CHARGE & CURRENT

v = charge vel.
Q_v = charge density

$I = \frac{dq}{dt} = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{Q}{\tau}$ A = cross section ⊥ to I

$\frac{Q}{\tau} = \frac{Q\,v}{L} = \frac{Q_v A L v}{L}$ $\tau = \frac{L}{v}$

$I = \frac{Q}{\tau} = Q_v A v$ $q(t) = \int_0^t i(t')\,dt'$ $Q_v = \frac{I}{vA}$

P,I,V,R CONVERSIONS

$V = IR$ $I = V/R$ $R = V/I$

$P = IV = I^2R = V^2/R$

DERIVATIVE DEFINTIONS

$I = dq/dt$ $q = \int_{-\infty}^{t} i\,dt$

$V_{AB} = dw/dq$ (dw: energy in J to move (+)q, b → a)

$P = dW/dt = \frac{dW}{dq}\cdot\frac{dq}{dt} = VI$

TRAVELING WAVES

$A \cdot \sin(kx \pm wt)$

k: wave vector = 2π/wavelength, $\lambda = w/2\pi$ [rad/m]

w: angular frequency = 2π/period = $2\pi f$ [rad/s]

v: phase velocity = $w/k = \lambda f$ [m/s]

STANDING WAVES

most general: $f(x,t) = A\sin(kx - wt) - B\sin(kx + wt)$

$= A[\sin(kx)\cos(wt) - \cos(kx)\sin(wt)]$

$+ B[\sin(kx)\cos(wt) + \cos(kx)\sin(wt)]$

salahuddin special: $f(x,t) = A[\sin(kx)\cos(wt) - \cos(kx)\sin(wt)]$

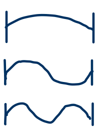
* $x = 0 \Rightarrow 0$ $= A[\sin(kx)\cos(wt)]$

* $(x = L \Rightarrow 0) \Rightarrow k = \frac{n\pi}{L}$ $= A[\sin(\frac{n\pi}{L}x)\cos(wt)]$, $n \in \mathbb{Z}$

1st harmonic

2nd harmonic

3rd harmonic



nth harmonic mode $k_{n-1} = \frac{n\pi}{L}$

harmonic node, n: $\lambda = 2L/n$

MOBILITY & RESISTIVITY

conductivity, $\sigma = nq\mu$ μ = electron mobility

resistivity, $\rho = \frac{1}{\sigma} = 1/nq\mu$ n = free electron conc.

resistance, $R = \rho L/A$ q = charge of elec.

mobility, $\mu = q\tau_{avg}/m_e$ $= 1.6 \cdot 10^{-19}$ C

m_e = mass of electron $= 9.1 \cdot 10^{-31}$ kg

$F = ma \to Eq = mv/\tau_{avg}$

$\to Eq\tau_{avg}/m = v$

E = electric field (V/m)

v = drift v of e (m/s)

τ_{avg} = avg time between 2 scattering events

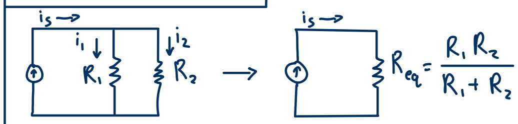
Q_v = charge density

UNITS

tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

resistance	R	Ω
resistivity	ρ	$\Omega \cdot m$
conductance	G	S or Ω^{-1}
conductivity	σ	S/m
impedence	Z	Ω
capacitance	C	F
inductance	L	H
mobility	μ	$m^2/V \cdot s$
charge den.	ρ	C/m^3
time const.	τ	s
wave vector	k	rad/m

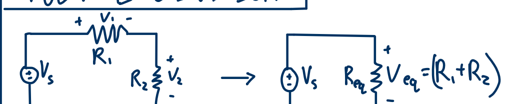
CURRENT DIVISION



$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

$i_1 = \left(\frac{R_2}{R_1 + R_2}\right) i_s$ $i_2 = \left(\frac{R_1}{R_1 + R_2}\right) i_s$

VOLTAGE DIVISION



$V_{eq} = (R_1 + R_2)\,i$

$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_s$ $V_2 = \left(\frac{R_2}{R_1 + R_2}\right) V_s$

MESH CURRENT ANALYSIS

** vs. only I sources

* when there are only independent V sources

NODAL ANALYSIS

G_{kk} = sum of all conductances ($\frac{1}{R}$) connected to node k

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & & & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

$G_{kl} = G_{lk}$ = negative of conductance(s) connecting k-l

V_k = voltage at node k

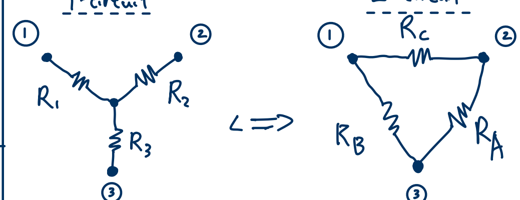
I_k = total of current sources entering node k (add current sources leaving node k as negative)

SOURCE TRANSFORMATION

you know this buddy!

Δ-Y TRANSFORMATION

Y-circuit Δ-circuit



Y → Δ

$A = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{1}$

$B = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{2}$

$C = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{3}$

Δ → Y

$1 = \frac{BC}{A+B+C}$

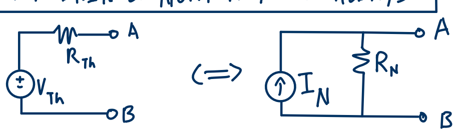
$2 = \frac{AC}{A+B+C}$

$3 = \frac{AB}{A+B+C}$

for $A = B = C$ ⇒ $1 = 2 = 3 = A/3$

for $R_1 = R_2 = R_3$ ⇒ $A = B = C = 3 \cdot R_1$

THEVENIN & NORTON EQUIVALENTS



$V_{Th} = V_A - V_B$ in open circuit

$R_{Th} = R_N$

$I_N = V_{Th}/R_{Th}$

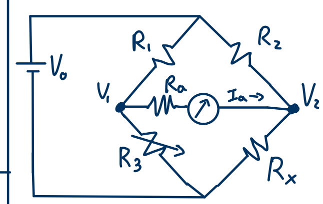
I_N = current flowing from A to B in short

R_{Th}: short all V sources
open all I sources

$R_{Th} = R_e$
$= V_{Th}/I_N$

WHEATSTONE BRIDGE

balanced + no approximation



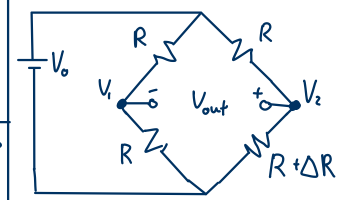
to determine R_x: adjust R_3 until $I_a = 0$

$\Rightarrow \frac{R_3 V_0}{R_1 + R_3} = \frac{R_x V_0}{R_2 + R_x}$

and $\frac{R_1 V_0}{R_1 + R_3} = \frac{R_2 V_0}{R_2 + R_x}$

$\Rightarrow R_x = \left(\frac{R_2}{R_1}\right) R_3$

sensor for small deviations from a ref. cond.



assuming $\Delta R/R \ll 1$

$V_1 = \frac{R}{R+R} V_0 = \frac{V_0}{2}$

$V_2 = \frac{R + \Delta R}{R + R + \Delta R} V_0$

$V_{out} = V_2 - V_1$
$\approx \frac{V_0}{4}\left(\frac{\Delta R}{R}\right)$

Capacitors:

$i = C\frac{dv}{dt}$

$w = \frac{1}{2}Cv^2$

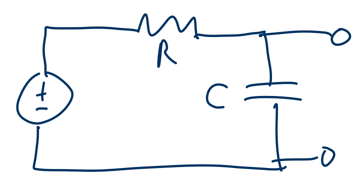
- v cannot charge instantaneously
- i **can** charge instantaneously (do not short circuit a charged capacitor)

Inductors:

$v = L\frac{di}{dt}$

$w = \frac{1}{2}Li^2$

- i cannot charge instantaneously
- v **can** charge instantaneously (do not open an inductor with current)



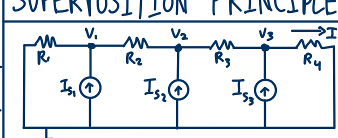
$Q = 0$

$$V_{out}(t) = e^{-t/RC}\int_0^t e^{\frac{t'}{RC}}\frac{V_{in}(t')}{RC}dt'$$

For any arbitrary Voltage source

when, $V_{in}(t) = V_0$, $V_{out}(t) = V_0(1 - e^{-t/RC})$

SUPERPOSITION PRINCIPLE



$V_3 = IR_4$

$I = \frac{R_1}{R_{1+2+3+4}} I_{S_1} + \frac{R_{1+2}}{R_{1+2+3+4}} I_{S_2} + \frac{R_{1+2+3}}{R_{1+2+3+4}} I_{S_3}$

CAPACITORS

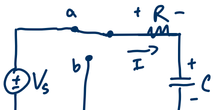
$C = \varepsilon A / d$ $\quad \varepsilon_0$, permittivity of vacuum $= 8.854 \times 10^{-12}$ F/m

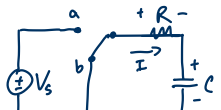
$\varepsilon = \varepsilon_r \varepsilon_0$

$Q = CV$

$i = \frac{dQ}{dt} = C\frac{dV}{dt}$

$\boxed{V(t) = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$





charging @ $t=0$

$V_s - IR - V_c = 0$

$V_s - RC\frac{dV_c}{dt} - V_c = 0$

$V_s - V_c = RC\frac{dV_c}{dt}$

$\frac{1}{V_s - V_c}\,dV_c = \frac{1}{RC}\,dt$

$\int_{V_c(0)}^{V_c(t)} \frac{1}{V_s - V_c} = \int_0^t \frac{1}{RC}\,dt$

$-\ln(V_s - V_c(t)) + \ln(V_s - V_c(0))$

$= \ln\left(\frac{V_s - V_c(0)}{V_s - V_c(t)}\right) = \ln\left(\frac{V_s}{V_s - V_c(t)}\right) = \frac{t}{RC}$

$\ln\left(\frac{V_s - V_c(t)}{V_s}\right) = -t/RC$

$1 - V_c(t)/V_s = e^{-t/RC}$

$V_c(t) = V_s(1 - e^{-t/RC})$

discharging @ $t = t'$

$-V_c - IR = 0$

$-V_c - RC\frac{dV_c}{dt} = 0$

$-V_c = RC\frac{dV_c}{dt}$

$\frac{1}{-V_c}\,dV_c = \frac{1}{RC}\,dt$

$\int_{V_c(t')}^{V_c(t)} \frac{1}{-V_c}\,dV_c = \int_{t'}^{t} \frac{1}{RC}\,dt$

$-\ln V_c(t) + \ln V_c(t')$

$= \ln\left(\frac{V_c(t')}{V_c(t)}\right) = \frac{t - t'}{RC}$

$\frac{V_c(t')}{V_c(t)} = e^{(t-t')/RC}$

$V_c(t) = V_c(t')\,e^{(t-t')/RC}$

$V_c(t) = V_c(t')\,e^{-(t-t')/RC}$

$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2}$ $\qquad C_{parallel} = C_1 + C_2$

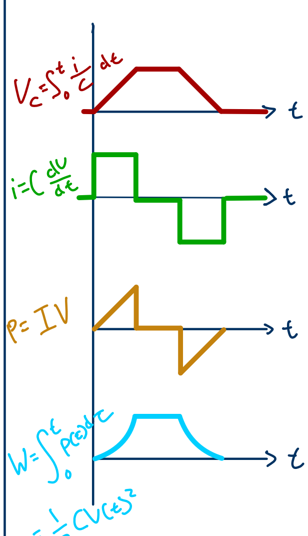
Power and energy

$P(t) = VI = V_c C\frac{dV}{dt}$

gives back while discharging

$\int P(t)\,dt = C\int V_c\,dV_c$

W (energy) $= \frac{1}{2}CV_c^2 \Rightarrow$ stored in capacitor



INDUCTORS

$L = \frac{\mu N^2 S}{\ell}$ $\quad N = \#$ coils, S = cross-sectional area

$\mu_r = \frac{\mu}{\mu_0}$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$\boxed{V_L = L\,di/dt}$

$\int_0^t \frac{V_L}{L}\,dt = \int_{i(0)}^{i(t)} 1\,di$

$\frac{1}{L}\int_0^t V_L\,dt = i(t) - i(0)$

$\boxed{i(t) = i(0) + \frac{1}{L}\int_0^t V_L\,dt}$

$L_{series} = L_1 + L_2$ $\qquad \frac{1}{L_{parallel}} = \frac{1}{L_1} + \frac{1}{L_2}$

$Z_R = R, \quad Z_L = j\omega L, \quad Z_C = \frac{1}{j\omega C}$

Phasor Notation

Convert everything to cosine

$\sin(x) = \cos(x - 90°)$

For each term:

$8\cos(6t - 45°) = 8e^{j-45°} = 8\angle -45°$

$8\angle -45° = 8(\cos(-45°) + j\sin(-45°))$

$= 5.66 - 5.66j$

Magnitude $= \sqrt{5.66^2 + (-5.66)^2}$

$= 8.00$

Phase Angle:

Draw real vs. imag graph $\theta = \tan^{-1}(-5.66/5.66) = -45°$ Final phasor:

$8.00\angle -45° = 8e^{j-45°}$

Power Stored in Capacitors

$P(t) = V \cdot i = V \cdot C\frac{dV}{dt}$

$\int_0^\infty P(t)\,dt = \int_0^{V_s} V \cdot C\,dV = \frac{1}{2}CV_2\Big|_0^{V_s} = \frac{1}{2}CV_s^2$

Time Constant

$\tau = R_{eq}C_{eq}$

Voltage to dB

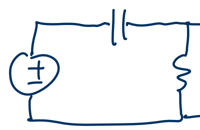
$dB = 20\log_{10}(V)$

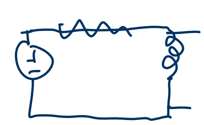
$V = 10^{dB/20}$

octave = 500–1000

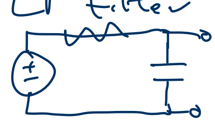
$|H(f)|_{dB} = 20\log_{10}|H(f)|$

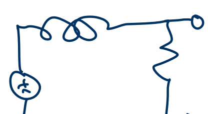
First order HP Filter





LP filter





TRANSFER FUNCTIONS

BODE PLOTS

1st order: denominator has ω deg 1

$|H_1(\omega_c)| = \frac{1}{\sqrt{2}}$ of max $|H_2(\omega)|$

filter roll off: ±20 dec/dB

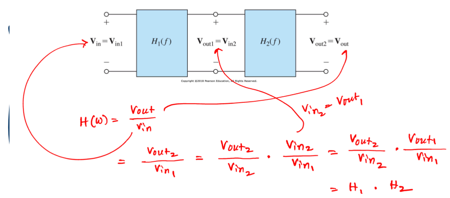
2nd order: denominator has ω deg 2

$|H_2(\omega_c)| = 1/2$ of max $|H_2(\omega)|$

filter roll off = ± 40 dec/dB

$\angle H(\omega) = \angle H_{numer}(\omega) - \angle H_{denom}$

$\angle H(\omega_c) = 90°$ always



$20\log(\omega^2) = 40\log(\omega)$

$20\log(\omega RL) = 20\log(RL) + 20\log(\omega)$

$20\log\left(\frac{A}{B}\right) = 20\log A - 20\log B$