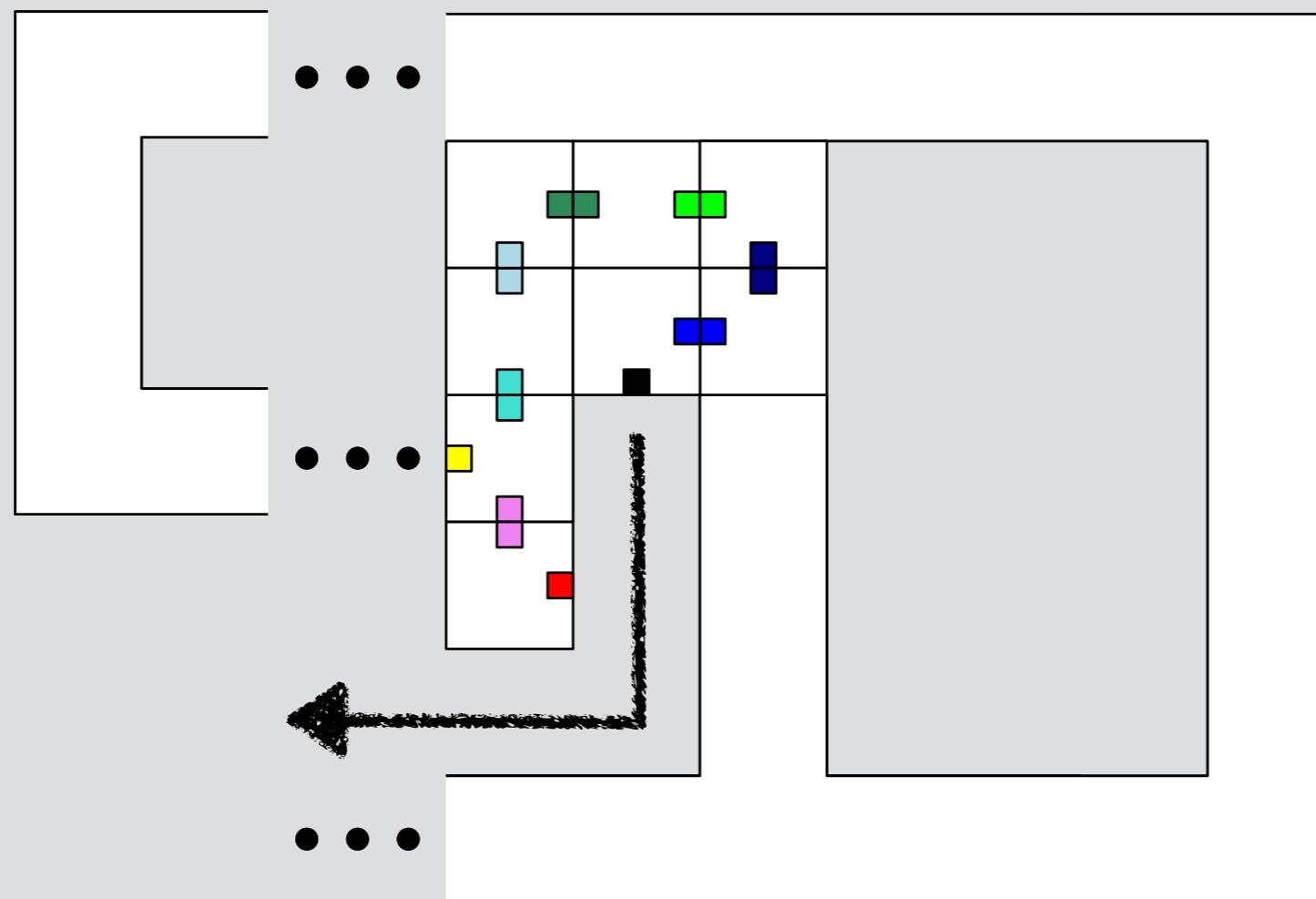
# Approach 4: Polyomino Tiles

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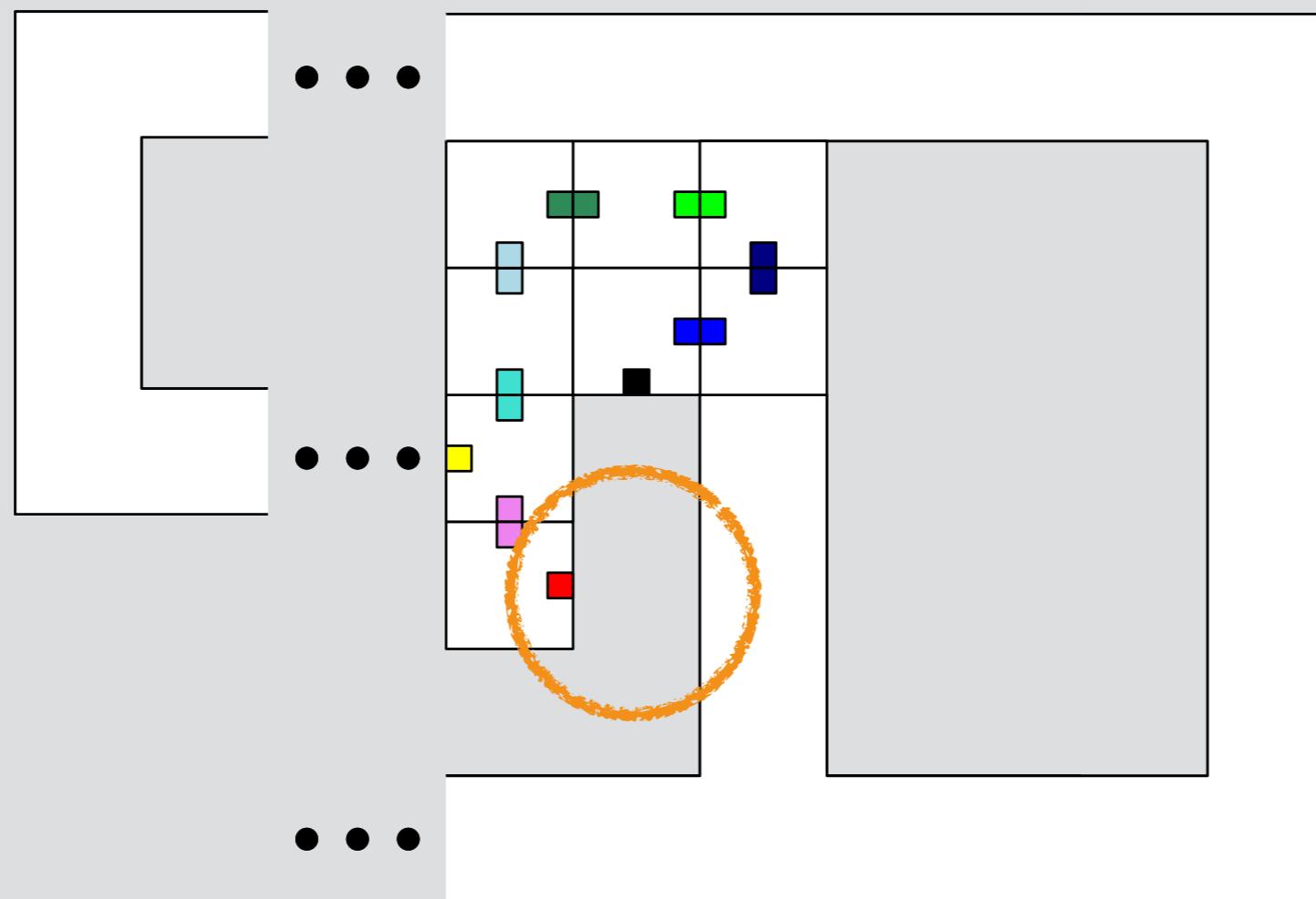
# Approach 4: Polyomino Tiles

[Hendricks et al. 2014], [Fekete et al. 2015]



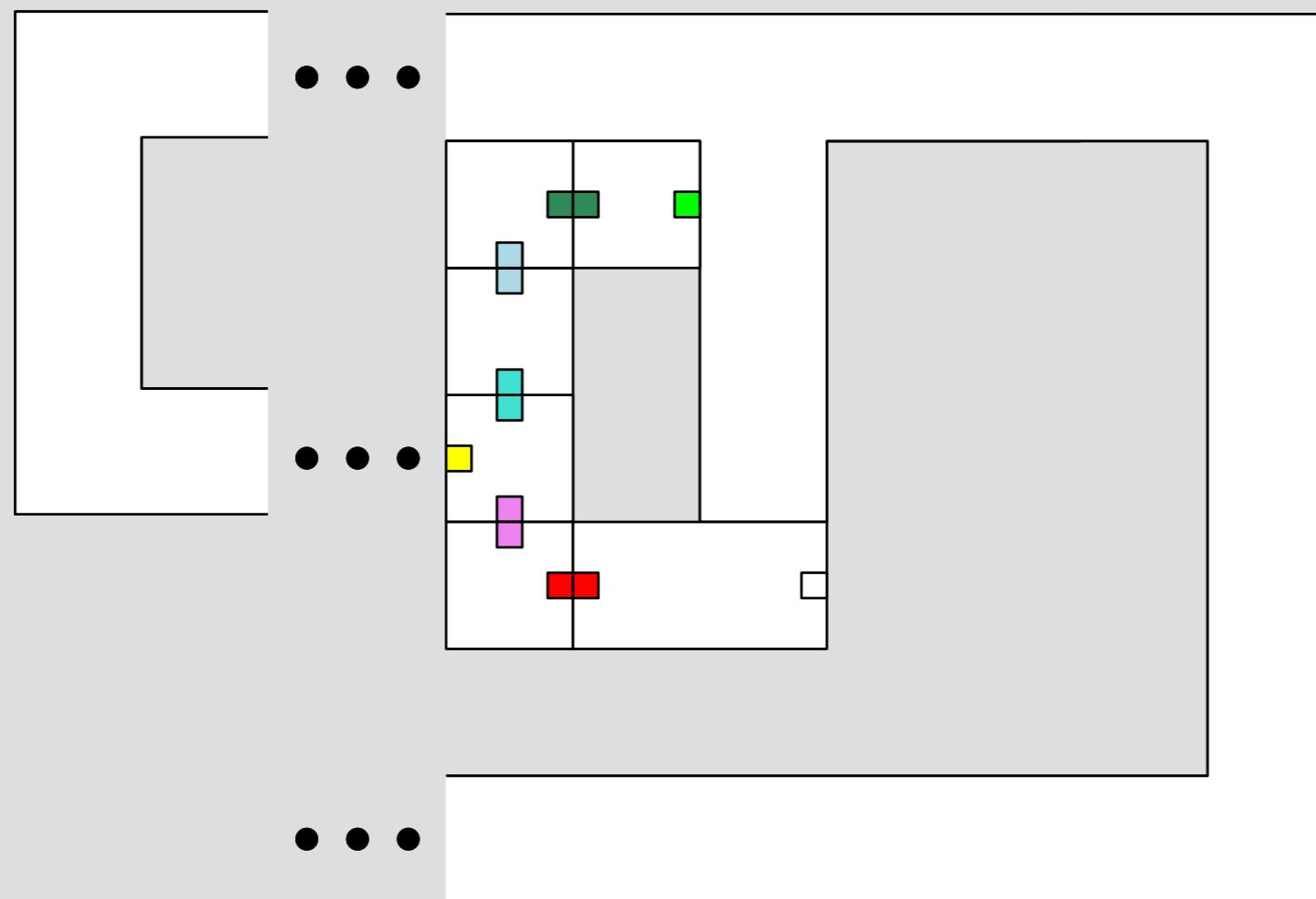
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[Hendricks et al. 2014], [Fekete et al. 2015]



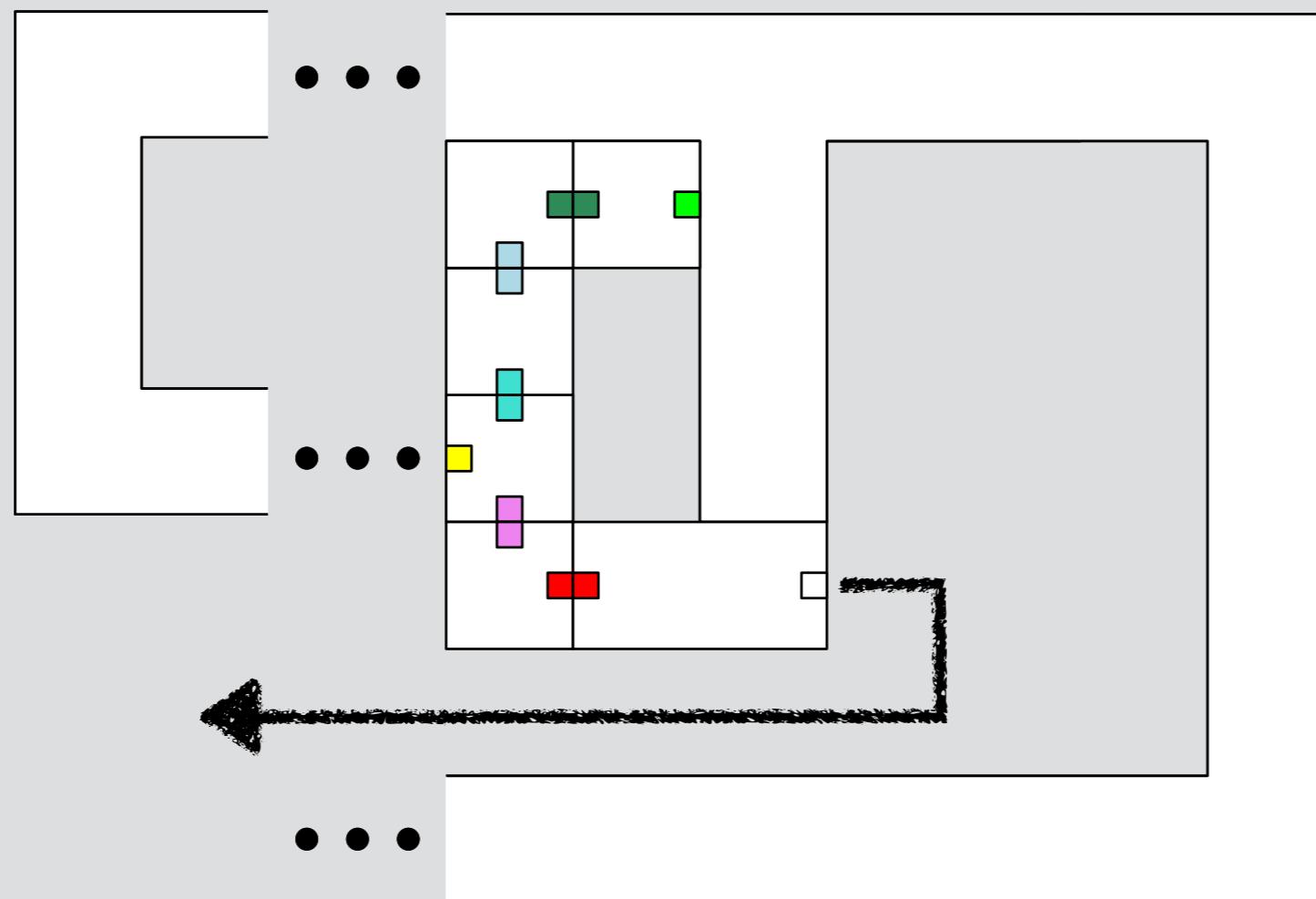
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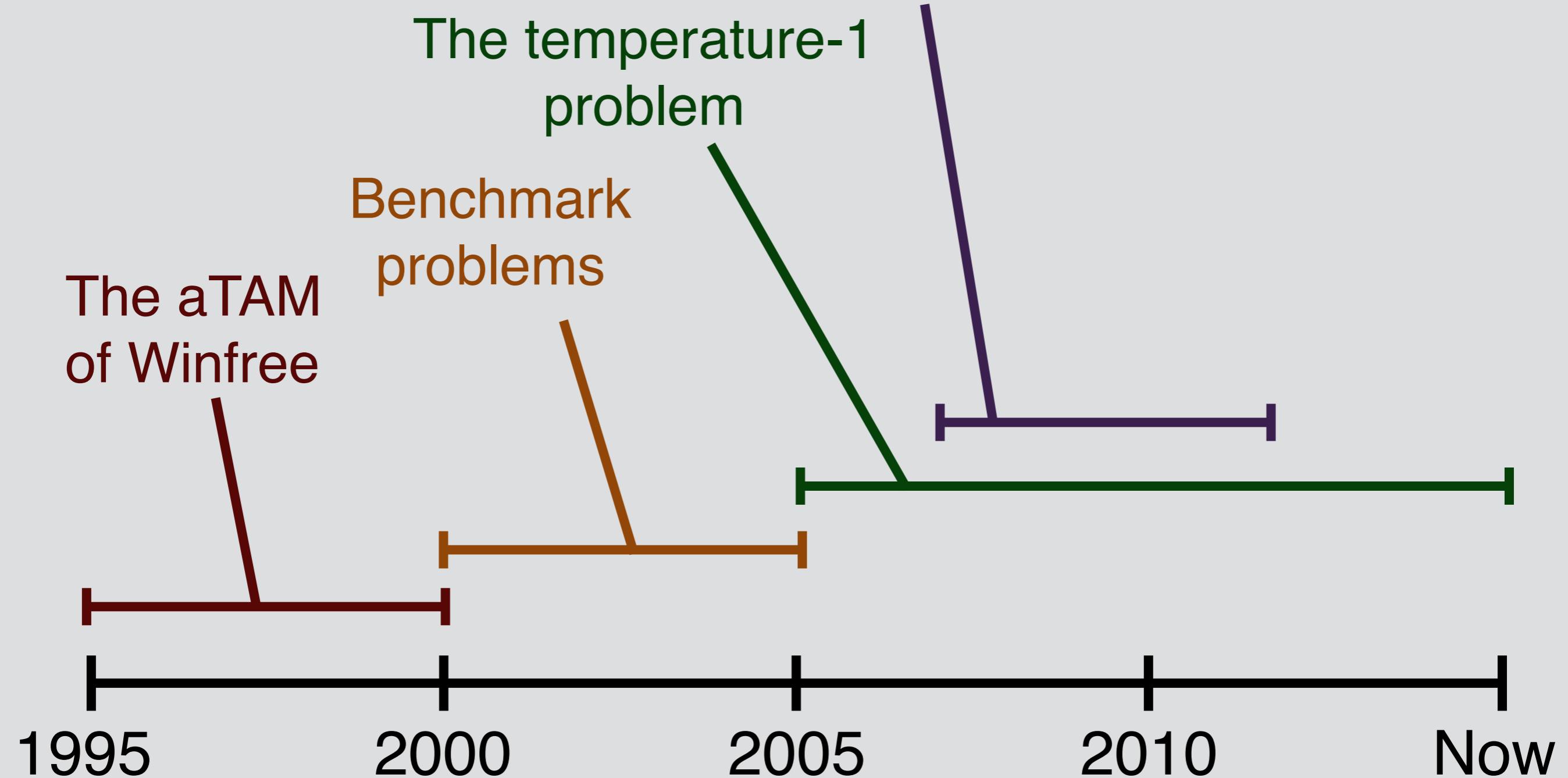


Universality via  
weak cooperation

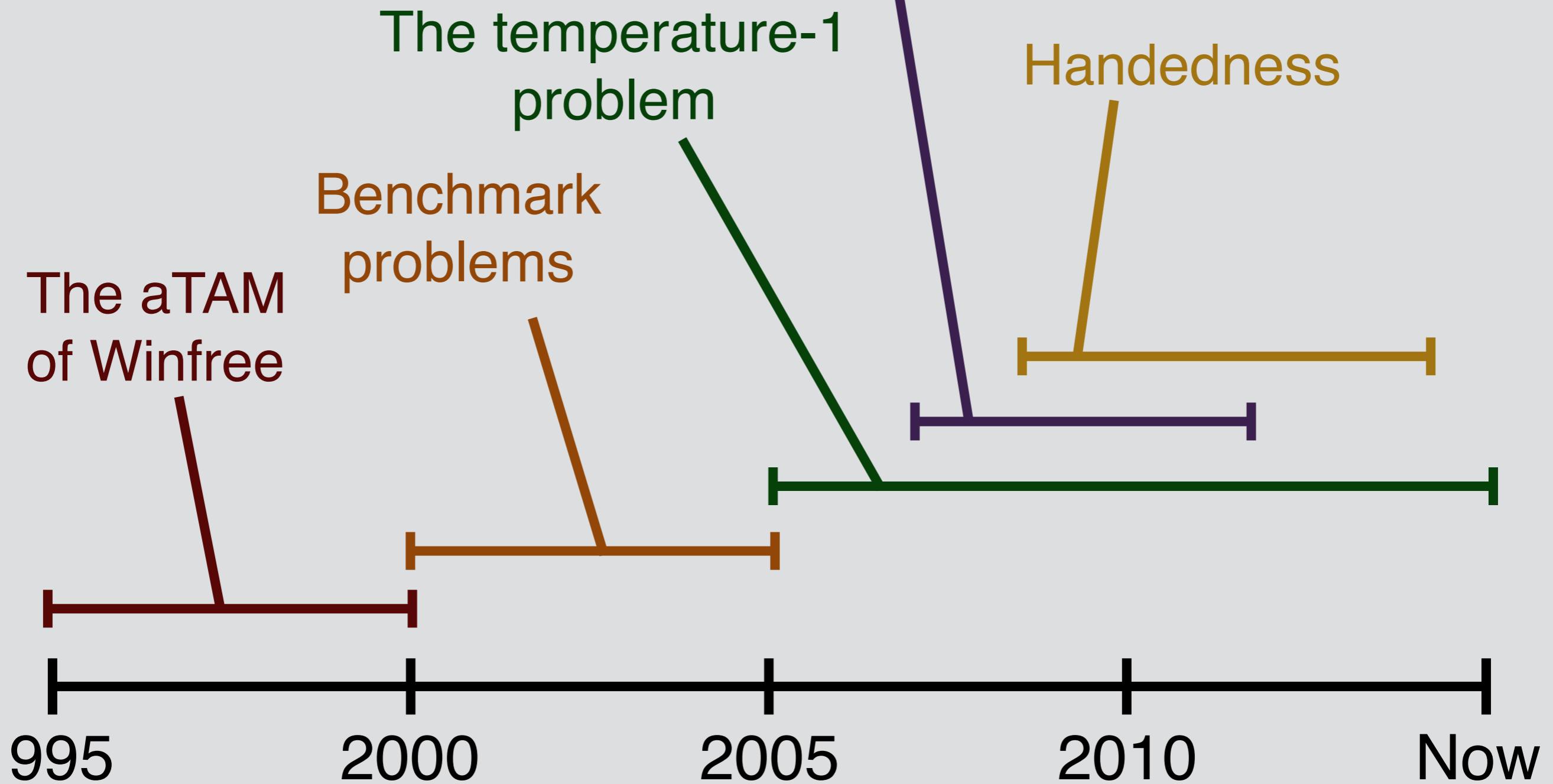
The temperature-1  
problem

Benchmark  
problems

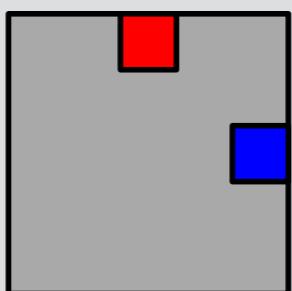
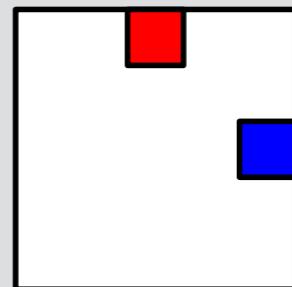
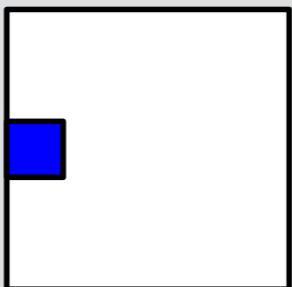
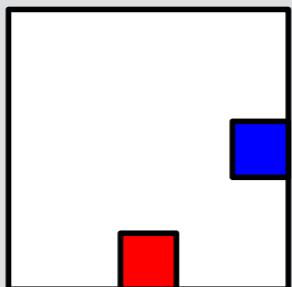
The aTAM  
of Winfree



Universality via  
weak cooperation

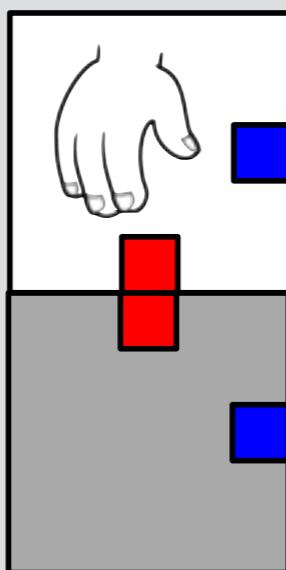
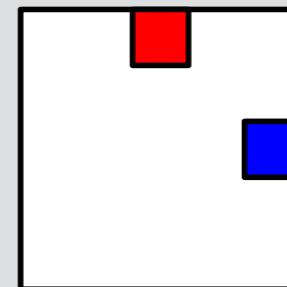
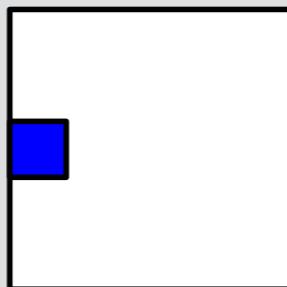
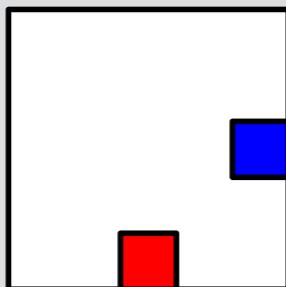


# One-handed assembly (aTAM)

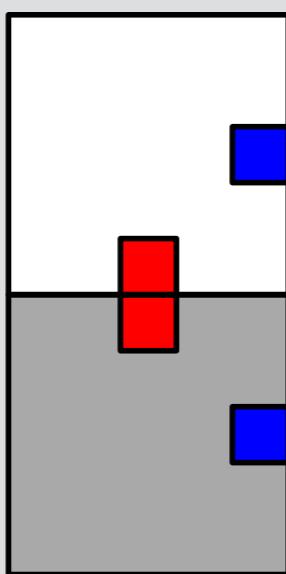
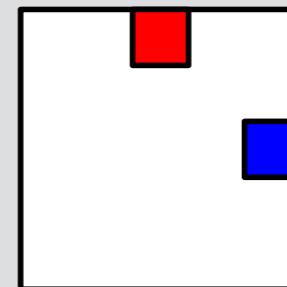
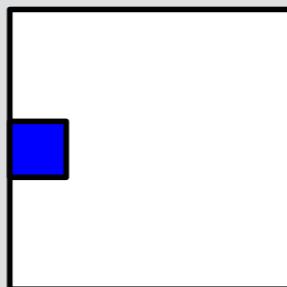
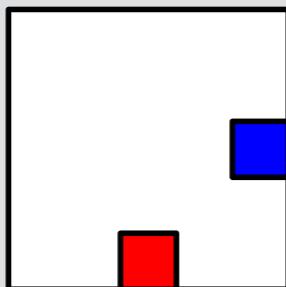


Temperature 1

# One-handed assembly (aTAM)

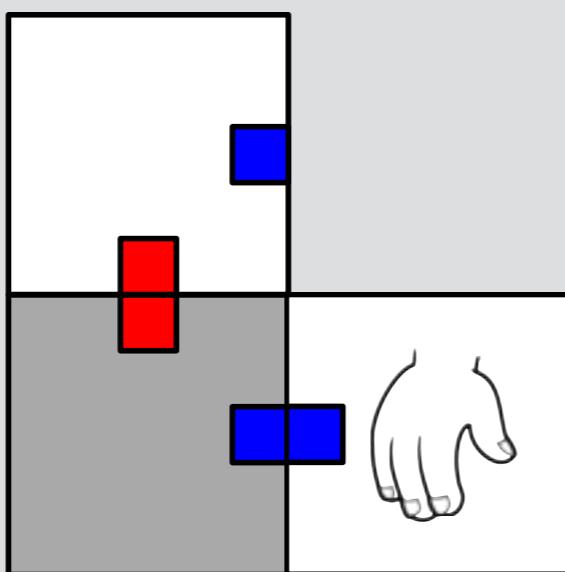
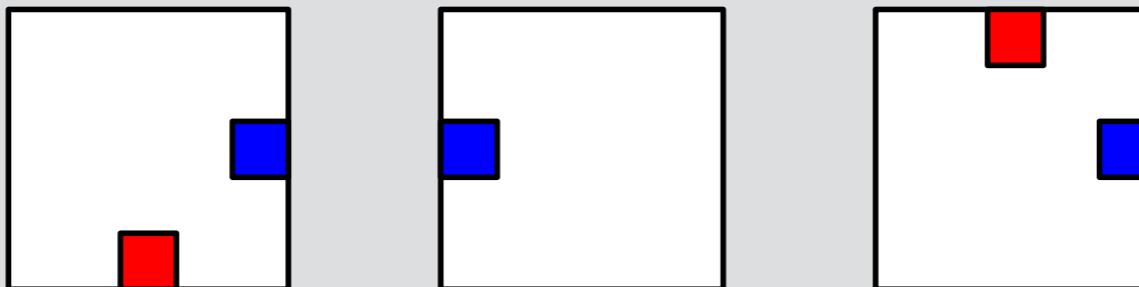


# One-handed assembly (aTAM)

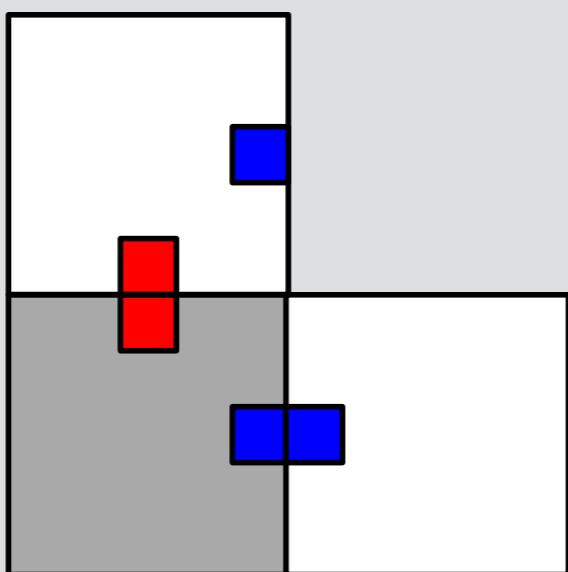
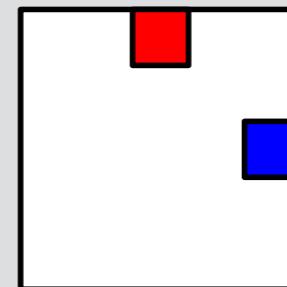
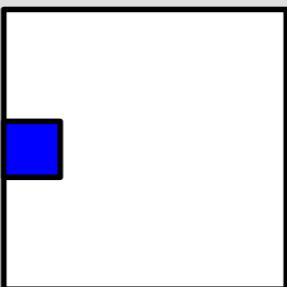
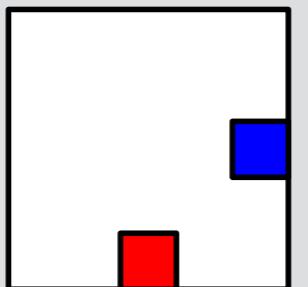


Temperature 1

# One-handed assembly (aTAM)

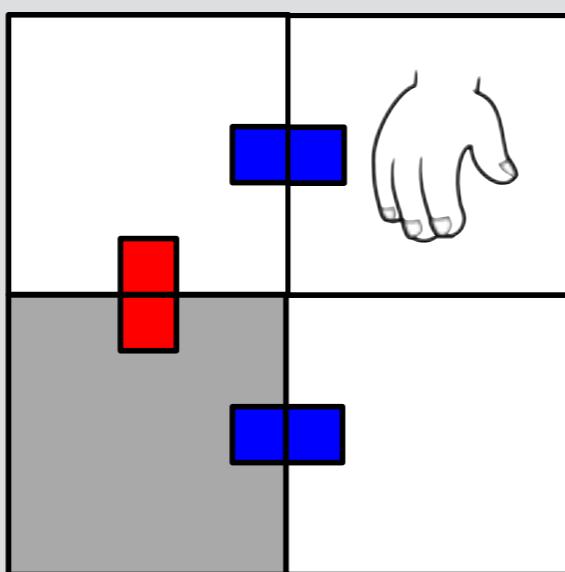
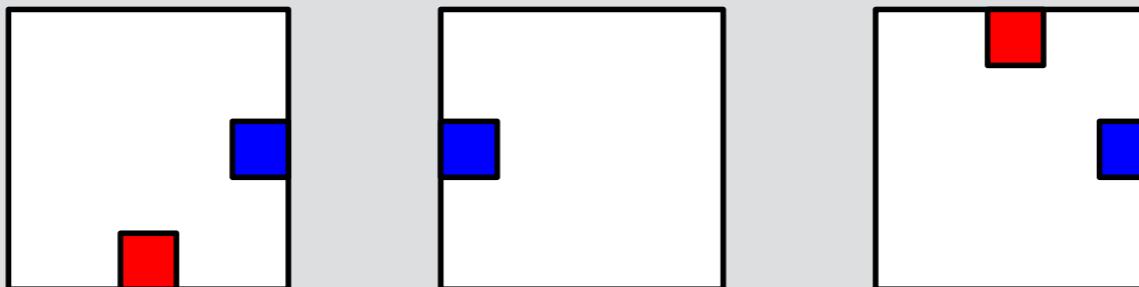


# One-handed assembly (aTAM)



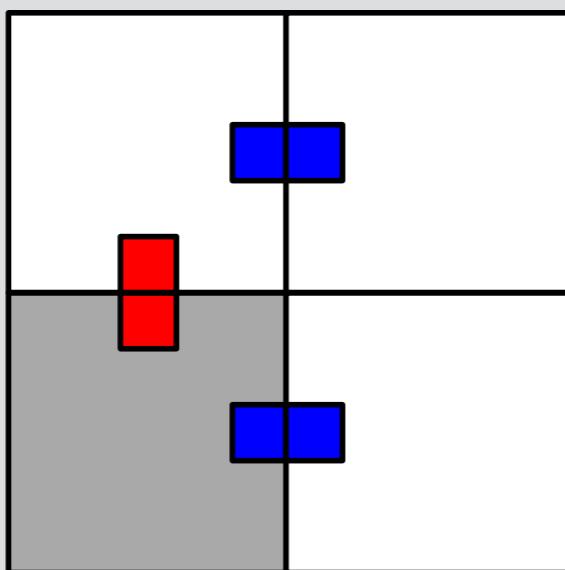
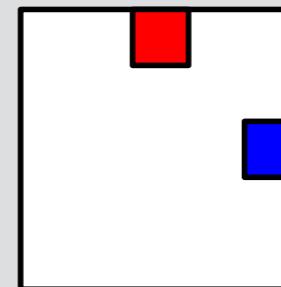
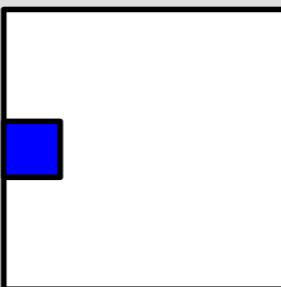
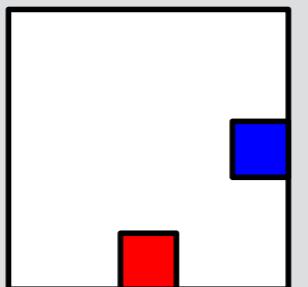
Temperature 1

# One-handed assembly (aTAM)



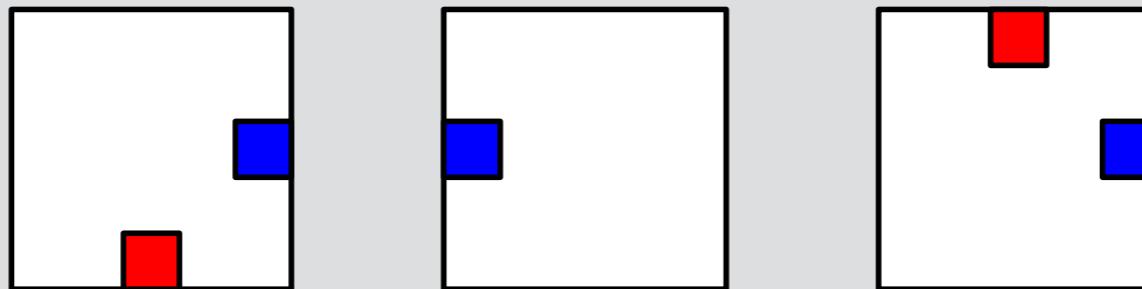
Temperature 1

# One-handed assembly (aTAM)

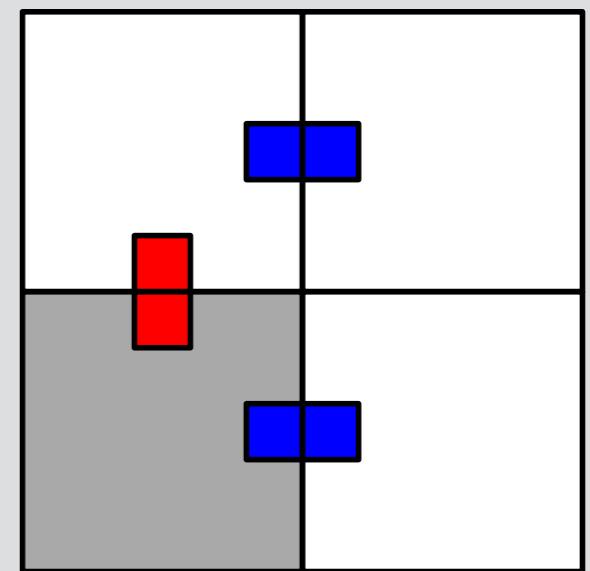
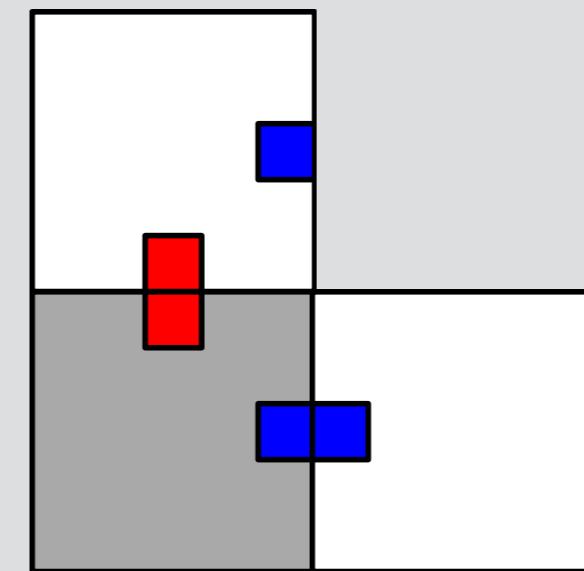
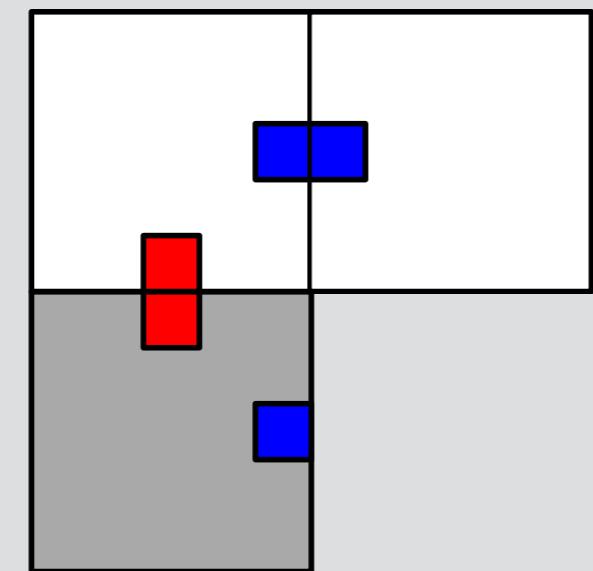
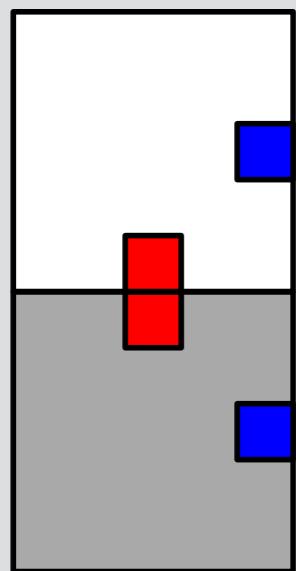
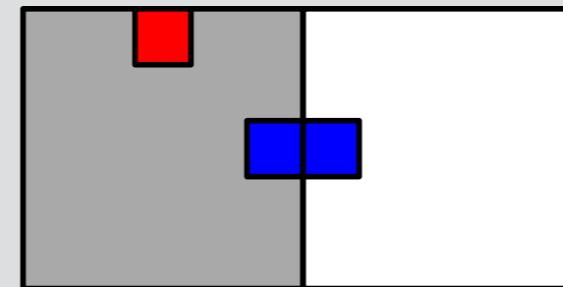
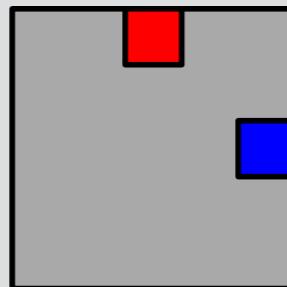


Temperature 1

# One-handed assembly (aTAM)

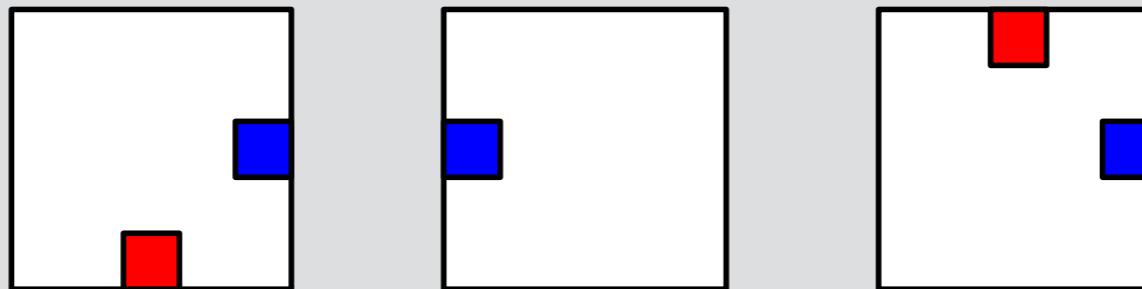


*Producible assemblies:*

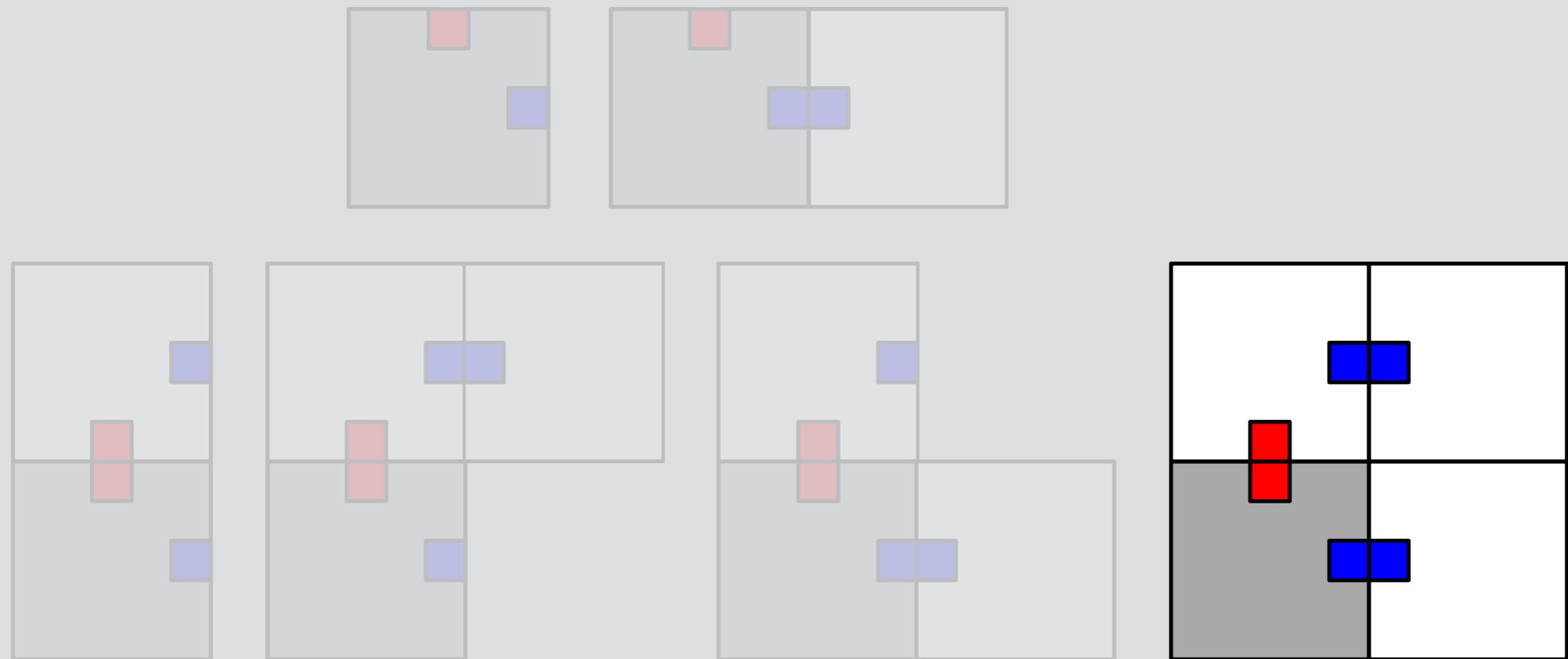


Temperature 1

# One-handed assembly (aTAM)



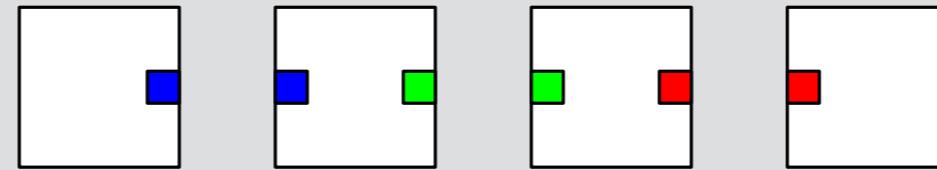
*Terminal assemblies:*



Temperature 1

# Two-handed assembly (2HAM)

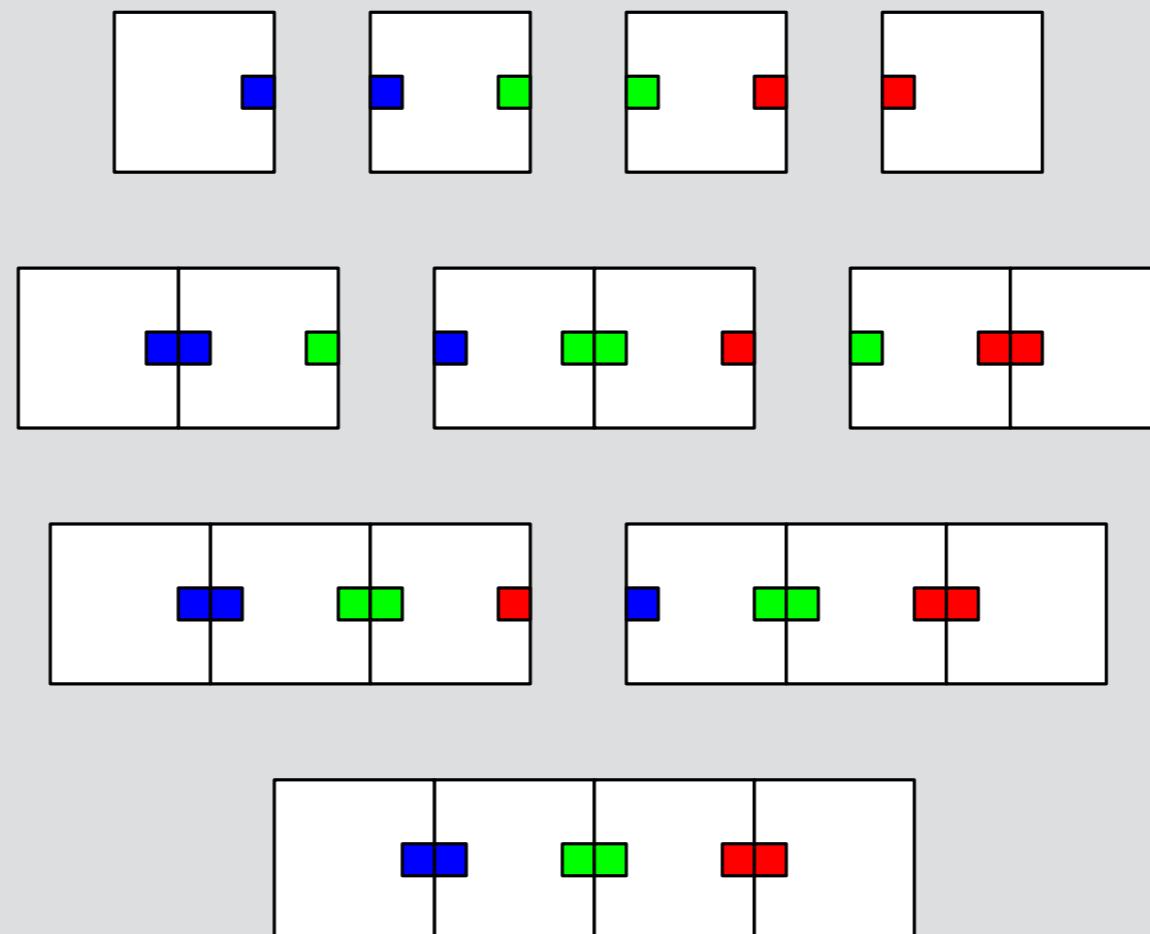
# Two-handed assembly (2HAM)



Temperature 1

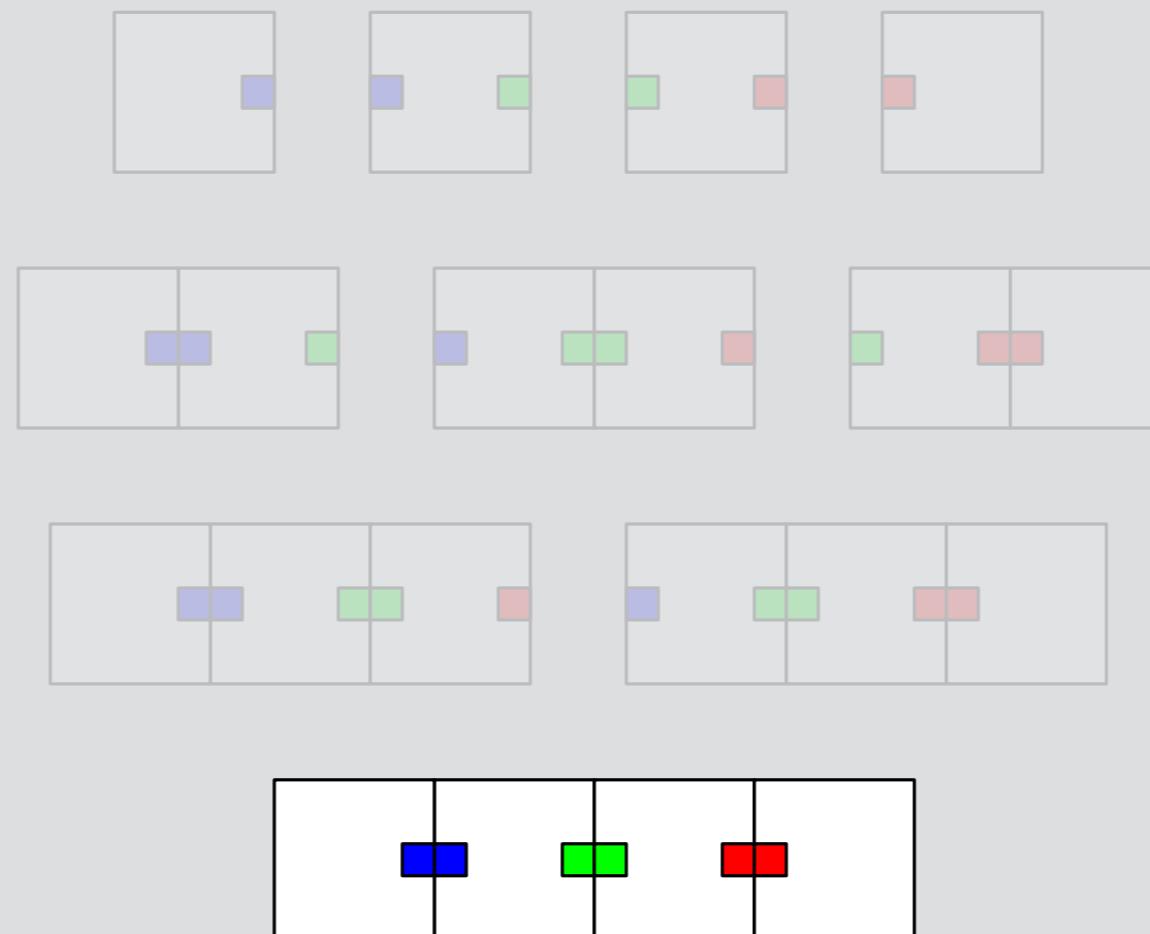
# Two-handed assembly (2HAM)

Producible assemblies:



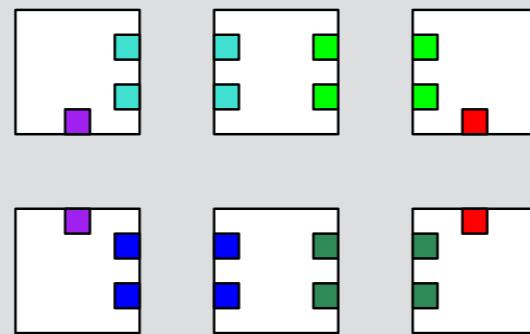
# Two-handed assembly (2HAM)

Terminal assemblies:



Temperature 1

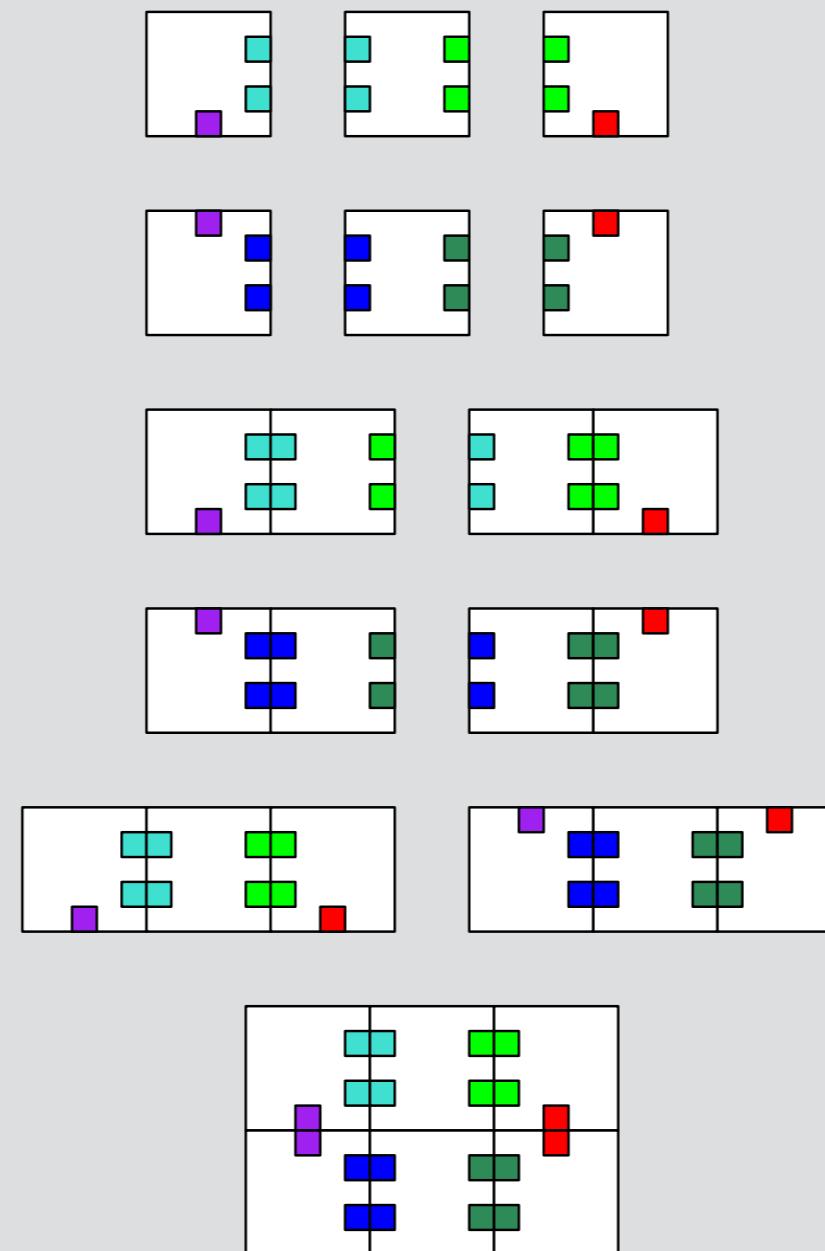
# Two-handed assembly (2HAM)



Temperature 2

# Two-handed assembly (2HAM)

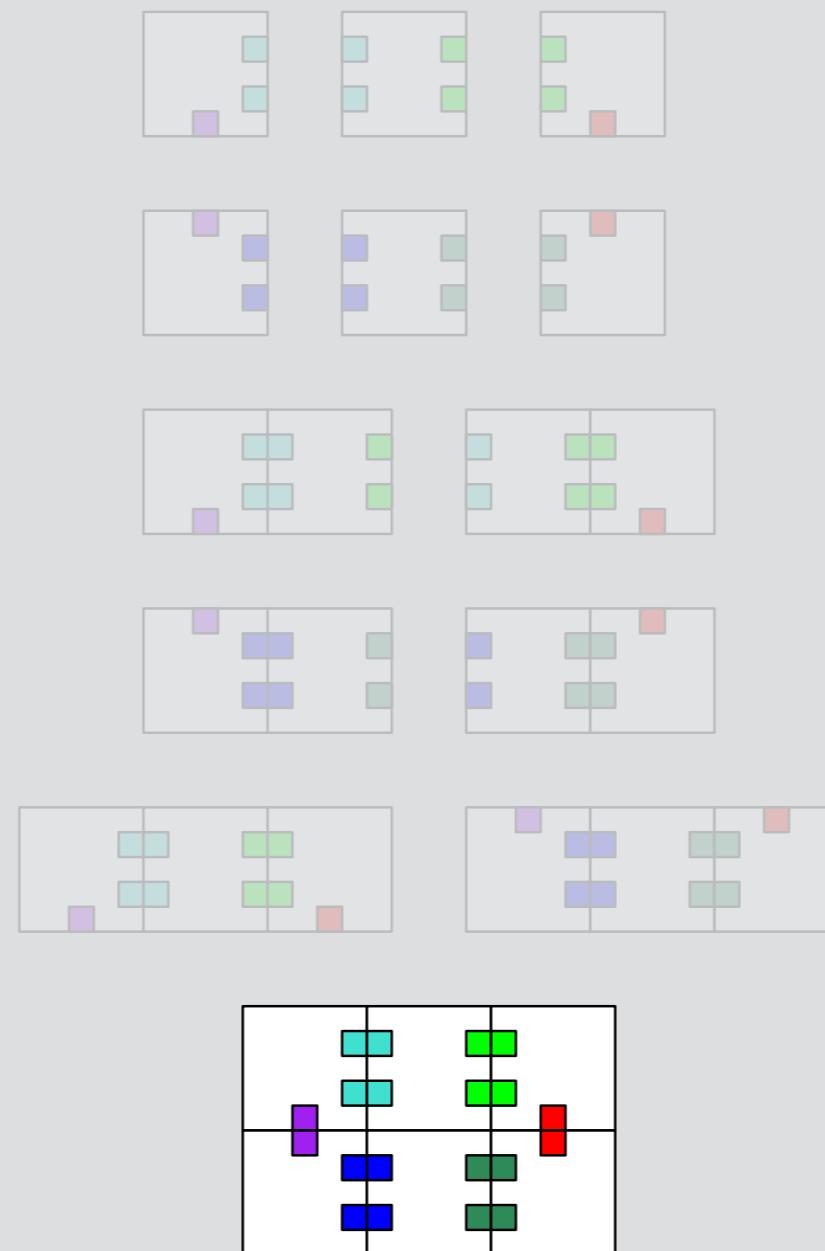
Producible assemblies:



Temperature 2

# Two-handed assembly (2HAM)

Terminal assemblies:



Temperature 2

# aTAM



Producible w/seed

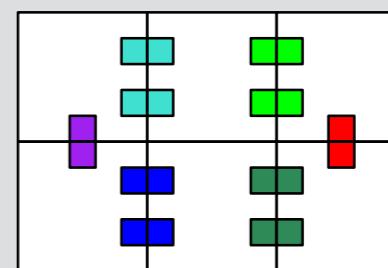
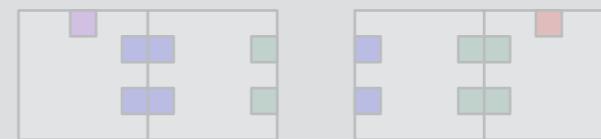
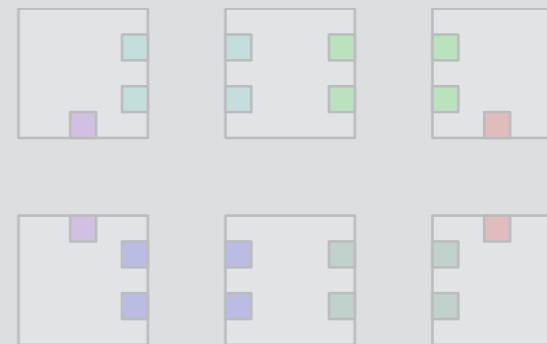


Producible w/seed



# 2HAM

Producible:



Temperature 2

# Is a seed necessary for some results?

no spurious nucleation

universal computation?  
efficient shape construction?

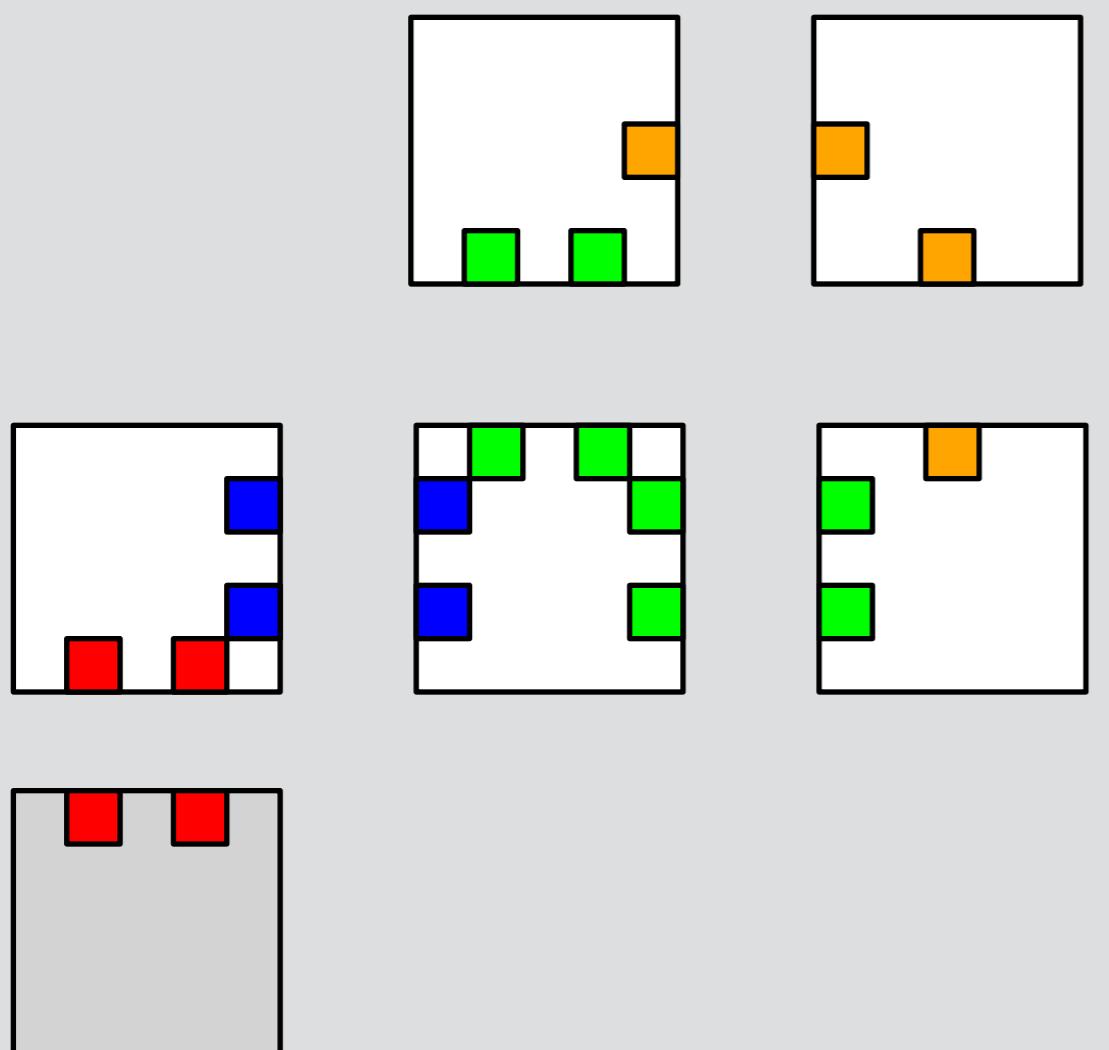
Is a seed necessary for some results? **No!**

no spurious nucleation

universal computation?  
efficient shape construction?

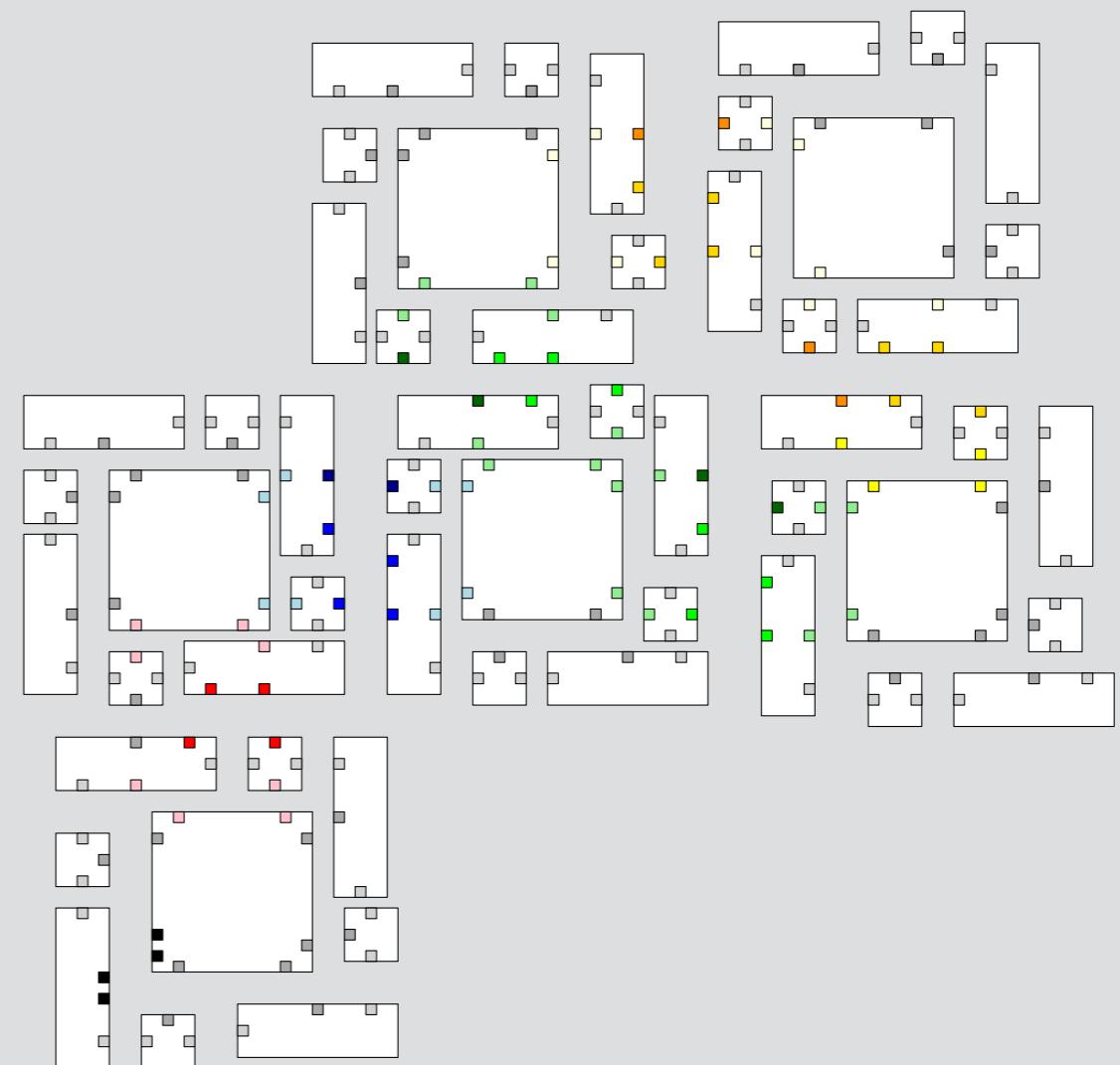
Every aTAM system can be  
*simulated* by a 2HAM system.

Every aTAM system can be  
*simulated* by a 2HAM system.



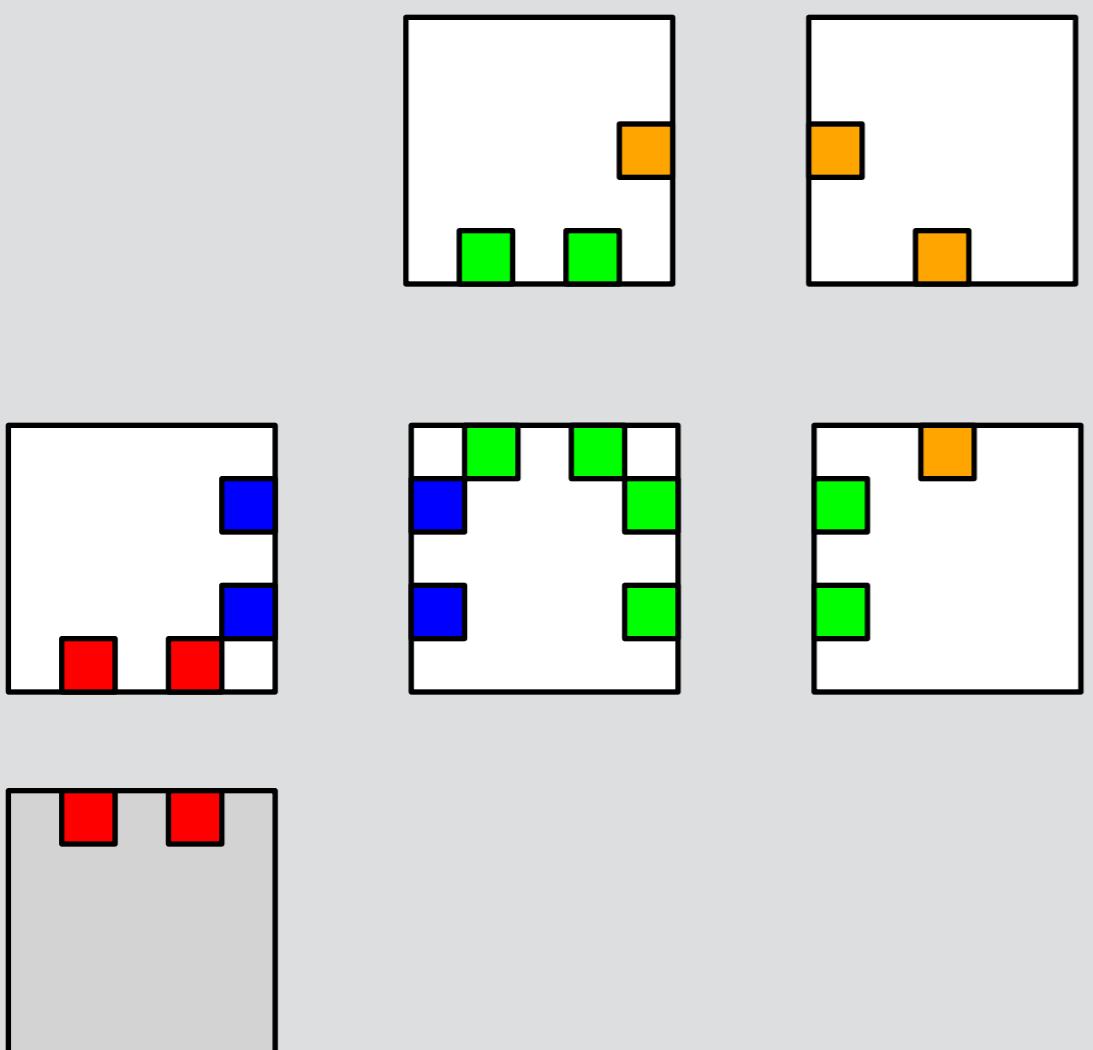
aTAM

Temperature 2



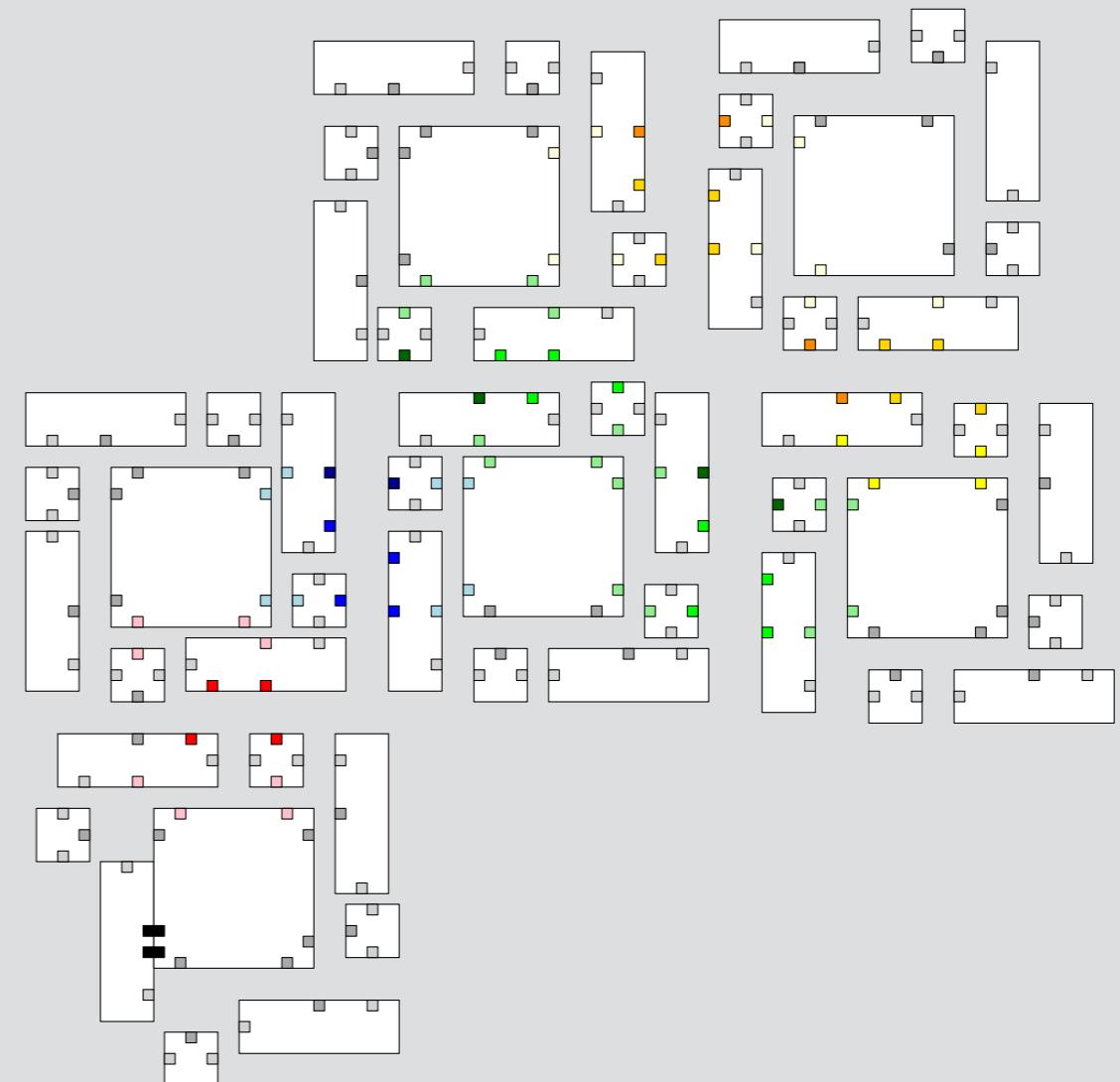
2HAM

Every aTAM system can be  
*simulated* by a 2HAM system.



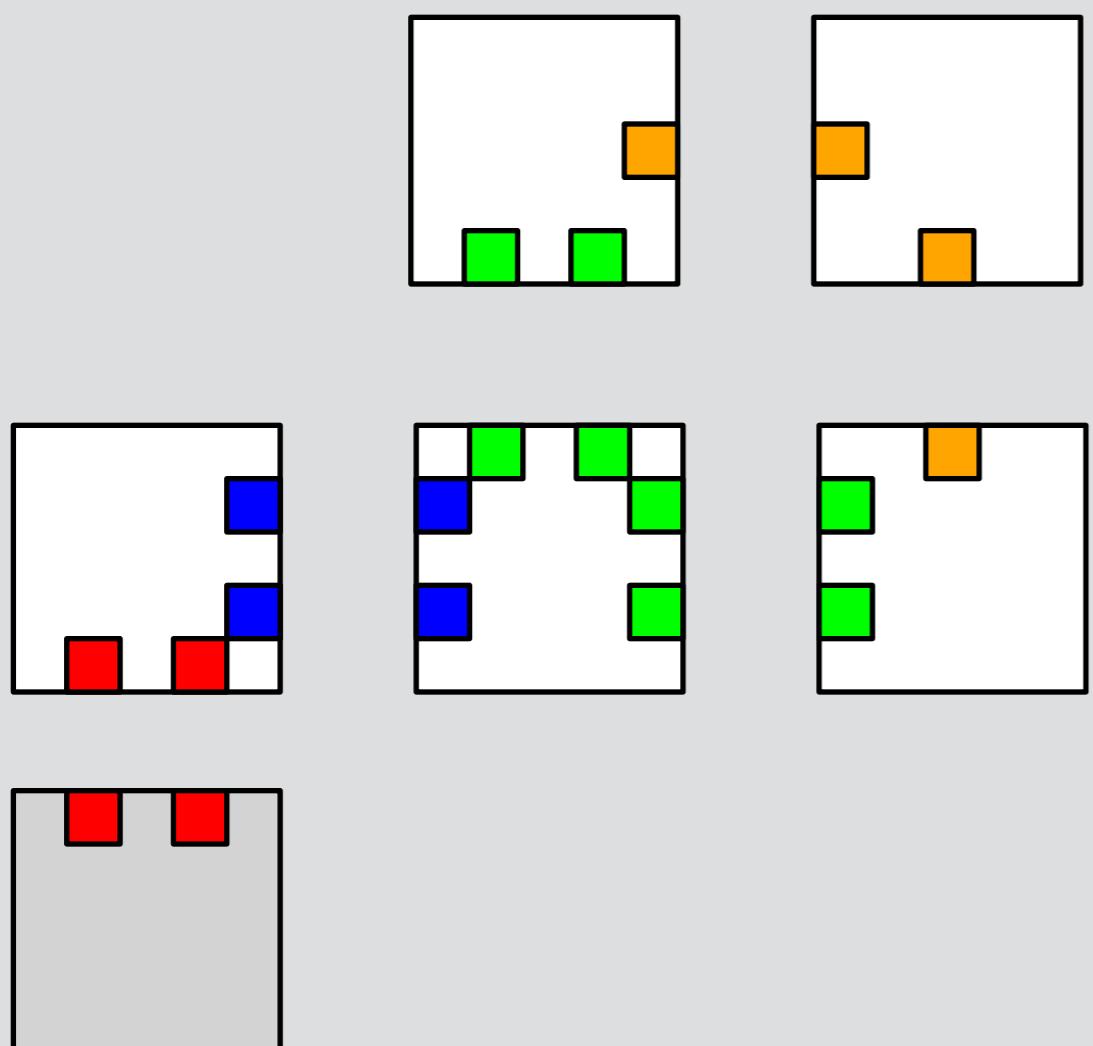
aTAM

Temperature 2



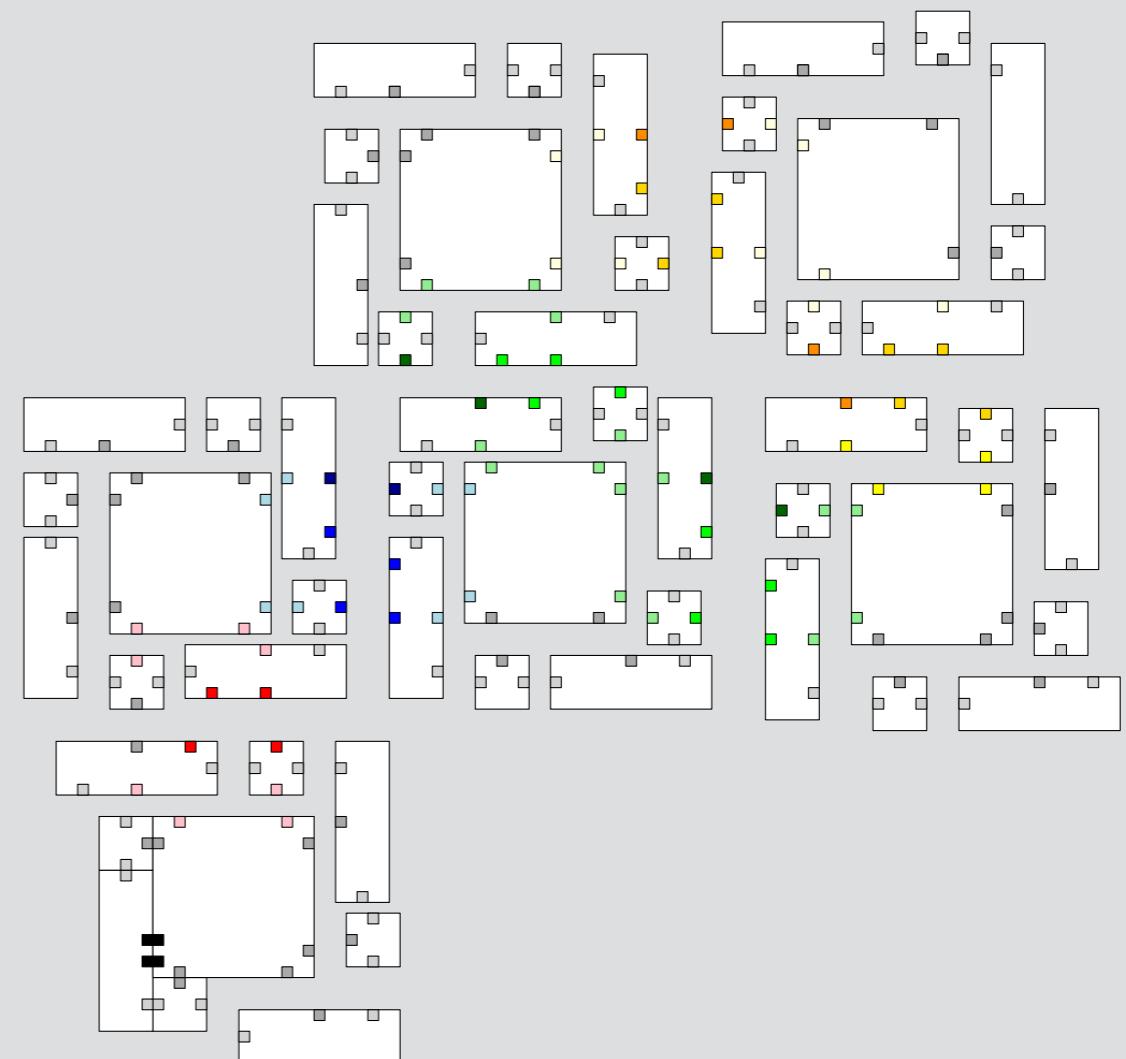
2HAM

Every aTAM system can be  
*simulated* by a 2HAM system.



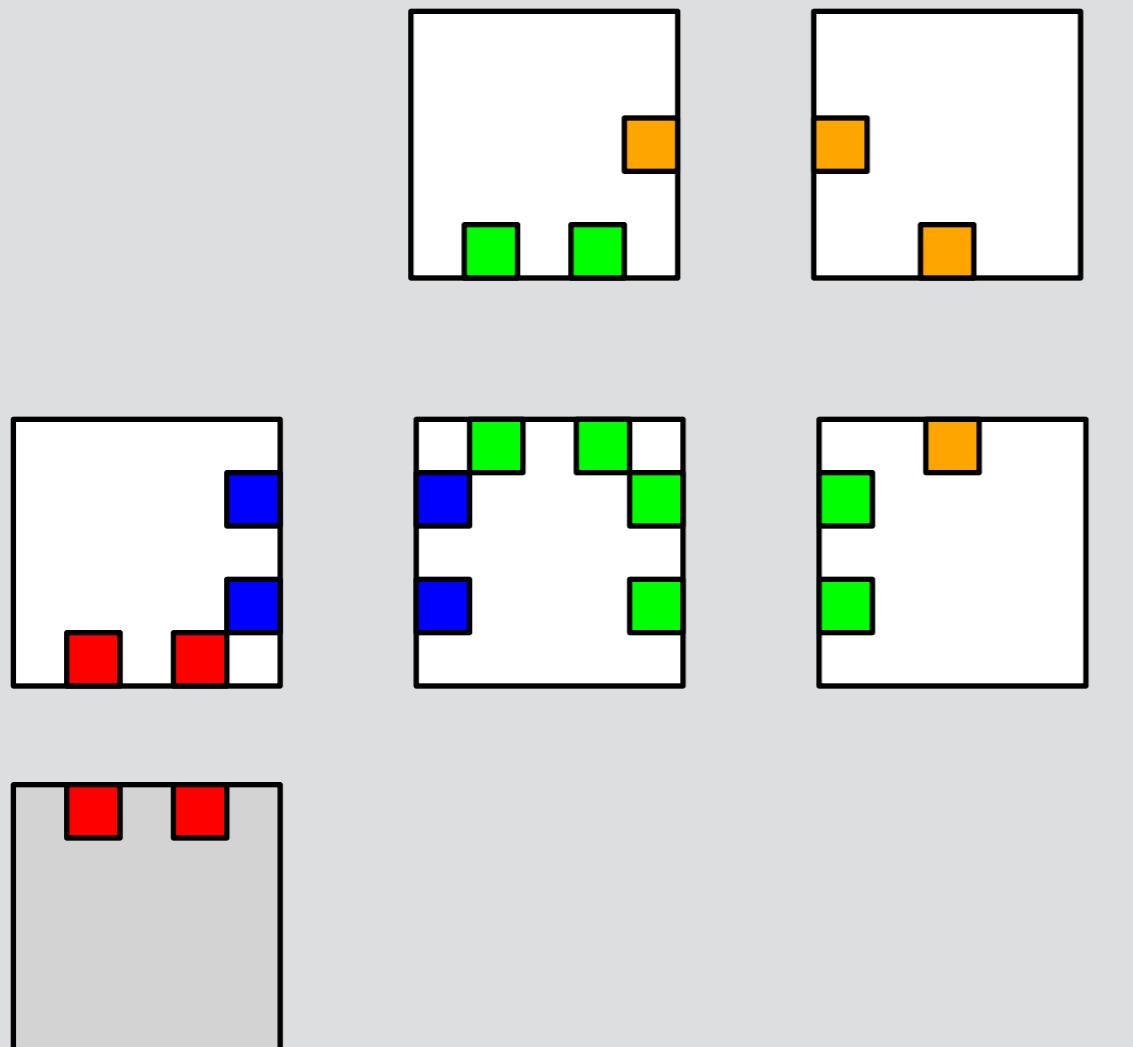
aTAM

Temperature 2



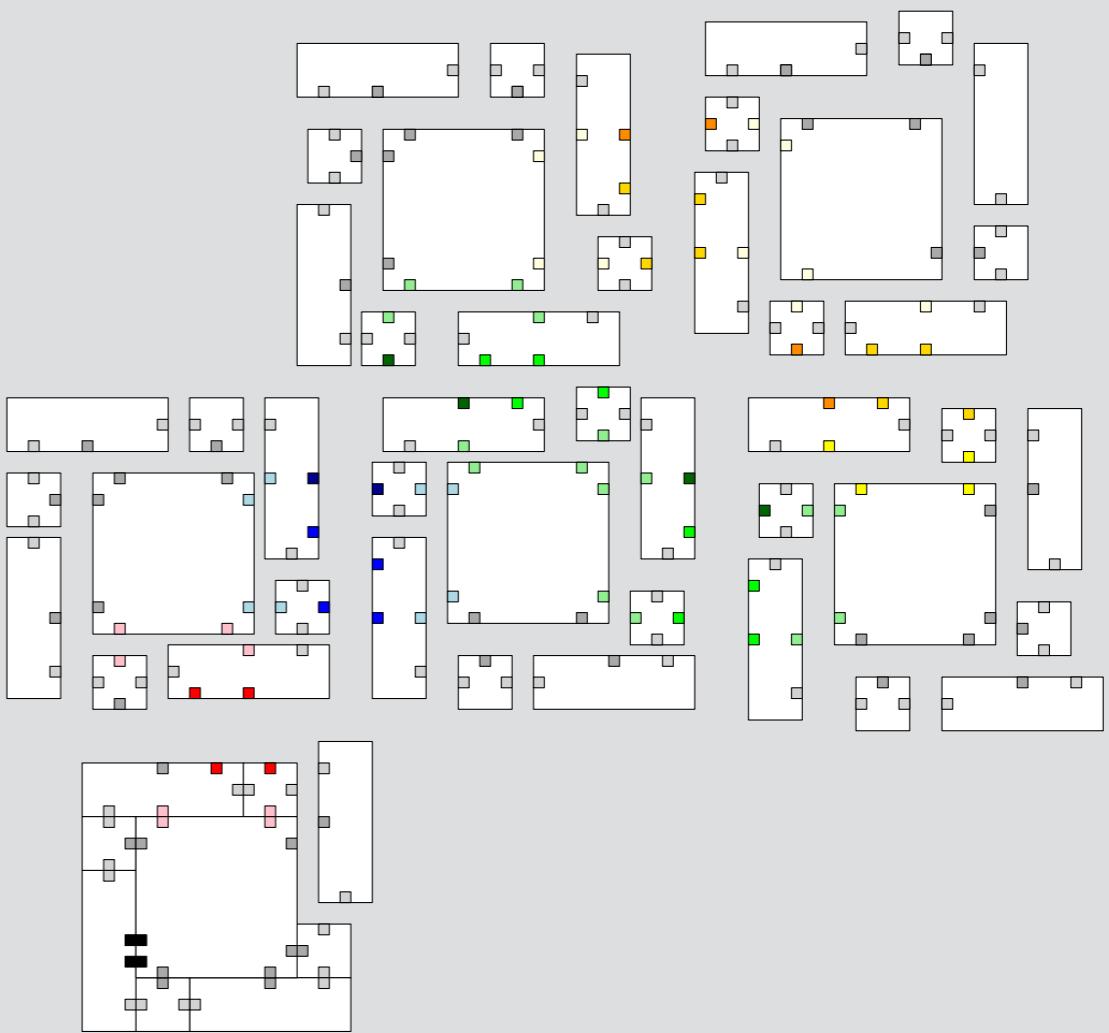
2HAM

Every aTAM system can be  
*simulated* by a 2HAM system.



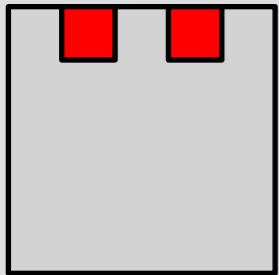
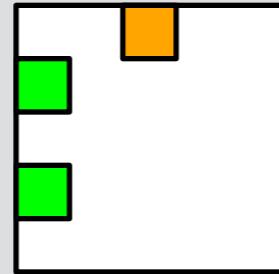
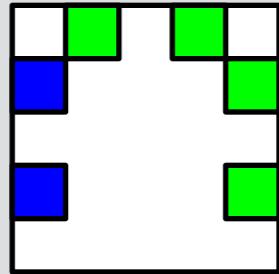
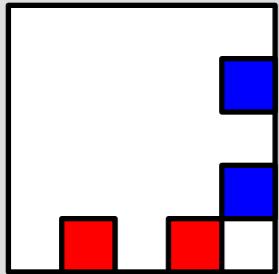
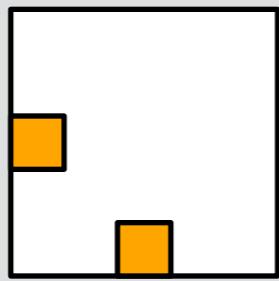
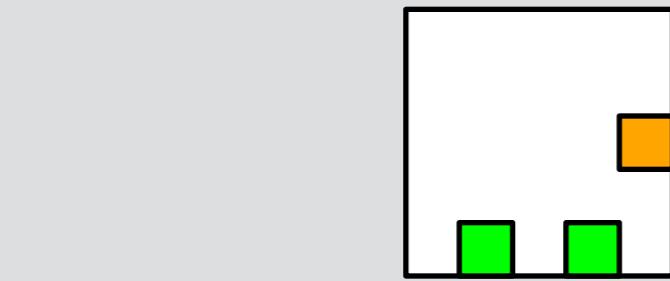
aTAM

Temperature 2



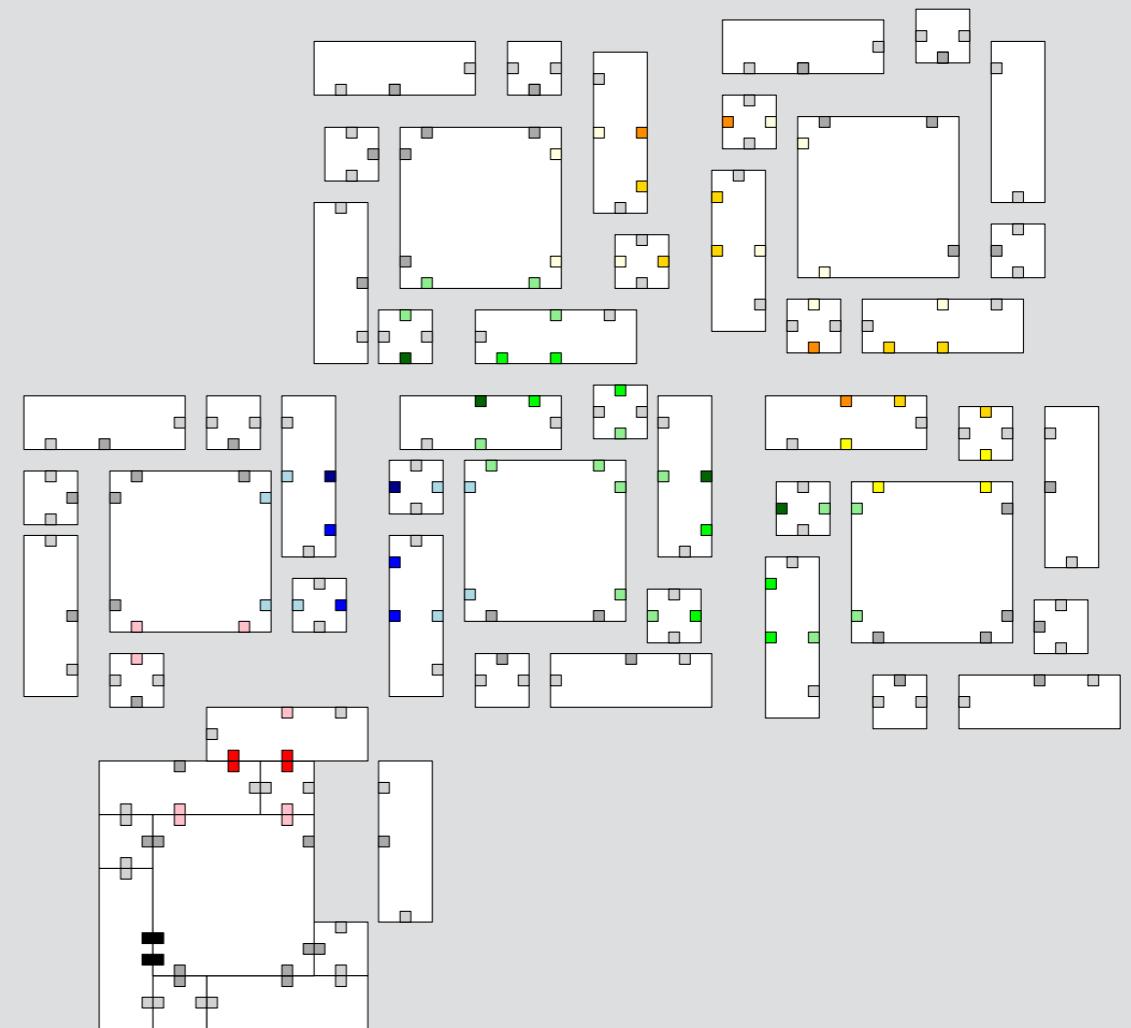
2HAM

Every aTAM system can be  
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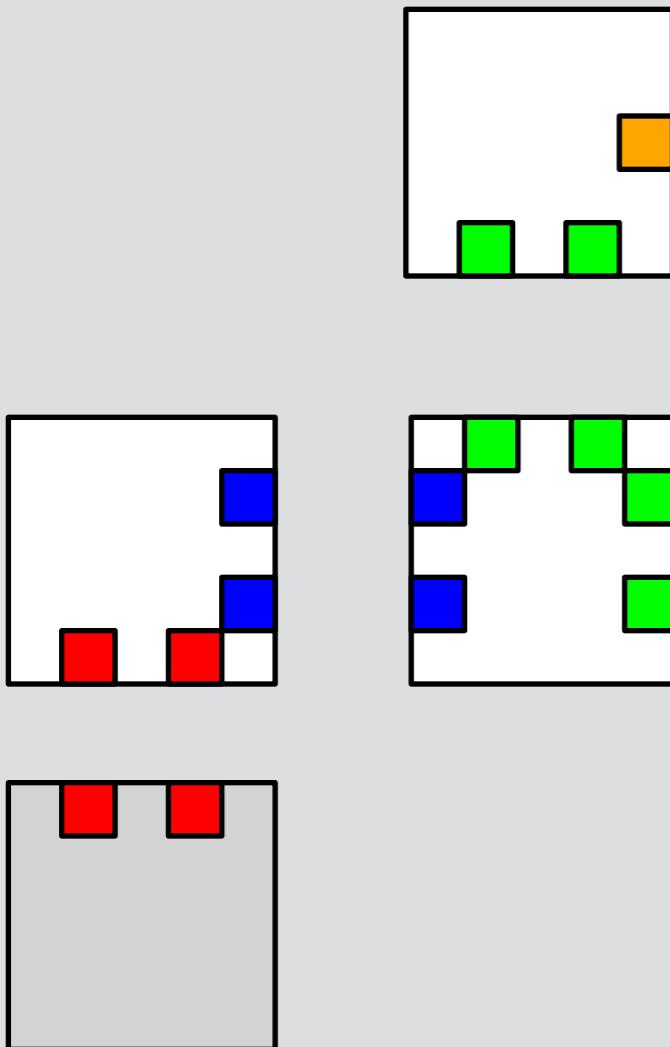
aTAM

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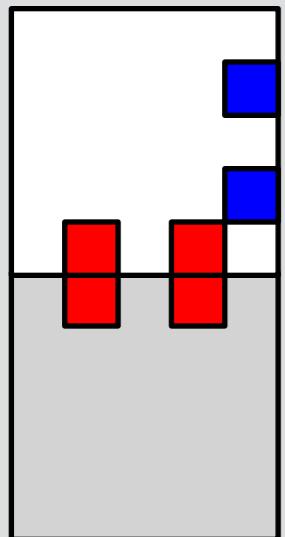
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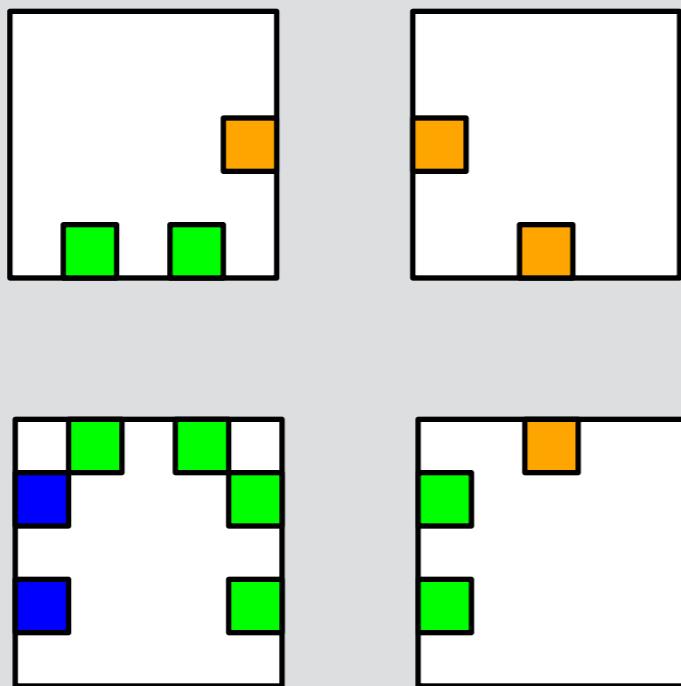
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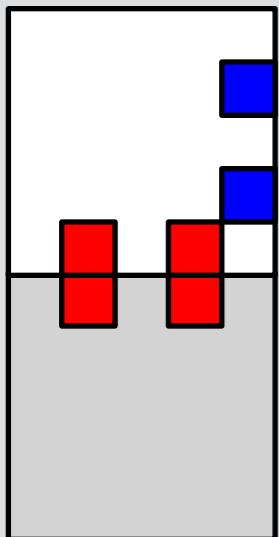


Temperature 2

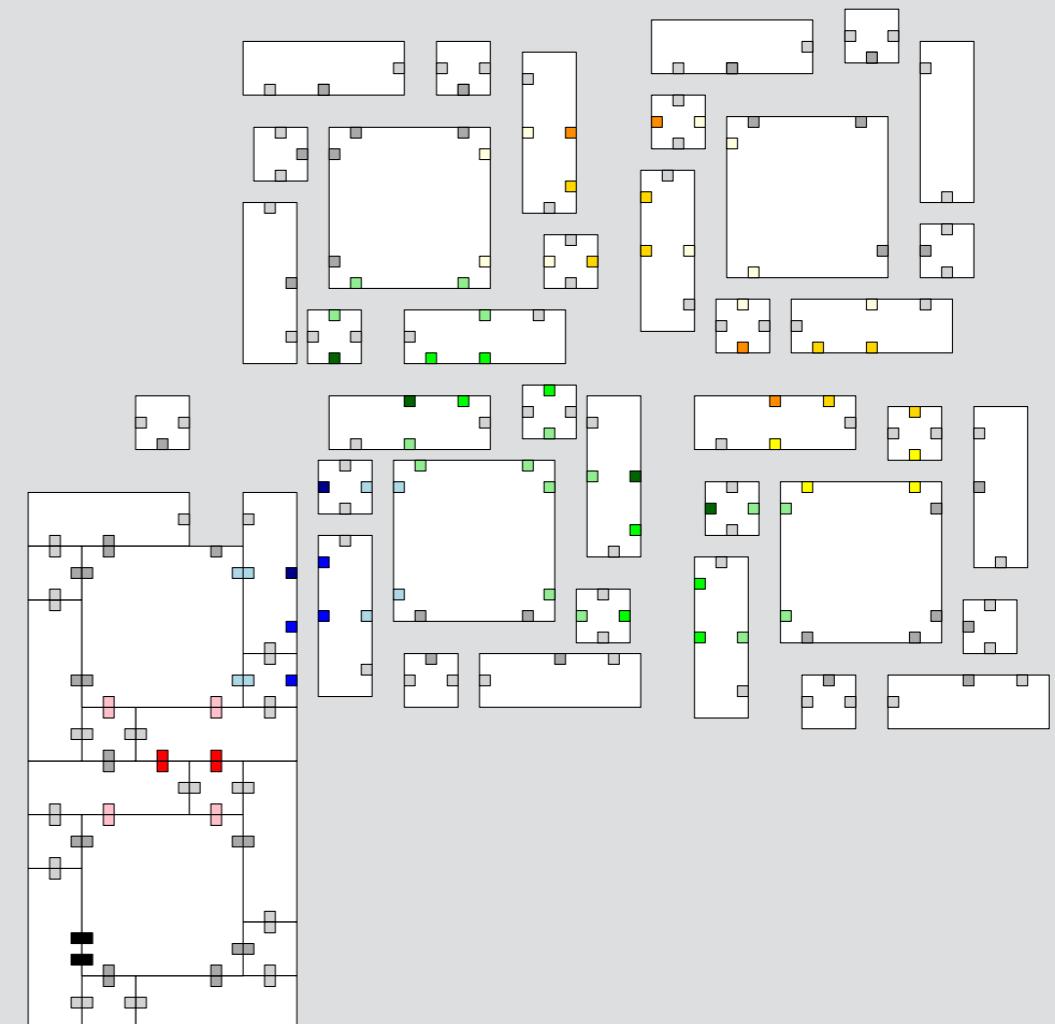
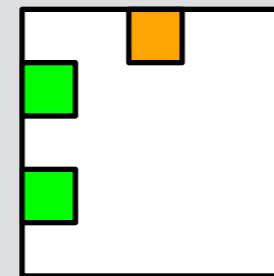
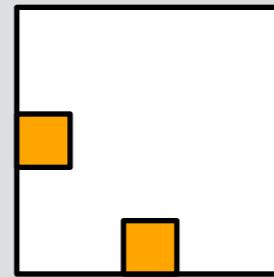
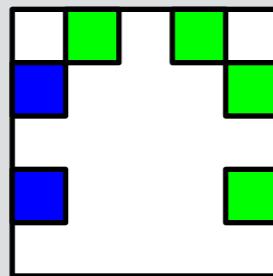
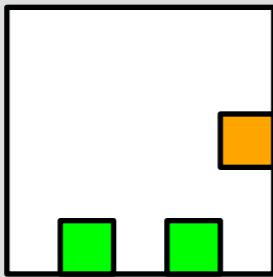


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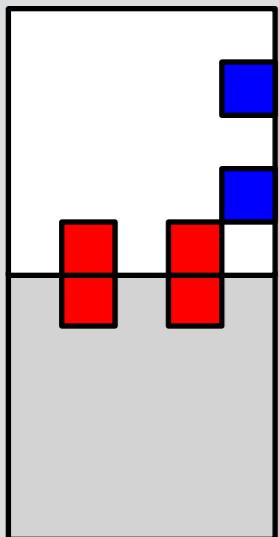
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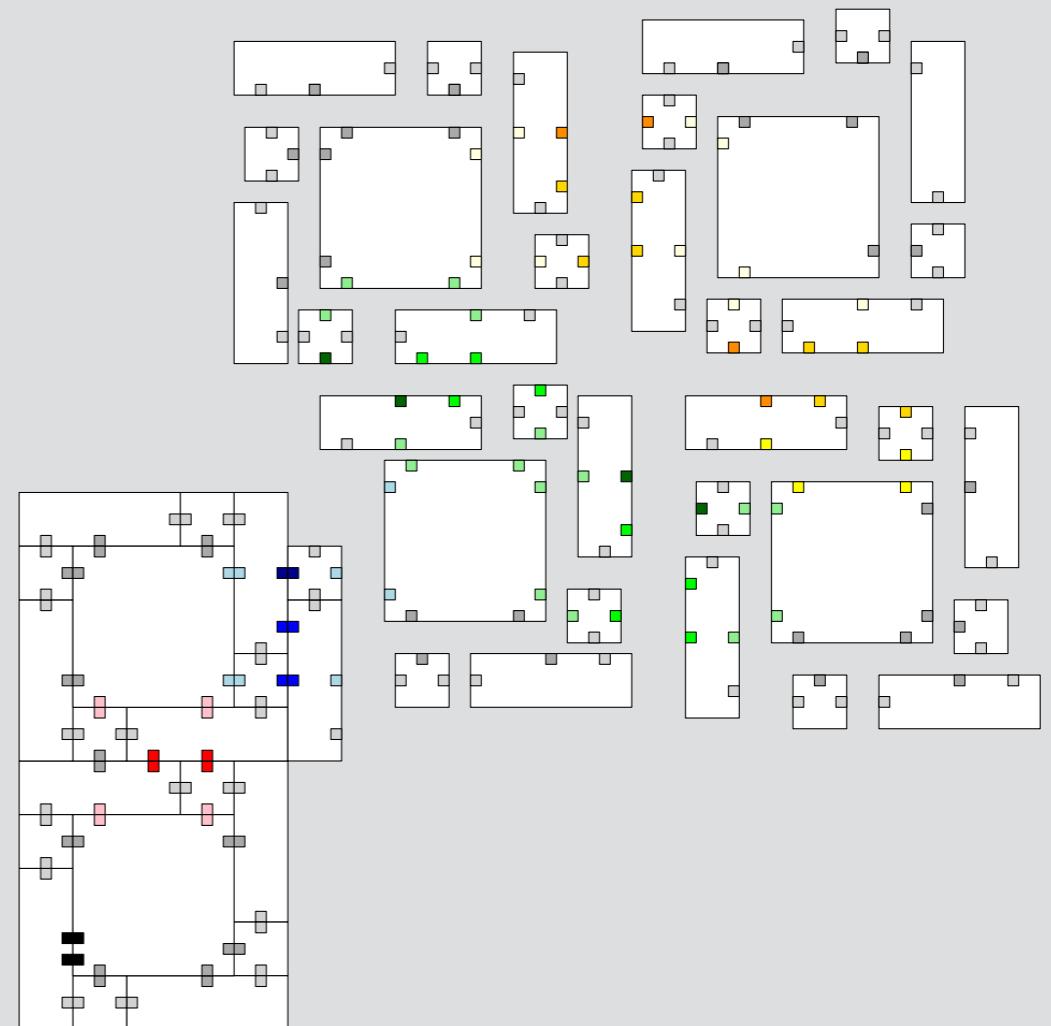
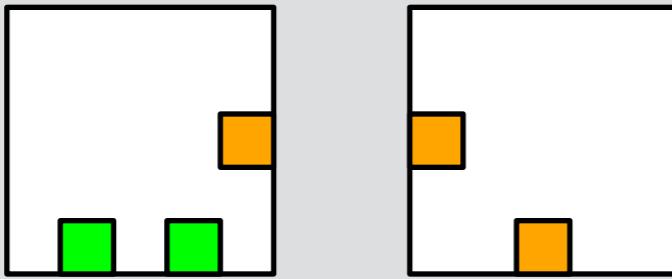
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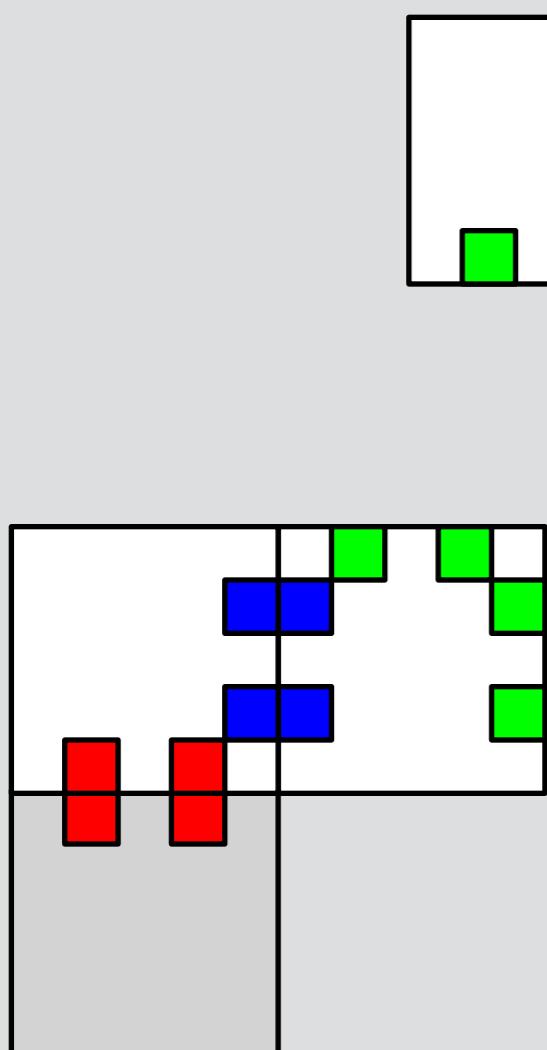
aTAM



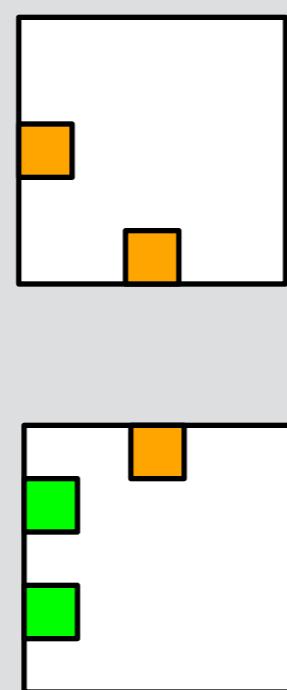
2HAM

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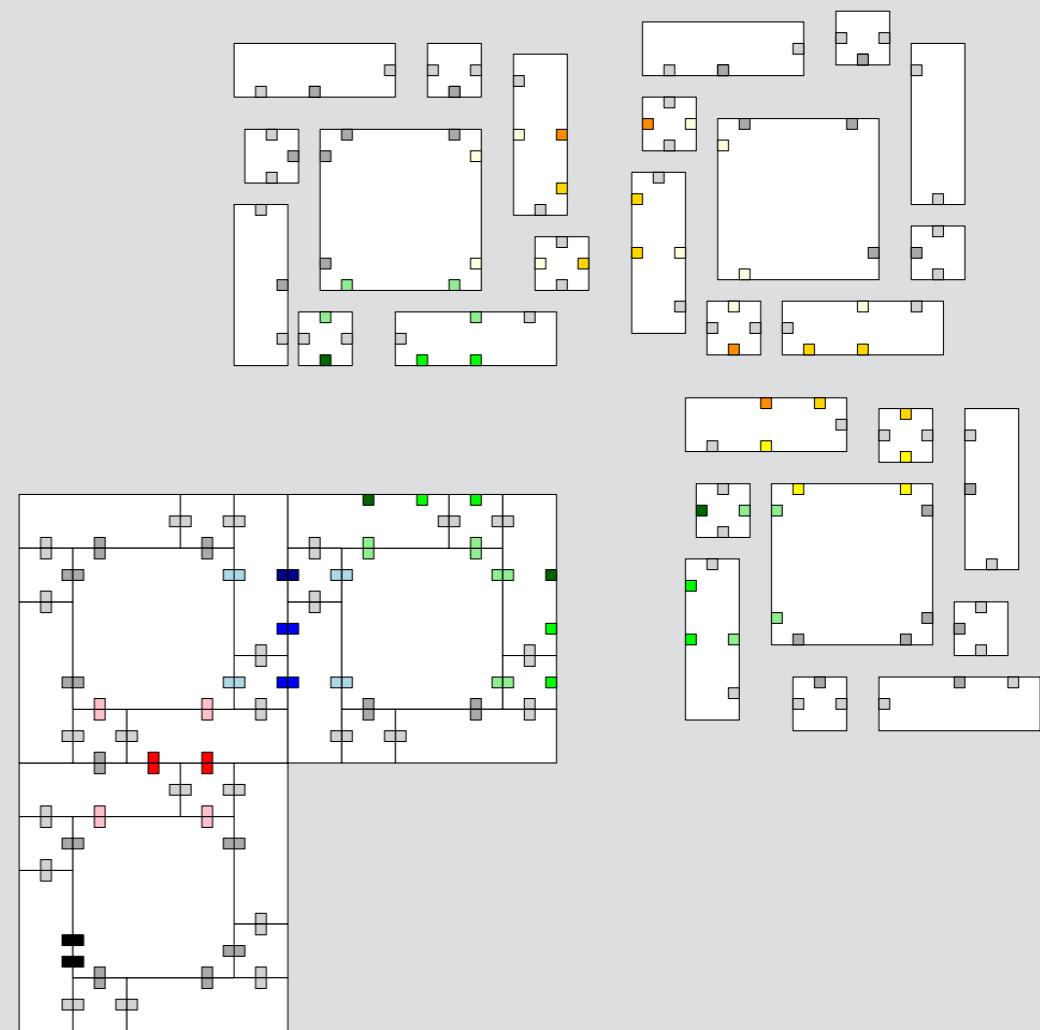
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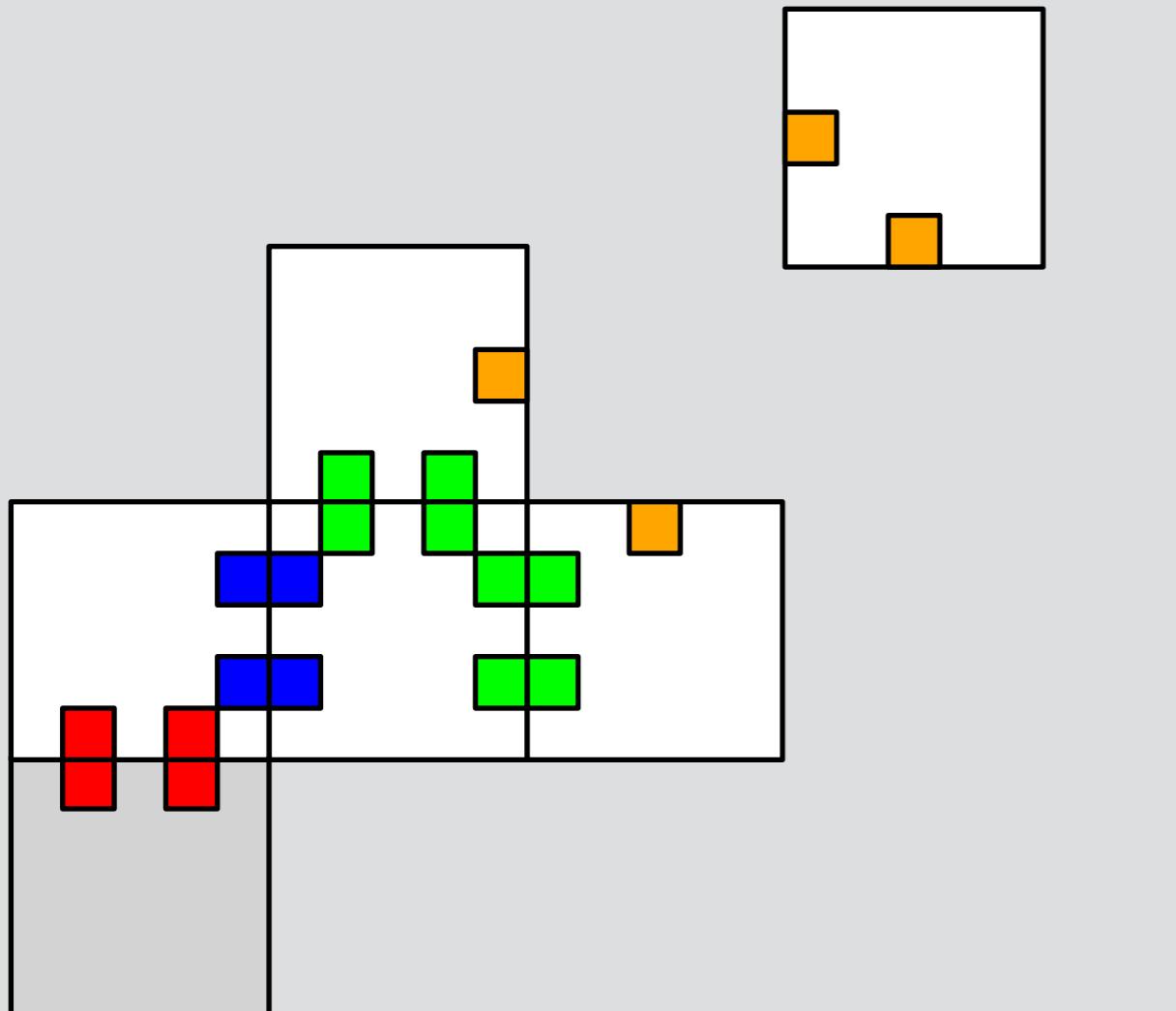


Temperature 2



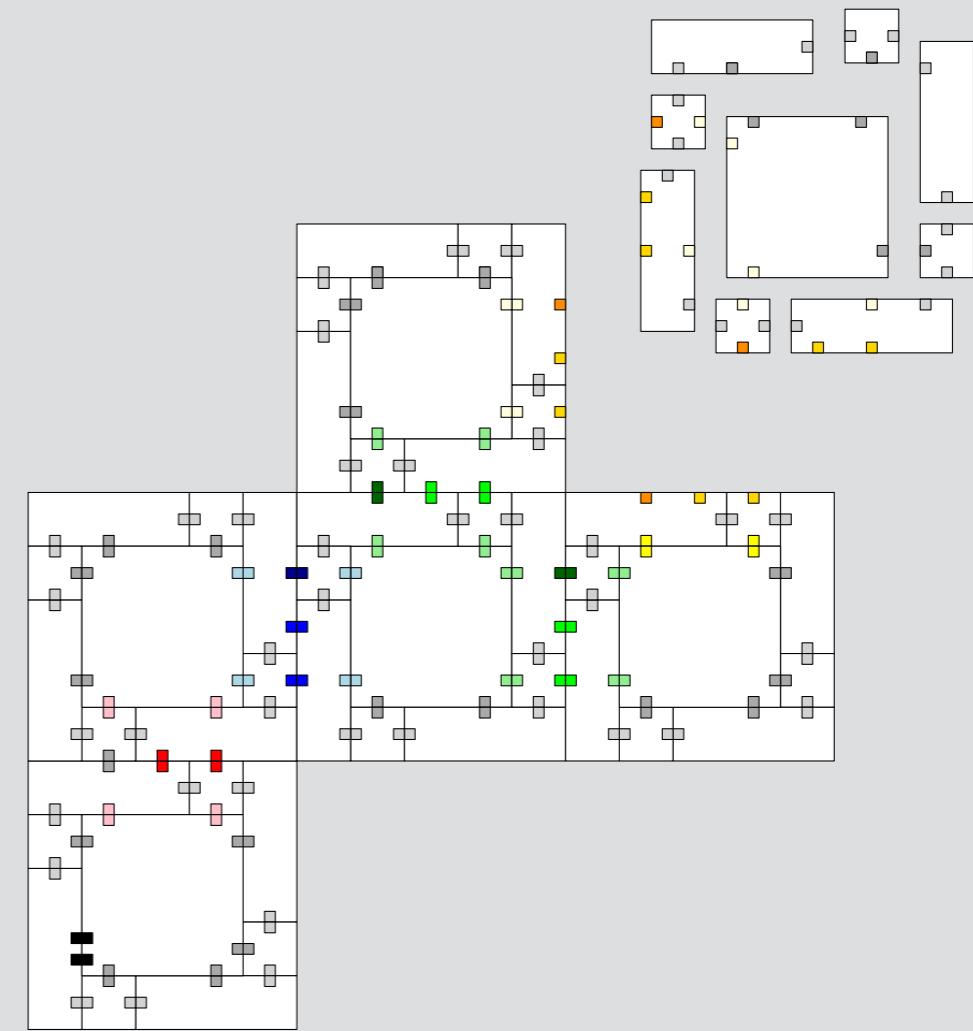
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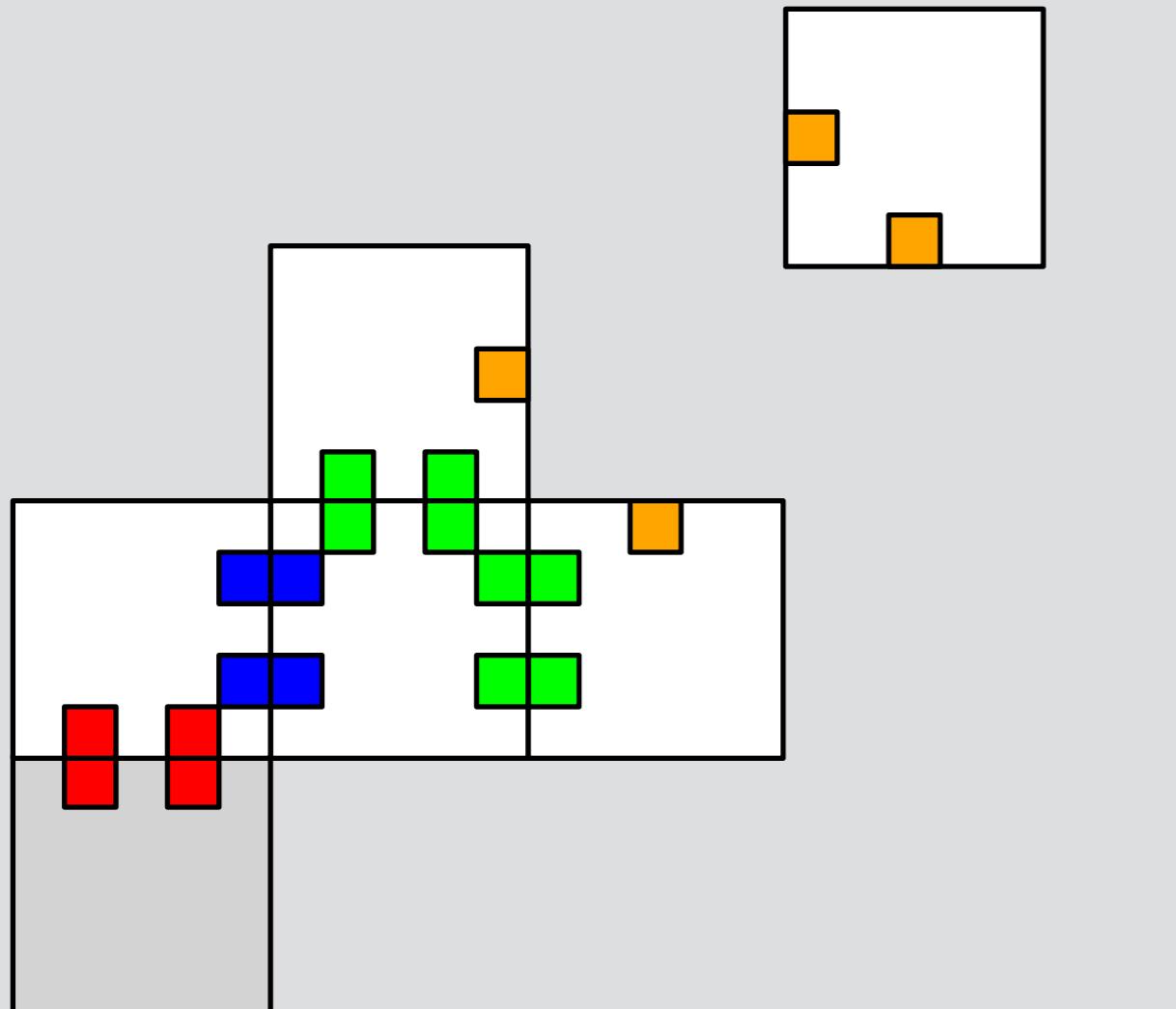
aTAM

Temperature 2



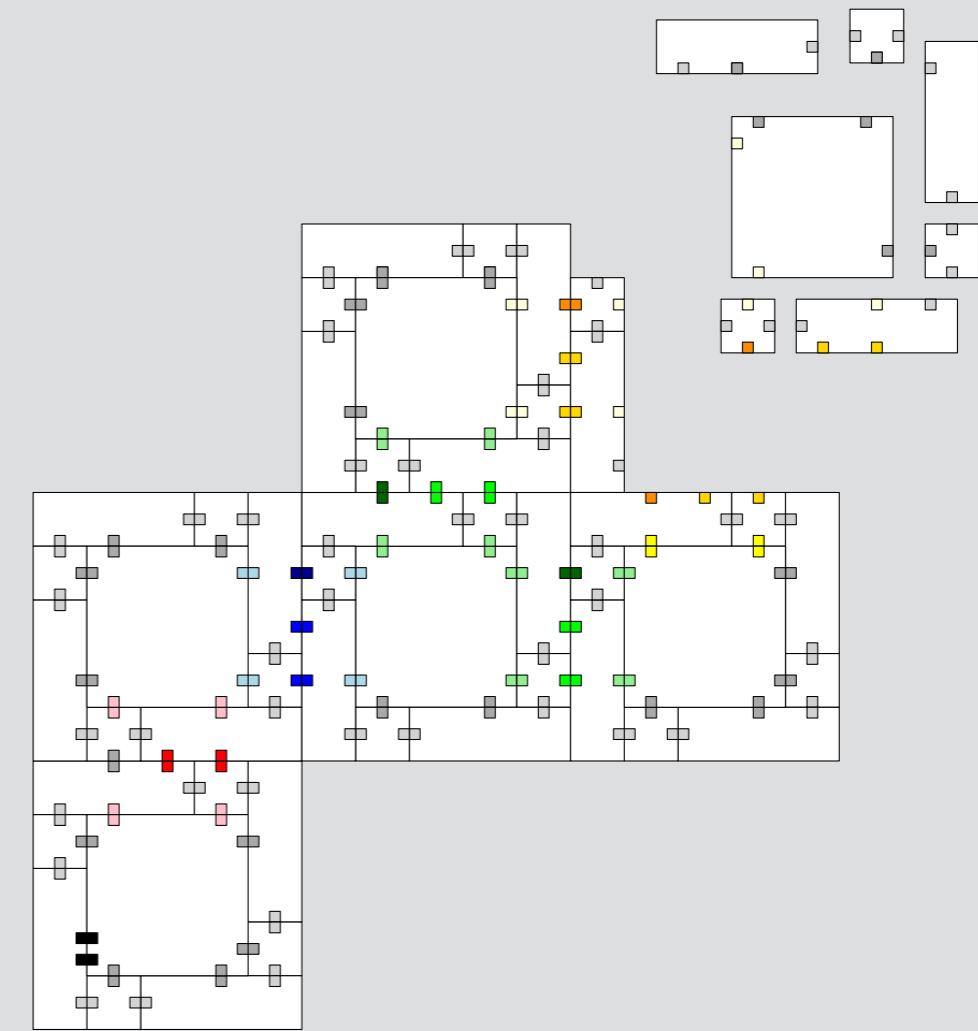
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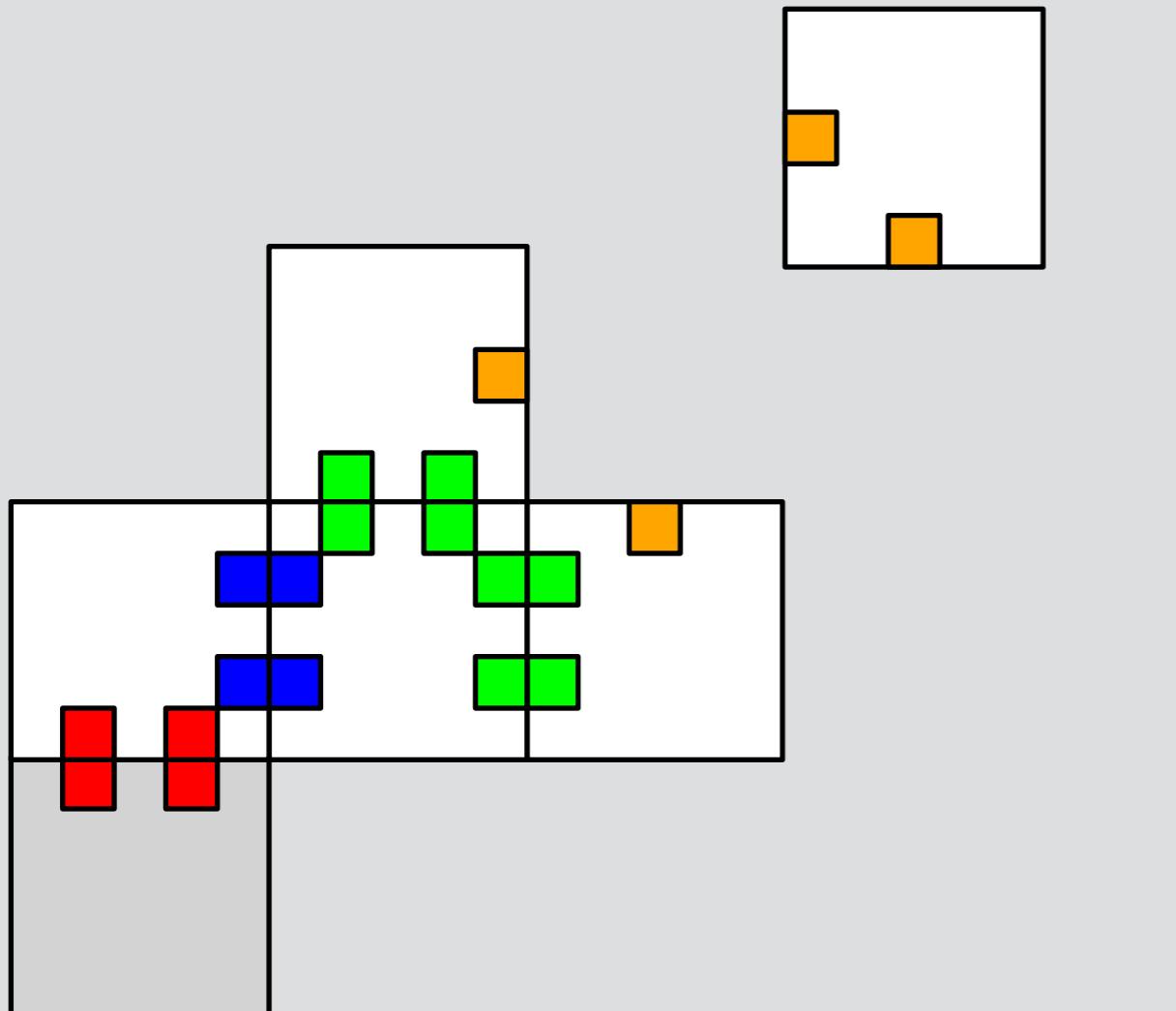
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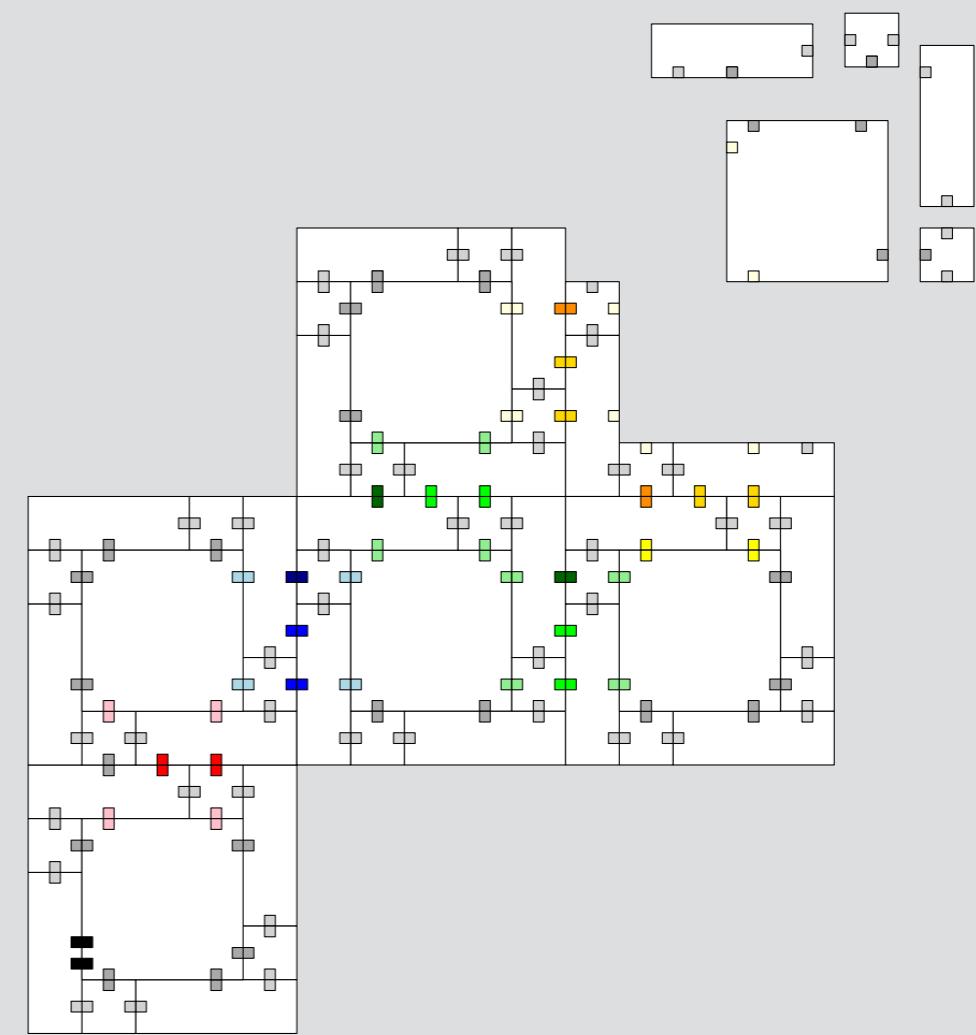
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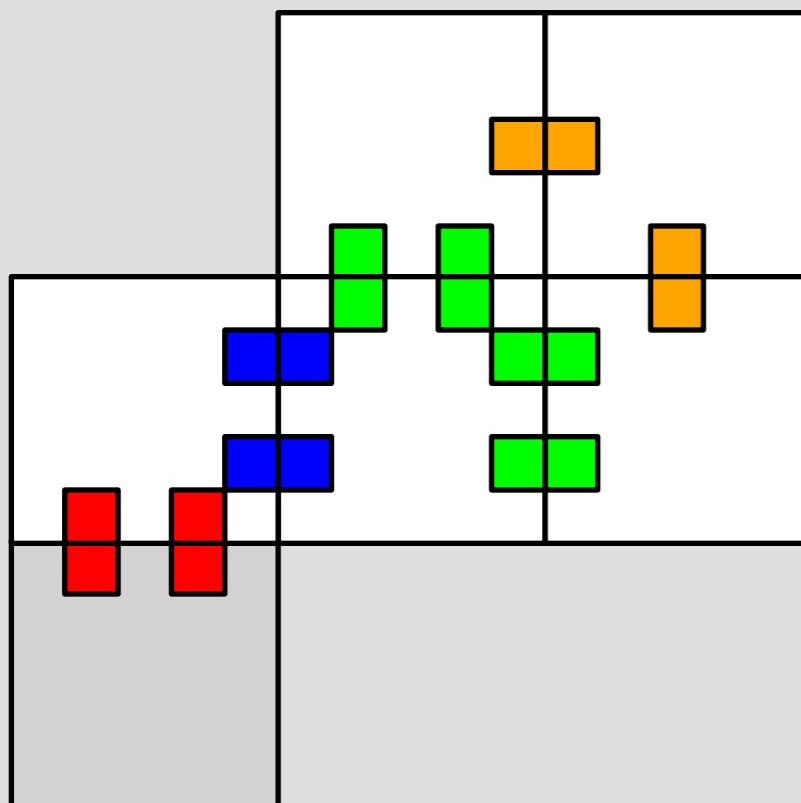
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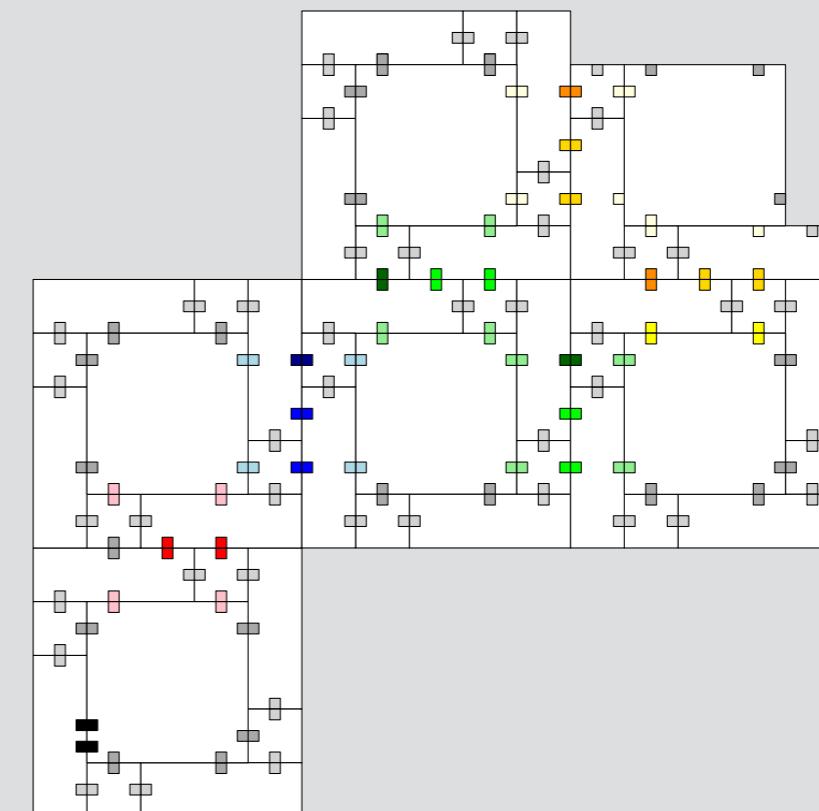


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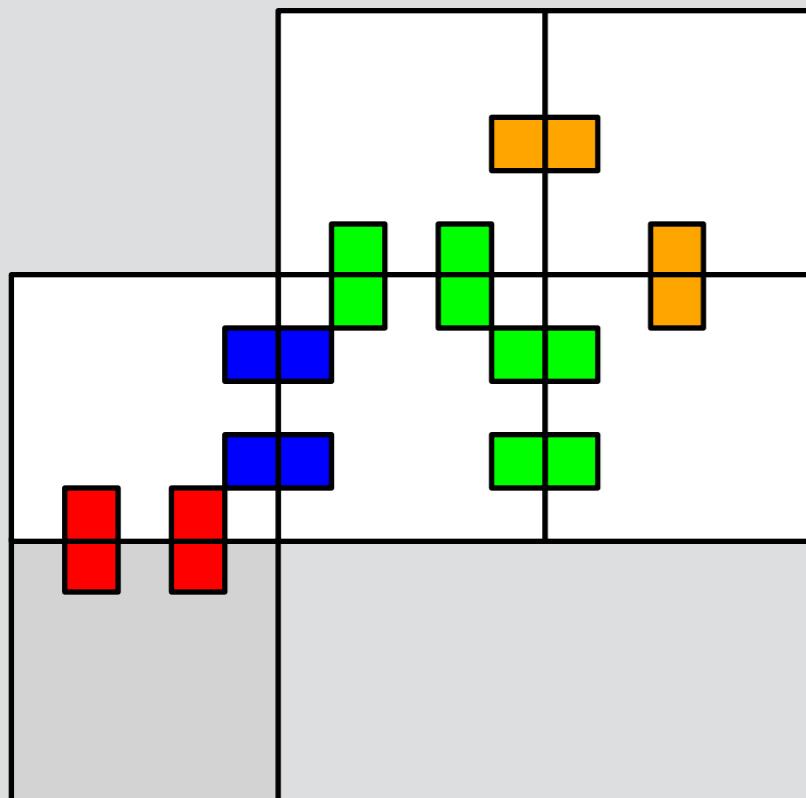
aTAM



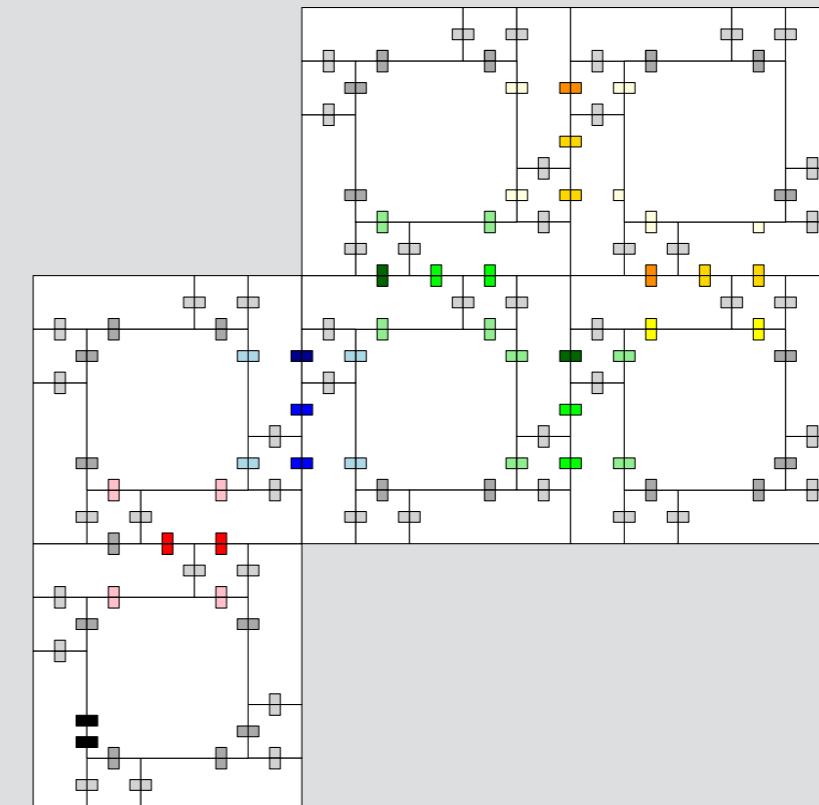
2HAM

Temperature 2

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[Cannon et al. 2013]



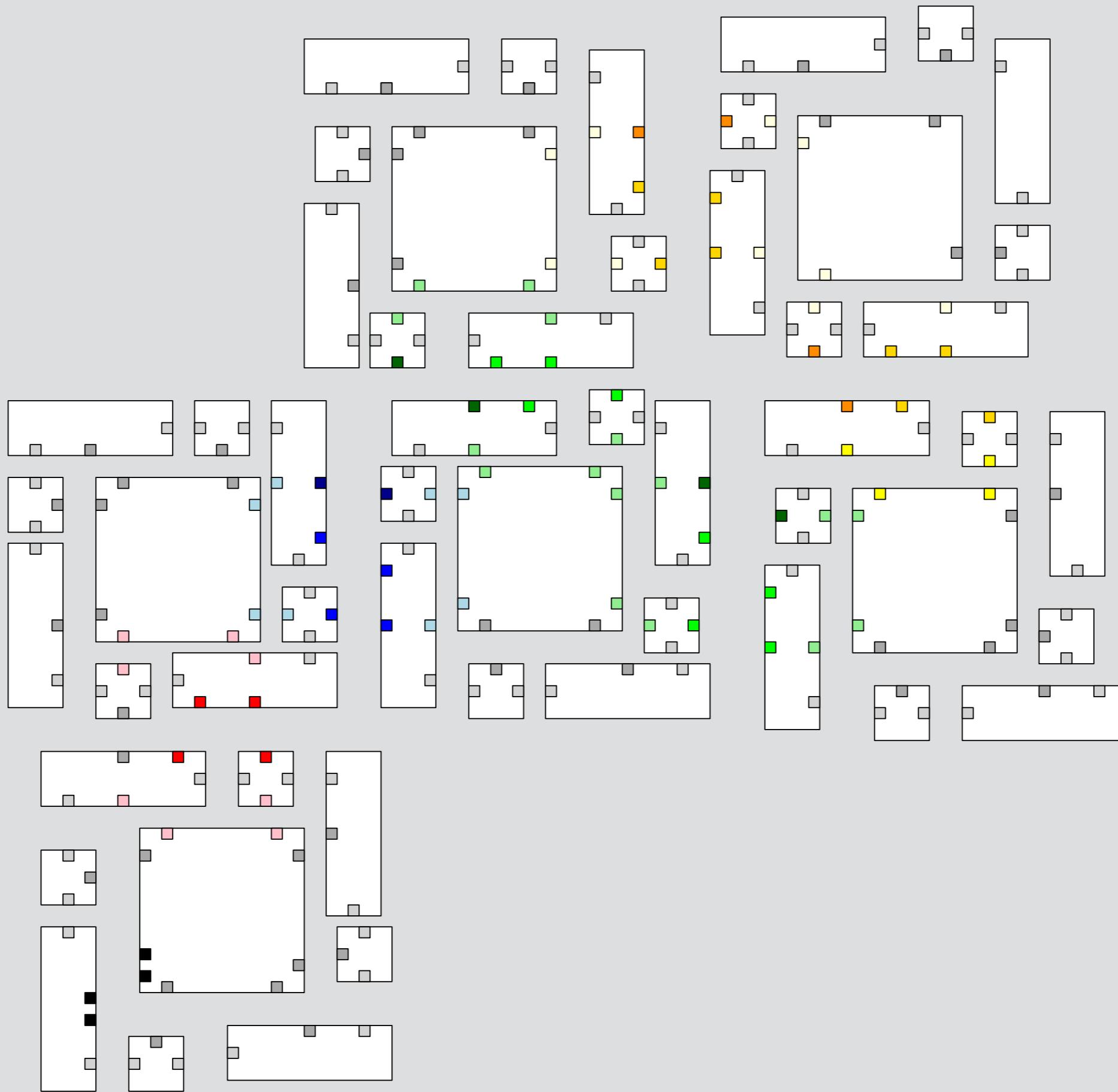
aTAM



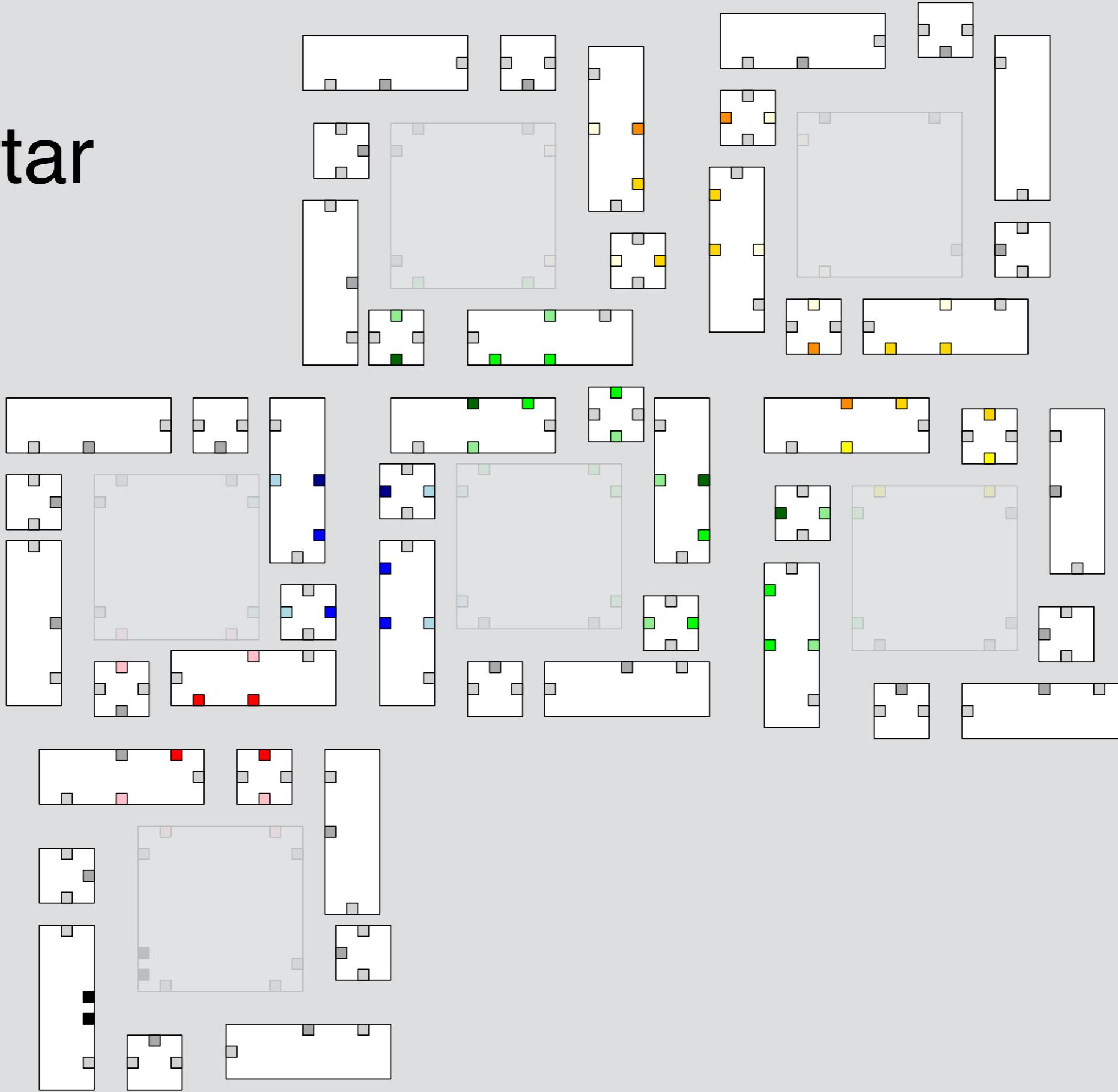
2HAM

Temperature 2

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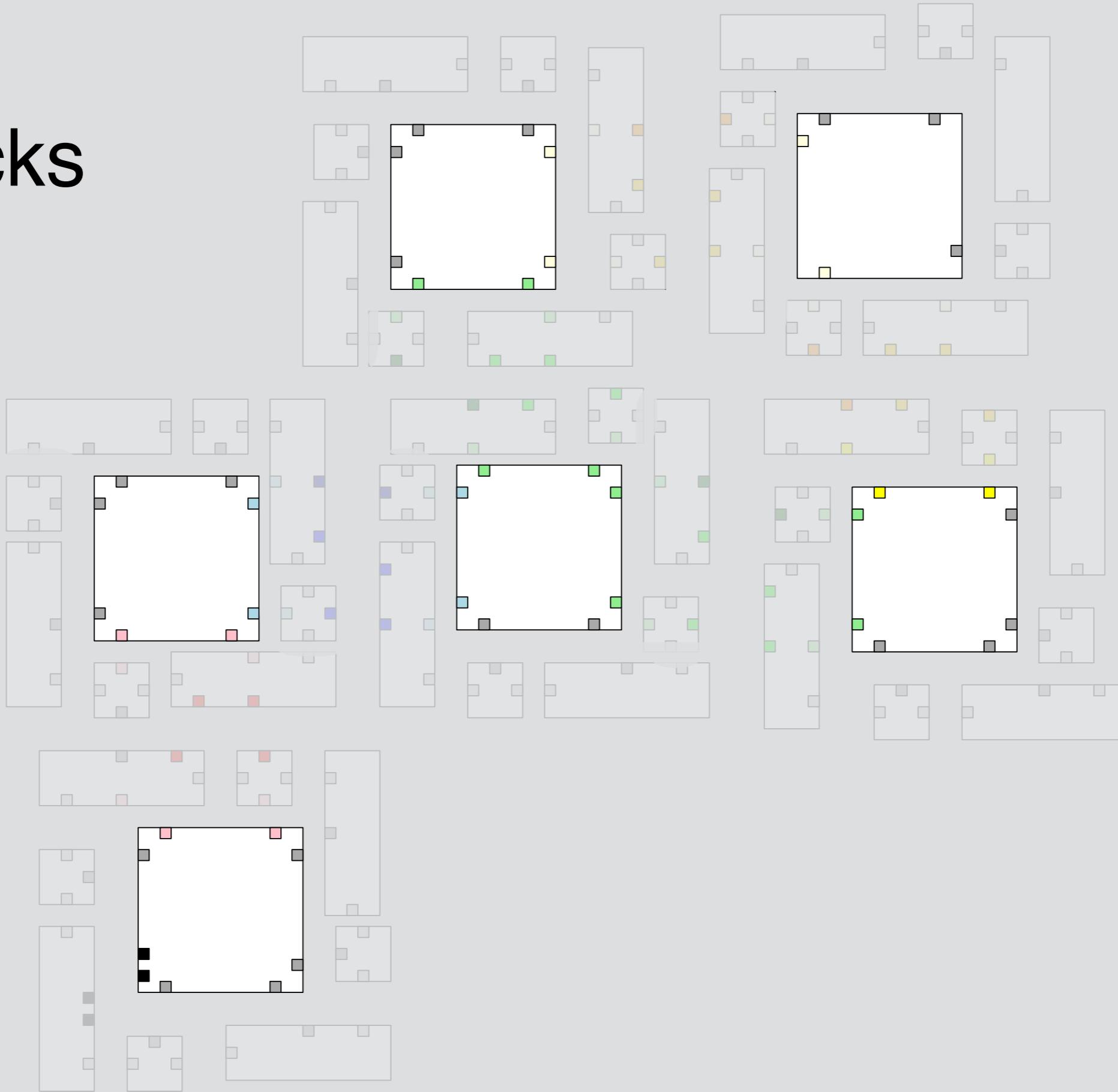


# Mortar

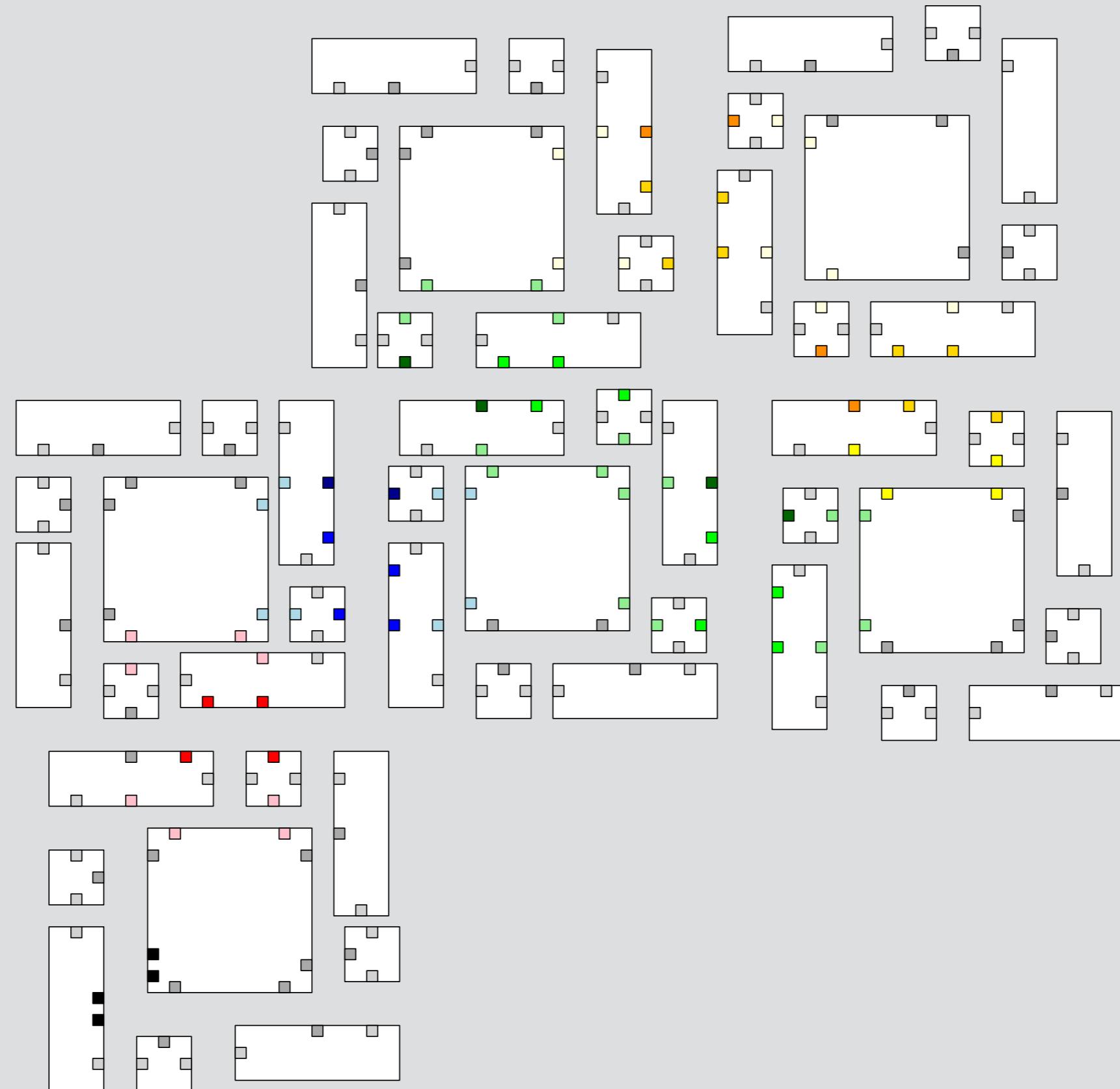


Temperature 2

# Bricks



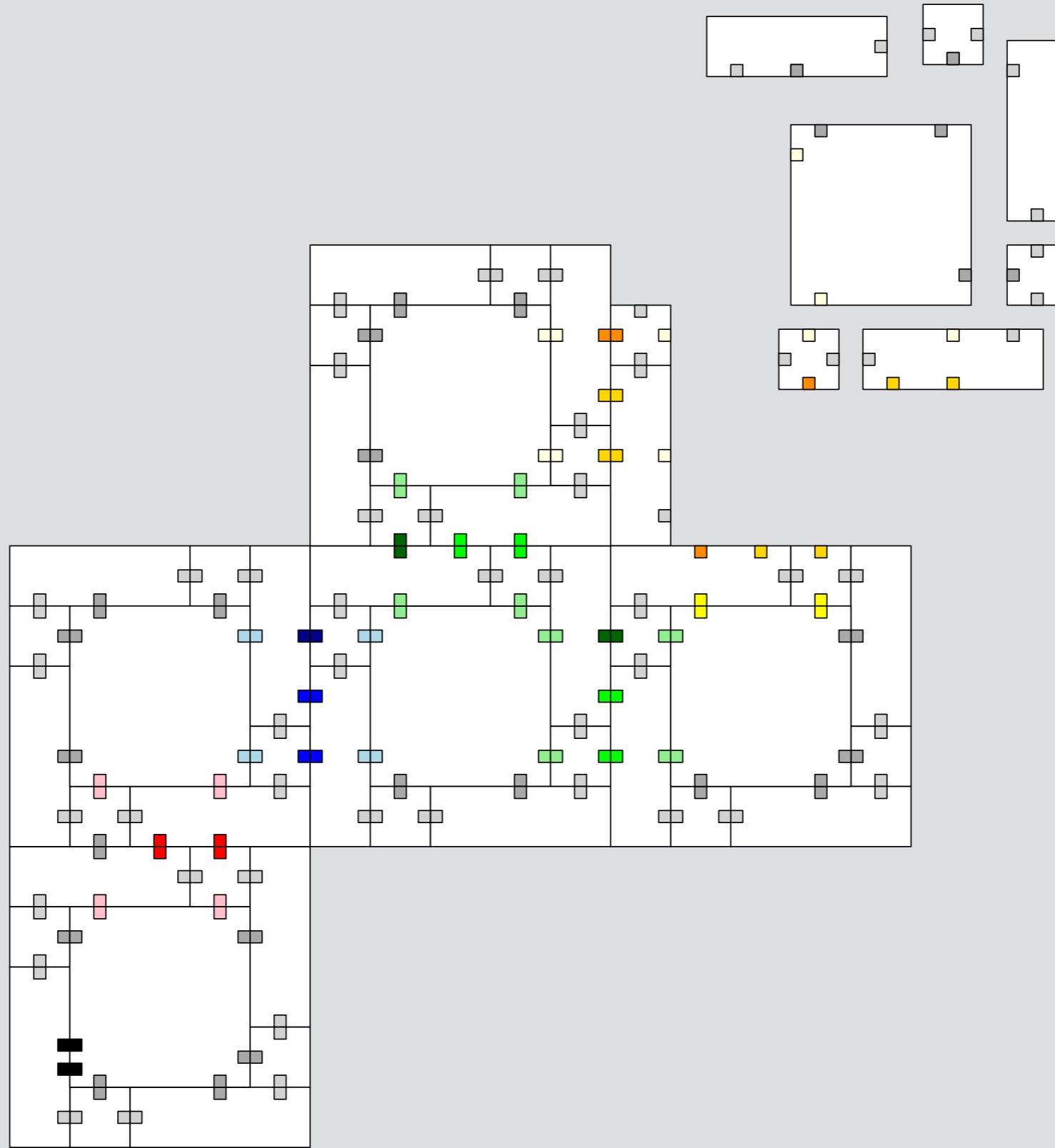
Temperature 2



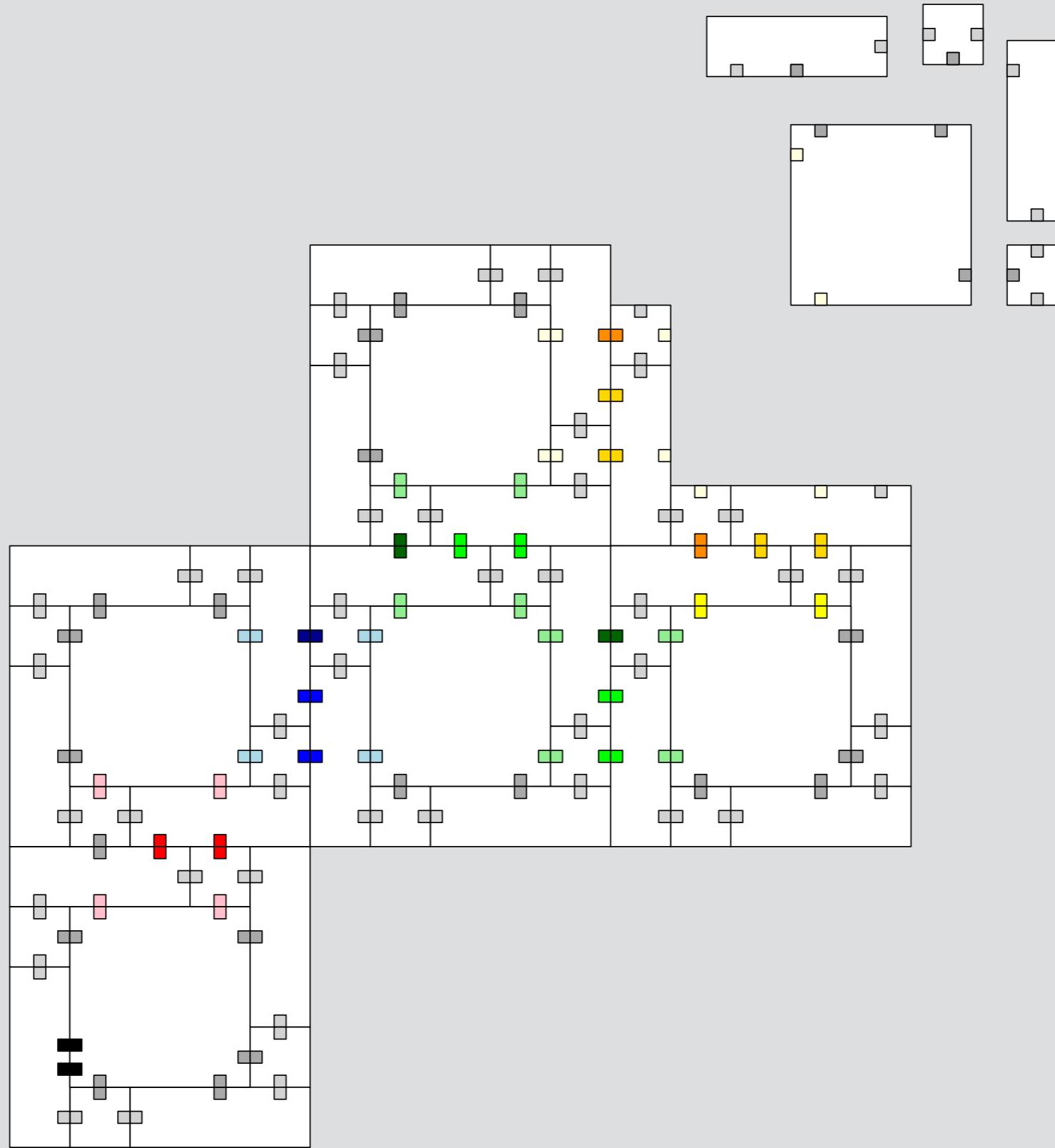
Temperature 2

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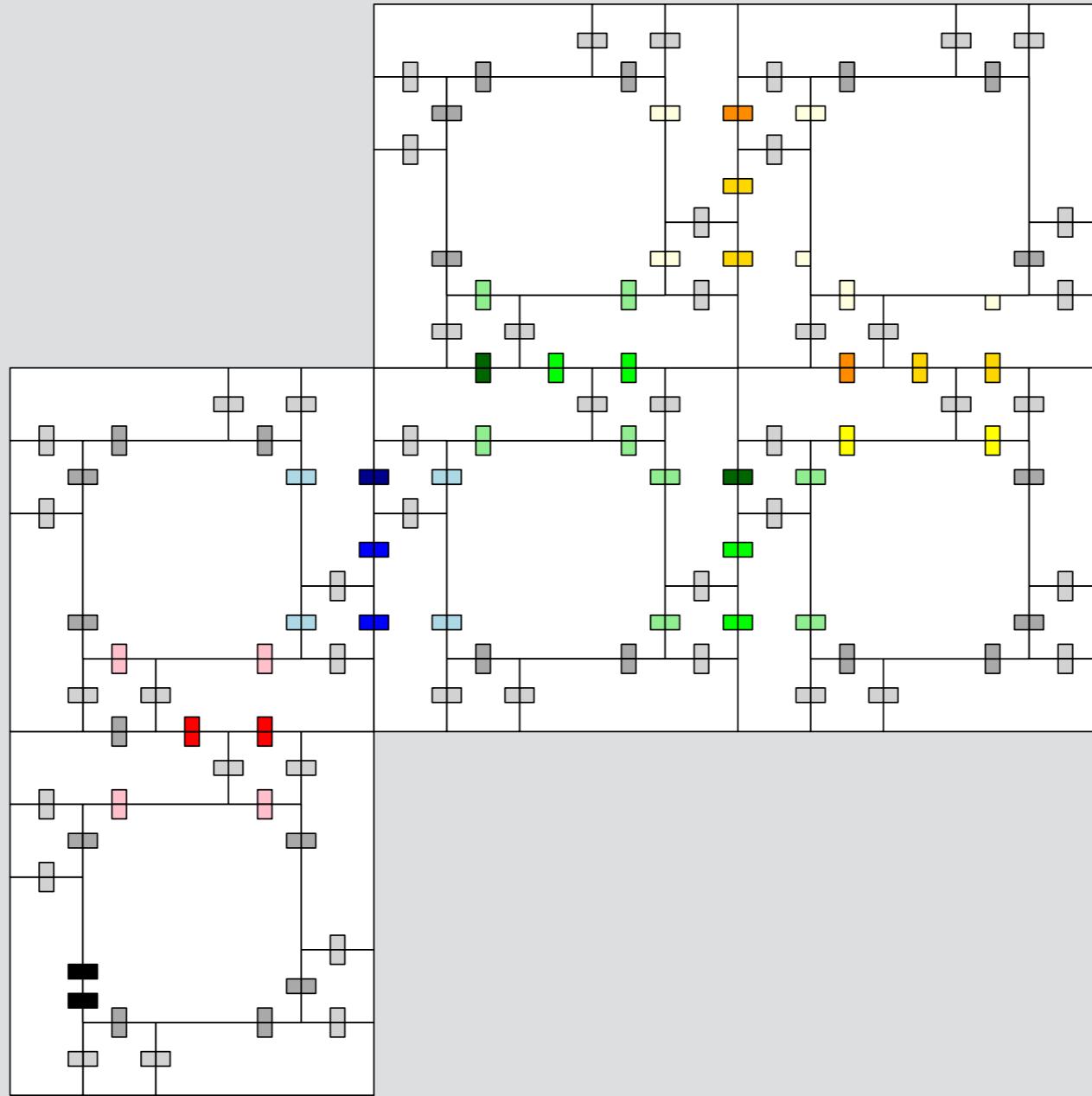




Temperature 2

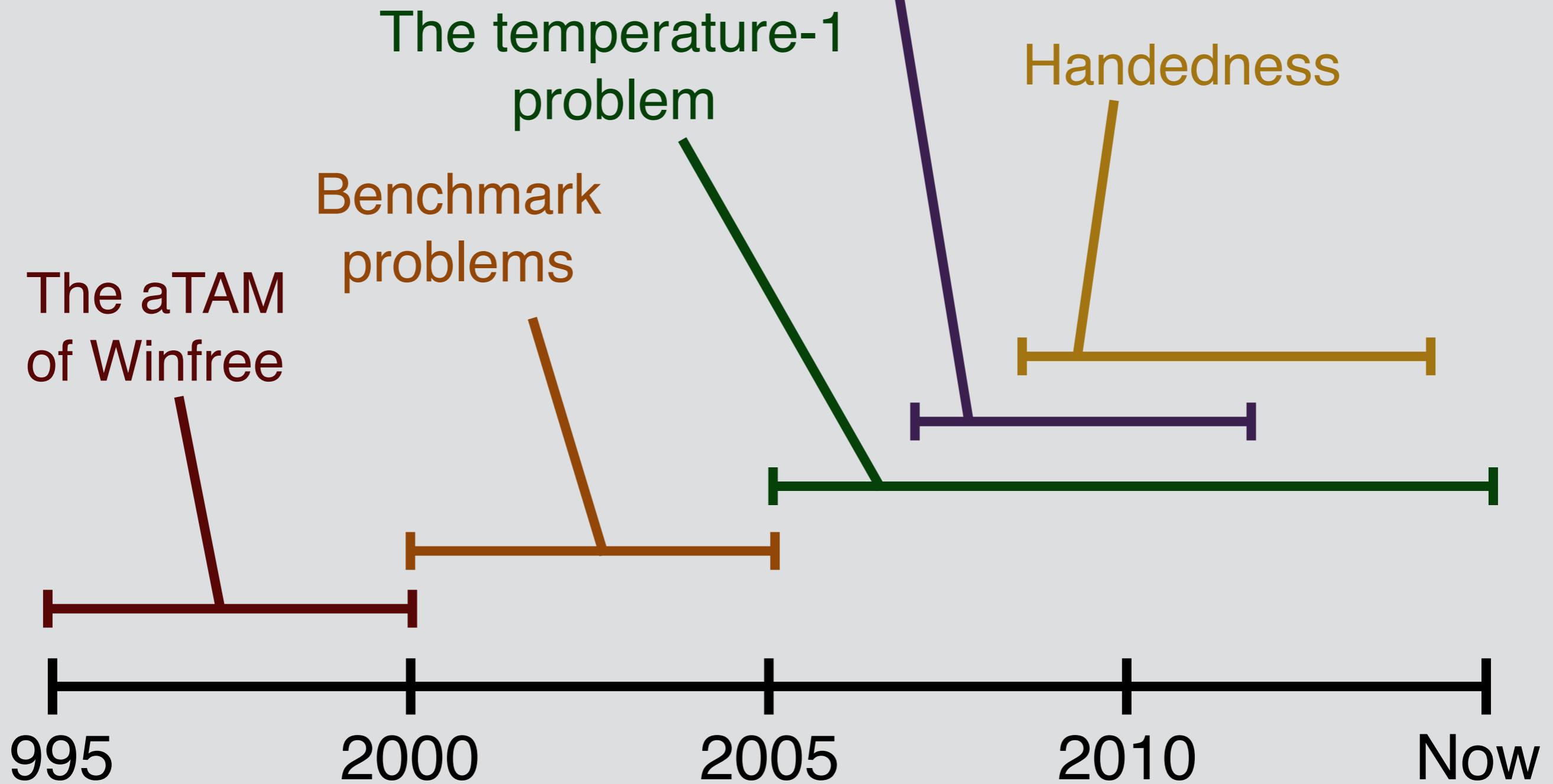


Temperature 2



Temperature 2

Universality via  
weak cooperation



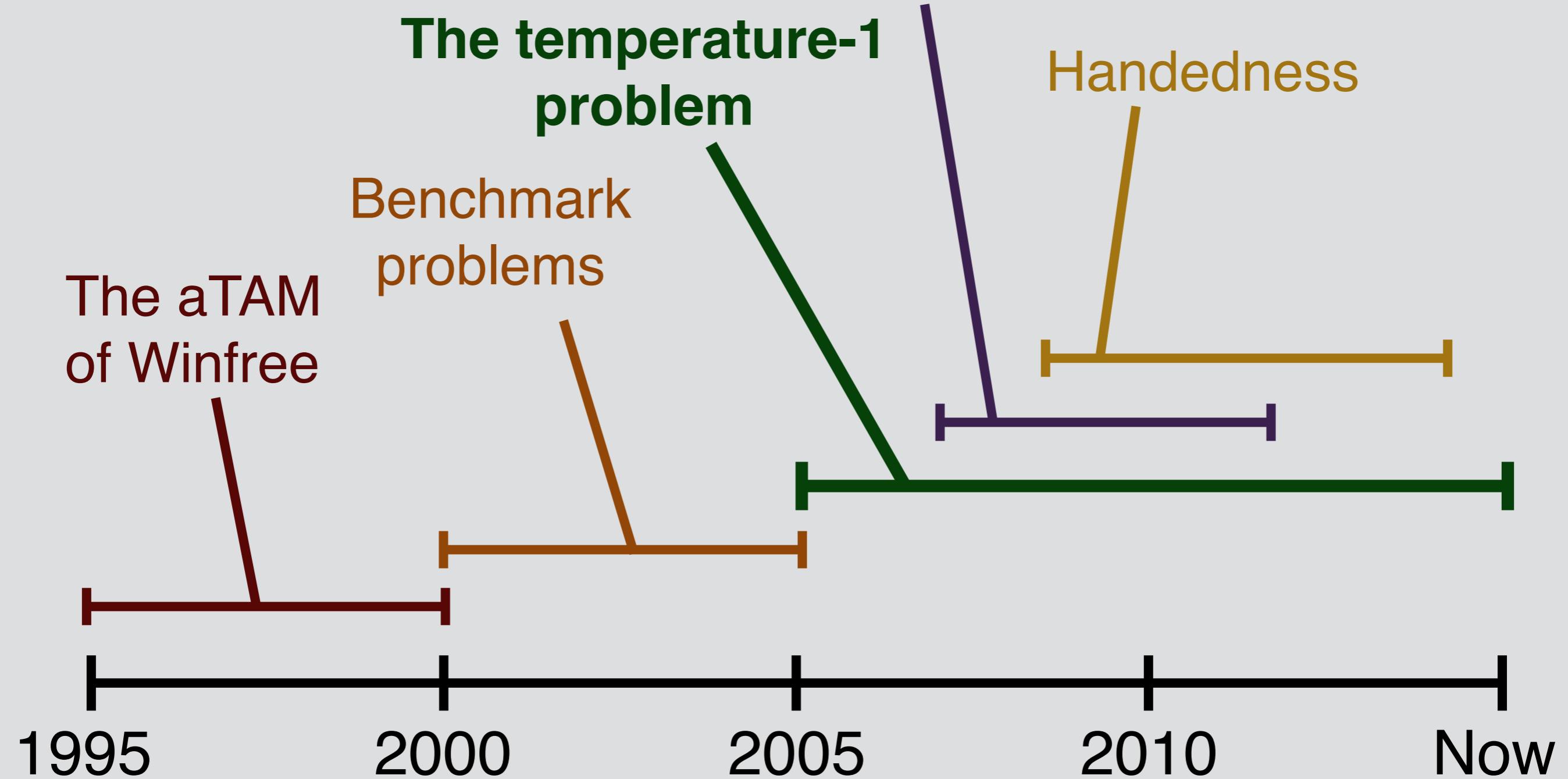
Universality via  
weak cooperation

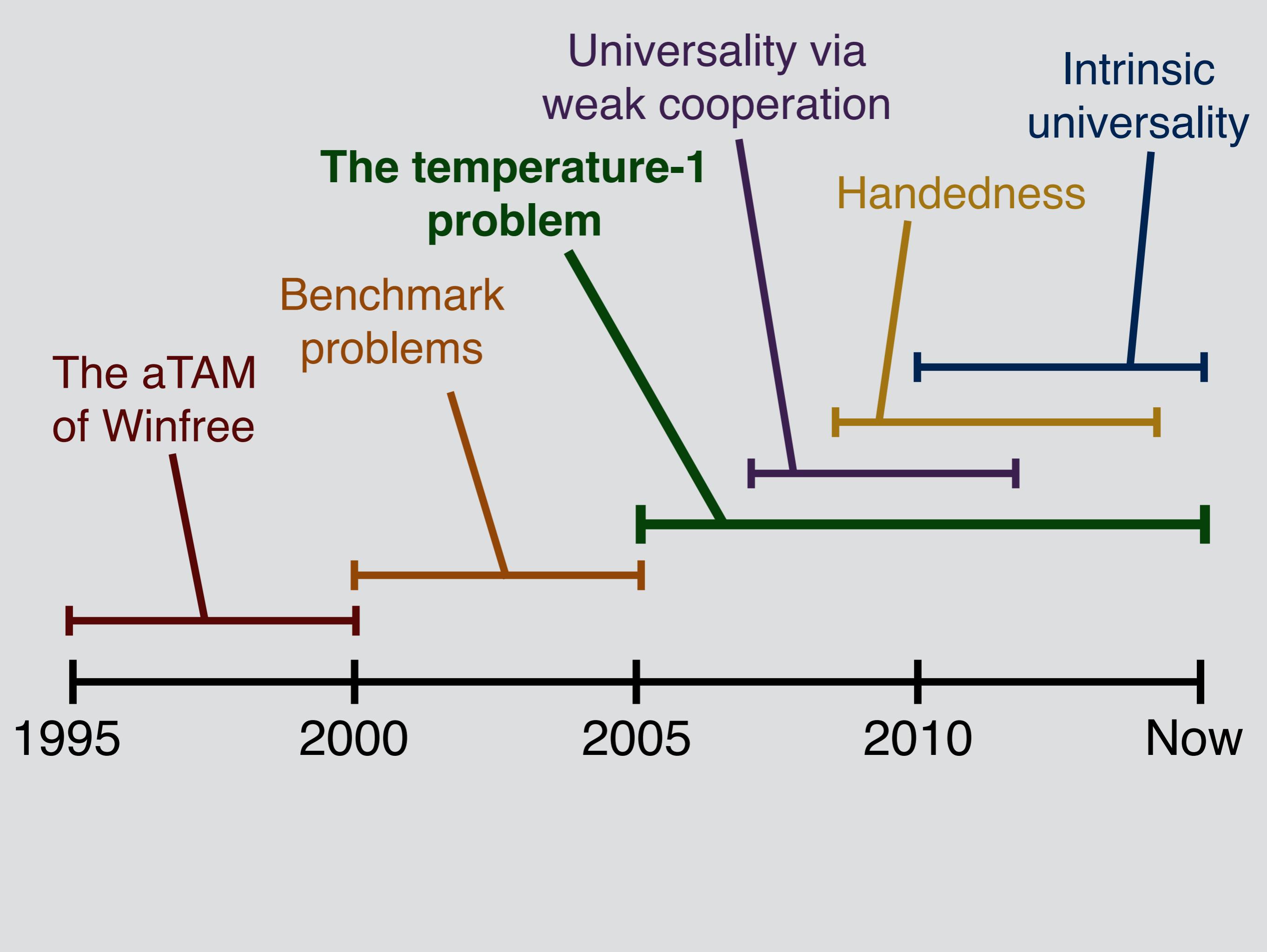
**The temperature-1  
problem**

The aTAM  
of Winfree

Benchmark  
problems

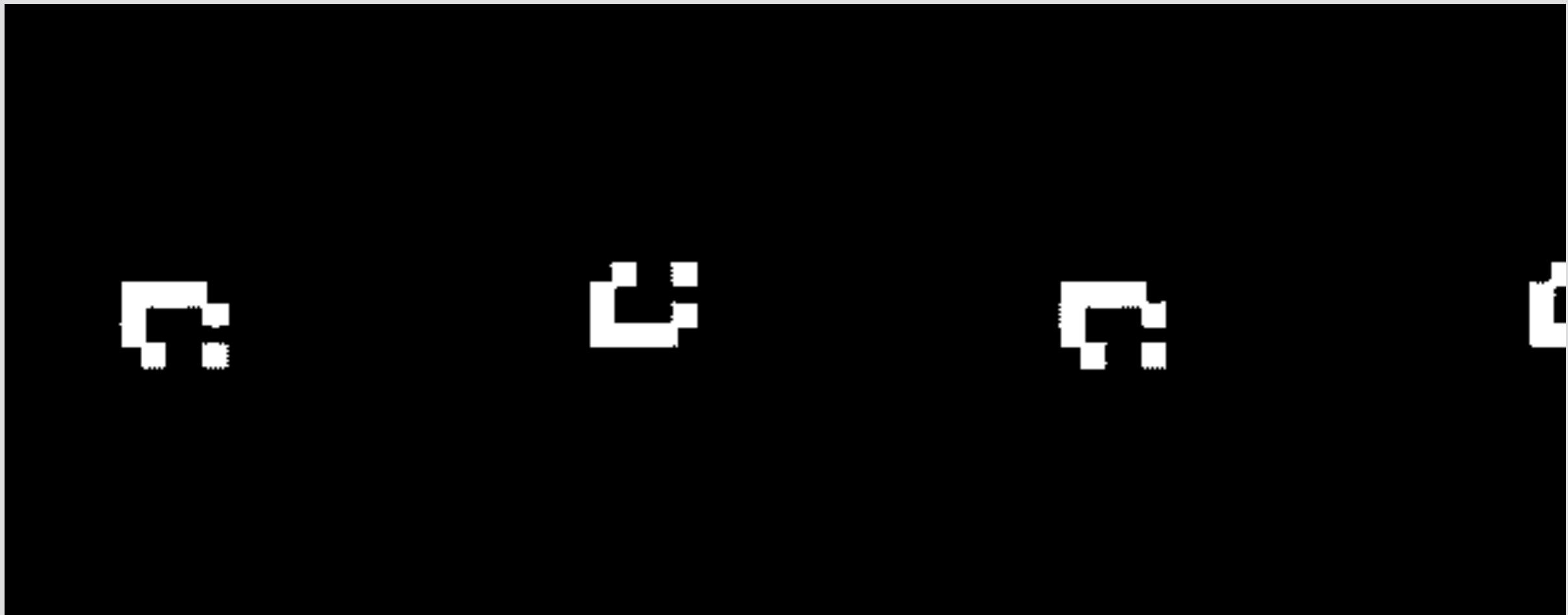
Handedness





# Intrinsic universality

In cellular automata:



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# State of aTAM computation:

(circa 2008)

Temperature-1 universal computation is open.

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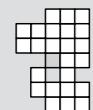
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Claim: temp-1 is *intrinsically universal* for temp-2.

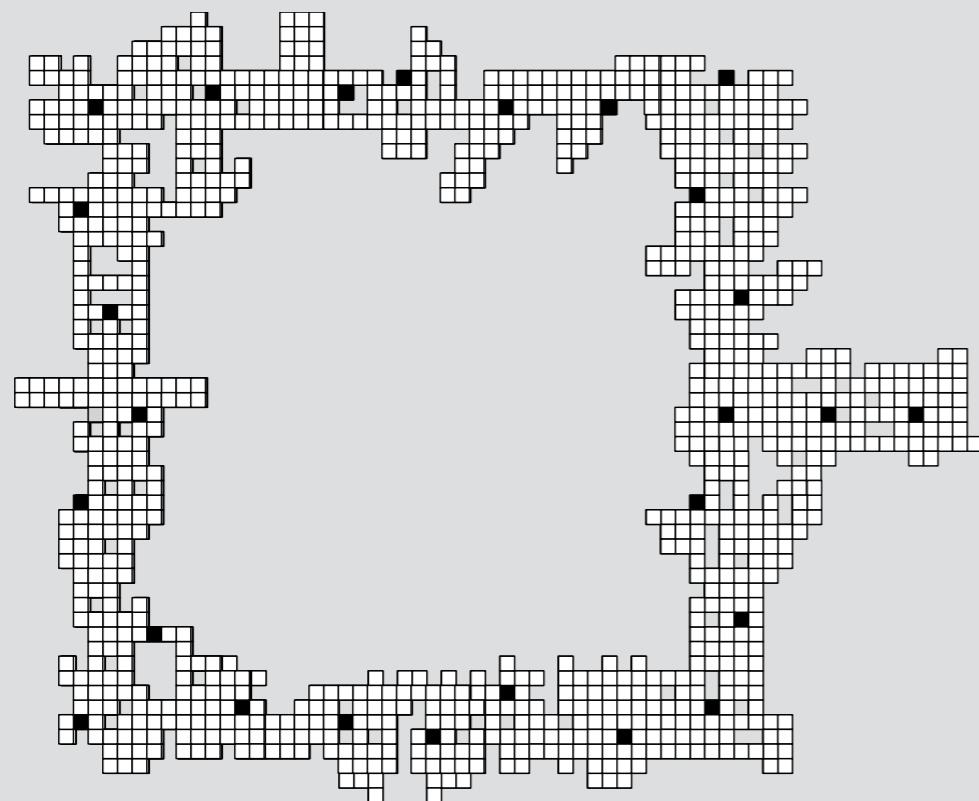
# Simulation: same behavior (with scale and fuzz)



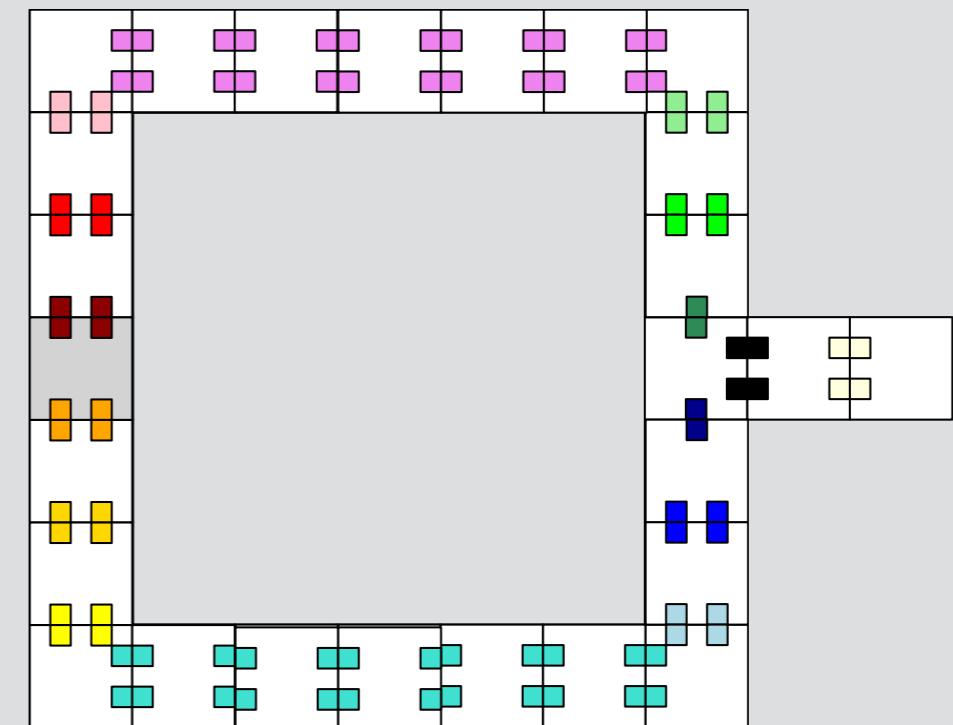
Temperature 1

Temperature 2

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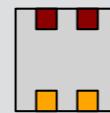
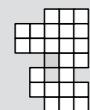


Temperature 1



Temperature 2

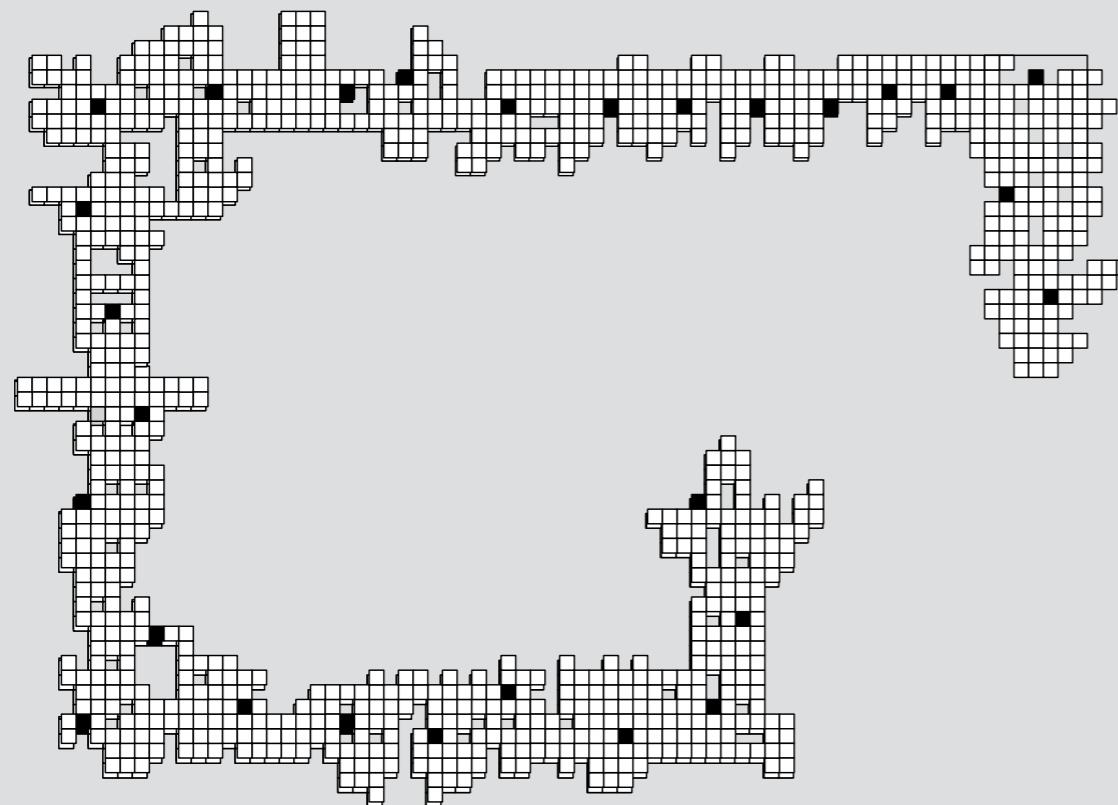
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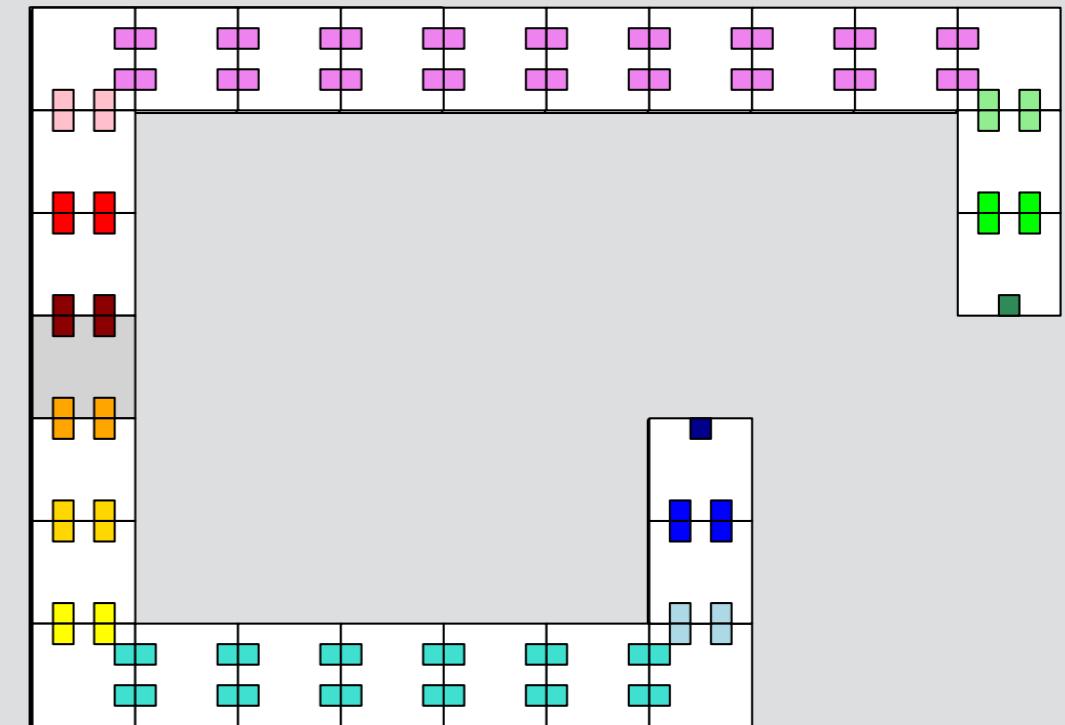
Temperature 1

Temperature 2

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Temperature 1



Temperature 2

# Is intrinsic universality too much to ask?

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No: temp-2 is intrinsically universal for all temps.

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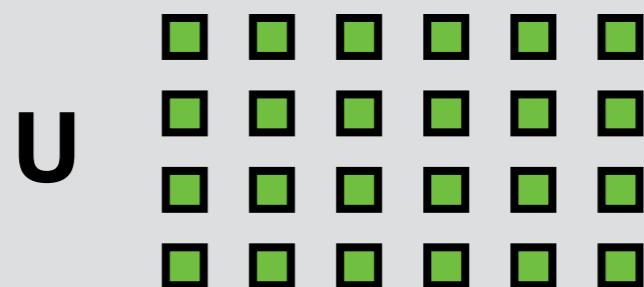
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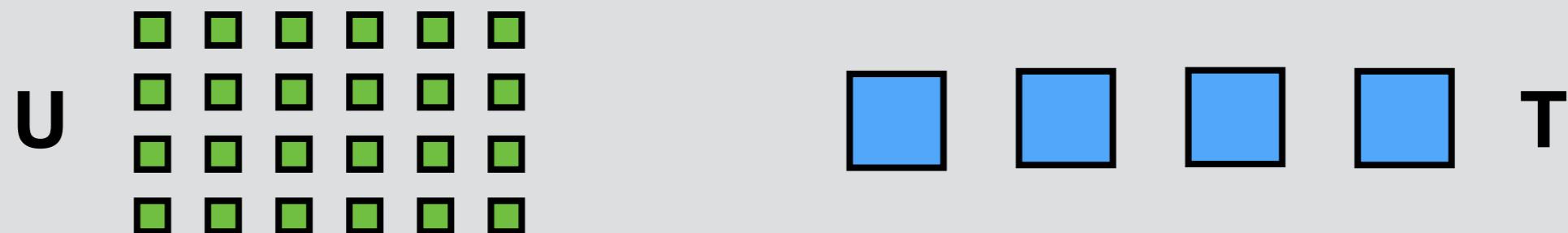
Temperature 2

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Temperature 2

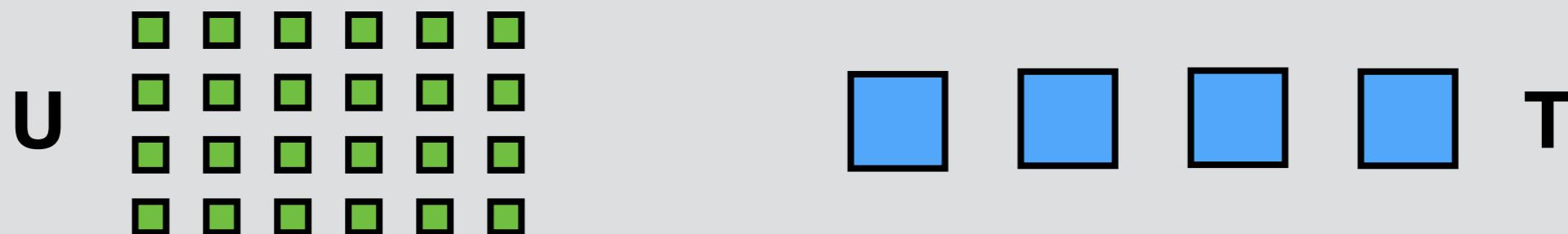
Temperature 6

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Temperature 2



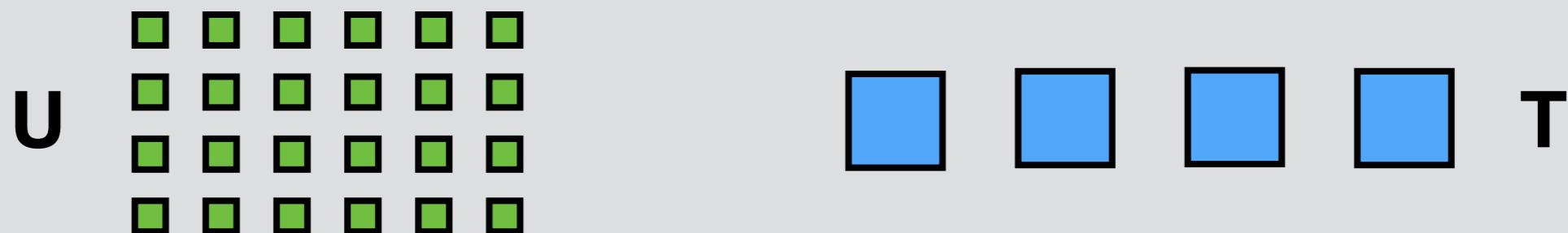
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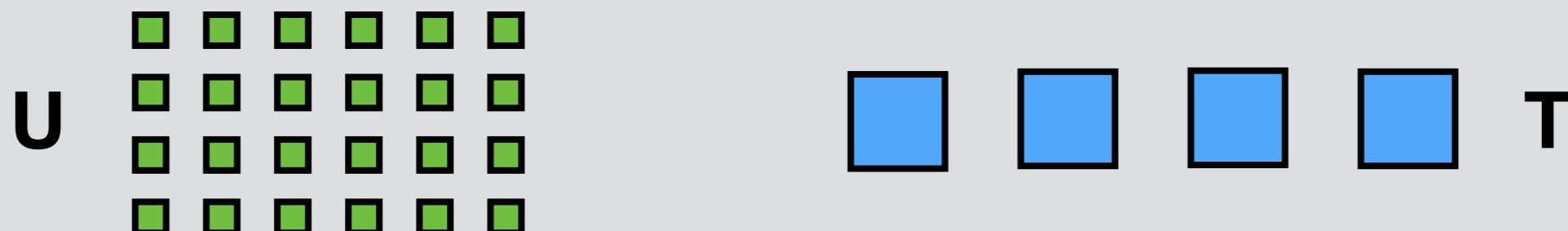
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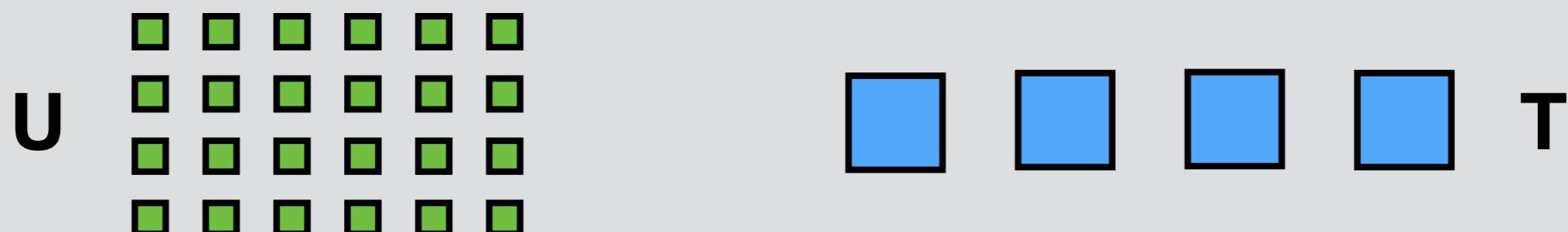
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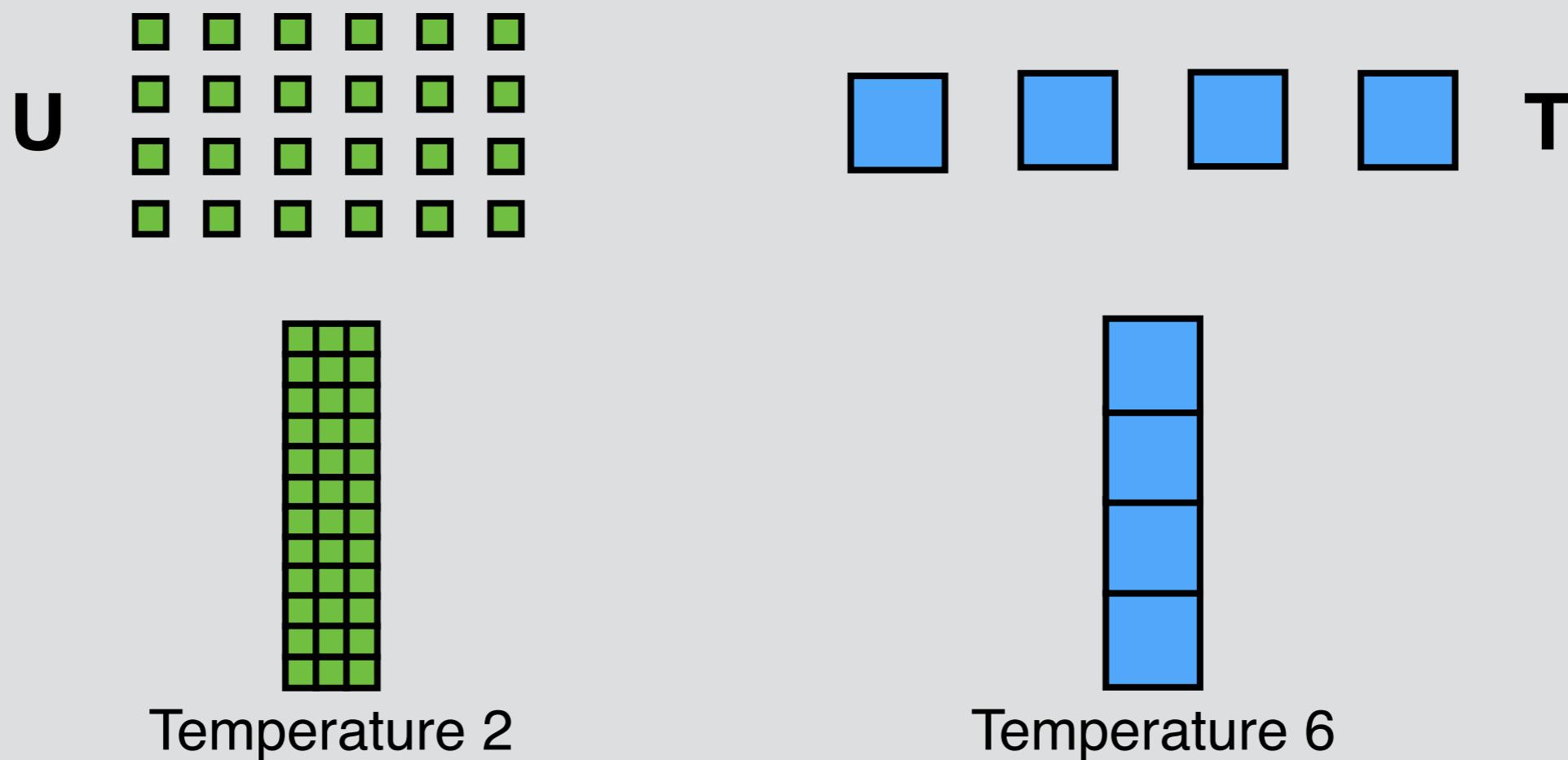
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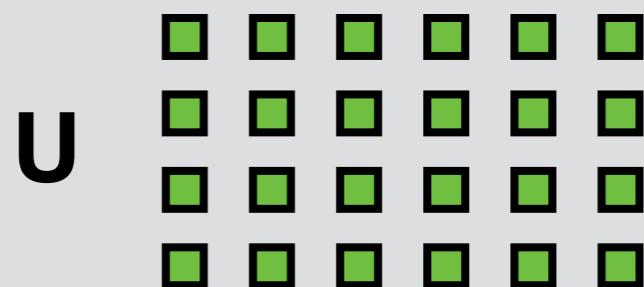


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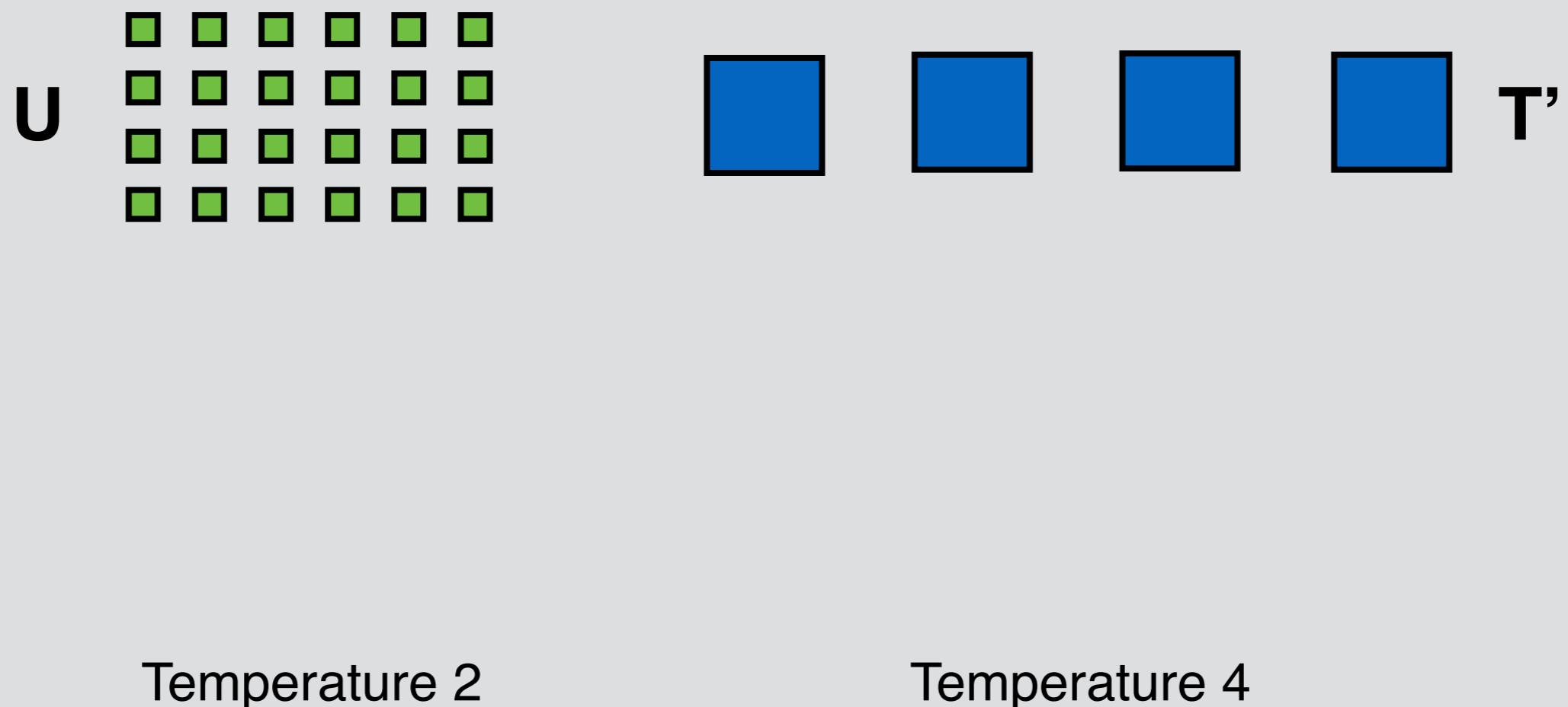
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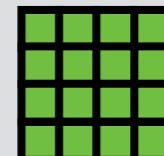
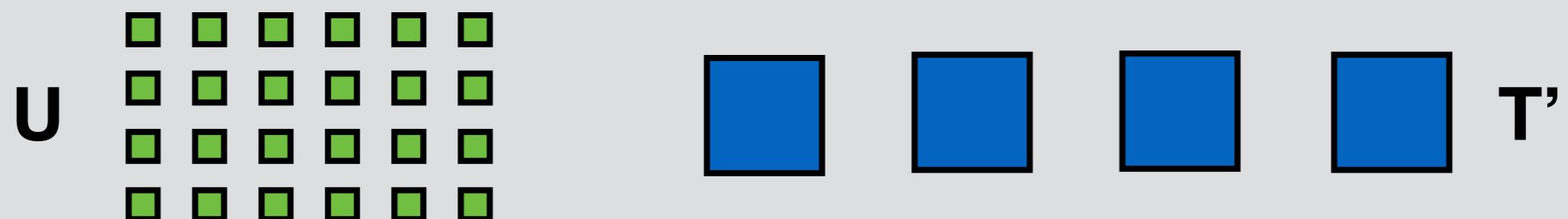


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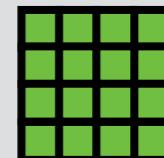
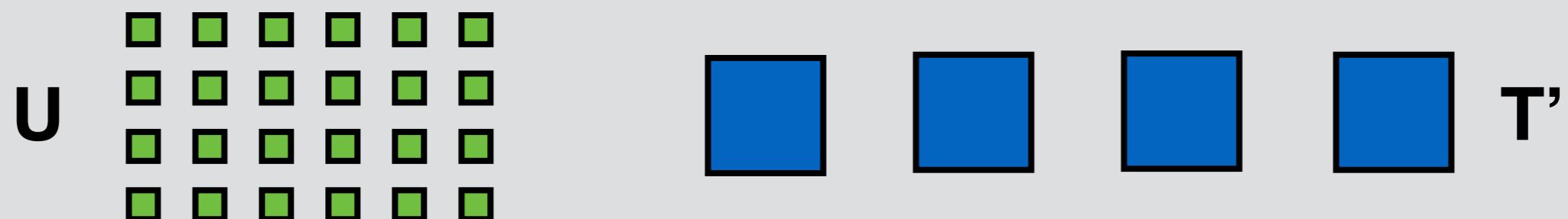
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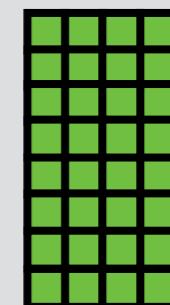
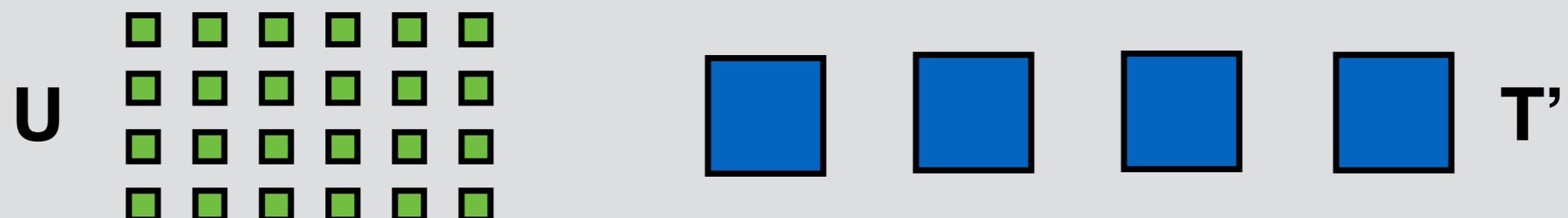
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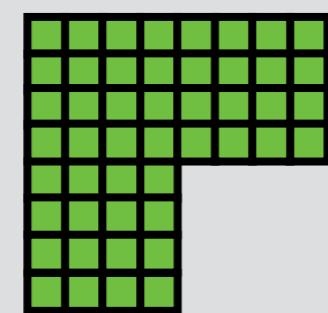
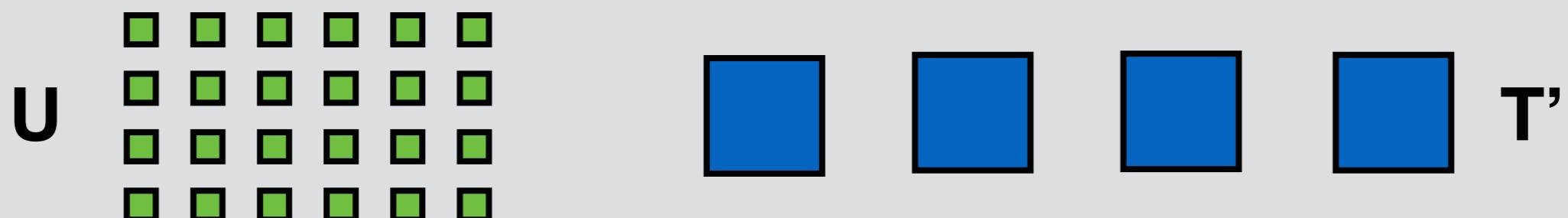
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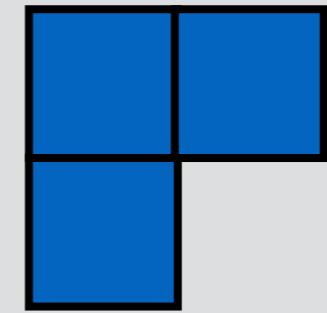
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Temperature 2



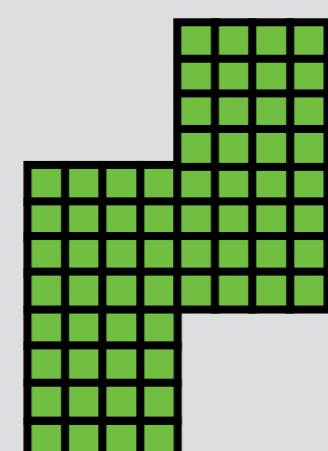
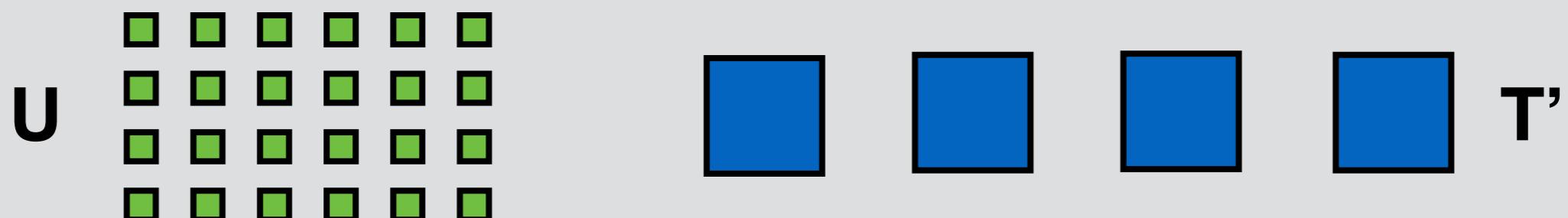
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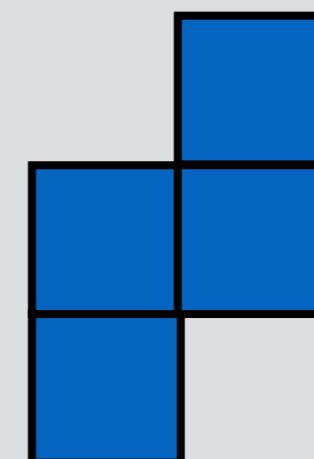
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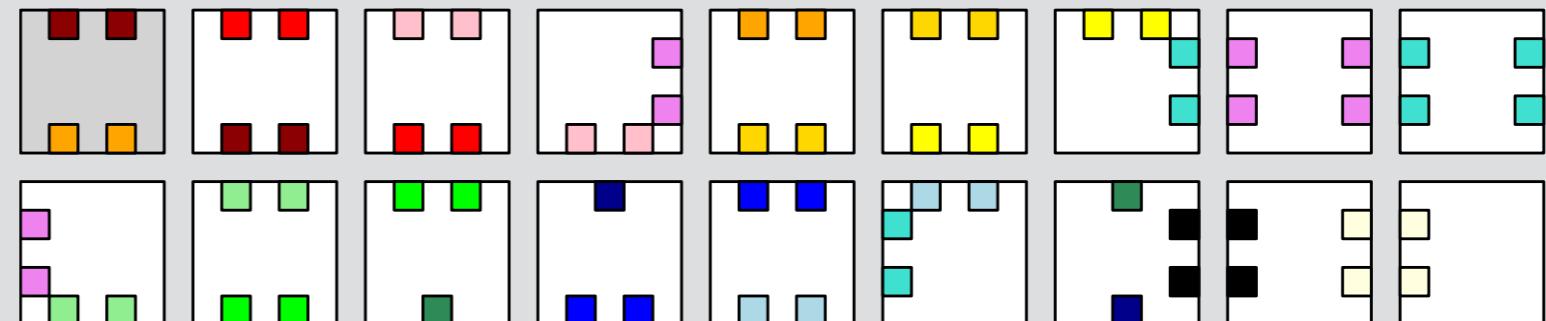
Temperature 4

Claim: temp-1 is *intrinsically universal* for temp-2.

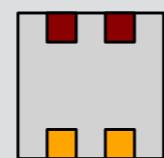
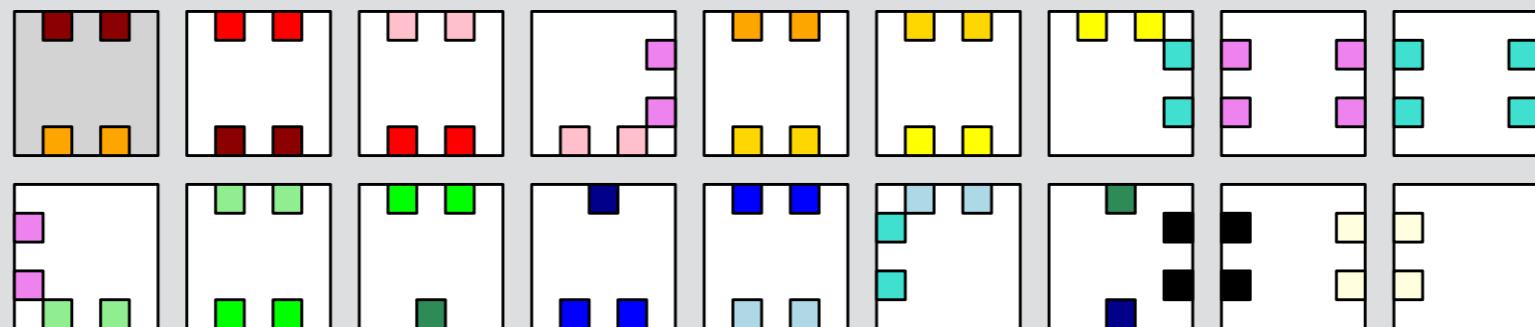
~~Claim: temp-1 is *intrinsically universal* for temp-2.~~

There is a temp-2 system  
not simulated by any temp-1 system.

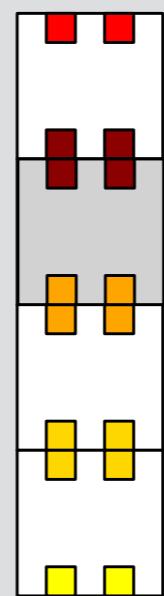
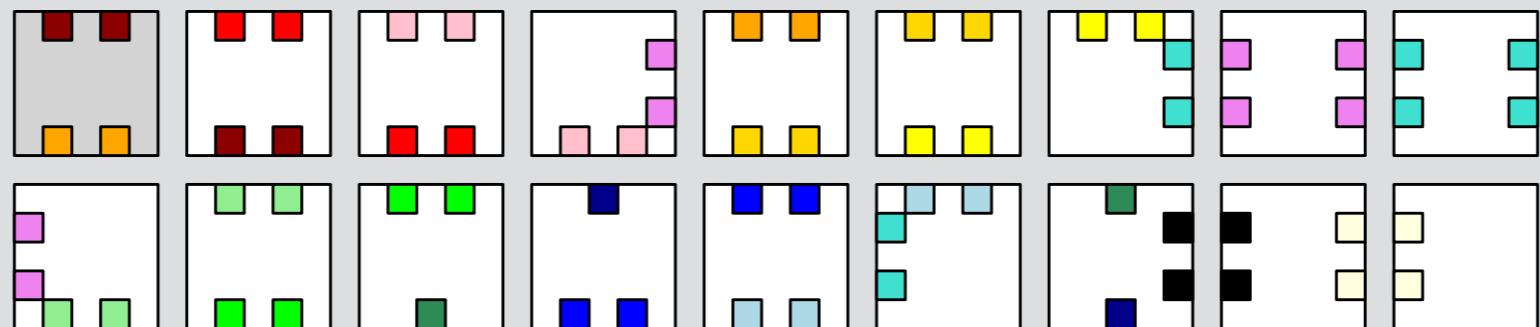
Specifically:



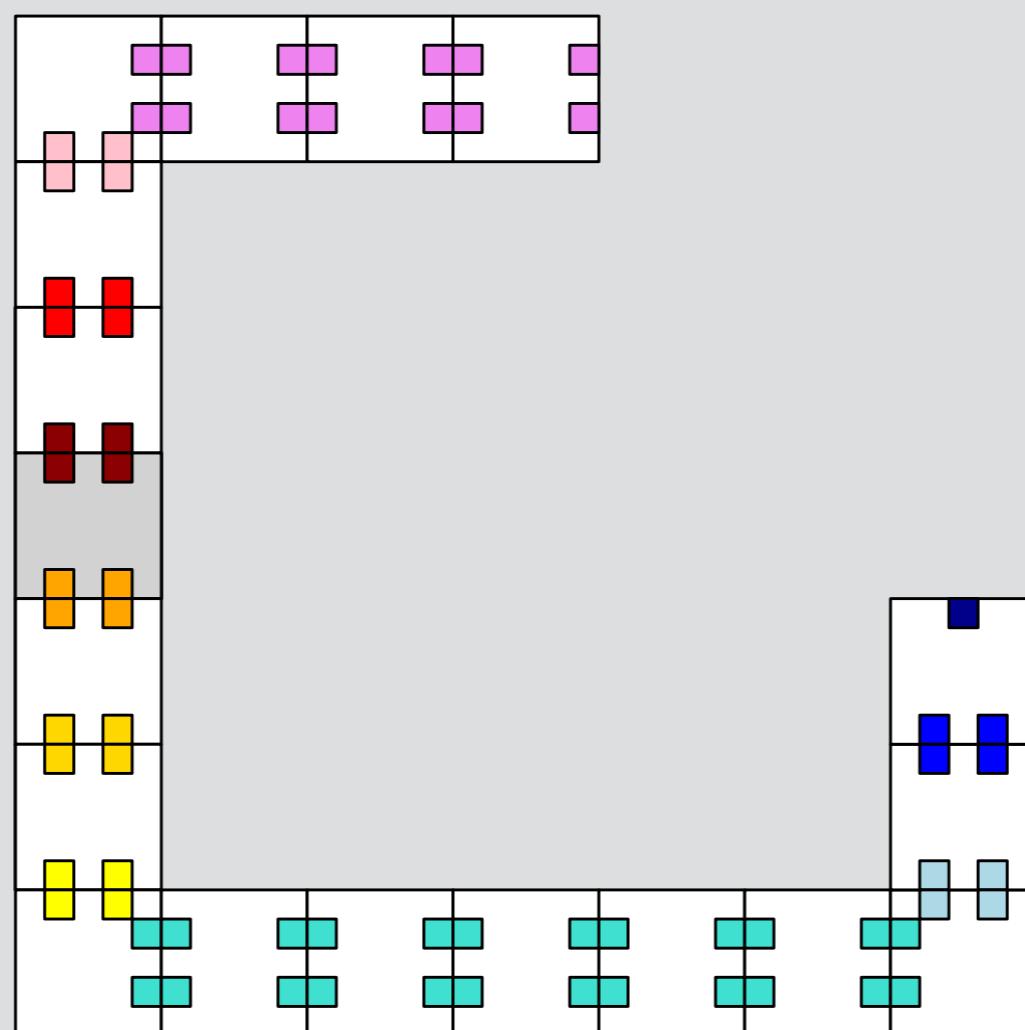
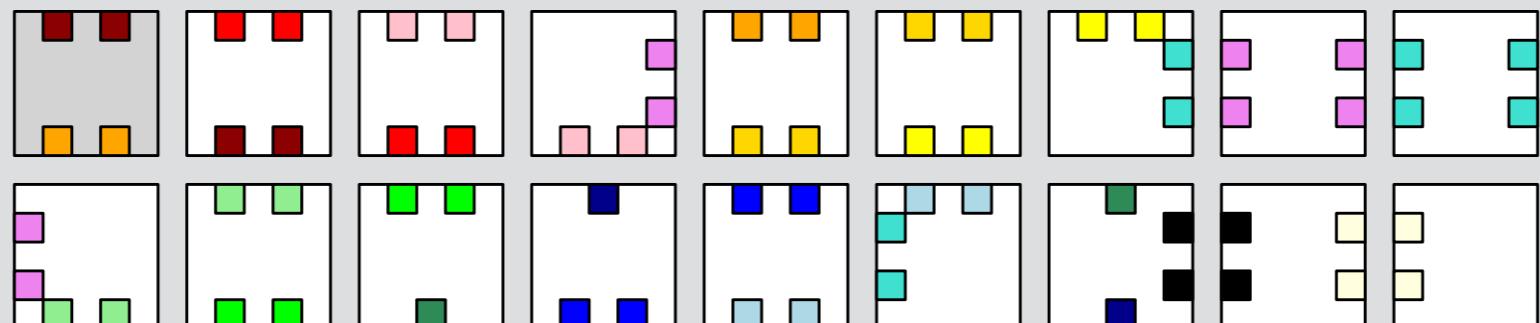
[Meunier et al. 2014]



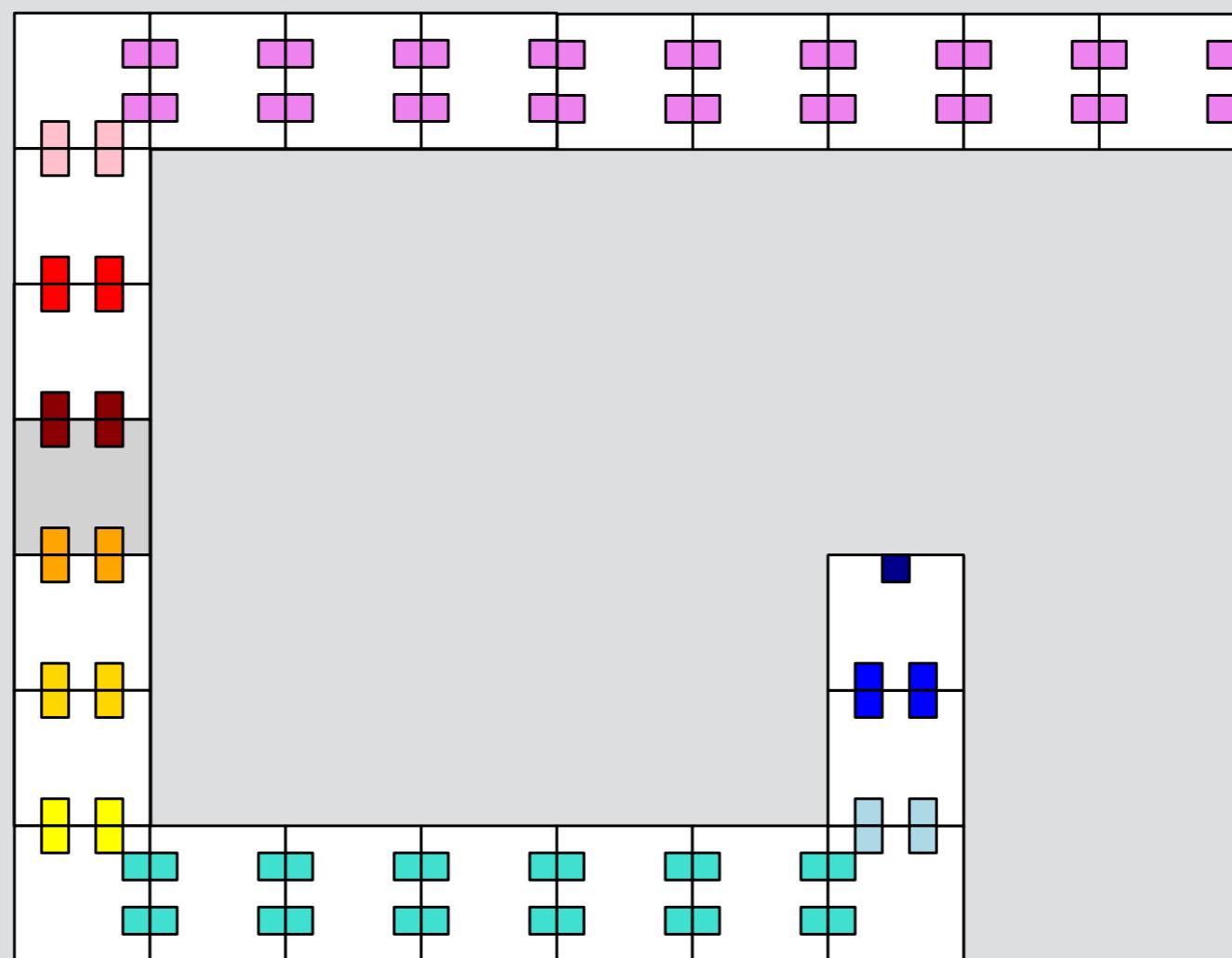
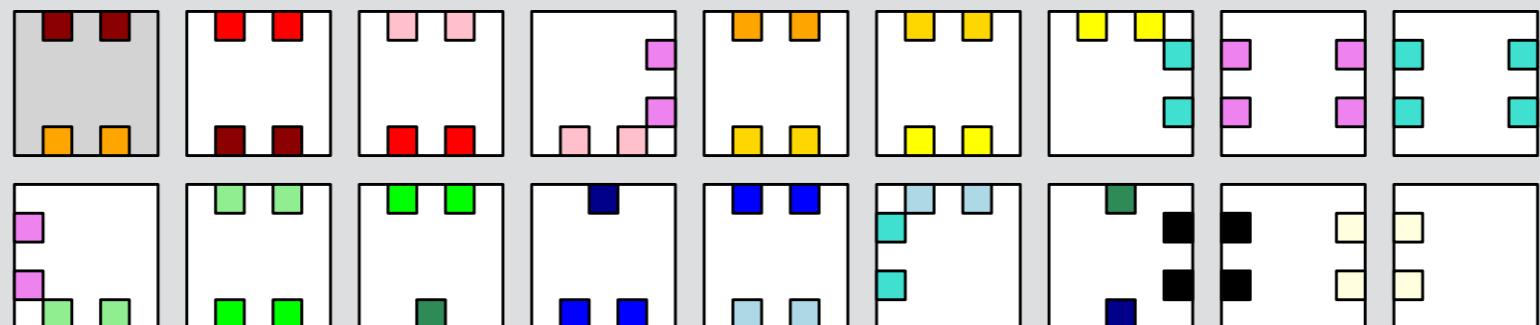
Temperature 2



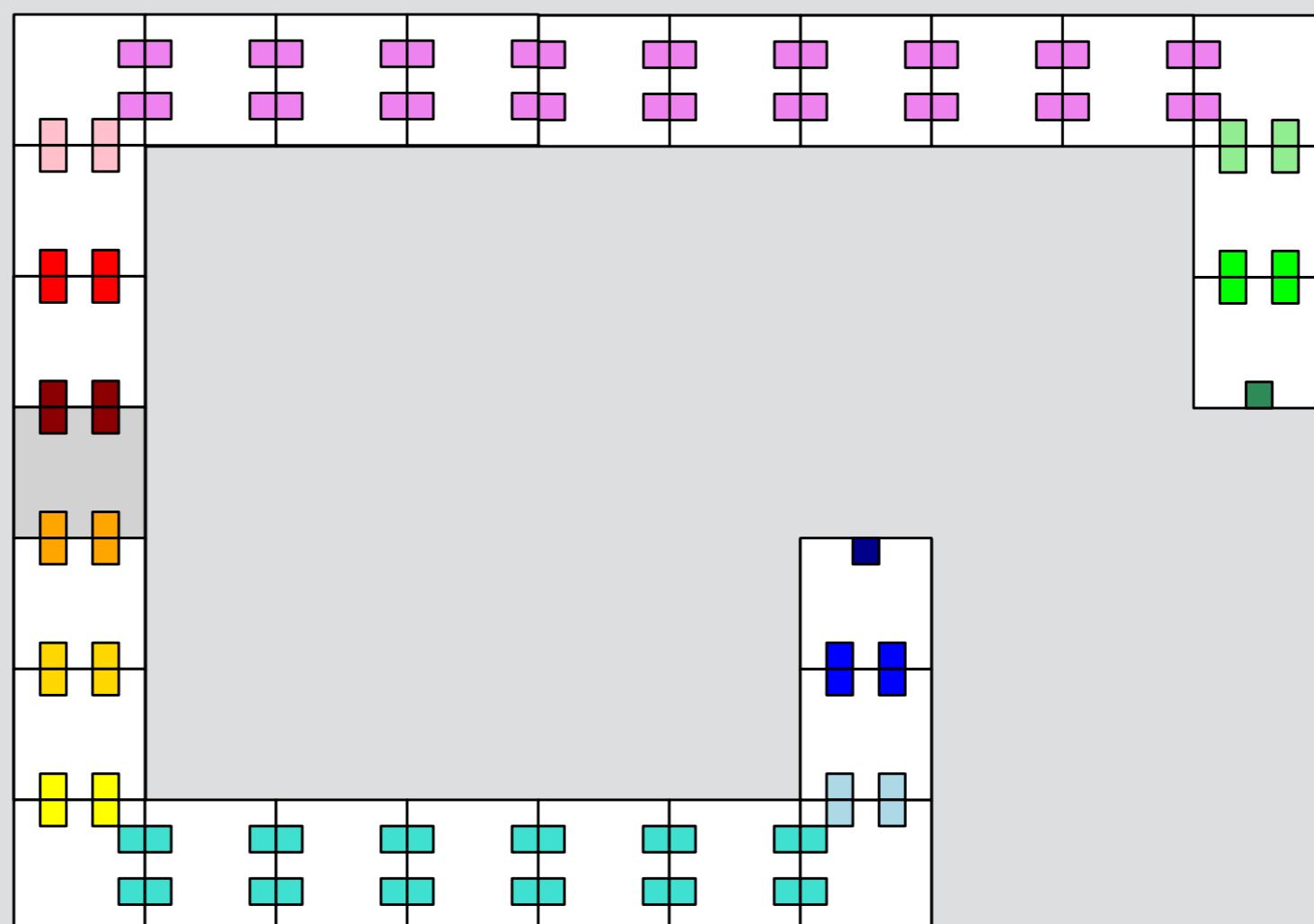
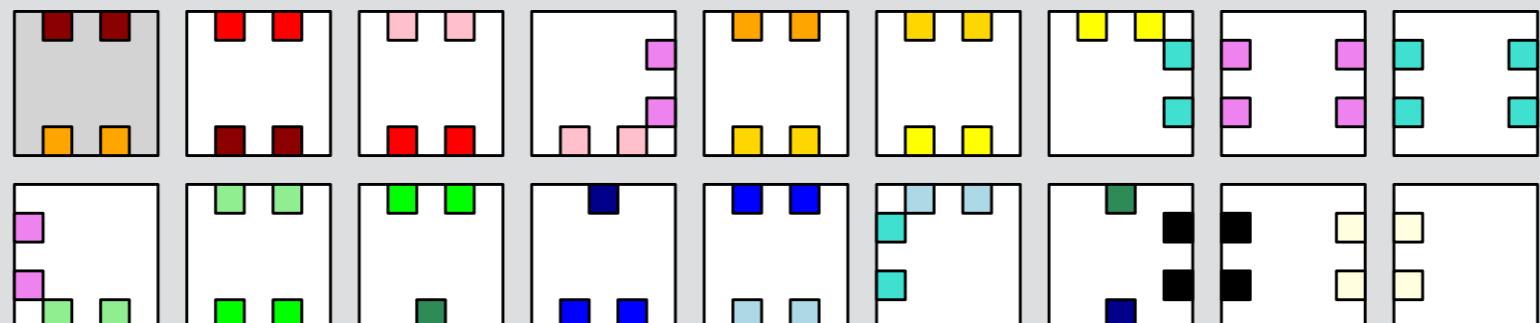
Temperature 2



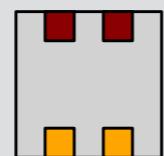
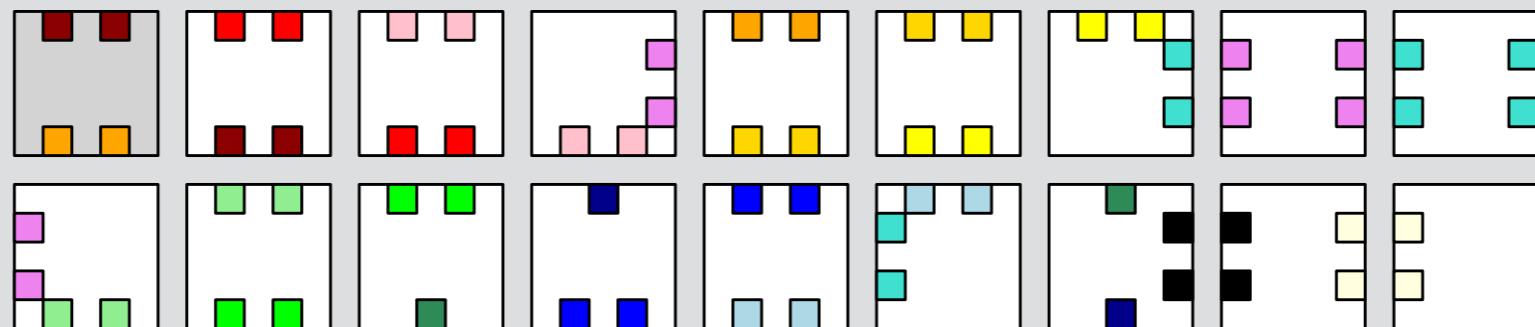
Temperature 2



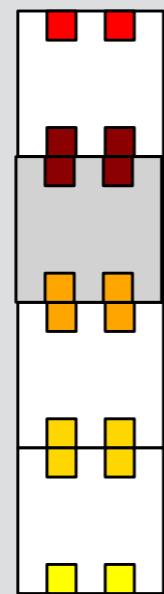
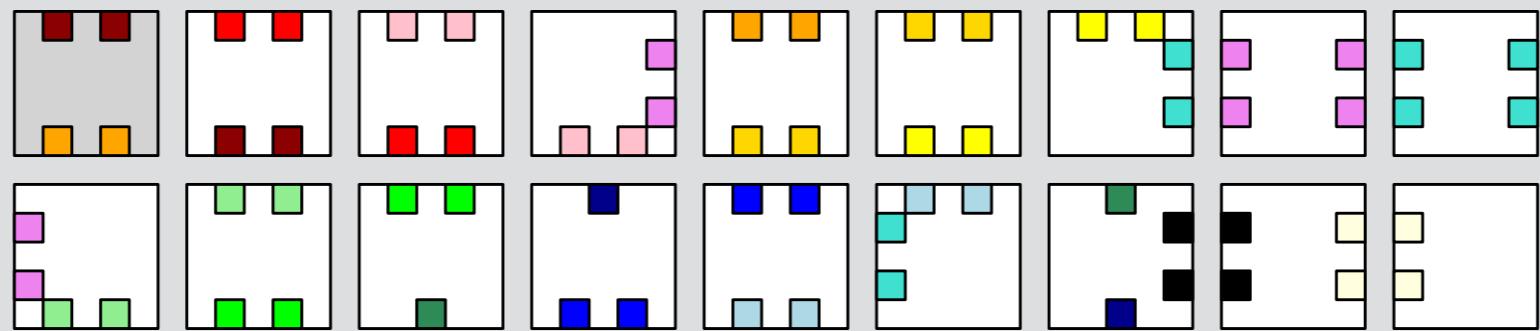
Temperature 2



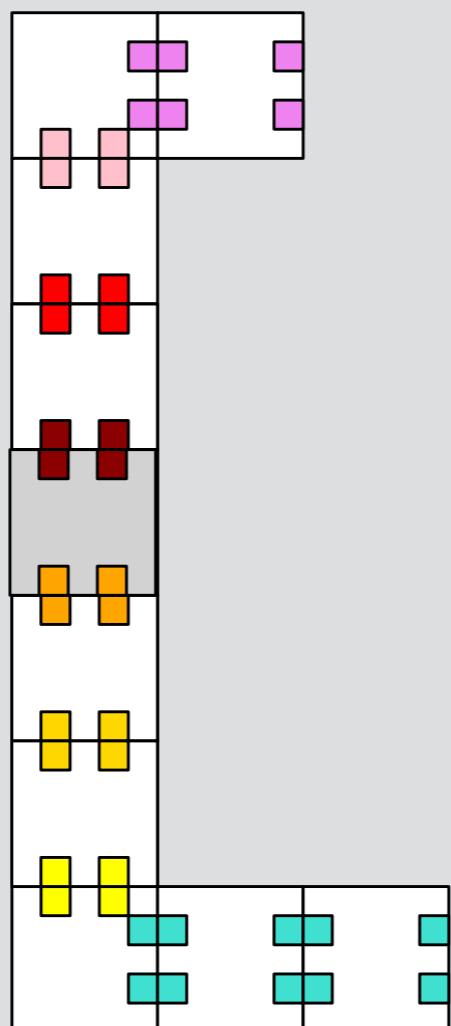
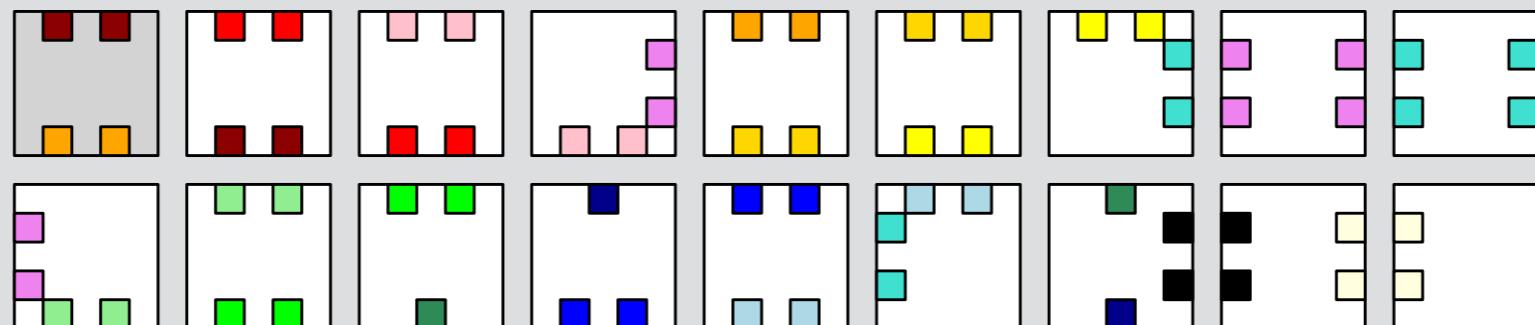
Temperature 2



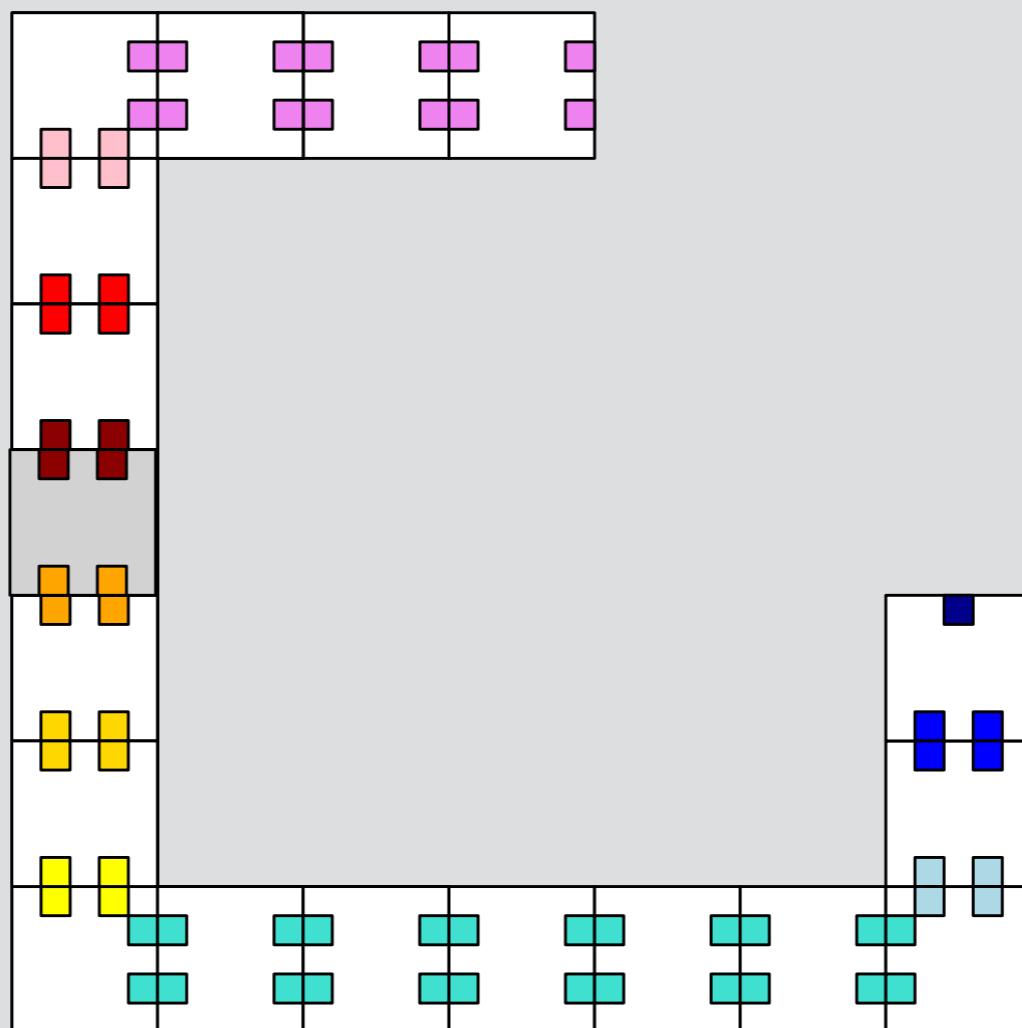
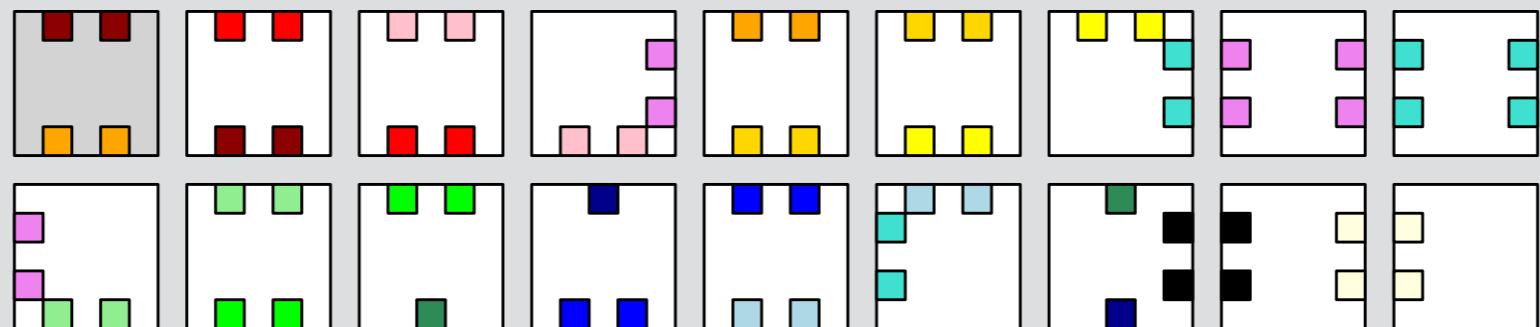
Temperature 2



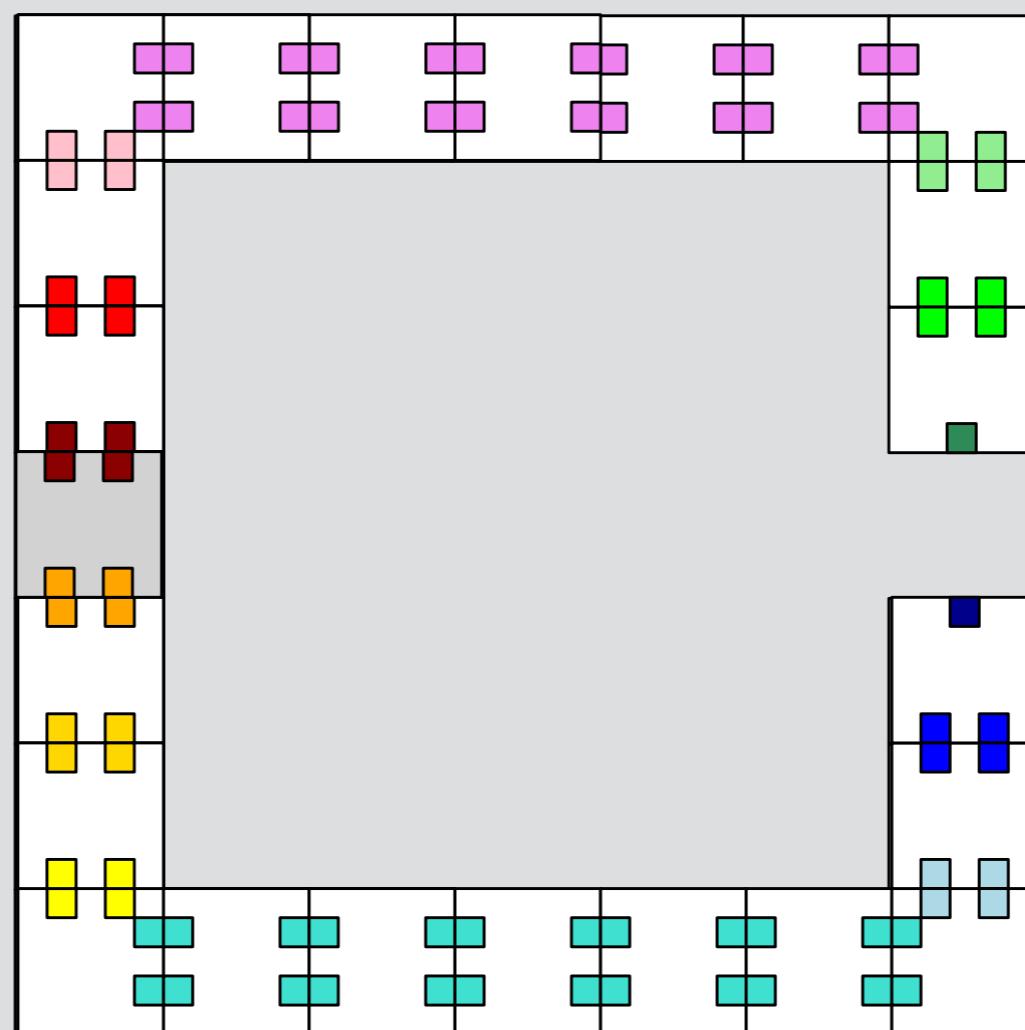
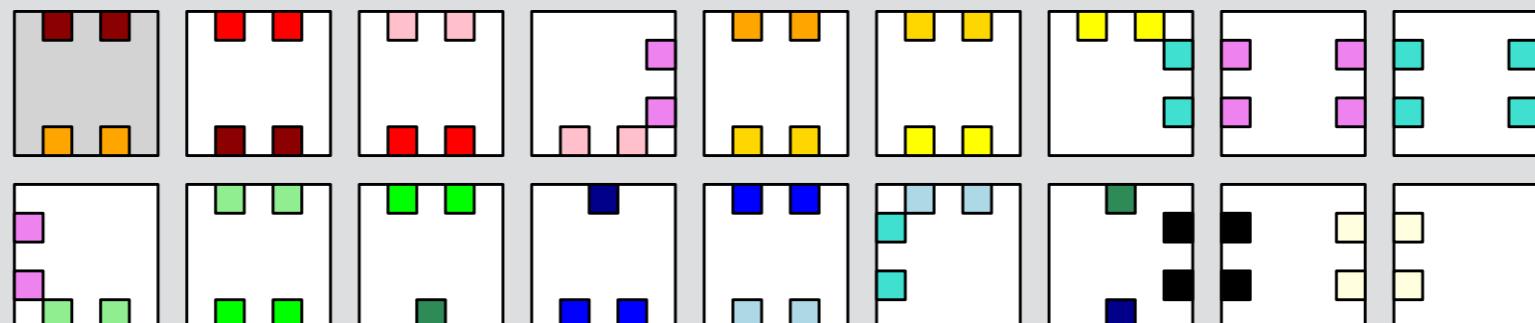
Temperature 2



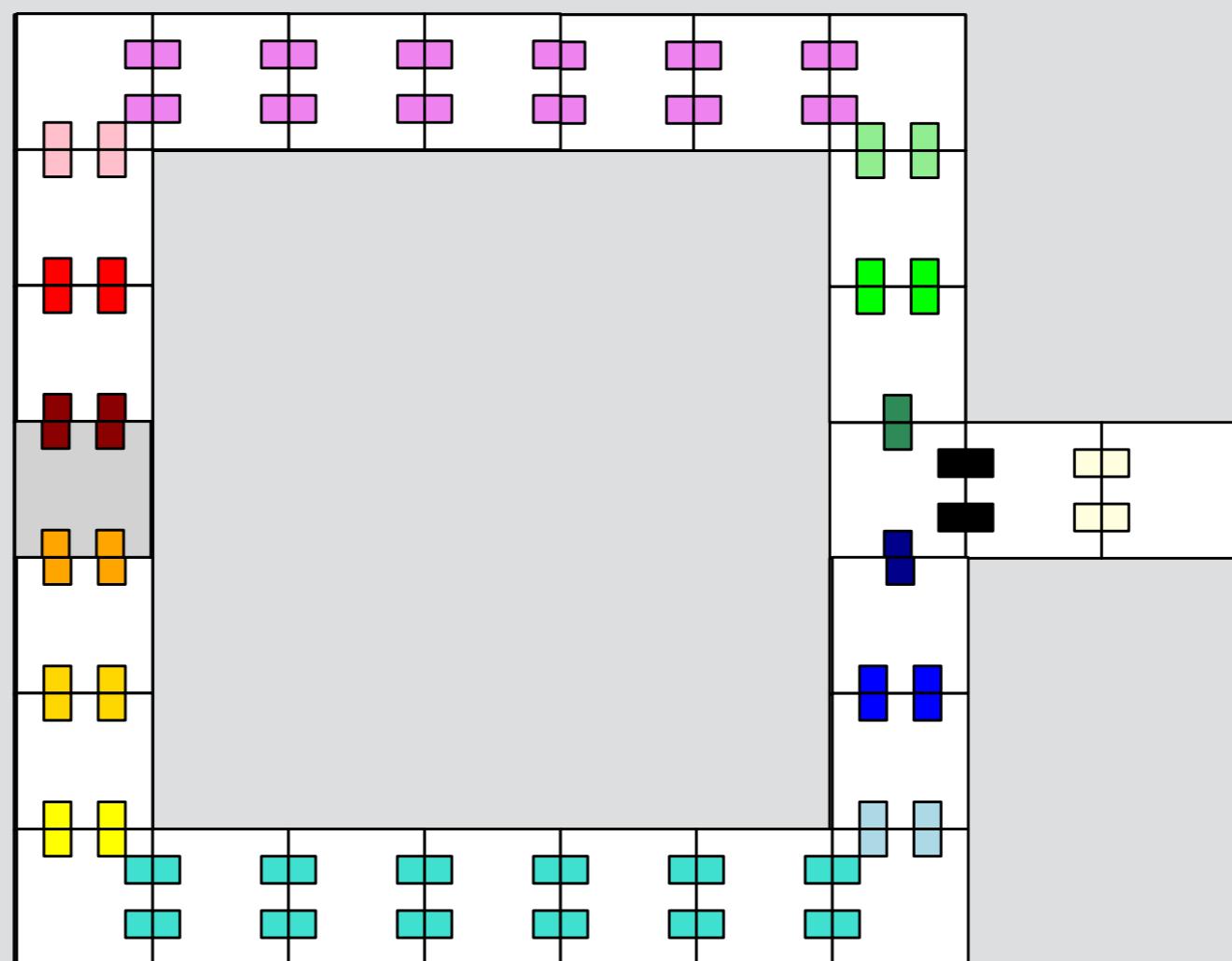
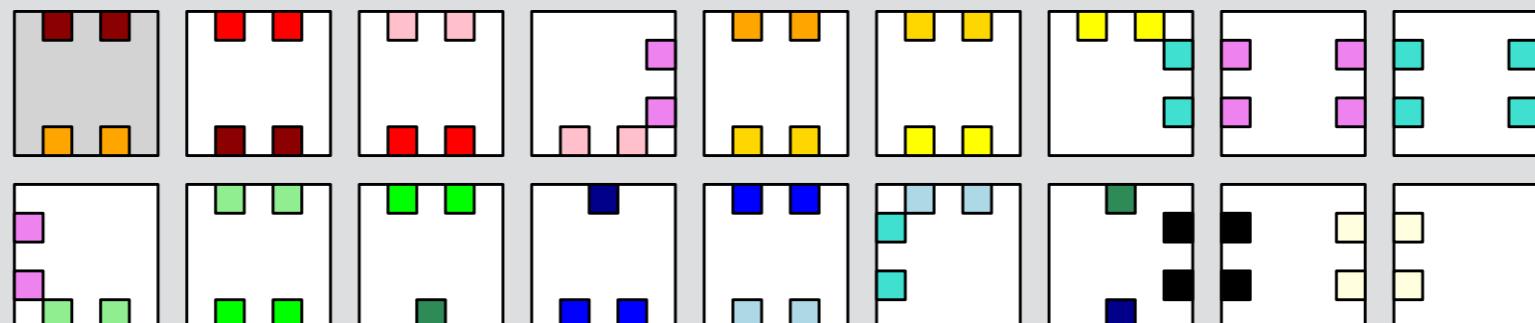
Temperature 2



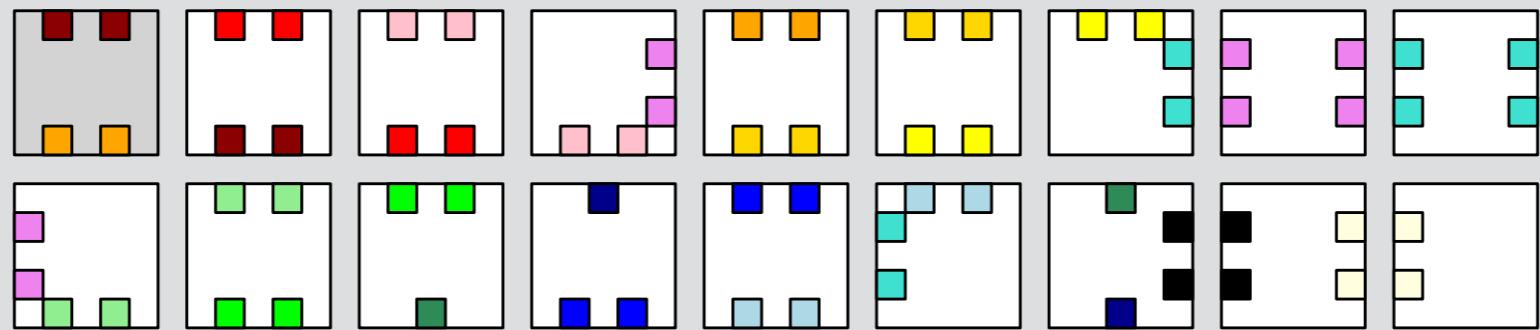
Temperature 2



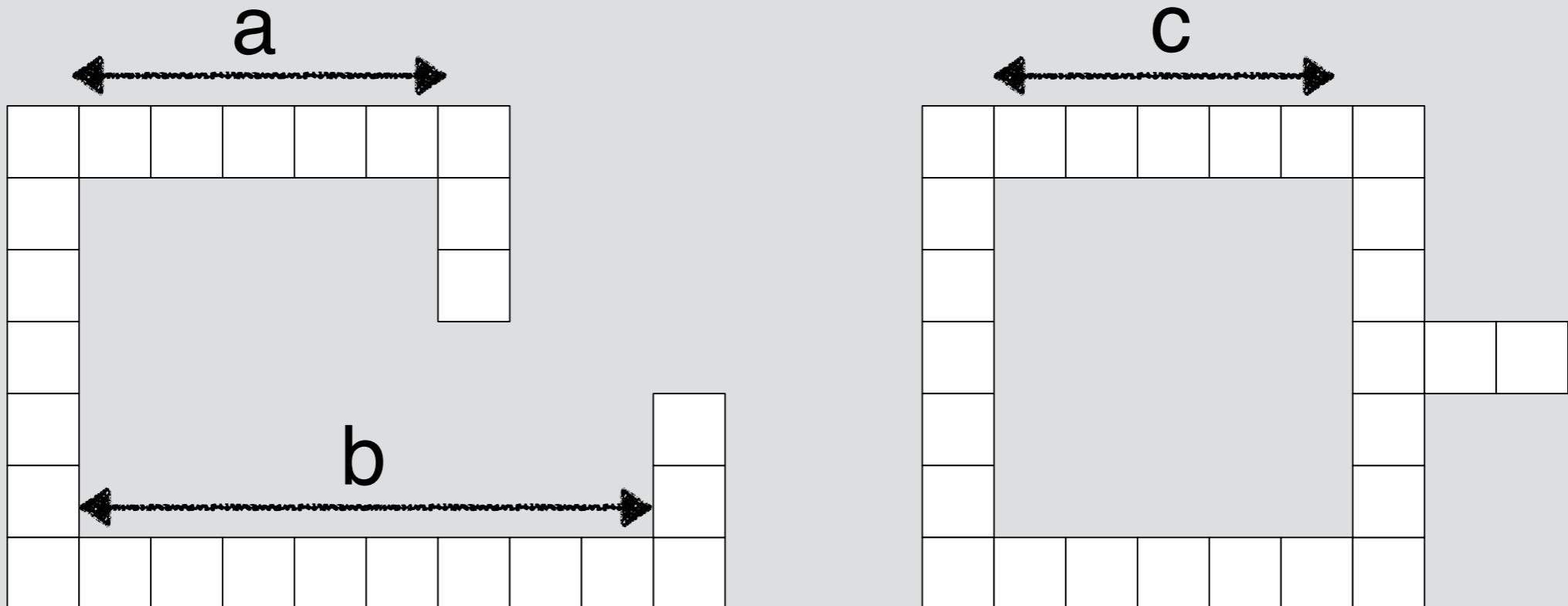
Temperature 2



Temperature 2

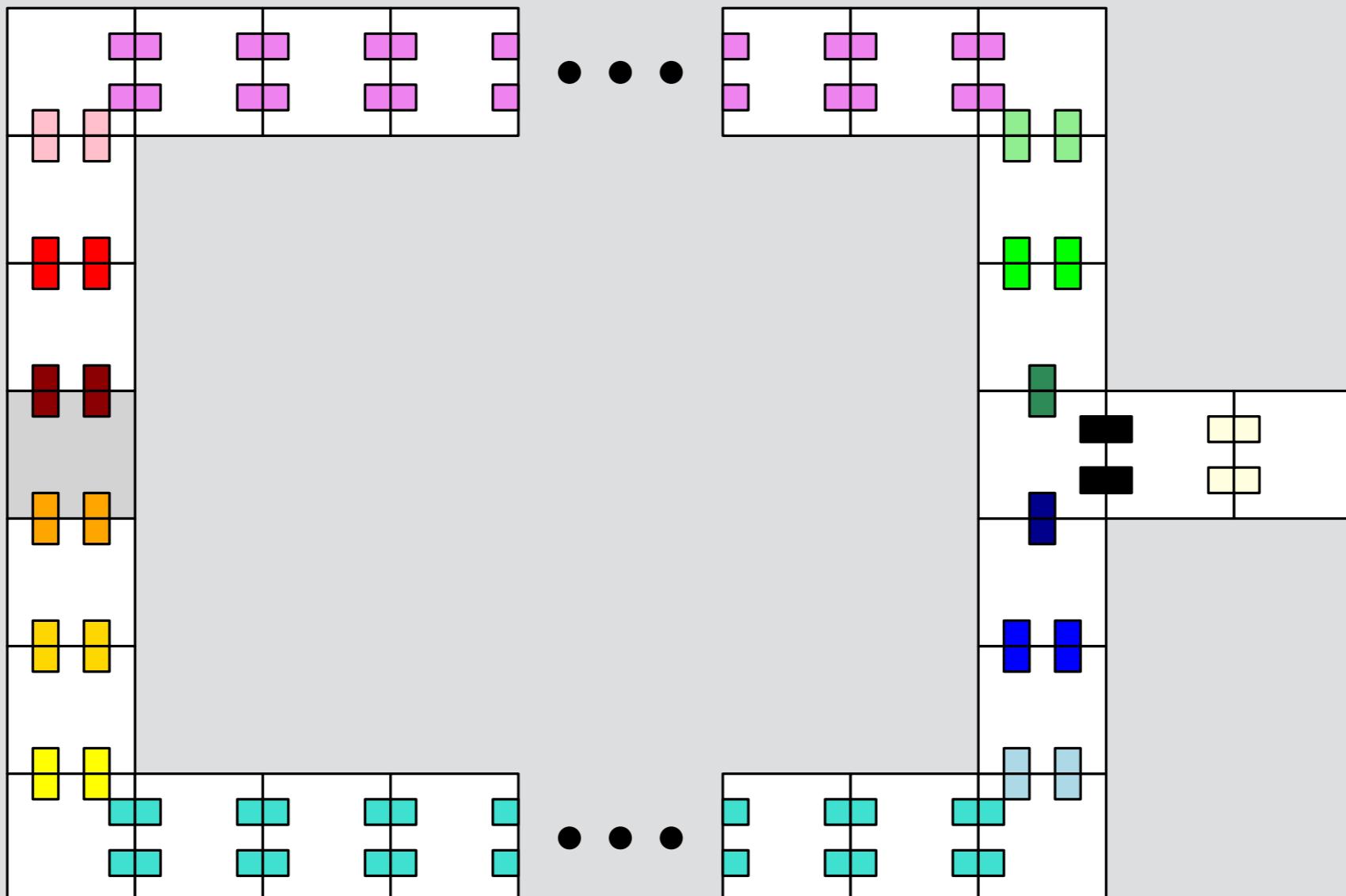


at temperature 2 assembles

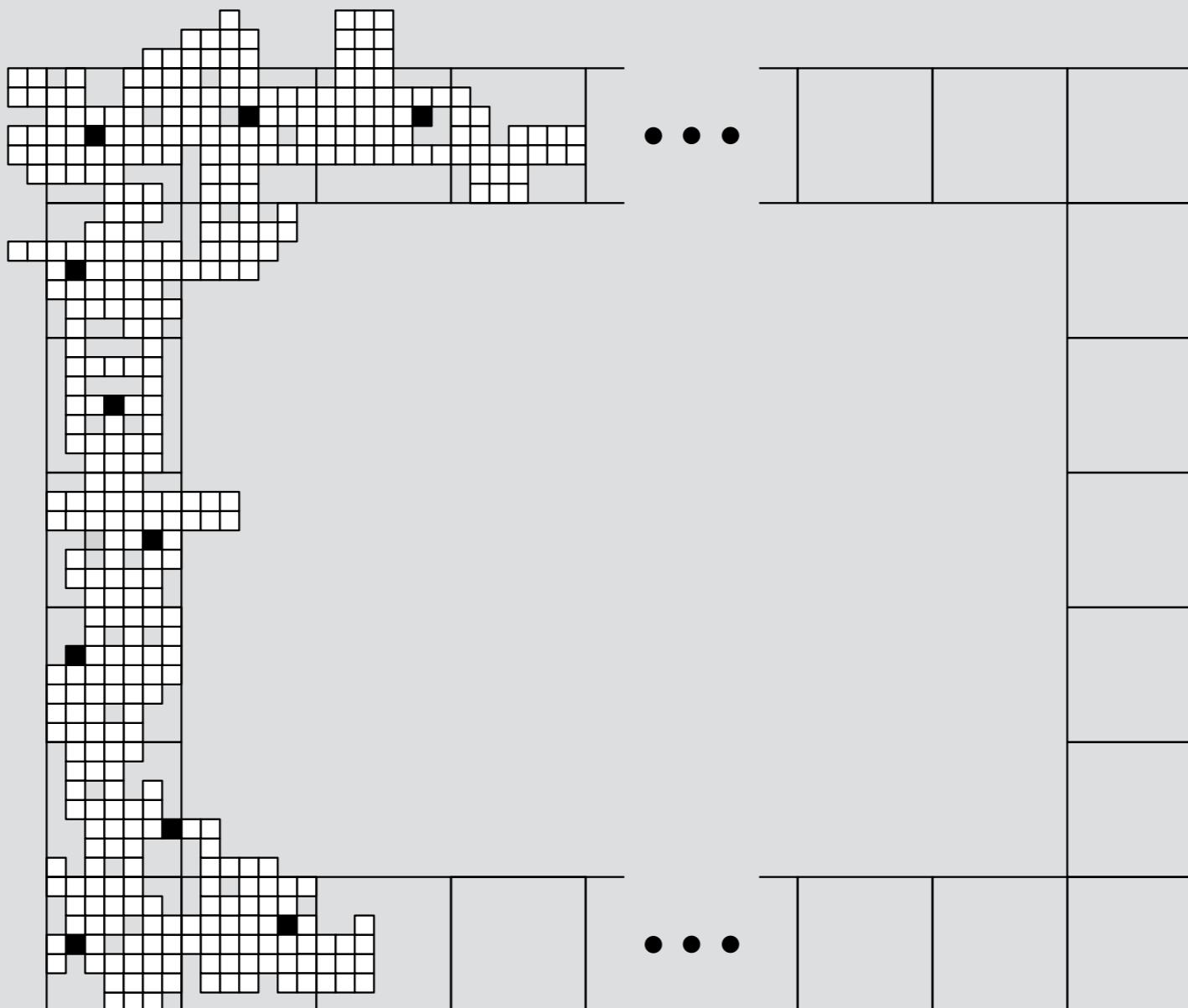


for all  $a, b, c \geq 0$

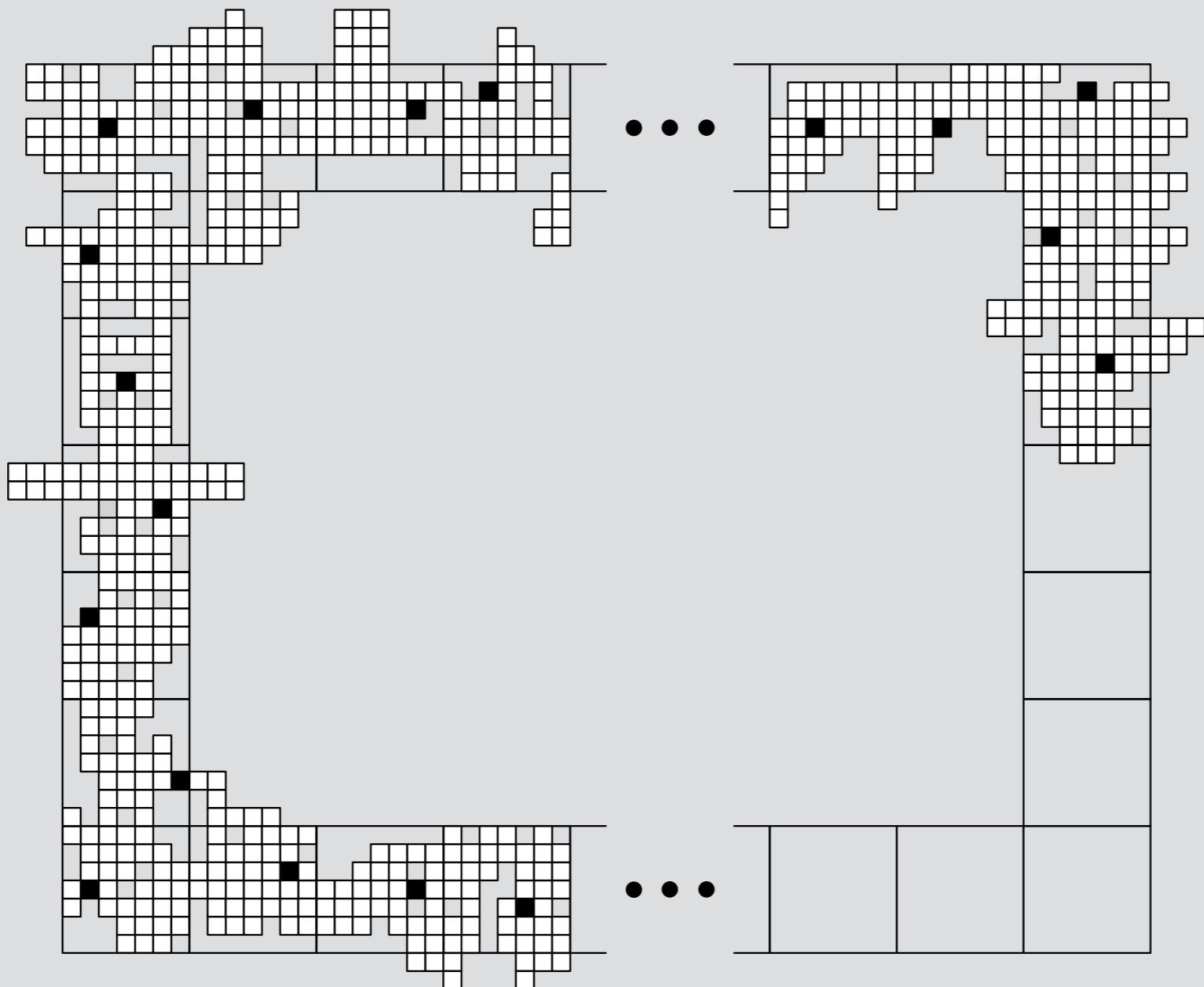
# Claim: simulation will fail.



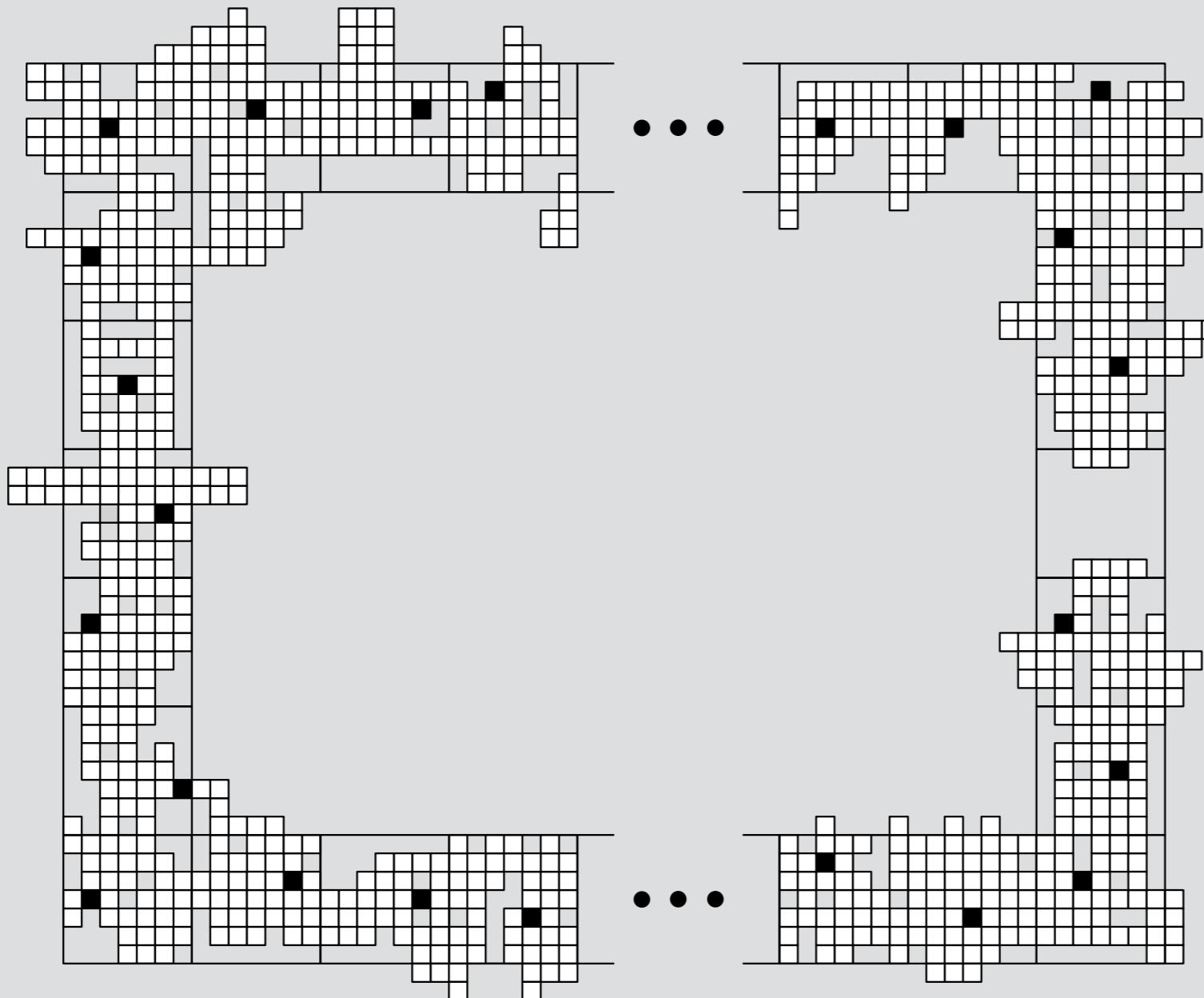
# Claim: simulation will fail.



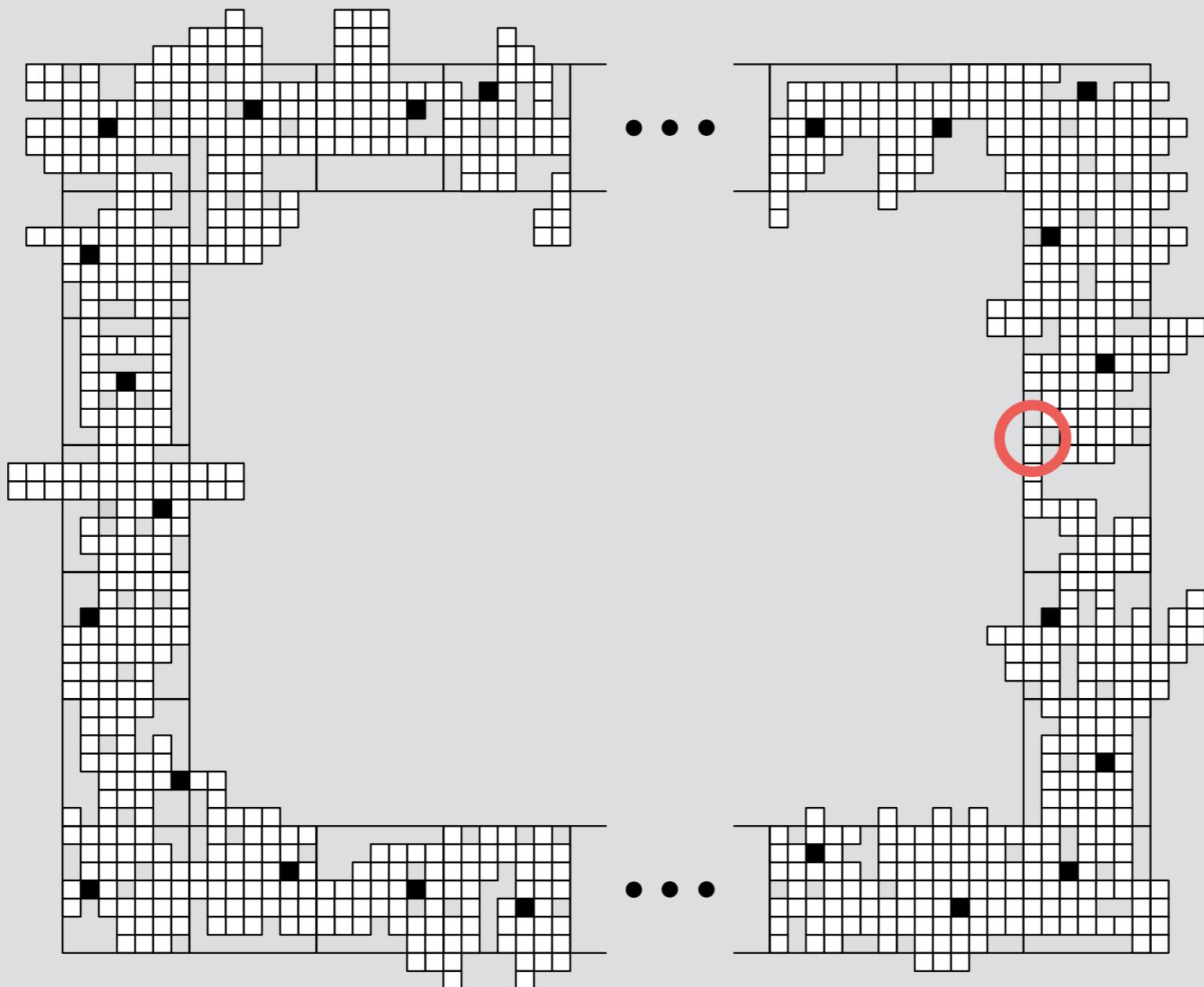
# Claim: simulation will fail.



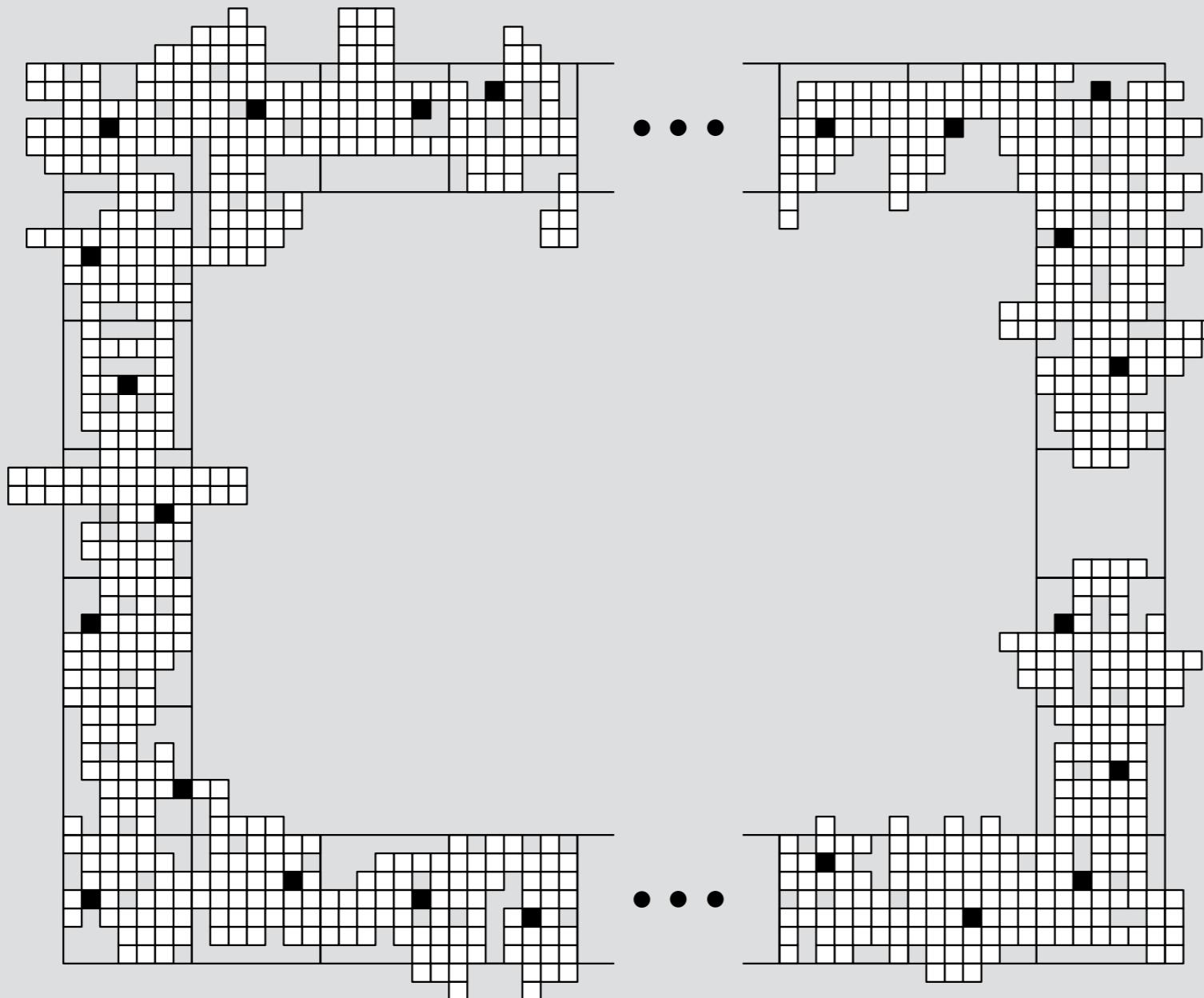
# Claim: simulation will fail.



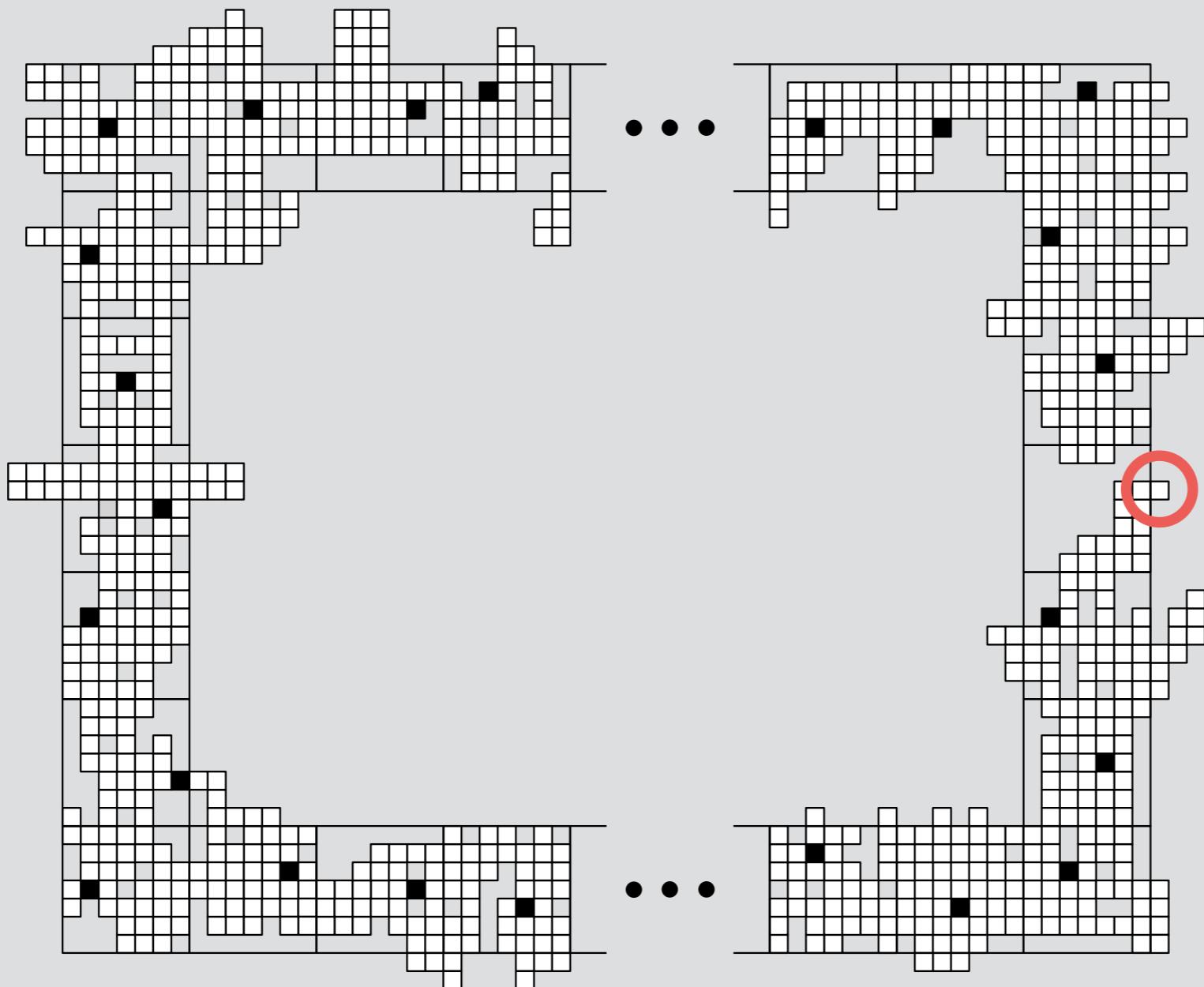
# Claim: simulation will fail.



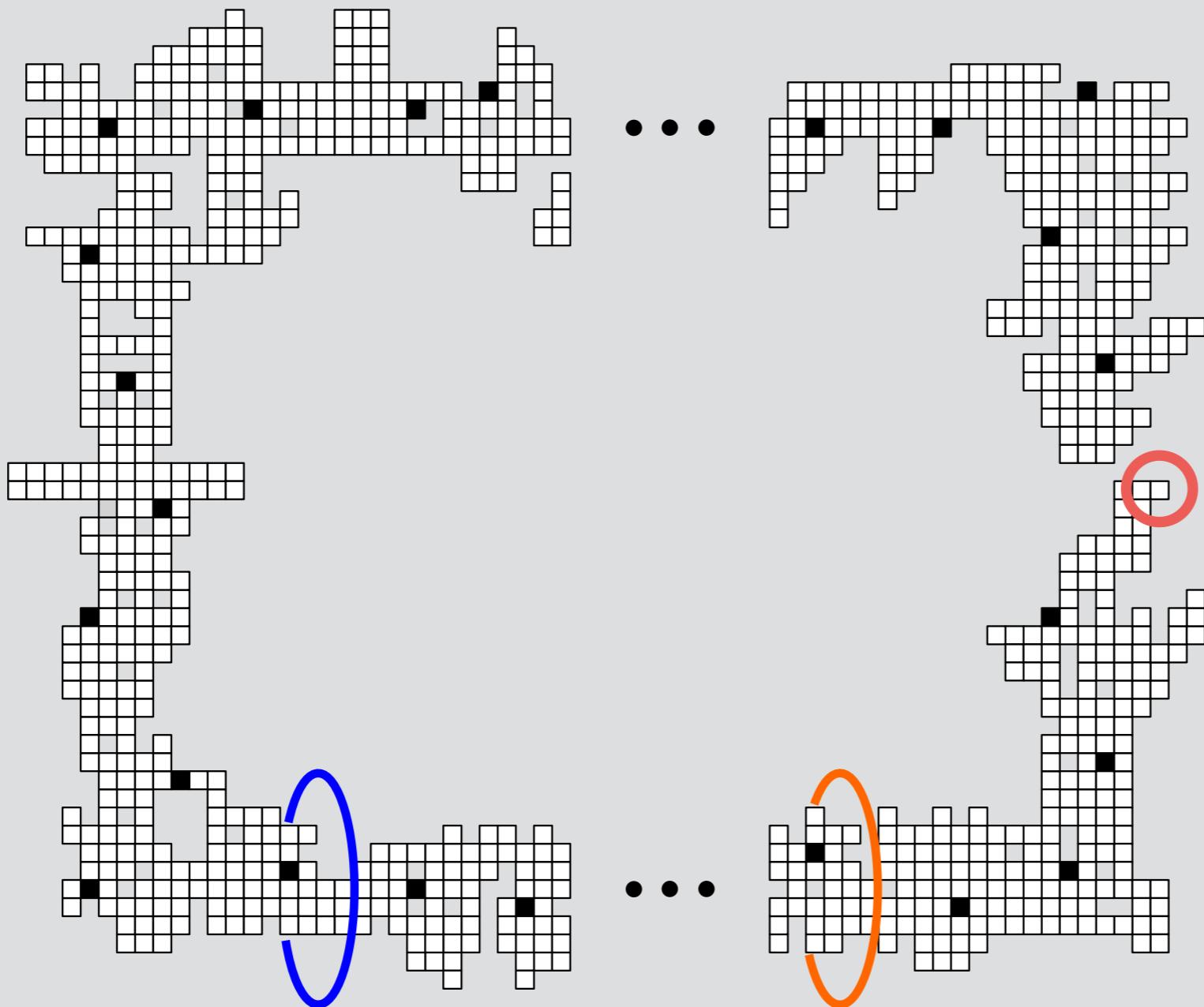
# Claim: simulation will fail.



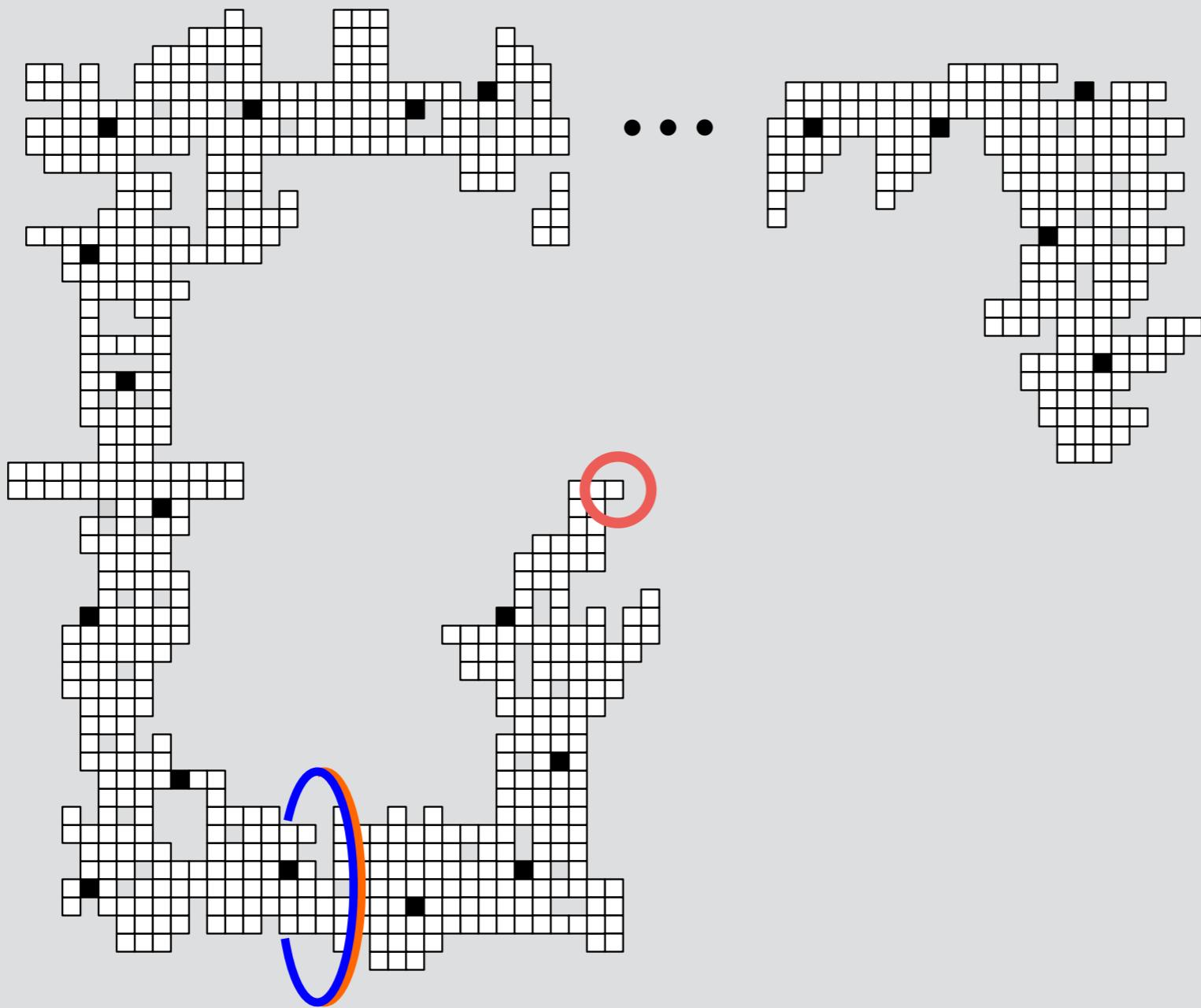
# Claim: simulation will fail.



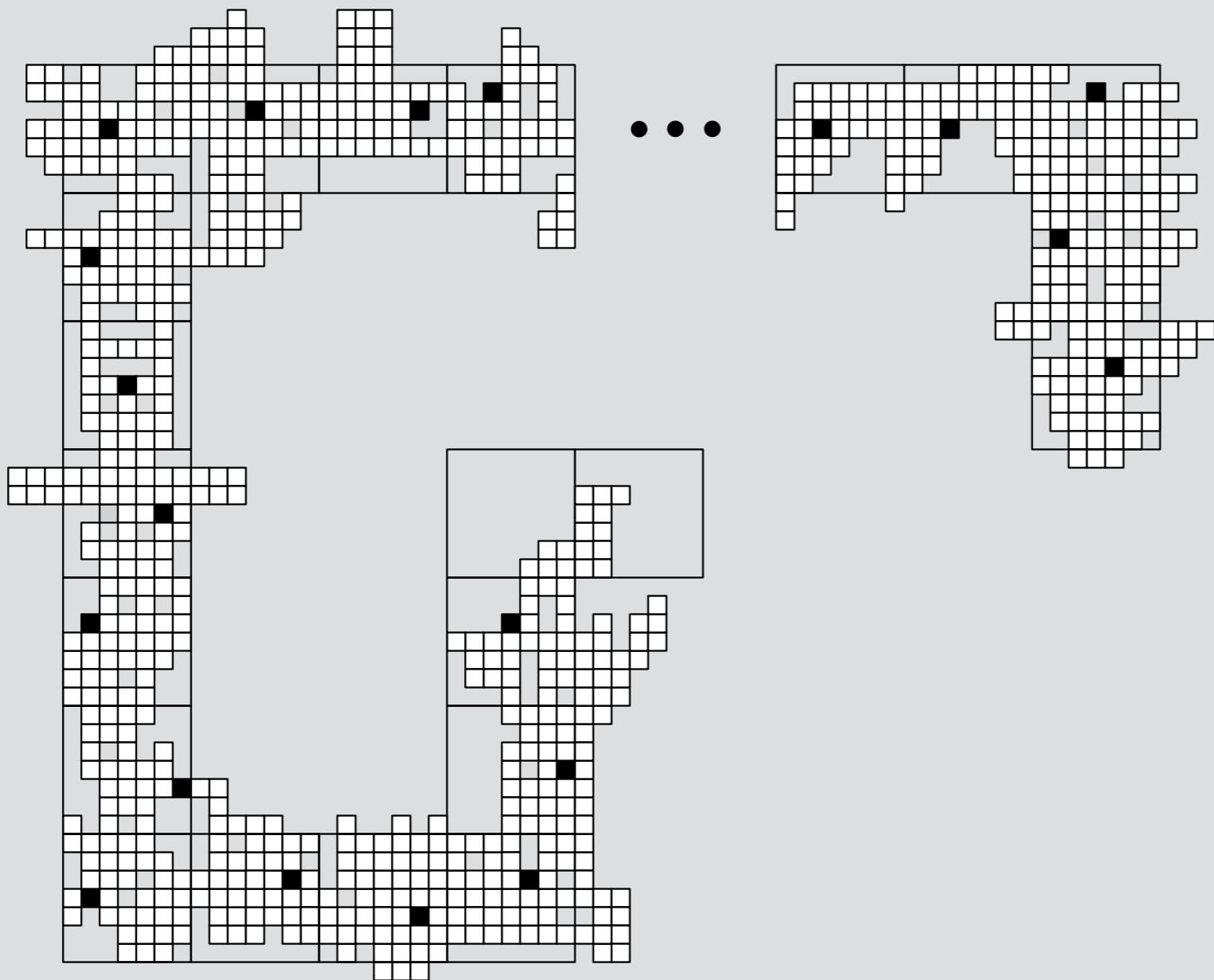
# Claim: simulation will fail.



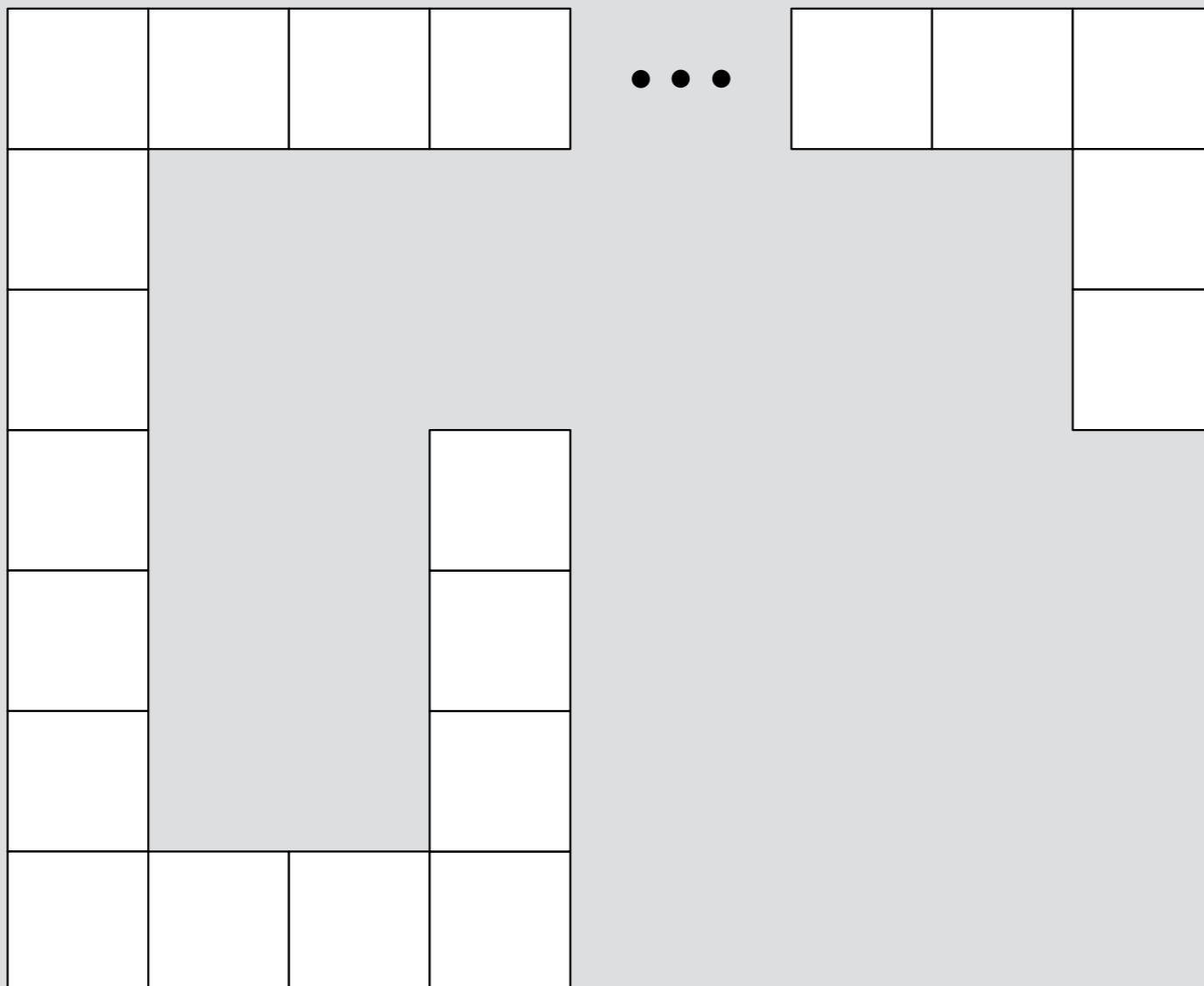
# Claim: simulation will fail.



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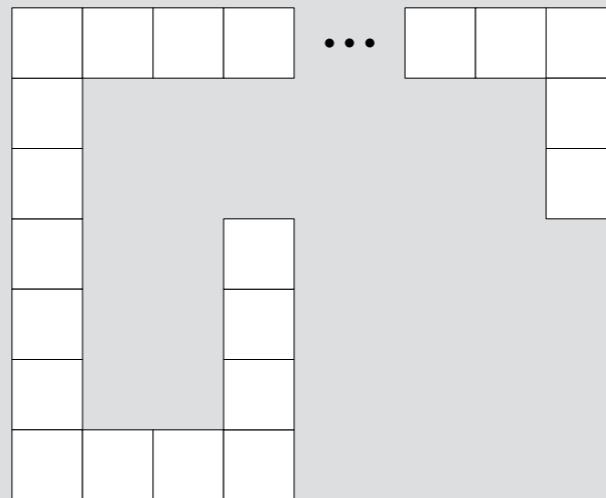


# Claim: simulation will fail.

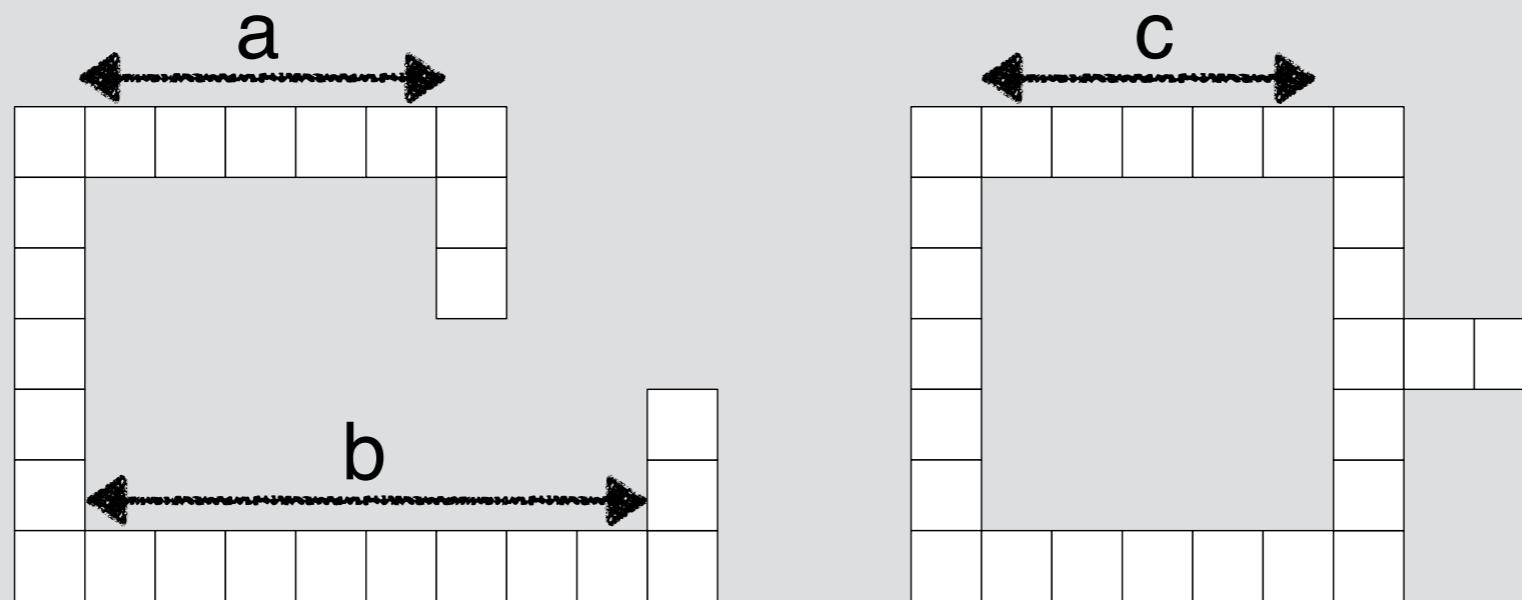


# Claim: simulation will fail.

Assembled by temperature-1 “simulation”:



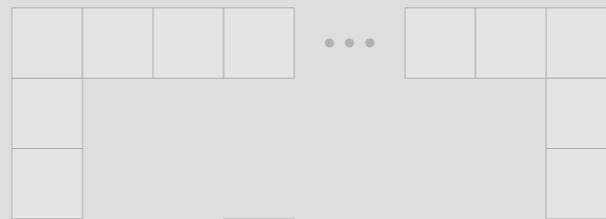
Shapes assembled by at temperature 2:



for all  $a, b, c \geq 0$

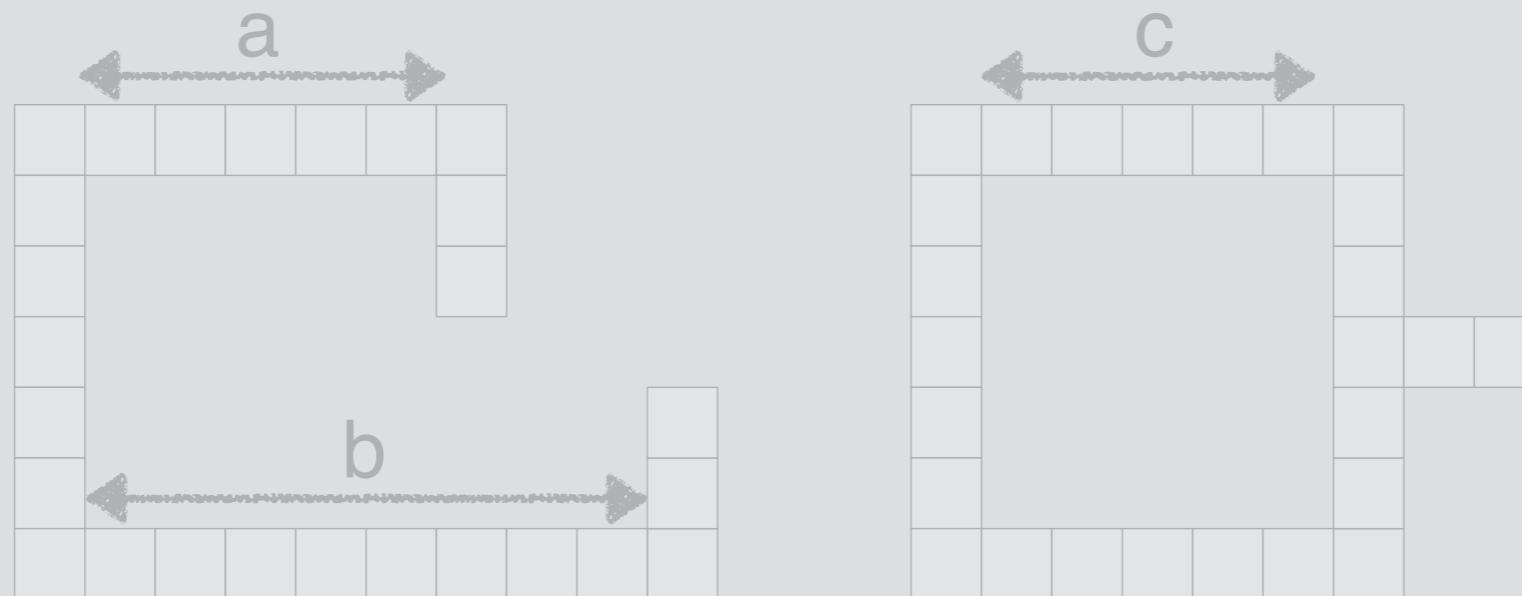
# Claim: simulation will fail.

Assembled by temperature-1 “simulation”:



# **SIMULATION FAILED**

Shapes assembled by  at temperature 2:



for all  $a, b, c \geq 0$

# Fruits of intrinsic universality

The tile assembly model is intrinsically universal

David Doty\*    Jack H. Lutz<sup>†</sup>    Matthew J. Patitz<sup>‡</sup>    Robert T. Schweller<sup>§</sup>  
Scott M. Summers<sup>¶</sup>    Damien Woods<sup>||</sup>

Intrinsic universality in tile self-assembly  
requires cooperation

Pierre-Etienne Meunier\*    Matthew J. Patitz<sup>†</sup>  
Scott M. Summers<sup>‡</sup>    Guillaume Theyssier<sup>§</sup>    Andrew Winslow<sup>¶</sup>  
Damien Woods<sup>||</sup>

The two-handed tile assembly model is not  
intrinsically universal

Erik D. Demaine\*    Matthew J. Patitz<sup>†</sup>    Trent A. Rogers<sup>‡</sup>  
Robert T. Schweller<sup>§</sup>    Scott M. Summers<sup>¶</sup>    Damien Woods<sup>||</sup>

**Signal Transmission across Tile Assemblies:  
3D Static Tiles Simulate Active Self-assembly  
by 2D Signal-Passing Tiles**

Jacob Hendricks<sup>1,\*</sup>, Jennifer E. Padilla<sup>2,\*\*</sup>,  
Matthew J. Patitz<sup>1,\*</sup>, and Trent A. Rogers<sup>3,\*</sup>

**One Tile to Rule Them All:  
Simulating Any Tile Assembly System  
with a Single Universal Tile<sup>\*,\*\*</sup>**

Erik D. Demaine<sup>1</sup>, Martin L. Demaine<sup>1</sup>, Sándor P. Fekete<sup>2</sup>,  
Matthew J. Patitz<sup>3</sup>, Robert T. Schweller<sup>4</sup>,  
Andrew Winslow<sup>5</sup>, and Damien Woods<sup>6</sup>

It's a Tough Nanoworld: in Tile Assembly,  
Cooperation is not (strictly) more Powerful than  
Competition

Florent Becker\*    Pierre-Étienne Meunier<sup>†</sup>

**The Simulation Powers and Limitations  
of Hierarchical Self-Assembly Systems**

Jacob Hendricks<sup>(✉)</sup>, Matthew J. Patitz, and Trent A. Rogers

Department of Computer Science and Computer Engineering,  
University of Arkansas, Fayetteville, AR, USA  
[{jhendric,patitz,tar003}@uark.edu](mailto:{jhendric,patitz,tar003}@uark.edu)

**Two Hands Are Better Than One  
(up to constant factors)**

Sarah Cannon\*    Erik D. Demaine<sup>†</sup>    Martin L. Demaine<sup>†</sup>    Sarah Eisenstat<sup>†</sup>  
Matthew J. Patitz<sup>‡</sup>    Robert Schweller<sup>‡</sup>    Scott M. Summers<sup>§</sup>    Andrew Winslow<sup>\*</sup>

# Fruits of intrinsic universality

The tile assembly m

David Doty\*    Jack H. Lutz†  
                  Scott M. Summers

Intrinsic universality requires

Pierre-Etienne Meunier  
Scott M. Summers‡    Guillaume Pajitnau  
                  Daniel Winograd

The two-handed t  
intrinsic universality

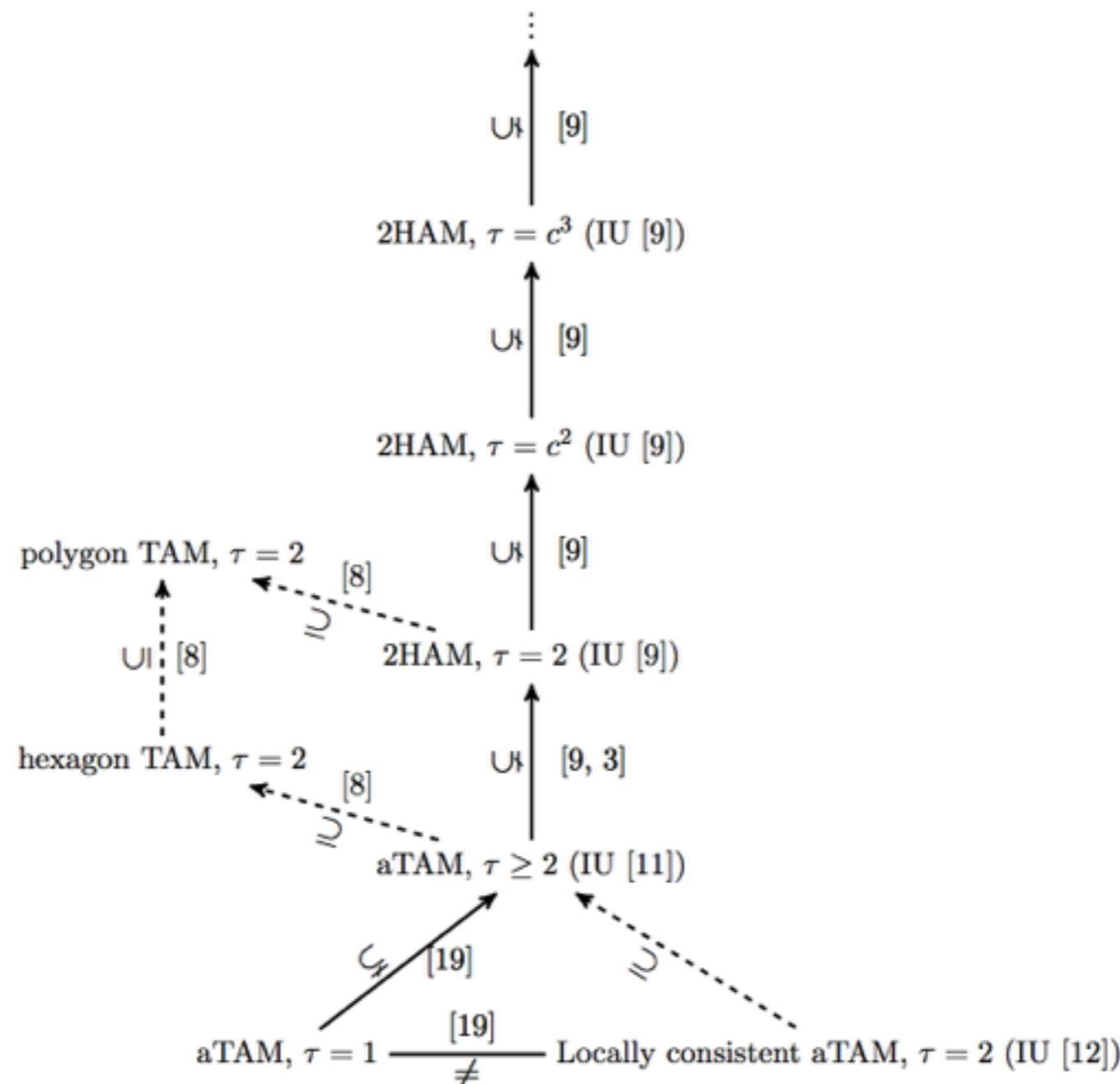
Erik D. Demaine\*    Matthew Patitz  
Robert T. Schweller§    Scott M. Summers

Signal Transmission  
3D Static Tiles Simulated by 2D Sig

Jacob Hendrickson  
Matthew J. Patitz

## Intrinsic universality and the computational power of self-assembly

Damien Woods\*



Them All:  
Assembly System  
Universal Tile\*,\*\*

aine<sup>1</sup>, Sándor P. Fekete<sup>2</sup>,  
rt T. Schweller<sup>4</sup>,  
Damien Woods<sup>6</sup>

: in Tile Assembly,  
) more Powerful than  
tion

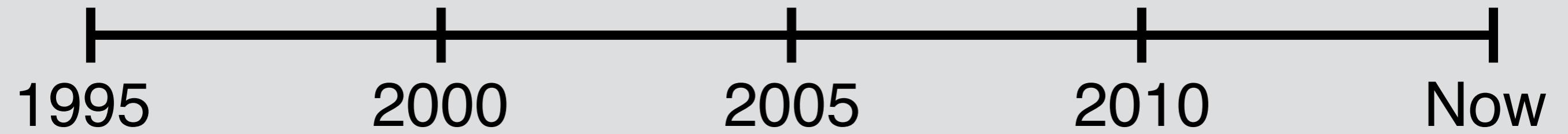
e-Étienne Meunier†

## Properties and Limitations of Self-Assembly Systems

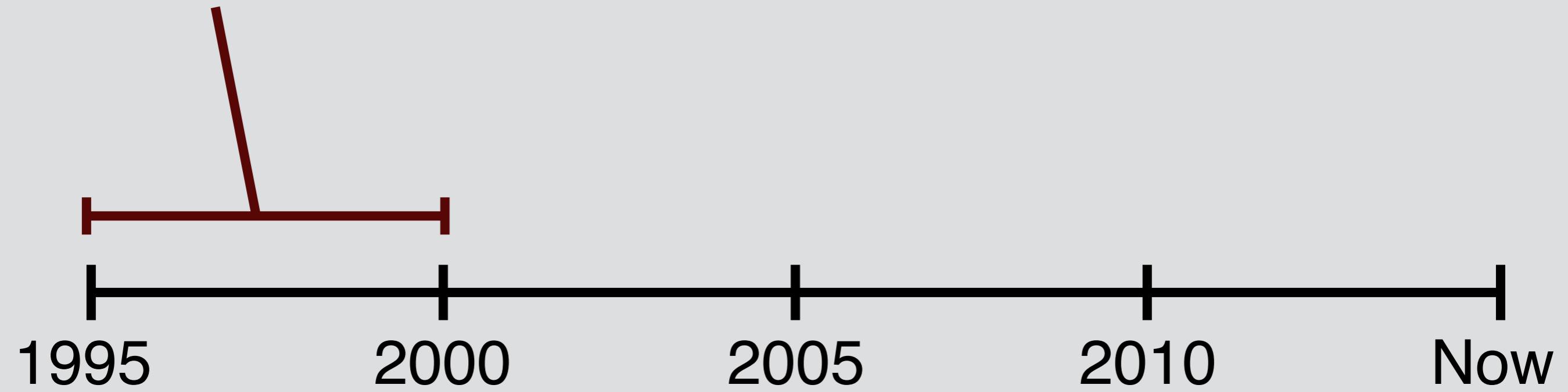
Patitz, and Trent A. Rogers  
and Computer Engineering,  
ayetteville, AR, USA  
003}@uark.edu

## er Than One factors)

Jordan L. Demaine†  
Matthew M. Summers§  
Sarah Eisenstat†  
Andrew Winslow\*

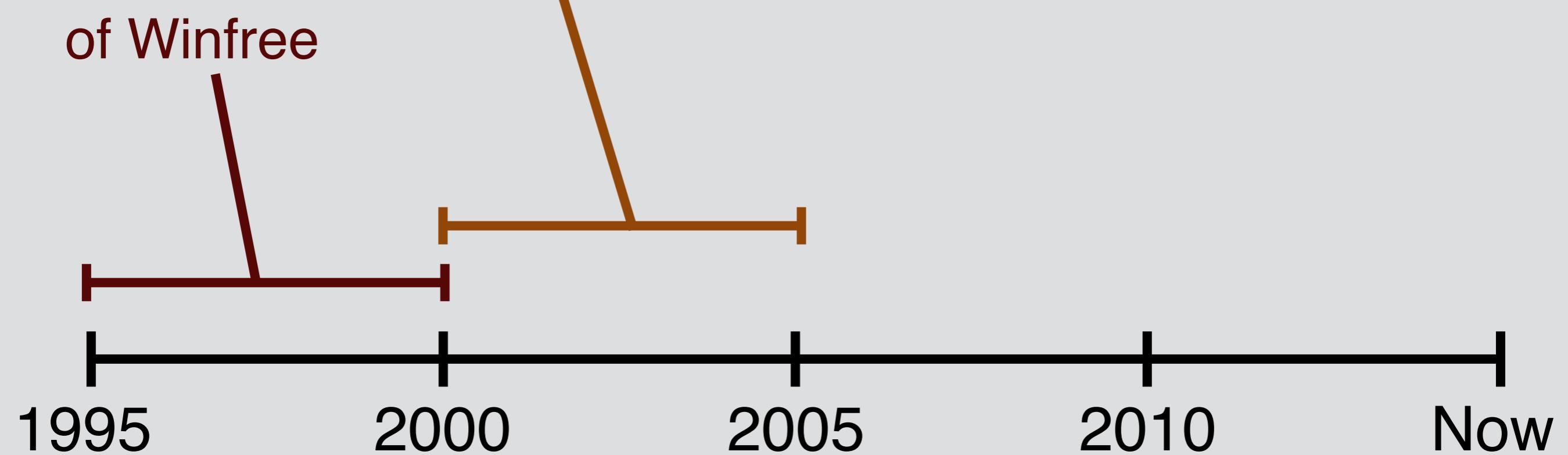


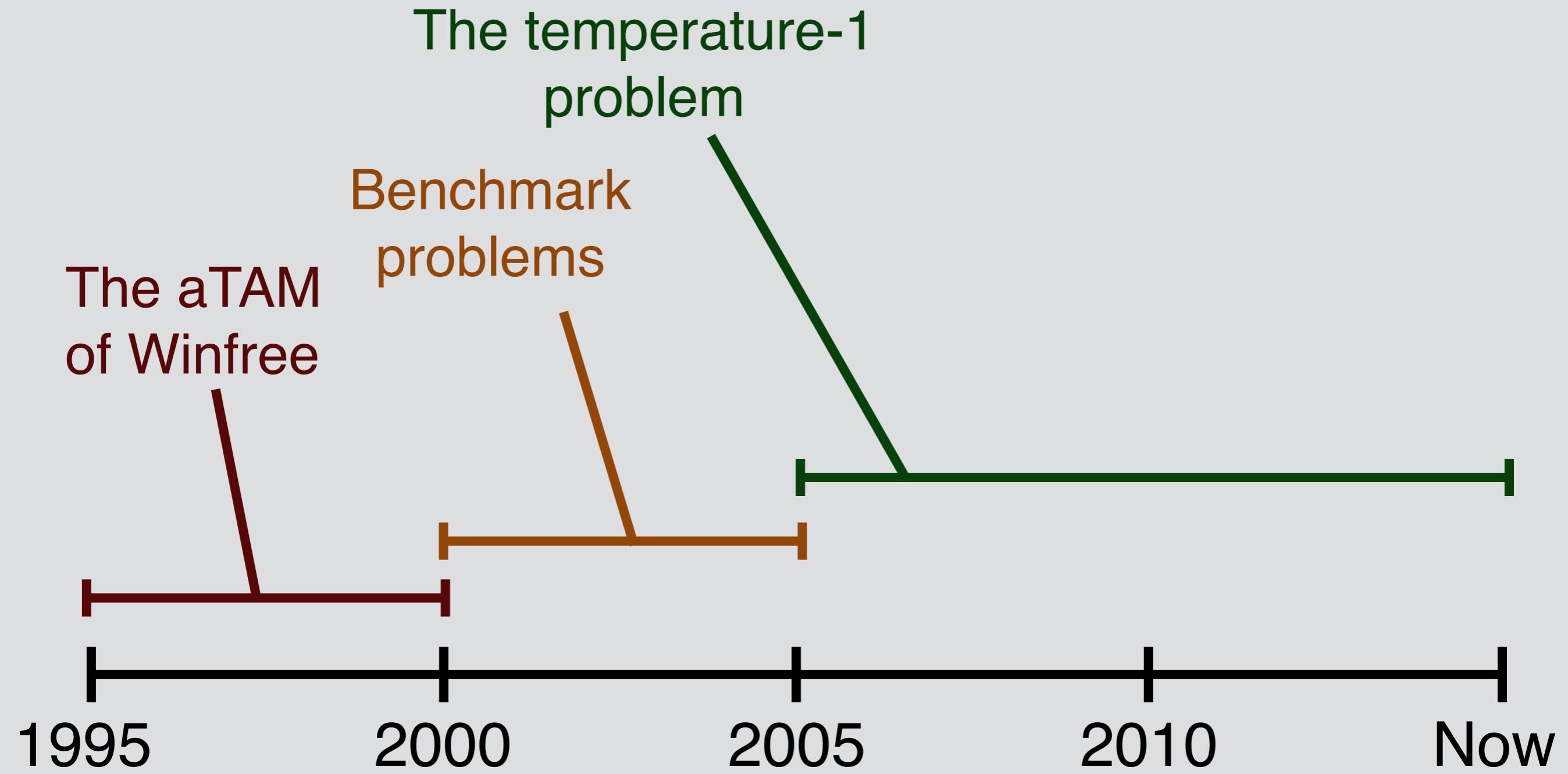
# The aTAM of Winfree



The aTAM  
of Winfree

Benchmark  
problems



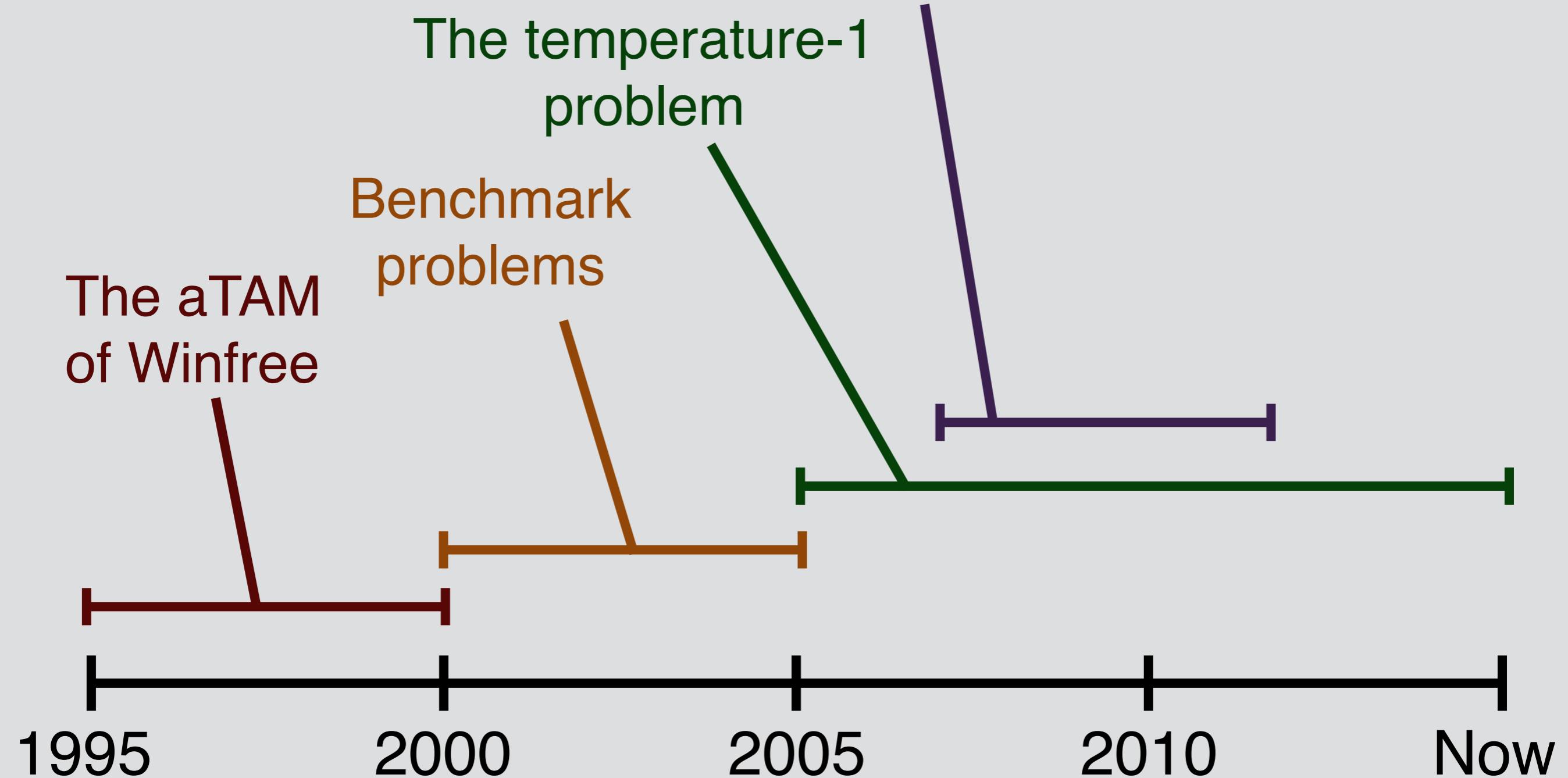


Universality via  
weak cooperation

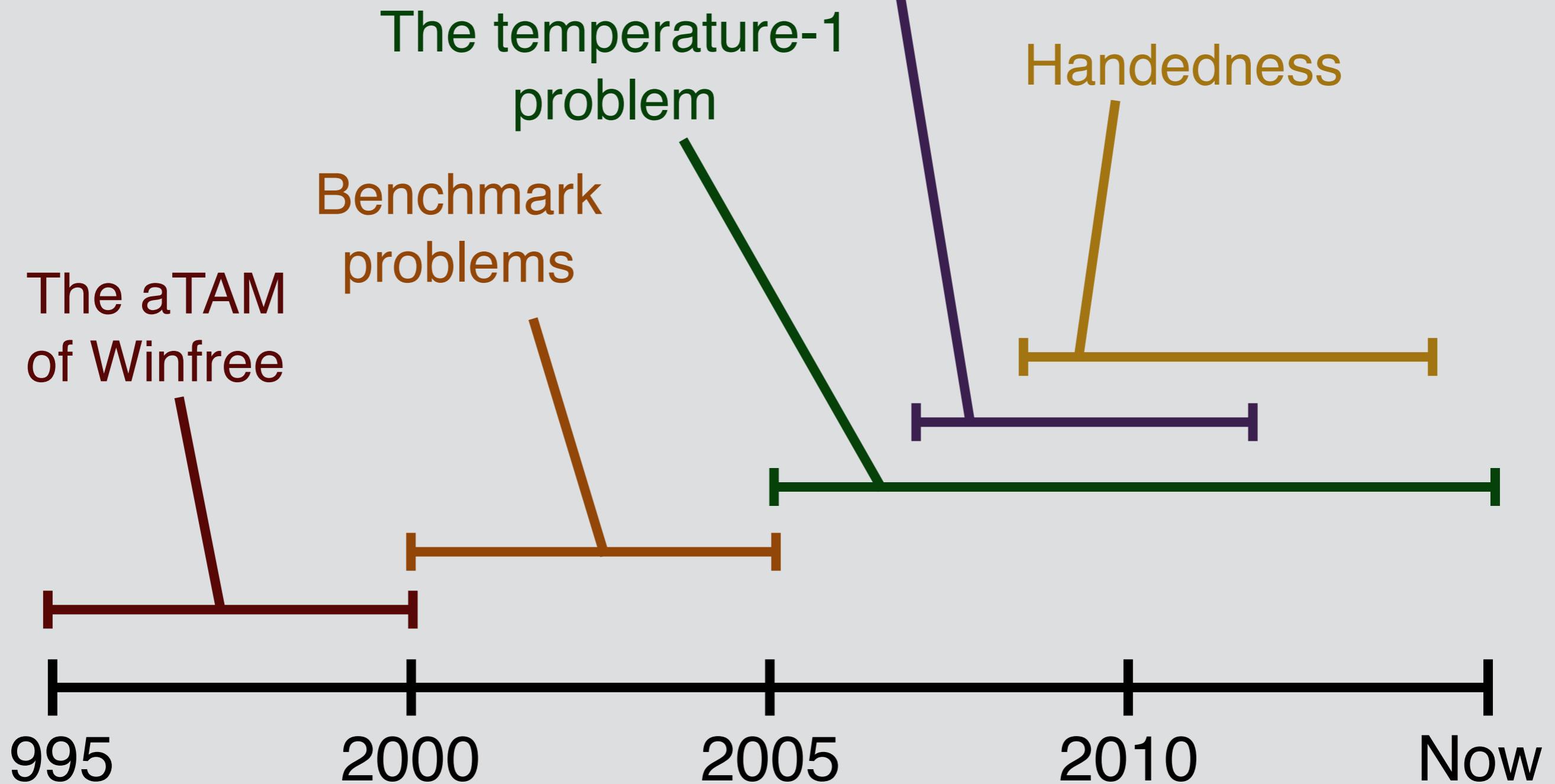
The temperature-1  
problem

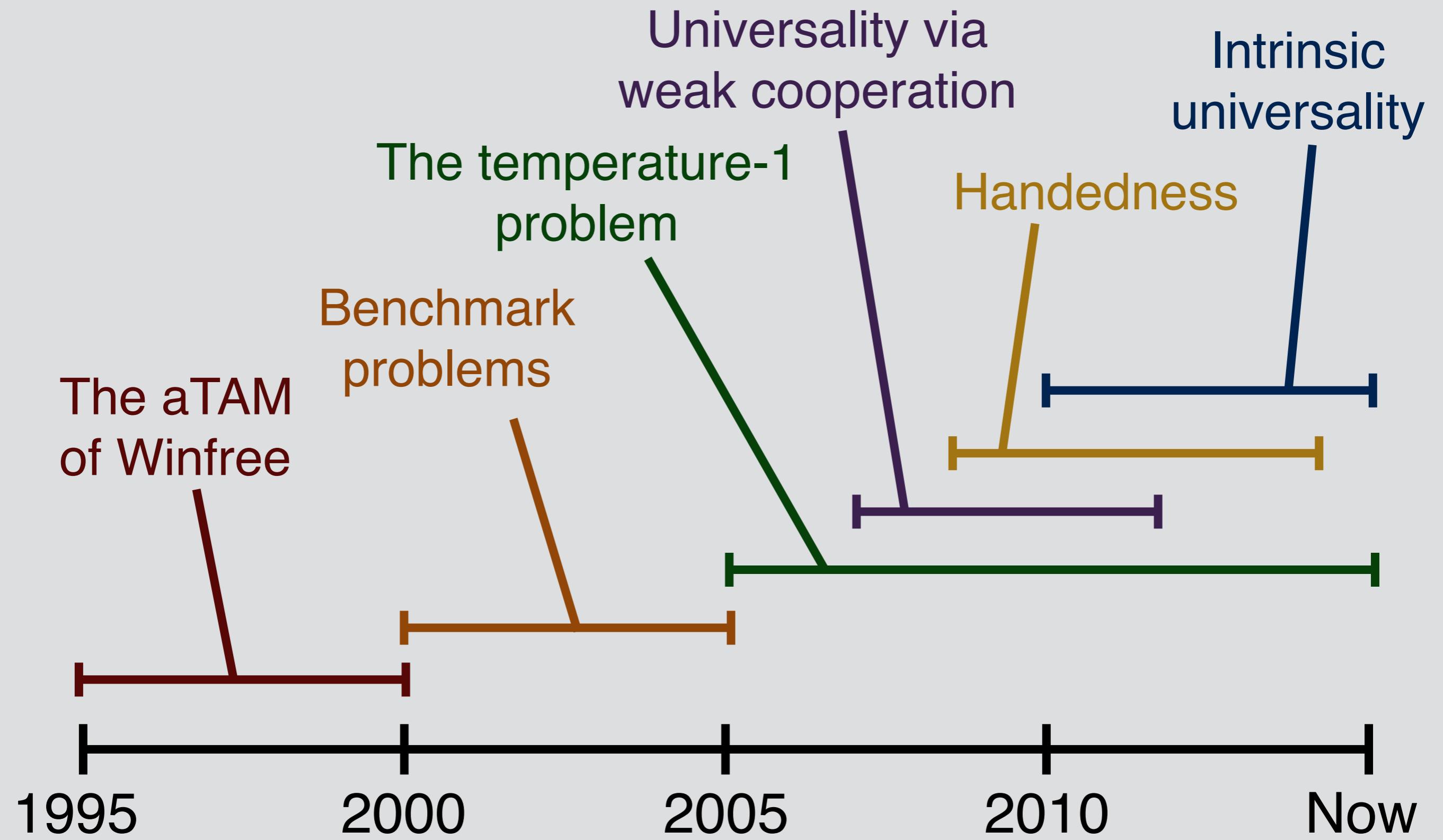
Benchmark  
problems

The aTAM  
of Winfree



Universality via  
weak cooperation





Complete references and more in:

# A Brief Tour of Theoretical Tile Self-Assembly

Andrew Winslow<sup>1</sup>

Université Libre de Bruxelles, Brussels, Belgium

[awinslow@ulb.ac.be](mailto:awinslow@ulb.ac.be)

**Abstract.** The author gives a brief historical tour of theoretical tile self-assembly via chronological sequence of reports on selected topics in the field. The result is to provide context and motivation for the these results and the field more broadly.

# Complete references and more in:



## A Brief Tour of Theoretical Tile Self-Assembly

Andrew Winslow<sup>1</sup>

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