# Harvard Movielens project

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## Introduction

Recomendation system is widely used in many different commercial applications, such as dating website, music streaming platform, or e-commerce website. The objective of the recommendation system is to predict the what the users want based on their preferences and others' previous likings on the platform.

In this project, I will use movielens to build a recommendation system using machine learning techniques to get the best possible residual mean square error (RMSE), which the goal is to reach RMSE $< 0.8649$. The project will show three analytical methods, basic linear regression, regularization, and matrix factorization.

This paper will present in this order, Exploratory data analysis, Modeling, Validation set, and Conculsion,

## Exploratory data analysis

### 1. Movielens dataset

The movielens dataset is an initiative created by the University of Minnesota GroupLens lab to study recommendation system. The website allows users to rate movies that is listed in the movielens website, which the complete dataset has about 26000000 ratings (from 2017 update). For this project, the dataset consists of total 10000054 observations and 6 variables, UserId, movieId, rating, timestamp, title, and genres.

### 2. Data setup

First of all, download the file and use fread() to read the rating.dat, then vectorized the string in movie data.dat by str_split_fixed() and readLines(). Later, combine both ratings and movies into movielens as a dataframe.

After all the process above, movielense dataset is ready to separate into two sets by 90% and 10%, edx for training and validation for testing. The edx is furture split into 2 sets by 80% and 20%, train_data and testing_data, the purpose here is to train the model, so it reach the target RMSE with the smaller set of edx then the model will be validate against the validation set.

```
ratings <- fread(text = gsub("::", "\t", readLines("ml-10M100K/ratings.dat")),
                 col.names = c("userId", "movieId", "rating", "timestamp"))
movies <- str_split_fixed(readLines("ml-10M100K/movies.dat"), "\\::", 3)
colnames(movies) <- c("movieId", "title", "genres")

# if using R 4.0 or later:
movies <- as.data.frame(movies) %>% mutate(movieId = as.numeric(movieId),
                                           title = as.character(title),
                                           genres = as.character(genres))
```

```r
movielens <- left_join(ratings, movies, by = "movieId")
# Validation set will be 10% of MovieLens data
set.seed(1, sample.kind="Rounding") # if using R 3.5 or earlier, use `set.seed(1)`
test_index <- createDataPartition(y = movielens$rating, times = 1, p = 0.1, list = FALSE)
edx <- movielens[-test_index,]
temp <- movielens[test_index,]

# Make sure userId and movieId in validation set are also in edx set
validation <- temp %>%
  semi_join(edx, by = "movieId") %>%
  semi_join(edx, by = "userId")

# Add rows removed from validation set back into edx set
removed <- anti_join(temp, validation)
edx <- rbind(edx, removed)

rm(dl, ratings, movies, test_index, temp, movielens, removed)

# My own training/testing set
# train and test data
set.seed(1, sample.kind="Rounding")
test_index <- createDataPartition(y = edx$rating,
                                  times = 1, p = 0.2, list = FALSE)
train_data <- edx[-test_index,]
temp <- edx[test_index,]

# Make sure userId and movieId in test set are also in train set
test_data <- temp %>%
  semi_join(train_data, by = "movieId") %>%
  semi_join(train_data, by = "userId")

# Add rows removed from test set back into train set
removed <- anti_join(temp, test_data)
train_data <- rbind(train_data, removed)

rm(test_index, temp, removed)
```

### 3. Exploration

```r
str(edx)
```

```
## Classes 'data.table' and 'data.frame':   9000055 obs. of  6 variables:
##  $ userId   : int  1 1 1 1 1 1 1 1 1 1 ...
##  $ movieId  : num  122 185 292 316 329 355 356 362 364 370 ...
##  $ rating   : num  5 5 5 5 5 5 5 5 5 5 ...
##  $ timestamp: int  838985046 838983525 838983421 838983392 838983392 838984474 838983653 838984885 8
##  $ title    : chr  "Boomerang (1992)" "Net, The (1995)" "Outbreak (1995)" "Stargate (1994)" ...
##  $ genres   : chr  "Comedy|Romance" "Action|Crime|Thriller" "Action|Drama|Sci-Fi|Thriller" "Action|A
##  - attr(*, ".internal.selfref")=<externalptr>
```

```r
head(edx)
```

```
##    userId movieId rating timestamp                         title
## 1:      1     122      5 838985046              Boomerang (1992)
## 2:      1     185      5 838983525               Net, The (1995)
## 3:      1     292      5 838983421               Outbreak (1995)
## 4:      1     316      5 838983392               Stargate (1994)
## 5:      1     329      5 838983392 Star Trek: Generations (1994)
## 6:      1     355      5 838984474       Flintstones, The (1994)
##                           genres
## 1:                Comedy|Romance
## 2:         Action|Crime|Thriller
## 3:  Action|Drama|Sci-Fi|Thriller
## 4:       Action|Adventure|Sci-Fi
## 5: Action|Adventure|Drama|Sci-Fi
## 6:       Children|Comedy|Fantasy
```

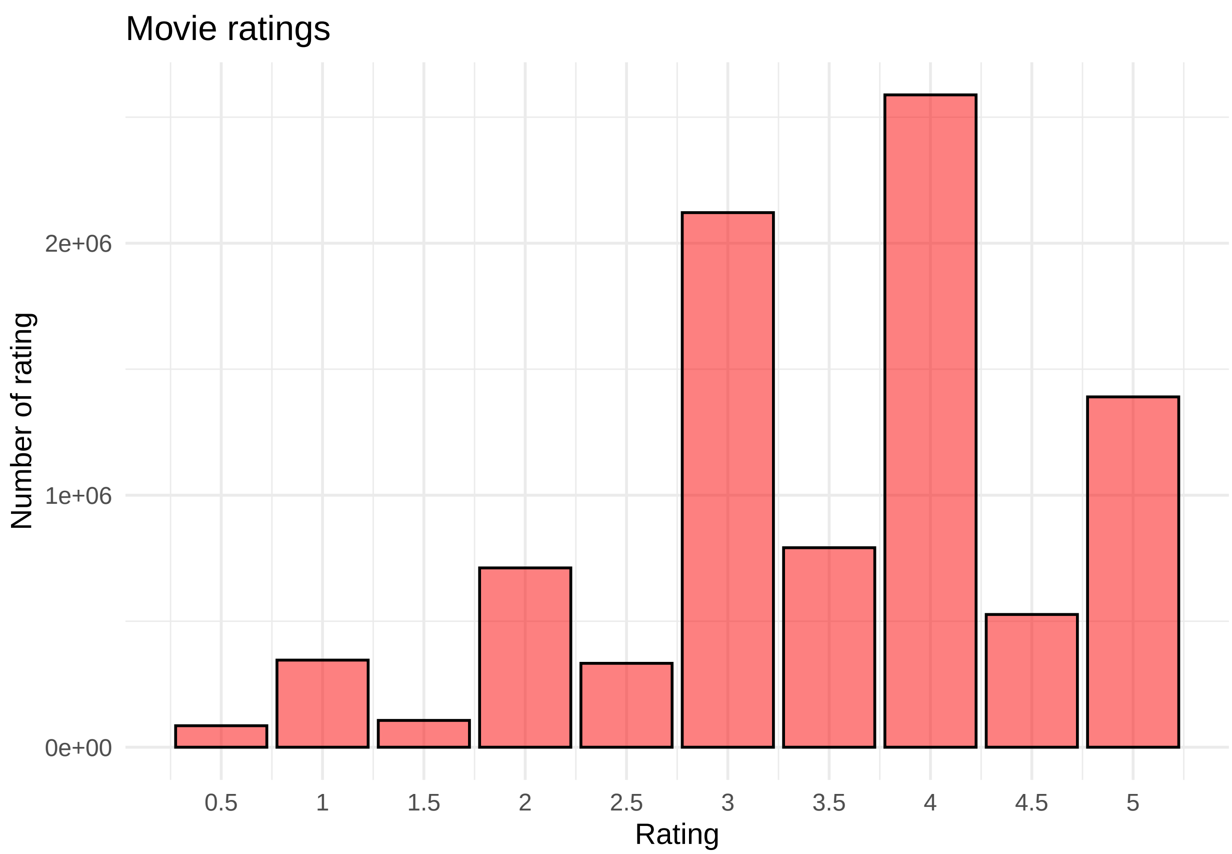
```r
summary(edx)
```

```
##      userId         movieId          rating        timestamp
##  Min.   :    1   Min.   :    1   Min.   :0.500   Min.   :7.897e+08
##  1st Qu.:18124   1st Qu.:  648   1st Qu.:3.000   1st Qu.:9.468e+08
##  Median :35738   Median : 1834   Median :4.000   Median :1.035e+09
##  Mean   :35870   Mean   : 4122   Mean   :3.512   Mean   :1.033e+09
##  3rd Qu.:53607   3rd Qu.: 3626   3rd Qu.:4.000   3rd Qu.:1.127e+09
##  Max.   :71567   Max.   :65133   Max.   :5.000   Max.   :1.231e+09
##     title              genres
##  Length:9000055     Length:9000055
##  Class :character   Class :character
##  Mode  :character   Mode  :character
##
##
##
```

### 3.1 Count of rating

From the bar chart below, you see that most ratings are located at 3 and 4.

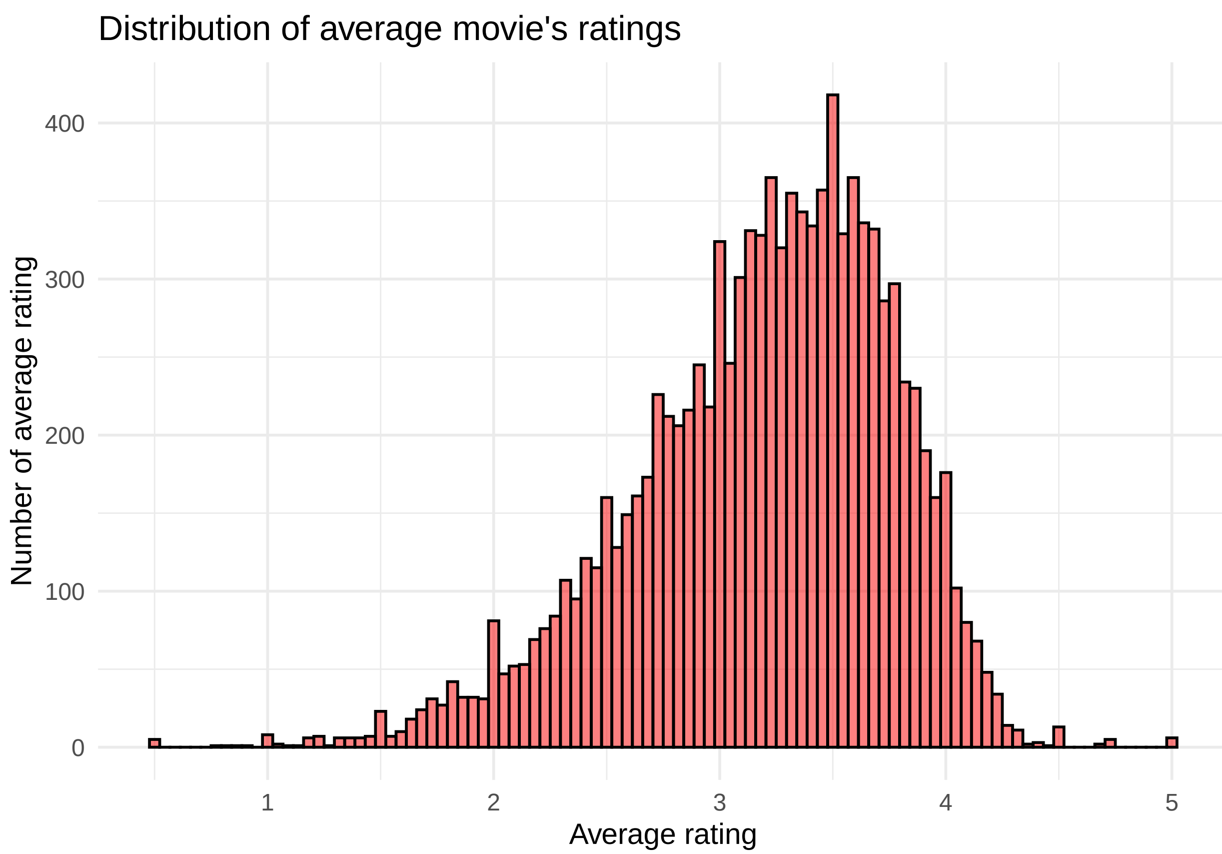
```r
edx %>% ggplot(aes(x = rating)) +
  geom_bar(fill="#fc0303", col = "black", alpha=0.5) +
  scale_x_continuous(breaks = seq(0.5,5, 0.5),
                     labels = seq(0.5,5, 0.5)) +
  labs(title = "Movie ratings", y = "Number of rating", x = "Rating") +
  theme_minimal()
```




### 3.2 Distribution of average movie's rating

The graph below show that the average movie's rating is more or less natural distributed with a left skewed, which means the mean < median < mode.

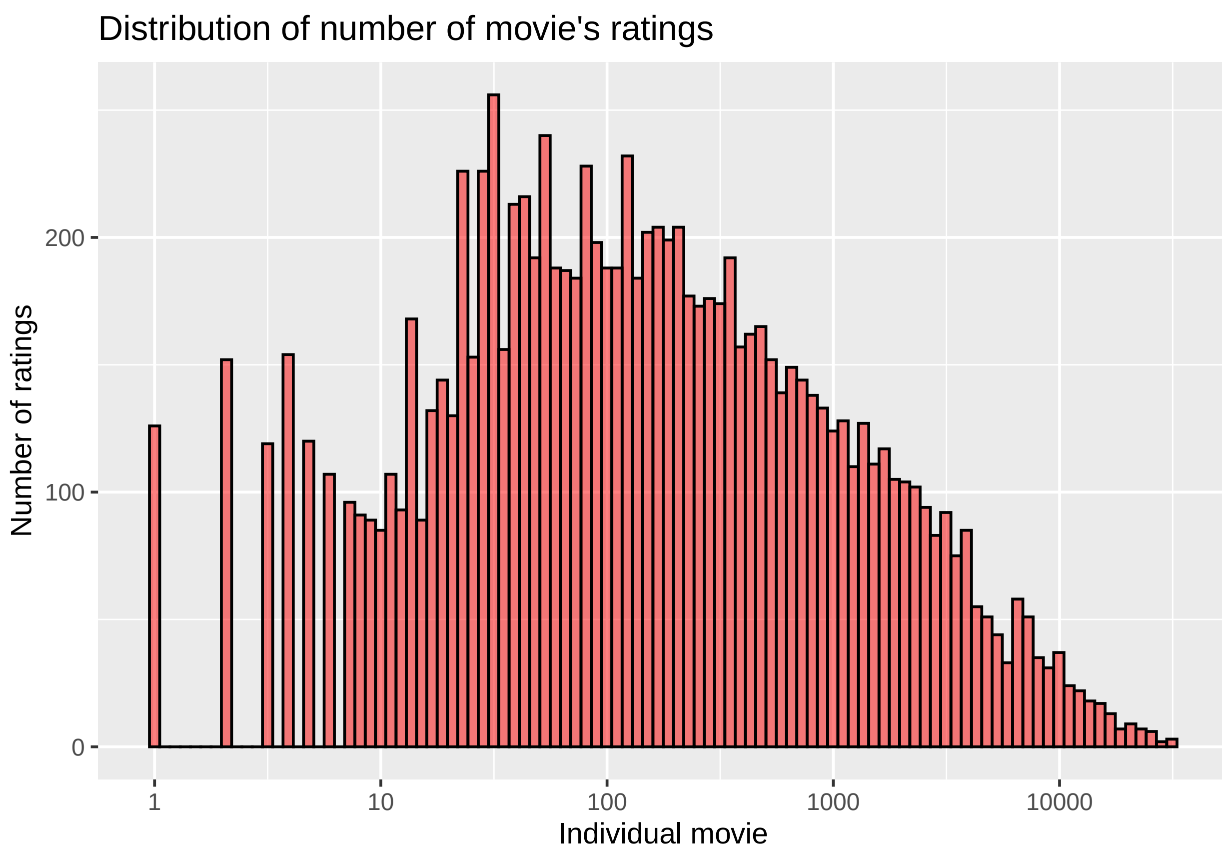
```
edx %>% count(movieId, rating) %>% mutate(score = rating*n) %>%
  group_by(movieId) %>% summarise(avg = sum(score)/sum(n)) %>%
  ggplot(aes(x = avg)) +
  theme_minimal() +
  labs(title = "Distribution of average movie's ratings",
       y = "Number of average rating", x = "Average rating") +
  geom_histogram(bins = 100, fill = "#fc0303", col = "black", alpha=0.5)
```




### 3.3 Distribution of movies

The first graph below show that not all movies are getting the equal amount of ratings, it's clearly that some movies get a lot more rating than others. From the list below show that top 15 rated movies are all blockbusters and the least 15 rated movies are more obscure than finding a needle in a haystack.

```
## some movies get rated more than others
edx %>%
  count(movieId) %>%
  ggplot(aes(n)) +
  geom_histogram(bins = 100, fill = "#fc0303", col = "black", alpha=0.5) +
  scale_x_log10() +
  labs(title = "Distribution of number of movie's ratings",
       y = "Number of ratings", x = "Individual movie")
```




```r
edx %>%
  count(movieId, title) %>% top_n(n = 15) %>%
  arrange(desc(n))
```

```
##      movieId                                                        title     n
##   1:     296                                          Pulp Fiction (1994) 31362
##   2:     356                                          Forrest Gump (1994) 31079
##   3:     593                             Silence of the Lambs, The (1991) 30382
##   4:     480                                         Jurassic Park (1993) 29360
##   5:     318                             Shawshank Redemption, The (1994) 28015
##   6:     110                                            Braveheart (1995) 26212
##   7:     457                                         Fugitive, The (1993) 25998
##   8:     589                            Terminator 2: Judgment Day (1991) 25984
##   9:     260 Star Wars: Episode IV - A New Hope (a.k.a. Star Wars) (1977) 25672
##  10:     150                                             Apollo 13 (1995) 24284
##  11:     592                                                Batman (1989) 24277
##  12:       1                                             Toy Story (1995) 23790
##  13:     780                             Independence Day (a.k.a. ID4) (1996) 23449
##  14:     590                                    Dances with Wolves (1990) 23367
##  15:     527                                      Schindler's List (1993) 23193
```

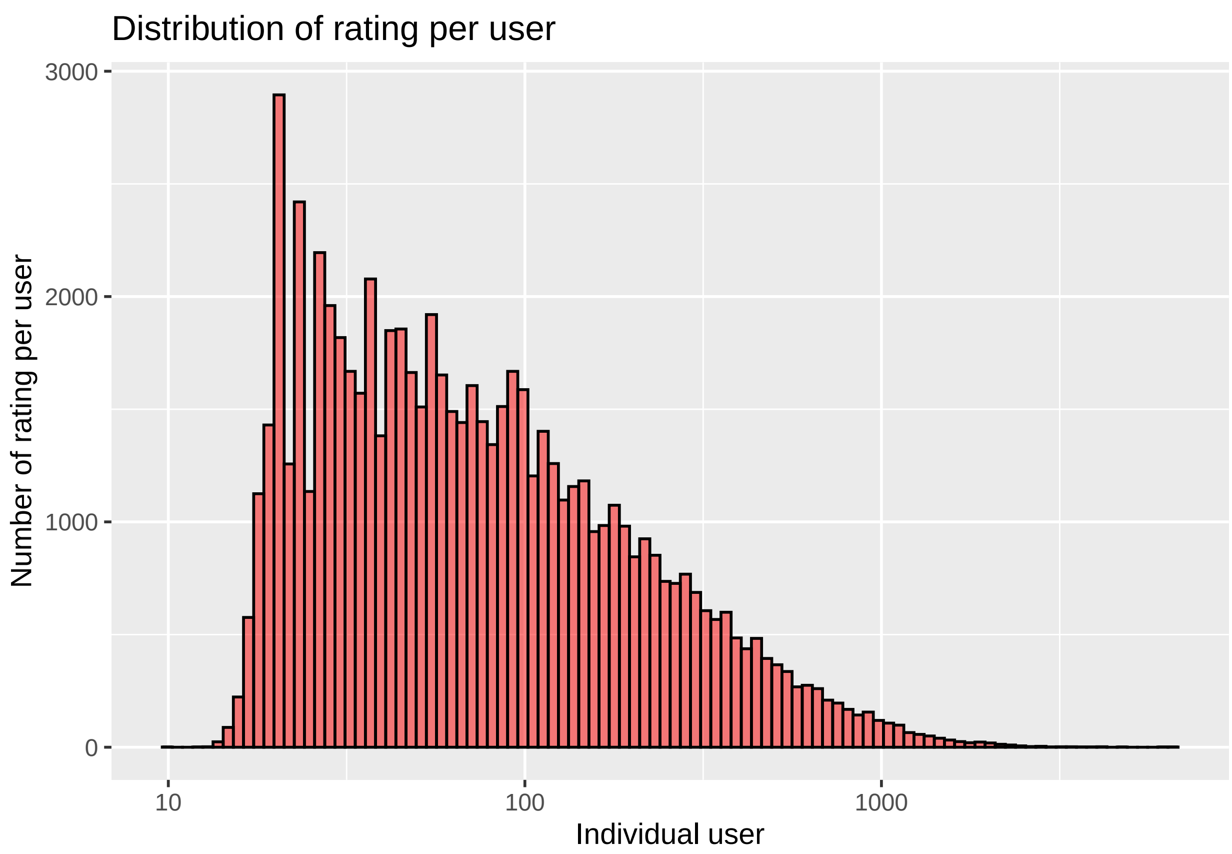
```r
edx %>%
  count(movieId, title) %>% top_n(n = -15) %>%
  arrange(desc(n))
```

```
##      movieId                           title n
##   1:    3191             Quarry, The (1998) 1
##   2:    3226  Hellhounds on My Trail (1999) 1
##   3:    3234 Train Ride to Hollywood (1978) 1
##   4:    3356          Condo Painting (2000) 1
##   5:    3383               Big Fella (1937) 1
##  ---
## 122:   64976                   Hexed (1993) 1
## 123:   65006                 Impulse (2008) 1
## 124:   65011          Zona Zamfirova (2002) 1
## 125:   65025         Double Dynamite (1951) 1
## 126:   65027         Death Kiss, The (1933) 1
```

### 3.4 Distribution of user

From the graph below, it shows that number of rating per user has an natural distribution with a right skewed, which means mean > median > mode. This clearly shows that only a few users rate a lot of movies and majorirty don't.

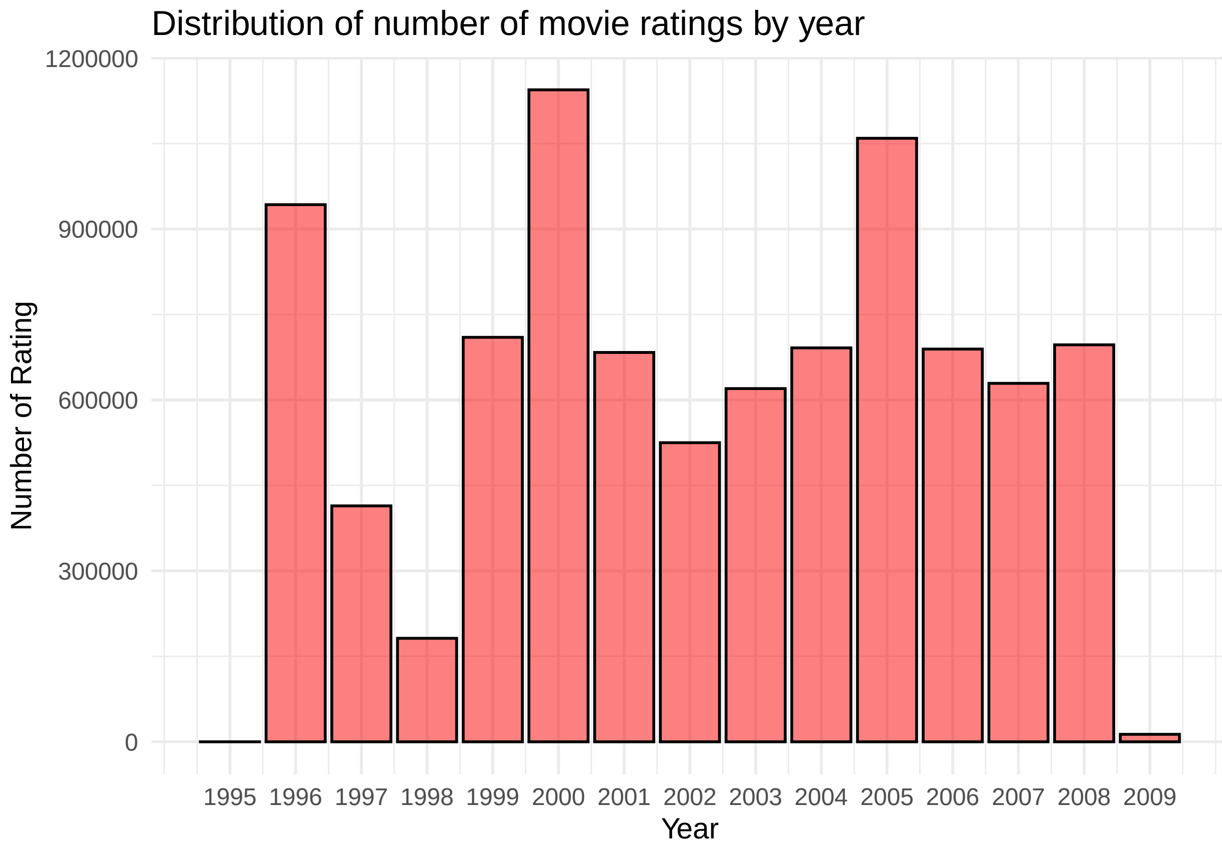
```
## some users rated more movies than the others, shows clearly that most
edx %>%
  count(userId) %>%
  ggplot(aes(x = n)) +
  geom_histogram(bins = 100, fill = "#fc0303", col = "black", alpha=0.5)+
  scale_x_log10() +
  labs(title = "Distribution of rating per user",
       y = "Number of rating per user", x = "Individual user")
```




### 3.5 Number of movies rating by year

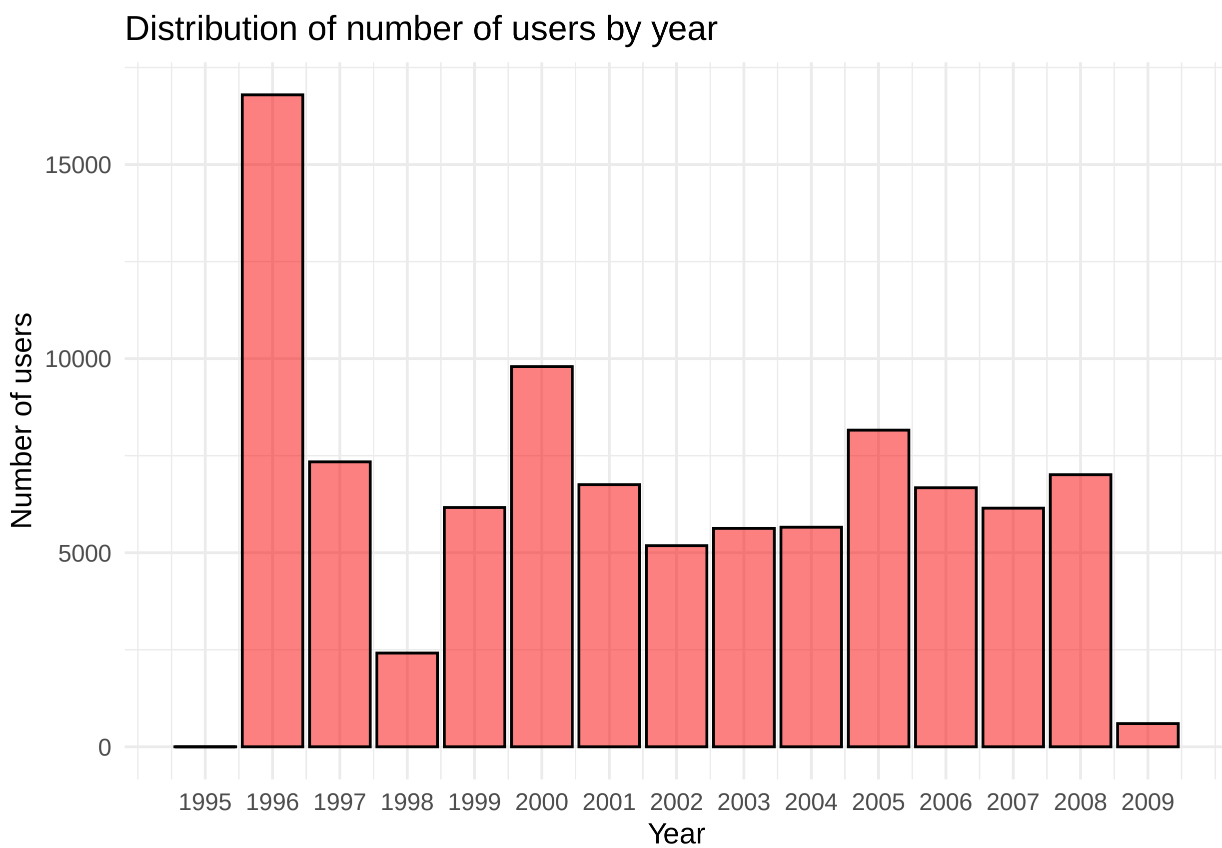
In ths graph shows that year 1996, 2000, and 2005 were all peak years for movie ratings compare to other years, but why is this the case? In the next graph I will try to explaim this phenomenon.

```
library(lubridate)
edx <- edx %>% mutate(year = year(as_datetime(timestamp, origin = "1970-01-01")))
edx %>% ggplot(aes(x = year)) +
        geom_bar(fill="#fc0303", col = "black", alpha=0.5) +
        scale_x_continuous(breaks=seq(1995,2009,1), labels=seq(1995,2009,1)) +
        labs(title = "Distribution of number of movie ratings by year",
             y = "Number of Rating", x = "Year") +
        theme_minimal()
```




From this graph below shows the distribution of number of users by year, which you can see that 1996 had a large amount of users influx follow by 2000 and 2005, but this still can't explain that year 1996 has less number of ratings than 2000 and 2005.

```
edx %>% count(year, userId) %>%
  ggplot(aes(x = year)) +
  geom_bar(fill = "#fc0303", col = "black", alpha=0.5) +
  scale_x_continuous(breaks=seq(1995,2009,1), labels=seq(1995,2009,1)) +
  labs(title = "Distribution of number of users by year",
       y = "Number of users", x = "Year") +
  theme_minimal()
```




In order to see why, first count the number of years which give you the total number of rating that year and save it as edx_1, then left join edx_1 to edx by year, count the total number of rating for individual users in each year follow by a group by year and total to count the total number of rating in each year. Lastly, mutate a new column for average number of rating per year by using the "total" column from edx_1.

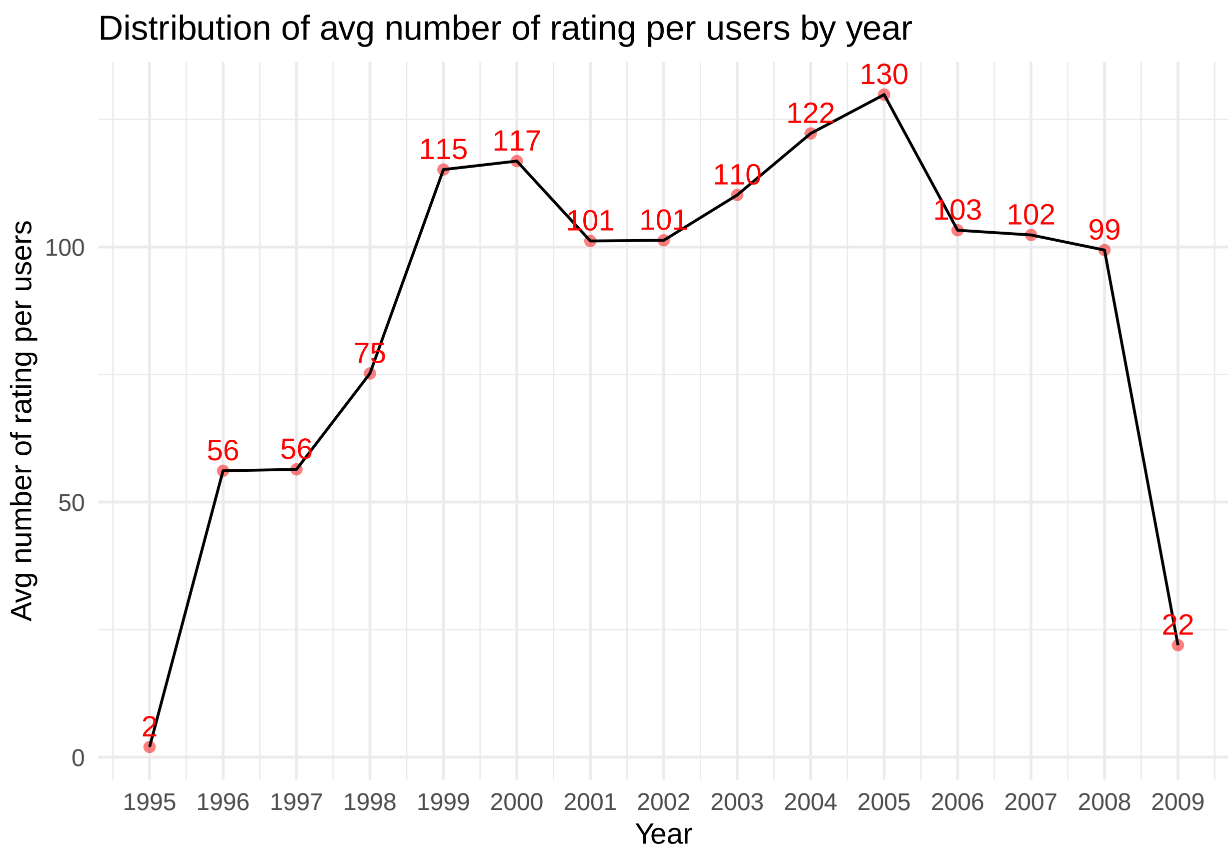
From the graph below shows that year 1996 actually had one of the lower average number of rating per user, where as year 2000 and 2005 were the top 3 highest average number of rating per user.

```
edx_1 <- edx %>% group_by(year) %>% count()
a <- edx %>% left_join(edx_1, by = "year") %>% rename(total = n) %>%
        count(year, userId, total) %>% group_by(year, total) %>%
        count() %>% mutate(avgn = total/n)
a
```

```
## # A tibble: 15 x 4
## # Groups:   year, total [15]
##     year   total     n  avgn
##    <dbl>   <int> <int> <dbl>
##  1  1995       2     1   2
##  2  1996  942772 16796  56.1
##  3  1997  414101  7341  56.4
##  4  1998  181634  2415  75.2
##  5  1999  709893  6164 115.
##  6  2000 1144349  9795 117.
##  7  2001  683355  6754 101.
##  8  2002  524959  5182 101.
##  9  2003  619938  5626 110.
```

```
## 10  2004  691429  5656 122.
## 11  2005 1059277  8157 130.
## 12  2006  689315  6674 103.
## 13  2007  629168  6147 102.
## 14  2008  696740  7010  99.4
## 15  2009   13123   598  21.9
```

```r
a %>% ggplot(aes(x = year, y = avgn)) +
      geom_point(col = "#fc0303", alpha=0.5) + geom_line() +
      geom_text(aes(label = round(avgn)), vjust = -0.5, col = "#fc0303") +
      scale_x_continuous(breaks=seq(1995,2009,1), labels=seq(1995,2009,1)) +
      labs(title = "Distribution of avg number of rating per users by year",
           y = "Avg number of rating per users", x = "Year") +
      theme_minimal()
```




## Modeling

### Loss function

How do one assess the accuracy of our model prediction? Residual mean square error (RMSE), but first one need to look at mean square error (MSE).

$$\mathrm{MSE} = \frac{1}{N}\sum_{u,i}\left(\hat{y}_{u,i} - y_{u,i}\right)^2$$

MSE measures the average of the square of the difference of predicted value and real value. In this method larger error will weight more than smaller error, if the error is 1 its squared error is also 1, but if the error is 0.1 the square error is 0.01, which the former sqaure error would be 100 times larger than the latter one. Another issue is that because the unit of the error is squared so it would be hard to interpret the results, hence I use RMSE which the sqaure root will return the same unit. However, there are other ways to meausre accuracy such as mean absolute error (MAE). Here denote $y_{u,i}$ as the rating of movie$i$ by user$u$, $\hat{y}_{u,i}$ as the prediction of the movie ratings, and $N$ as total combination movie$i$ and user$u$.

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{u,i}\left(\hat{y}_{u,i} - y_{u,i}\right)^2}$$

```r
RMSE <- function(true_ratings, predicted_ratings){
  sqrt(mean((true_ratings - predicted_ratings)^2))
}
```

### 1.1 Base model

Since I am trying to predict movie ratings, what would be the a good initial guess for all the ratings in the dataset? I could pick any number, but I am trying to get the lowest RMSE and the average would minimize RMSE, hence the base model is the average of all movie ratings in traing set (the observed value) $\mu$ + the independent error $\varepsilon$.

$$Y_{u,i} = \mu + \varepsilon_{u,i}$$

### 1.2 Movie effect

From the data exploration in section 3, it shows that some movies are highly rated some not, this variability can be explain by the mean difference of observed rating $Y_{u,i}$ and average rating of all movies $\mu$.

$$b_i = \frac{1}{N}\sum_i (Y_{u,i} - \mu)$$

From the above $b_i$, then add it back to the base model.

$$Y_{u,i} = \mu + b_i + \varepsilon_{u,i}$$

### 1.3 User effect

Similar to movie effect, user variable also show that some give a lot of high rating and some not, this variability can be eplain by the mean difference of observed rating $Y_{u,i}$, average rating of all movies $\mu$, and movie effect $b_i$.

$$b_u = \frac{1}{N}\sum_i (Y_{u,i} - \mu - b_i)$$

From the above $b_u$, then add it back the movie effect model.

$$Y_{u,i} = \mu + b_i + b_u + \varepsilon_{u,i}$$

### 1.4 Result

Now the model is ready to be tested, let's see how it perform. From the rmse dataframe below shows that the RMSE of the base model is 1.06, with movie effect it improves to 0.94 and with user effect it further improve RMSE to 0.87. The linear model improve RMSE about 18%, but this is still not enough to reach the target RMSE < 0.8649, so how do one improve from here?

```r
# First Model: Overall average rating
mu <- mean(train_data$rating)
rmse <- tibble(Method = "Base Model",
               RMSE = RMSE(test_data$rating, mu))
# Second model: movie effect
movie_avgs <- train_data %>%
  group_by(movieId) %>%
  summarize(b_i = mean(rating - mu))

predicted_ratings <- test_data %>%
  left_join(movie_avgs, by='movieId') %>%
  mutate(pred = mu +b_i) %>%
  pull(pred)
rmse <- bind_rows(rmse,
                  tibble(Method = "Base + b_i",
                         RMSE = RMSE(test_data$rating, predicted_ratings)))
# Third model: user effect
user_avgs <- train_data %>%
  left_join(movie_avgs, by='movieId') %>%
  group_by(userId) %>%
  summarize(b_u = mean(rating - mu - b_i))

predicted_ratings <- test_data %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  mutate(pred = mu + b_i + b_u) %>%
  pull(pred)

rmse <- bind_rows(rmse,
                  tibble(Method = "Base + b_i + b_u",
                         RMSE = RMSE(test_data$rating, predicted_ratings)))

print.data.frame(rmse, digits = 6)
```

```
##            Method     RMSE
## 1       Base Model 1.059904
## 2       Base + b_i 0.943743
## 3 Base + b_i + b_u 0.865932
```

### 1.4 Regularization

#### 1.4.1 Residuals

Before getting into regularization, let's check what is the top mistake rating the linear model predicted. "From Justin to Kelly" a rather obscure movie getting a 5 rating with a large positive residual indicates that this user rated this movie a lot higher than others, where as "The Shawshank Redemption" a well known critically acclaimed movie getting a 0.5 rating with a small negative residual show that this user rated this movie a lot lower than others.

```r
## checking the model
train_data %>%
  left_join(movie_avgs, by = "movieId") %>%
  mutate(residual = rating - (mu + b_i)) %>%
  arrange(desc(abs(residual))) %>%
  slice(1:10)
```

```
##     userId movieId rating  timestamp                           title
##  1:  26423    6483    5.0 1097653302      From Justin to Kelly (2003)
##  2:   2507     318    0.5 1111809069 Shawshank Redemption, The (1994)
##  3:   7708     318    0.5 1136745552 Shawshank Redemption, The (1994)
##  4:   9568     318    0.5 1165693345 Shawshank Redemption, The (1994)
##  5:   9975     318    0.5 1165764304 Shawshank Redemption, The (1994)
##  6:  10749     318    0.5 1137685578 Shawshank Redemption, The (1994)
##  7:  13496     318    0.5 1167308184 Shawshank Redemption, The (1994)
##  8:  24312     318    0.5 1096667629 Shawshank Redemption, The (1994)
##  9:  25239     318    0.5 1097222585 Shawshank Redemption, The (1994)
## 10:  26260     318    0.5 1224724866 Shawshank Redemption, The (1994)
##              genres        b_i  residual
##  1: Musical|Romance -2.6672397  4.154762
##  2:           Drama  0.9448401 -3.957318
##  3:           Drama  0.9448401 -3.957318
##  4:           Drama  0.9448401 -3.957318
##  5:           Drama  0.9448401 -3.957318
##  6:           Drama  0.9448401 -3.957318
##  7:           Drama  0.9448401 -3.957318
##  8:           Drama  0.9448401 -3.957318
##  9:           Drama  0.9448401 -3.957318
## 10:           Drama  0.9448401 -3.957318
```

Now let's check the 10 best movie base on our movie effect $b_i$, without even googling these title I think it is fair to say these movies are definitely obscure, "Hellhounds on My Trail" is the movie effect model best rated movie, from IMDB as a reference this documentary film only has 28 ratings.

```r
# top 10 best movie
movie_title <- train_data %>% select(movieId, title) %>% distinct()
movie_avgs %>% left_join(movie_title, by = "movieId") %>%
  arrange(desc(b_i)) %>%
  slice(1:10) %>%
  pull(title)
```

```
##  [1] "Hellhounds on My Trail (1999)"
##  [2] "Who's Singin' Over There? (a.k.a. Who Sings Over There) (Ko to tamo peva) (1980)"
##  [3] "Satan's Tango (Sátántangó) (1994)"
##  [4] "Shadows of Forgotten Ancestors (1964)"
##  [5] "Money (Argent, L') (1983)"
##  [6] "Fighting Elegy (Kenka erejii) (1966)"
##  [7] "Sun Alley (Sonnenallee) (1999)"
##  [8] "Aerial, The (La Antena) (2007)"
##  [9] "Blue Light, The (Das Blaue Licht) (1932)"
## [10] "More (1998)"
```

This is the 10 worst movie, also again these titles are beyond a hipster's taste, but I must admit that I would want to watch "SuperBabies: Baby Geniuses 2" and "Da Hip Hop Witch", the former had John Volt starting and the latter had Eminem.

```
# top 10 worst movie
movie_avgs %>% left_join(movie_title, by = "movieId") %>%
  arrange(b_i) %>%
  slice(1:10) %>%
  pull(title)
```

```
##  [1] "Besotted (2001)"
##  [2] "Hi-Line, The (1999)"
##  [3] "Accused (Anklaget) (2005)"
##  [4] "Confessions of a Superhero (2007)"
##  [5] "War of the Worlds 2: The Next Wave (2008)"
##  [6] "SuperBabies: Baby Geniuses 2 (2004)"
##  [7] "From Justin to Kelly (2003)"
##  [8] "Legion of the Dead (2000)"
##  [9] "Disaster Movie (2008)"
## [10] "Hip Hop Witch, Da (2000)"
```

So are these movies even get rated that much? From the two printed lists below, it showed that all of these movies are obscure titles most only get rated once. These predictions are untrustworthy, a large residual returns large RMSE, which is not what the model is aiming for. Hence, this is where the regularization comes in to penalized large estimates that are from small sample sizes.

```
# number of rating, best movie
train_data %>% count(movieId) %>%
  left_join(movie_avgs, by="movieId") %>%
  left_join(movie_title, by="movieId") %>%
  arrange(desc(b_i)) %>%
  slice(1:10) %>%
  pull(n)
```

```
##  [1] 1 3 2 1 1 1 1 1 1 6
```

```
# number of rating, worst movie
train_data %>% count(movieId) %>%
  left_join(movie_avgs, by="movieId") %>%
  left_join(movie_title, by="movieId") %>%
  arrange(b_i) %>%
  slice(1:10) %>%
  pull(n)
```

```
##  [1]   1   1   1   1   2  40 168   4  28  11
```

### 1.4.2 Regularization model

How do one model regularization? Without going into too much detail of math, regularization is to minimize the variability of the effect sizes, in this case would be movie effect $b_i$ and user effect $b_u$.

The first term is the least square of movie effect + the penalty added to the movie effect.

$$\sum_{u,i}\left(y_{u,i}-\mu-b_i\right)^2+\lambda\sum_i b_i^2$$

By using calculus it can show the equation that minimize $b_i$, $n_i$ here denotes the number of ratings per movie $i$. When $n_i$ gets really large, the penalty $\lambda$ can be ignored because a very large $n_i+\lambda\approx n_i$, but if $n_i$ is very small $\hat{b}_i(\lambda)$ will be reduced towards 0, becuase the larger $\lambda$ the more it reduce.

$$\hat{b}_i(\lambda)=\frac{1}{\lambda+n_i}\sum_{u=1}^{n_i}\left(Y_{u,i}-\hat{\mu}\right)$$

The first term is the least square of user effect + the penalty added to the user effect, $n_u$ here denotes the number of ratings per user $u$. The following equation are the same as above beside adding user effect.

$$\sum_{u,i}\left(y_{u,i}-\mu-b_i-b_u\right)^2+\lambda\sum_u b_u^2$$

$$\hat{b}_u(\lambda)=\frac{1}{\lambda+n_u}\sum_{i=1}^{n_u}\left(Y_{u,i}-\hat{\mu}-\hat{b}_i\right)$$

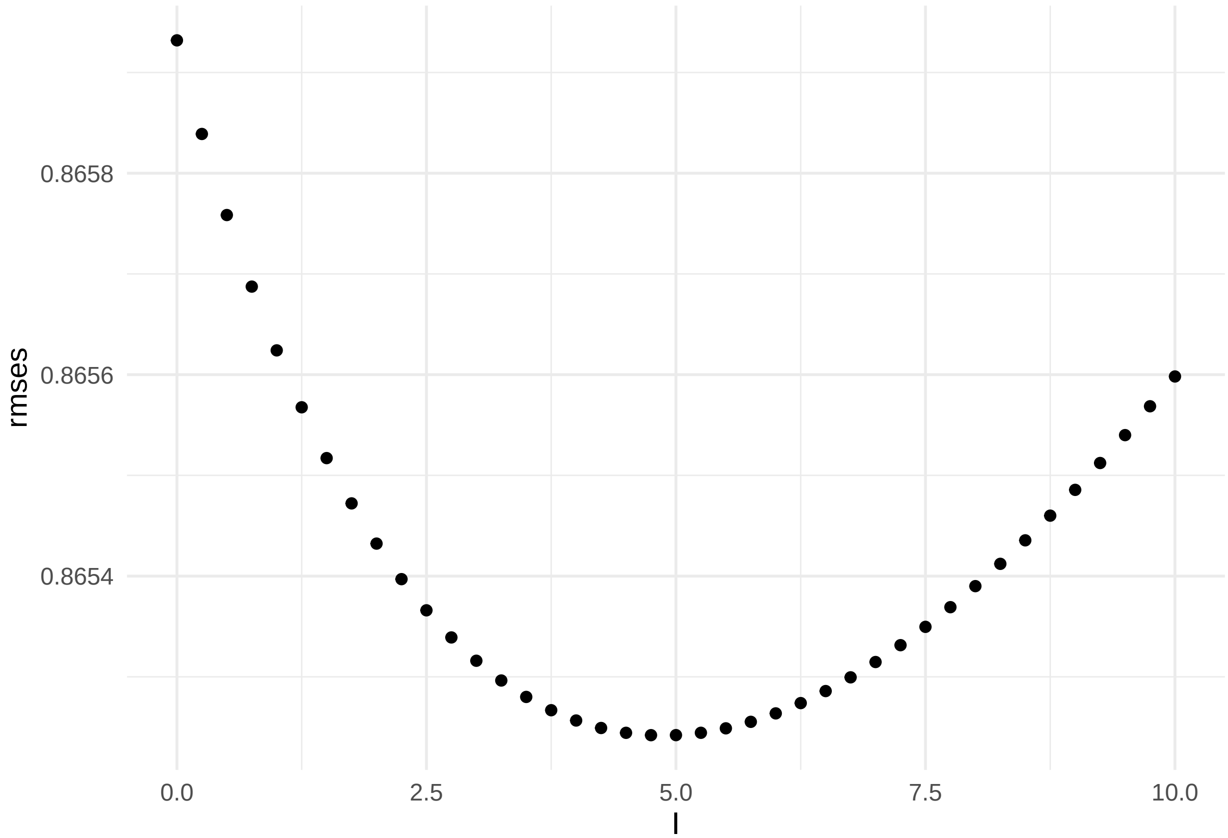
### 1.4.3 Result

After the above explaination, it is ready to test the linear model with regularization.

```
lambdas <- seq(0,10, 0.25)

regular <- sapply(lambdas, function(l){
  mu <- mean(train_data$rating)
  b_i <- train_data %>%
    group_by(movieId) %>%
    summarize(b_i = sum(rating - mu)/(n()+l))
  b_u <- train_data %>%
    left_join(b_i, by="movieId") %>%
    group_by(userId) %>%
    summarize(b_u = sum(rating - b_i - mu)/(n()+l))
  predicted_ratings <-
    test_data %>%
    left_join(b_i, by = "movieId") %>%
    left_join(b_u, by = "userId") %>%
    mutate(pred = mu + b_i + b_u) %>%
    pull(pred)
  return(RMSE(predicted_ratings, test_data$rating))
})

tibble(l = lambdas, rmses = regular) %>%
  ggplot(aes(x = l, y = rmses)) +
  geom_point() +
  theme_minimal()
```




```
l <- lambdas[which.min(regular)]

mu <- mean(train_data$rating)
b_i <- train_data %>%
  group_by(movieId) %>%
  summarize(b_i = sum(rating - mu)/(n()+l))
b_u <- train_data %>%
  left_join(b_i, by="movieId") %>%
  group_by(userId) %>%
  summarize(b_u = sum(rating - b_i - mu)/(n()+l))
predicted_ratings <-
  test_data %>%
  left_join(b_i, by = "movieId") %>%
  left_join(b_u, by = "userId") %>%
  mutate(pred = mu + b_i + b_u) %>%
  pull(pred)
rmse <- bind_rows(rmse,
                  tibble(Method = "Regularization",
                         RMSE = RMSE(test_data$rating, predicted_ratings)))
print.data.frame(rmse, digits = 6)
```

```
##              Method     RMSE
## 1        Base Model 1.059904
## 2        Base + b_i 0.943743
## 3 Base + b_i + b_u 0.865932
## 4   Regularization 0.865242
```

### 1.5 Matrix Factorization

Matrix factorization (MF) is widely use in tackling recommender system problem when predicting unobserved rating based on the observed rating. The basic idea is to reduce the rating matrix $R_{m\times n}$ to user matrix $P_{k\times m}$ and movie $Q_{k\times n}$ so that $R \approx P \times Q$. From the 4x4 matrix below shows a simple example of what MF transform the original data. From more detail mathematical explaination please check recosystem and Supplementary Materials for "LIBMF", as this project is intend to focus on the appling techniques of machine learning.

```
m <- cbind(1, c(1,NA,3, NA), c(4,5,NA,3), c(NA,2,3,4))
colnames(m) <- c("movie.1","movie.2", "movie.3", "movie.4")
rownames(m) <- rownames(m, do.NULL = FALSE, prefix = "user.")
m
```

```
##        movie.1 movie.2 movie.3 movie.4
## user.1       1       1       4      NA
## user.2       1      NA       5       2
## user.3       1       3      NA       3
## user.4       1      NA       3       4
```

Here, I use recosystem package to utilize the power of multilcore CPUs for computing speed.

The recosystem prove a step by step guide: 1. Transform your train and test data with data_memory() 2. Create a model object with Reco() 3. Tuning the parameters with $tune() 4. Train model with $train 5. Predict with $predict

Doing the MF process will take quite a bit time from my laptop takes about 30 to 50 minutes, from the rmse table MF method significantly reduce RMSE to 0.7859012 which is about 9% reduce from the regularization model and it also reach our target RMSE < 0.8649.

```
library(recosystem)
# transform train data
train_reco <- with(train_data, data_memory(user_index = userId,
                                           item_index = movieId,
                                           rating = rating))
# transform test data
test_reco <- with(test_data, data_memory(user_index = userId,
                                         item_index = movieId,
                                         rating = rating))
# create model object
r <-  recosystem::Reco()
# tuning parameter
opts <- r$tune(train_reco, opts = list(dim = c(10, 20, 30),
                                       lrate = c(0.1, 0.2),
                                       costp_l2 = c(0.01, 0.1),
                                       costq_l2 = c(0.01, 0.1),
                                       nthread  = 4, niter = 10))
# training model
r$train(train_reco, opts = c(opts$min, nthread = 4, niter = 20))
```

```
## iter      tr_rmse          obj
##    0       0.9922   1.0018e+07
##    1       0.8786   8.0864e+06
##    2       0.8465   7.5094e+06
```

```
##    3       0.8244   7.1541e+06
##    4       0.8073   6.8976e+06
##    5       0.7949   6.7232e+06
##    6       0.7843   6.5867e+06
##    7       0.7750   6.4734e+06
##    8       0.7670   6.3834e+06
##    9       0.7600   6.3054e+06
##   10       0.7537   6.2383e+06
##   11       0.7481   6.1814e+06
##   12       0.7431   6.1307e+06
##   13       0.7386   6.0875e+06
##   14       0.7345   6.0497e+06
##   15       0.7306   6.0128e+06
##   16       0.7272   5.9828e+06
##   17       0.7239   5.9559e+06
##   18       0.7210   5.9298e+06
##   19       0.7182   5.9059e+06
```

```r
# testing model
y_hat_reco <-  r$predict(test_reco, out_memory())
# RMSE
rmse <- bind_rows(rmse,
                  tibble(Method = "MF", RMSE = RMSE(test_data$rating, y_hat_reco)))
print.data.frame(rmse, digits = 6)
```

```
##             Method     RMSE
## 1       Base Model 1.059904
## 2       Base + b_i 0.943743
## 3 Base + b_i + b_u 0.865932
## 4   Regularization 0.865242
## 5               MF 0.790601
```

## Validation set

In the predictions above are all using the 20% of edx as testing data, I done training the model, it is ready to employ on validation set. From the RMSE reports from each method below, besides base model and base model + $b_i$ are performing slightly worst than the test_data, all other models performs better and are able to reach the target RMSE $0.7826 < 0.8649$.

### 1. Linear model

```r
# First Model: Overall average rating
mu <- mean(edx$rating)
valid <- tibble(Method = "Base Model",
                RMSE = RMSE(validation$rating, mu))
print.data.frame(valid, digits = 6)
```

```
##       Method   RMSE
## 1 Base Model 1.0612
```

```r
# Second model: movie effect
movie_avgs <- edx %>%
  group_by(movieId) %>%
  summarize(b_i = mean(rating - mu))

predicted_ratings <- validation %>%
  left_join(movie_avgs, by='movieId') %>%
  mutate(pred = mu +b_i) %>%
  pull(pred)

valid <- bind_rows(valid,
                  tibble(Method = "Base + b_i",
                         RMSE = RMSE(validation$rating, predicted_ratings)))
print.data.frame(valid, digits = 6)
```

```
##       Method     RMSE
## 1 Base Model 1.061202
## 2 Base + b_i 0.943909
```

```r
# Third model: user effect
user_avgs <- edx %>%
  left_join(movie_avgs, by='movieId') %>%
  group_by(userId) %>%
  summarize(b_u = mean(rating - mu - b_i))

predicted_ratings <- validation %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  mutate(pred = mu + b_i + b_u) %>%
  pull(pred)

valid <- bind_rows(valid,
                  tibble(Method = "Base + b_i + b_u",
                         RMSE = RMSE(validation$rating, predicted_ratings)))

print.data.frame(valid, digits = 6)
```

```
##             Method     RMSE
## 1       Base Model 1.061202
## 2       Base + b_i 0.943909
## 3 Base + b_i + b_u 0.865349
```

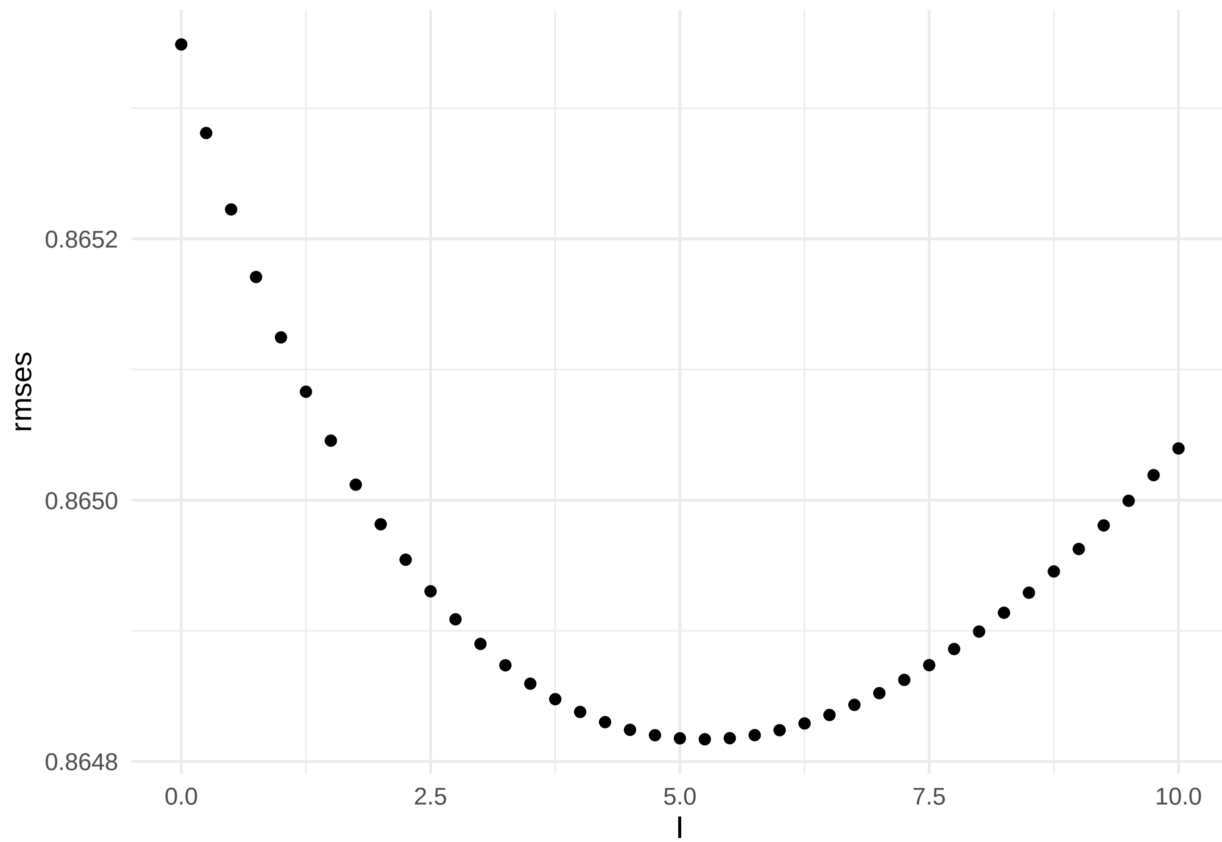
### 2. Regularization

```r
lambdas <- seq(0,10, 0.25)

regular <- sapply(lambdas, function(l){
  mu <- mean(edx$rating)
  b_i <- edx %>%
    group_by(movieId) %>%
    summarize(b_i = sum(rating - mu)/(n()+l))
  b_u <- edx %>%
```

```
    left_join(b_i, by="movieId") %>%
    group_by(userId) %>%
    summarize(b_u = sum(rating - b_i - mu)/(n()+l))
  predicted_ratings <-
    validation %>%
    left_join(b_i, by = "movieId") %>%
    left_join(b_u, by = "userId") %>%
    mutate(pred = mu + b_i + b_u) %>%
    pull(pred)
  return(RMSE(validation$rating, predicted_ratings))
})

tibble(l = lambdas, rmses = regular) %>%
  ggplot(aes(x = l, y = rmses)) +
  geom_point() +
  theme_minimal()
```




```
l <- lambdas[which.min(regular)]

mu <- mean(edx$rating)
b_i <- edx %>%
  group_by(movieId) %>%
  summarize(b_i = sum(rating - mu)/(n()+l))
b_u <- edx %>%
  left_join(b_i, by="movieId") %>%
```

```r
  group_by(userId) %>%
  summarize(b_u = sum(rating - b_i - mu)/(n()+1))
predicted_ratings <-
  validation %>%
  left_join(b_i, by = "movieId") %>%
  left_join(b_u, by = "userId") %>%
  mutate(pred = mu + b_i + b_u) %>%
  pull(pred)
valid <- bind_rows(valid,
                  tibble(Method = "Regularization",
                         RMSE = RMSE(validation$rating, predicted_ratings)))
print.data.frame(valid, digits = 6)
```

```
##              Method     RMSE
## 1        Base Model 1.061202
## 2        Base + b_i 0.943909
## 3 Base + b_i + b_u 0.865349
## 4    Regularization 0.864817
```

### 3. Matrix Factorization

```r
# transform train data
train_edx <- with(edx, data_memory(user_index = userId,
                                   item_index = movieId,
                                   rating = rating))
# transform test data
test_vali <- with(validation, data_memory(user_index = userId,
                                          item_index = movieId,
                                          rating = rating))
# create model object
r <-  recosystem::Reco()
# tuning parameter
opts <- r$tune(train_edx, opts = list(dim = c(10, 20, 30),
                                      lrate = c(0.1, 0.2),
                                      costp_l2 = c(0.01, 0.1),
                                      costq_l2 = c(0.01, 0.1),
                                      nthread  = 4, niter = 10))
# training model
r$train(train_edx, opts = c(opts$min, nthread = 4, niter = 20))
```

```
## iter      tr_rmse          obj
##    0       0.9713   1.2000e+07
##    1       0.8717   9.8756e+06
##    2       0.8383   9.1622e+06
##    3       0.8166   8.7458e+06
##    4       0.8013   8.4746e+06
##    5       0.7894   8.2771e+06
##    6       0.7798   8.1236e+06
##    7       0.7717   8.0034e+06
##    8       0.7649   7.9064e+06
##    9       0.7591   7.8271e+06
##   10       0.7540   7.7612e+06
```

```
## 11       0.7494   7.7049e+06
## 12       0.7454   7.6531e+06
## 13       0.7416   7.6098e+06
## 14       0.7382   7.5714e+06
## 15       0.7351   7.5383e+06
## 16       0.7322   7.5046e+06
## 17       0.7295   7.4760e+06
## 18       0.7270   7.4536e+06
## 19       0.7247   7.4284e+06
```

```
# testing model
y_hat_edx <-  r$predict(test_vali, out_memory())
# RMSE
valid <- bind_rows(valid,
                  tibble(Method = "MF", RMSE = RMSE(validation$rating, y_hat_edx)))
print.data.frame(valid, digits = 6)
```

```
##             Method     RMSE
## 1       Base Model 1.061202
## 2       Base + b_i 0.943909
## 3 Base + b_i + b_u 0.865349
## 4   Regularization 0.864817
## 5               MF 0.782935
```

## Conclusion

The initial base model is the mean of all movie ratings, a rather simple approach without catching other variabilities and effects in movies and users, which RMSE is about 1.061. Once I added movie and user effect, the RMSE is reduced to 0.8653, with regularization I added penalty term for both movie and user effect the RMSE is reduced to 0.8648. By using regularization the RMSE reaches the initial target, RMSE $< 0.8649$, but to further improve the model I employed matrix factorization with recosystem, this method reduced RMSE massively to 0.7826.

Matrix Factorization comes with a cost that it needs a machine equipped with strong computational CPUs and memories to run, considering the movielense dataset in this project only has about 10 million observations and it took about 30 minutes to process let alone a dataset that is much larger. Also, my model only have two features, where as the model implemented by Netflix or Amazon have a lot more features to predict user's preferences, such as movie actors, genres, groups, etc.

Since this a project about applying machine learning techniques, I did not get in depth into the mathematic behind regularization and matrix factorization. In addition, I did not touch on two other commonly used filtering systems, content based and collaborative filtering. Do check out recommendarlab, and recommender system on wikipedia for further studies.

## References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3. Michael Hahsler. (2019), recommendationlab
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6. Xie, Allaire, Grolemund (2020), R Markdown: The Definitive Guide