CHARGE & CURRENT

v = charge vel.
Q_v = charge density

$I = \frac{dq}{dt} = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{Q}{\tau}$ A = cross section ⊥ to I

$\frac{Q}{\tau} = \frac{Q\,v}{L} = \frac{Q_v A L v}{L}$ $\tau = \frac{L}{v}$

$I = \frac{Q}{\tau} = Q_v A v$ | $q(t) = \int_0^t i(t')\,dt'$ | $Q_v = \frac{I}{vA}$

P,I,V,R CONVERSIONS

$V = IR$ $I = V/R$ $R = V/I$

$P = IV = I^2R = V^2/R$

DERIVATIVE DEFINTIONS

$I = dq/dt$ $q = \int_{-\infty}^{t} i\,dt$

$V_{AB} = dw/dq$ (dw: energy in J to move (+)q, b→a)

$P = dW/dt = dW/dq \cdot dq/dt = VI$

TRAVELING WAVES

$A \cdot \sin(kx \pm \omega t)$

k: wave vector = 2π/wavelength, λ = w/2π [rad/m]

w: angular frequency = 2π/period = 2πf [rad/s]

v: phase velocity = w/k = λf [m/s]

STANDING WAVES

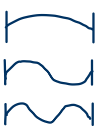
most general: $f(x,t) = A(\sin(kx - wt)) - B(\sin(kx + wt))$

$= A[\sin(kw)\cos(wt) - \cos(kx)\sin(wt)]$

$+ B[\sin(kw)\cos(wt) + \cos(kx)\sin(wt)]$

salahuddin special: $f(x,t) = A[\sin(kx)\cos(wt) - \cos(kx)\sin(wt)]$

* $x = 0 \Rightarrow 0$ $= A[\sin(kx)\cos(wt)]$
* $(x = L \Rightarrow 0) \Rightarrow k = \frac{n\pi}{L}$ $= A[\sin(\frac{n\pi}{L}x)\cos(wt)],\ n \in \mathbb{Z}$



n^{th} harmonic mode $k_{n-1} = \frac{n\pi}{L}$

harmonic node, n: $\lambda = 2L/n$

MOBILITY & RESISTIVITY

conductivity, $\sigma = nq\mu$ μ = electron mobility

resistivity, $\rho = \frac{1}{\sigma} = 1/nq\mu$ n = free electron conc.

resistance, $R = \rho L/A$ q = charge of elec. $= 1.6 \cdot 10^{-19}$ C

mobility, $\mu = q\tau_{avg}/m_e$ m_e = mass of electron $= 9.1 \cdot 10^{-31}$ kg

$F = ma \to Eq = m\,v/\tau_{avg}$

$\to Eq\,\tau_{avg}/m = v$

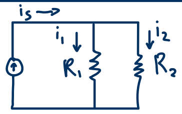
τ_{avg} = avg time between 2 scattering events

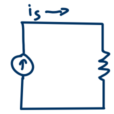
E = electric field (V/m)

v = drift v of e (m/s)

Q_v = charge density

CURRENT DIVISION

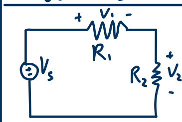


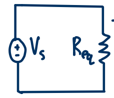


$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i_s$ $i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$

VOLTAGE DIVISION





$V_{eq} = (R_1 + R_2)\,i$

$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_s$ $V_2 = \left(\frac{R_2}{R_1 + R_2}\right) V_s$

MESH CURRENT ANALYSIS

** vs. only I sources

* when there are only independent V sources

NODAL ANALYSIS

G_{kk} = sum of all conductances $(\frac{1}{R})$ connected to node k

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & & & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

$G_{kl} = G_{lk}$ = negative of conductance(s) connecting k,l

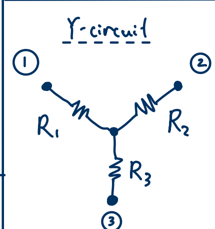
V_k = voltage at node k

I_k = total of current sources entering node k (add current sources leaving node k as negative)

SOURCE TRANSFORMATION

you know this buddy!

Δ-Y TRANSFORMATION



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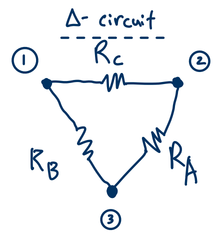


Y→Δ

$A = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{1}$

$B = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{2}$

$C = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{3}$

Δ→Y

$1 = \frac{BC}{A+B+C}$

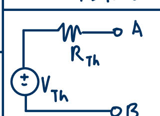
$2 = \frac{AC}{A+B+C}$

$3 = \frac{AB}{A+B+C}$

for $A = B = C \Rightarrow 1 = 2 = 3 = A/3$

for $R_1 = R_2 = R_3 \Rightarrow A = B = C = 3 \cdot R_1$

THEVENIN & NORTON EQUIVALENTS



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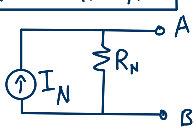


$V_{Th} = V_A - V_B$ in open circuit

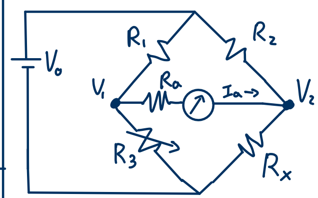
$R_{Th} = R_N$

$I_N = V_{Th}/R_{Th}$

I_N = current flowing from A to B in short

WHEATSTONE BRIDGE

balanced + no approximation



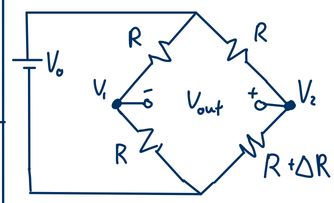
to determine R_x: adjust R_3 until $I_a = 0$

$\Rightarrow \frac{R_3 V_0}{R_1 + R_3} = \frac{R_x V_0}{R_2 + R_x}$

and $\frac{R_1 V_0}{R_1 + R_3} = \frac{R_2 V_0}{R_2 + R_x}$

$\Rightarrow R_x = \left(\frac{R_2}{R_1}\right) R_3$

sensor for small deviations from a ref. cond.



assuming $\Delta R/R << 1$

$V_1 = \frac{R}{R+R} V_0 = \frac{V_0}{2}$

$V_2 = \frac{R + \Delta R}{R + R + \Delta R} V_0$

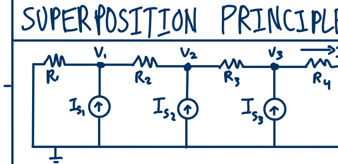
$V_{out} = V_2 - V_1$

$\approx \frac{V_0}{4}\left(\frac{\Delta R}{R}\right)$

conceptual:

- wheatstone bridge (WB) can be used as a high precision ohmmeter
- by adding a second voltage divider branch in parallel with the first branch and taking the difference between branch output voltages as the output, the WB nulls common mode voltages to accentuate differential voltages resulting from changes to one or more sense resistors
    - common mode voltages: the same voltage that appears on both sides of the WB
- in the context of strain guages, the WB can null the changes in resistance due to temperature so readers of the WB output can more accurately determine the change in resistance due to strain on the resistor, the value they are interested in

SUPERPOSITION PRINCIPLE



$V_3 = I R_4$

$I = \frac{R_1}{R_{1+2+3+4}} I_{s_1} + \frac{R_{1+2}}{R_{1+2+3+4}} I_{s_2} + \frac{R_{1+2+3}}{R_{1+2+3+4}} I_{s_3}$