CHARGE & CURRENT

v = charge vel.
Q_v = charge density
A = cross section $\perp$ to I
$\tau = \frac{L}{v}$

$I = \frac{dq}{dt} = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{Q}{\tau}$

$\frac{Q}{\tau} = \frac{Q\,v}{L} = \frac{Q_v A L v}{L}$

$I = \frac{Q}{\tau} = Q_v A v$ $\quad q(t) = \int_0^t i(t')\,dt'$ $\quad Q_v = \frac{I}{vA}$

P,I,V,R CONVERSIONS

$V = IR \quad I = V/R \quad R = V/I$

$P = IV = I^2R = V^2/R$

DERIVATIVE DEFINTIONS

$I = dq/dt \qquad q = \int_{-\infty}^{t} i\,dt$

$V_{AB} = dw/dq$ (dw: energy in J to move (+)q, b→a)

$P = dW/dt = \frac{dW}{dq}\cdot\frac{dq}{dt} = VI$

TRAVELING WAVES

$A \cdot \sin(kx \pm wt)$

k: wave vector $= 2\pi/\text{wavelength}, \lambda = w/2\pi$ [rad/m]

w: angular frequency $= 2\pi/\text{period} = 2\pi f$ [rad/s]

v: phase velocity $= w/k = \lambda f$ [m/s]

STANDING WAVES

most general: $f(x,t) = A(\sin(kx-wt)) - B(\sin(kx+wt))$

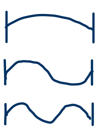
$= A[\sin(kx)\cos(wt) - \cos(kx)\sin(wt)]$

$+ B[\sin(kx)\cos(wt) + \cos(kx)\sin(wt)]$

salahuddin special: $f(x,t) = A[\sin(kx)\cos(wt) - \cos(kx)\sin(wt)]$

* $x = 0 \Rightarrow 0$ $\qquad = A[\sin(kx)\cos(wt)]$
* $(x = L \Rightarrow 0) \Rightarrow k = \frac{n\pi}{L}$ $\qquad = A[\sin(\frac{n\pi}{L}x\ \cos(wt)], n \in \mathbb{Z}$

1st harmonic
2nd harmonic
3rd harmonic



nth harmonic mode $k_{n-1} = \frac{n\pi}{L}$

harmonic node, n: $\lambda = 2L/n$

MOBILITY & RESISTIVITY

conductivity, $\sigma = nq\mu$ $\qquad \mu$ = electron mobility

resistivity, $\rho = \frac{1}{\sigma} = 1/nq\mu$ $\qquad n$ = free electron conc.

resistance, $R = \rho L/A$ $\qquad q$ = charge of elec.

mobility, $\mu = q\tau_{avg}/m_e$ $\qquad = 1.6\cdot10^{-19}$ C

m_e = mass of electron $= 9.1\cdot10^{-31}$ kg

τ_{avg} = avg time between 2 scattering events

Q_v = charge density

$F = ma \to E_q = m v/\tau_{avg}$

$\to Eq\tau_{avg}/m = v$

E = electric field (V/m)

v = drift v of e (m/s)

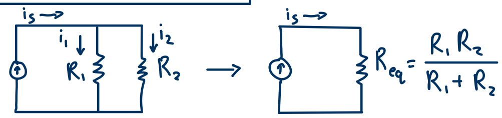
UNITS

tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

resistance	R	Ω
resistivity	ρ	$\Omega\cdot$m
conductance	G	S or Ω^{-1}
conductivity	σ	S/m
impedence	Z	Ω
capacitance	C	F
inductance	L	H
mobility	μ	$m^2/V\cdot s$
charge den.	ρ	C/m^3

time const.	τ	s
wave vector	k	rad/m

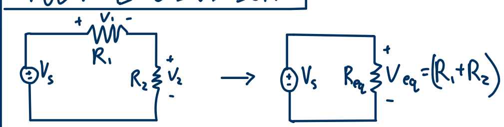
CURRENT DIVISION



$R_{eq} = \frac{R_1R_2}{R_1+R_2}$

$i_1 = \left(\frac{R_2}{R_1+R_2}\right)i_s \qquad i_2 = \left(\frac{R_1}{R_1+R_2}\right)i_s$

VOLTAGE DIVISION



$V_{eq} = (R_1+R_2)i$

$V_1 = \left(\frac{R_1}{R_1+R_2}\right)V_s \qquad V_2 = \left(\frac{R_2}{R_1+R_2}\right)V_s$

MESH CURRENT ANALYSIS

** vs. only I sources

* when there are only independent V sources

NODAL ANALYSIS

G_{kk} = sum of all conductances ($\frac{1}{R}$) connected to node k

$$\begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & & & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

$G_{kl} = G_{lk}$ = negative of conductance(s) connecting k-l

V_k = voltage at node k $\qquad$ [skip a node (=)]

I_k = total of current sources entering node k

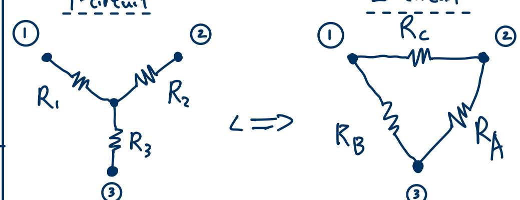
(add current sources leaving node k as negative)

SOURCE TRANSFORMATION

you know this buddy!

Δ-Y TRANSFORMATION

Y-circuit $\qquad$ Δ-circuit



Y→Δ

$A = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{1}$

$B = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{2}$

$C = \frac{1\cdot2 + 2\cdot3 + 3\cdot1}{3}$

Δ→Y

$1 = \frac{BC}{A+B+C}$

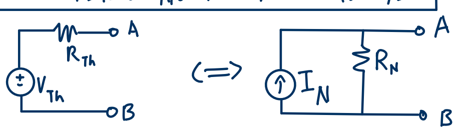
$2 = \frac{AC}{A+B+C}$

$3 = \frac{AB}{A+B+C}$

for $A = B = C \Rightarrow 1 = 2 = 3 = A/3$

for $R_1 = R_2 = R_3 \Rightarrow A = B = C = 3\cdot R_1$

THEVENIN & NORTON EQUIVALENTS



$V_{Th} = V_A - V_B$ in open circuit

$R_{Th} = R_N$

$I_N = V_{Th}/R_{Th}$

I_N = current flowing from A to B in short

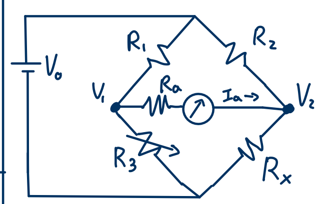
R_{Th}: short all V sources
open all I sources

$R_{Th} = R_e$
$= V_{Th}/I_N$

WHEATSTONE BRIDGE

to determine R_x:
adjust R_3 until $I_a = 0$

balanced + no approximation

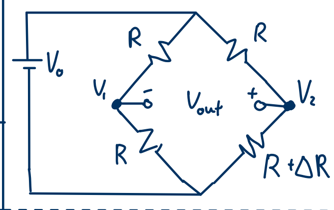


$\Rightarrow \frac{R_3V_0}{R_1+R_3} = \frac{R_xV_0}{R_2+R_x}$

and $\frac{R_1V_0}{R_1+R_3} = \frac{R_2V_0}{R_2+R_x}$

$\Rightarrow R_x = \left(\frac{R_2}{R_1}\right)R_3$

sensor for small deviations from a ref. cond.



assuming $\Delta R/R \ll 1$

$V_1 = \frac{R}{R+R}V_0 = \frac{V_0}{2}$

$V_2 = \frac{R+\Delta R}{R+R+\Delta R}V_0$

$V_{out} = V_2 - V_1$

$\approx \frac{V_0}{4}\left(\frac{\Delta R}{R}\right)$

Capacitors:

$i = C\frac{dv}{dt}$

$w = \frac{1}{2}Cv^2$

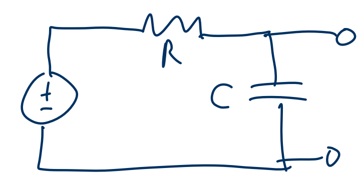
* v can't change instantly
* i can change instantly

Inductors:

$v = L\frac{di}{dt}$

$w = \frac{1}{2}Li^2$

* i can't change instantly
* v can change instantly



$Q = 0$

$V_{out}(t) = e^{-t/RC}\int_0^t e^{\frac{t'}{RC}}\frac{V_{in}(t')}{RC}dt$

$V_{out} = V_0(1 - e^{-t/RC})$

$V_C(t=0^+) = V_C(t=0^-)$

$i_L(t=0^+) = i_L(t=0^-)$

$i_C(t=\infty) = C\frac{dV_C}{dt} = 0 \to$ open

$V_L(t=\infty) = L\frac{di_L}{dt} = 0 \to$ short

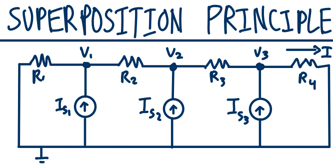
capacitor @ $t = 0^+ \to$ short

inductor @ $t = 0^+ \to$ open

$V_C(t=0^+) = V_C(t=0^-)$	$i_C(\infty) = C\frac{dV_C}{dt} = 0 \to open$
$i_L(t=0^+) = i_L(t=0^-)$	$V_L(\infty) = L\frac{di_L}{dt} = 0 \to short$

SUPERPOSITION PRINCIPLE

works for voltage too



$V_3 = IR_4$ $\qquad$ short V $\qquad$ open I sources

$I = \frac{R_1}{R_{1+2+3+4}}I_{S_1} + \frac{R_{1+2}}{R_{1+2+3+4}}I_{S_2} + \frac{R_{1+2+3}}{R_{1+2+3+4}}I_{S_3}$

CAPACITORS

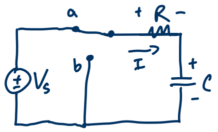
$C = \varepsilon A/d$ $\quad \varepsilon_0$, permittivity of vacuum $= 8.854 \times 10^{-12}$ F/m

$\varepsilon = \varepsilon_r \varepsilon_0$

$Q = CV$

$i = \frac{dQ}{dt} = C\frac{dV}{dt}$

$$\boxed{V(t) = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)}$$



charging @ $t=0$

$V_s - IR - V_c = 0$

$V_s - RC\frac{dV_c}{dt} - V_c = 0$

$V_s - V_c = RC\frac{dV_c}{dt}$

$\frac{1}{V_s - V_c}\,dV_c = \frac{1}{RC}\,dt$

$\int_{V_c(0)}^{V_c(t)} \frac{1}{V_s - V_c} = \int_0^t \frac{1}{RC}\,dt$

$-\ln(V_s - V_c(t)) + \ln(V_s - V_c(0))$

$= \ln\left(\frac{V_s - V_c(0)}{V_s - V_c(t)}\right) = \frac{t}{RC}$

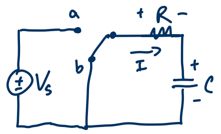
$\ln\left(\frac{V_s - V_c(t)}{V_s - V_c(0)}\right) = -t/RC$

$\frac{V_s - V_c(t)}{V_s - V_c(0)} = e^{-t/RC}$

$V_s - V_c(t) = (V_s - V_c(0))e^{-t/RC}$

$V_c(t) = V_s - (V_s - V_c(0))e^{-t/RC}$

$V_c(t) = V_s(1 - e^{-t/RC})$



discharging @ $t = t'$

$-V_c - IR = 0$

$-V_c - RC\frac{dV_c}{dt} = 0$

$-V_c = RC\frac{dV_c}{dt}$

$\frac{1}{-V_c}\,dV_c = \frac{1}{RC}\,dt$

$\int_{V_c(t')}^{V_c(t)} \frac{1}{-V_c}\,dV_c = \int_{t'}^{t} \frac{1}{RC}\,dt$

$-\ln V_c(t) + \ln V_c(t')$

$= \ln\left(\frac{V_c(t')}{V_c(t)}\right) = \frac{t - t'}{RC}$

$\frac{V_c(t')}{V_c(t)} = e^{(t-t')/RC}$

$V_c(t) = V_c(t')e^{(t-t')/RC}$

$V_c(t) = V_c(t')e^{-(t-t')/RC}$

$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2}$ $\qquad C_{parallel} = C_1 + C_2$

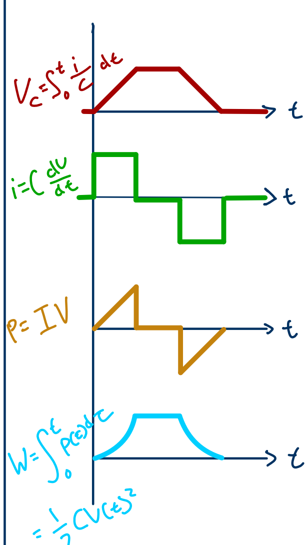
Power and energy

$P(t) = VI = V_c C\frac{dV_c}{dt}$

$\int_0^\infty P(t)\,dt = C\int_0^{V_s} V_c\,dV_c = \frac{1}{2}CV_c^2$

gives back while discharging

$W_{(energy)} = \frac{1}{2}CV_c^2 \Rightarrow$ stored in capacitor



$I_L(t) = I_L(\infty) + (I_L(t_0) - I_L(\infty))e^{-\frac{t-t_0}{\tau}}$

$V_c(t) = V_c(\infty) + (V_c(t_0) - V_c(\infty))e^{-\frac{t-t_0}{\tau}}$

$\tau = R_{eq}C$ or $\frac{L}{R_{eq}}$

INDUCTORS

$L = \frac{\mu N^2 S}{\ell}$ $\quad N = \#$ coils, S = cross-sectional area

$\mu_r = \frac{\mu}{\mu_0}$ $\quad \mu_0 = 4\pi \cdot 10^{-7}$ H/m

$$\boxed{V_L = L\,\frac{di}{dt}}$$

$\int_0^t \frac{V_L}{L}\,dt = \int_{i(0)}^{i(t)} 1\,di$

$\frac{1}{L}\int_0^t V_L\,dt = i(t) - i(0)$

$$\boxed{i(t) = i(0) + \frac{1}{L}\int_0^t V_L\,dt}$$

$L_{series} = L_1 + L_2$ $\qquad \frac{1}{L_{parallel}} = \frac{1}{L_1} + \frac{1}{L_2}$

$Z_R = R$, $\quad Z_L = j\omega L$, $\quad Z_c = \frac{1}{j\omega C}$

Phasor Notation

Convert everything to cosine

$\sin(x) = \cos(x - 90°)$

For each term:

$8\cos(6t - 45°) = 8e^{j(-45°)} = 8\angle -45°$

$8\angle -45° = 8(\cos(-45°) + j\sin(-45°))$

$= 5.66 - 5.66j$

Magnitude $= \sqrt{5.66^2 + (-5.66)^2}$

$= 8.00$

Phase Angle:

Draw real vs. imag graph $\theta = \tan^{-1}(-5.66/5.66) = -45°$ Final phasor:

$8.00\angle -45° = 8e^{j\cdot -45°}$

Power Stored in Capacitors

$P(t) = V \cdot i = V \cdot C\frac{dV}{dt}$

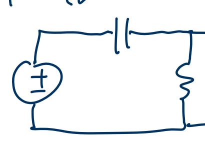
$\int_0^\infty P(t)\,dt = \int_0^{V_s} V \cdot C\,dV = \frac{1}{2}CV_c^2\Big|_0^{V_s} = \frac{1}{2}CV_s^2$

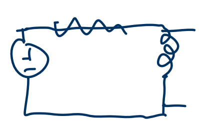
Time Constant	Voltage to dB
$\tau = R_{eq}C_{eq}$	$dB = 20\log_{10}(V)$ $V = 10^{dB/20}$

****

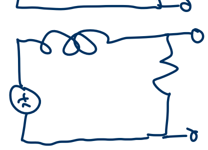
octave = 500–1000

First order HP Filter





LP Filter



TRANSFER FUNCTIONS

BODE PLOTS

1st order: denominator has ω deg 1

$|H_1(\omega_c)| = \frac{1}{\sqrt{2}}$ of max $|H_2(\omega)|$

filter roll off: ±20 dec/dB

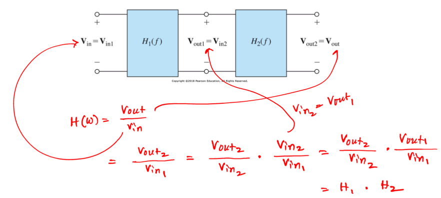
2nd order: denominator has ω deg 2

$|H_2(\omega_c)| = 1/2$ of max $|H_2(\omega)|$

filter roll off = ±40 dec/dB

$\angle H(\omega) = \angle H_{numer}(\omega) - \angle H_{denom}$

$\angle H(\omega_c) = 90°$ always



$20\log(\omega^2) = 40\log(\omega)$

$20\log(\omega RL) = 20\log(RL) + 20\log(\omega)$

$20\log\left(\frac{A}{B}\right) = 20\log A - 20\log B$

Factor	Bode Magnitude	Bode Phase
Constant K	$20\log K$; 0 dB; ω	(i) $-90°/\pm180°$ if $K<0$; $0°$ (i) $90°/0°$ if $K>0$; ω
Zero @ Origin $(j\omega)^N$	0 dB; 1; slope = 20N dB/dec; ω	$0°$; $(90N)°$; ω
Pole @ Origin $(j\omega)^{-N}$	0 dB; 1; slope = −20N dB/dec; ω	$0°$; $(-90N)°$; ω
Simple Zero $(1 + j\omega/\omega_c)^N$	0 dB; ω_c; slope = 20N dB/dec; ω	$0°$; $0.1\omega_c$; ω_c; $10\omega_c$; $(90N)°$; ω
Simple Pole $\left(\frac{1}{1 + j\omega/\omega_c}\right)^N$	0 dB; ω_c; slope = −20N dB/dec; ω	$0°$; $0.1\omega_c$; ω_c; $10\omega_c$; $(-90N)°$; ω
Quadratic Zero $[1 + j2\zeta\omega/\omega_c + (j\omega/\omega_c)^2]^N$	0 dB; ω_c; slope = 40N dB/dec; ω	$0°$; $0.1\omega_c$; ω_c; $10\omega_c$; $(180N)°$; ω
Quadratic Pole $\frac{1}{[1 + j2\zeta\omega/\omega_c + (j\omega/\omega_c)^2]^N}$	0 dB; ω_c; slope = −40N dB/dec; ω	$0°$; $0.1\omega_c$; ω_c; $10\omega_c$; $(-180N)°$; ω

Second-Order DEs:

$$\frac{d^2x(t)}{dt^2} + 2\alpha\frac{dx(t)}{dt} + \omega_0^2 x(t) = 0$$

- α: Damping Factor
- ω_0: Undamped Natural Frequency

- Characteristic Polynomial:
  - $s^2 + 2\alpha s + \omega_0^2 = 0$
- Homogeneous Solution:
  - $x_h(t) = Ae^{s_1 t} + Be^{s_2 t}$
  - $s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

1. [Overdamped]: $\alpha > \omega_0 \rightarrow s_1, s_2$ are real and neg.
   $x_h(t) = Ae^{-(\alpha - \sqrt{\alpha^2 - \omega_0^2})t} + Be^{-(\alpha + \sqrt{\alpha^2 - \omega_0^2})t}$
2. [Underdamped]: $\alpha < \omega_0 \rightarrow s_1, s_2$ are complex
   $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
   $x_h(t) = e^{-\alpha t}[C\cos(\omega_d t) + D\sin(\omega_d t)]$. (decaying sinusoid)
3. [Critically Damped] $\alpha = \omega_0 \rightarrow s_1 = s_2 = -\alpha$
   $x_h(t) = Ae^{-\alpha t} + Bte^{-\alpha t}$

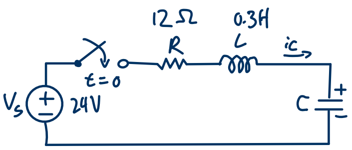
- Particular Solution
  $x_p(t) = x(t = \infty)$
- General Solution
  $x(t) = x_h(t) + x_p(t)$

- use $x(0)$, $x'(0)$ to solve for A, B, C, D

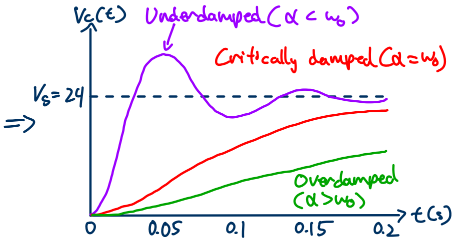
$A\cos(\omega t - x)$
$= -A\cos(\omega t - x + 180°)$
$= -A\cos(\omega t - (x - 180))$

~~****~~ $(A\angle B°)(C\angle D°) = (AC)\angle(B° + D°)$

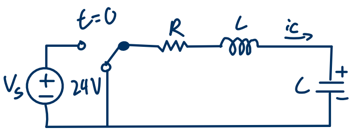
$\alpha = R/2L = 20$ Np/s



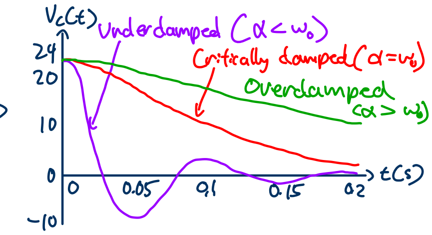
$\Rightarrow$



ⓐ charging up C

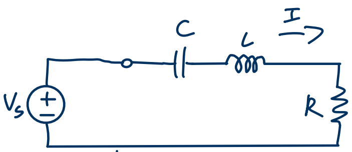


$\Rightarrow$



ⓑ Discharging C

RLC circuit with V_0 on R: Band pass filter



$$H(\omega) = \frac{V_R}{V_s} = \frac{RI}{V_s}$$

$$I = \frac{V_s}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{j\omega C V_s}{(1 - \omega^2 LC) + j\omega RC}$$

$$= \frac{j\omega RC}{(1 - \omega^2 LC) + j\omega RC}$$

$$|H(\omega)| = \frac{\omega RC}{\sqrt{(1 - \omega^2 LC)^2 + \omega^2 R^2 C^2}}$$

$$\phi(\omega) = 90° - \tan\left[\frac{\omega RC}{1 - \omega^2 LC}\right]$$

$$\omega_0 = 1/\sqrt{LC}$$

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

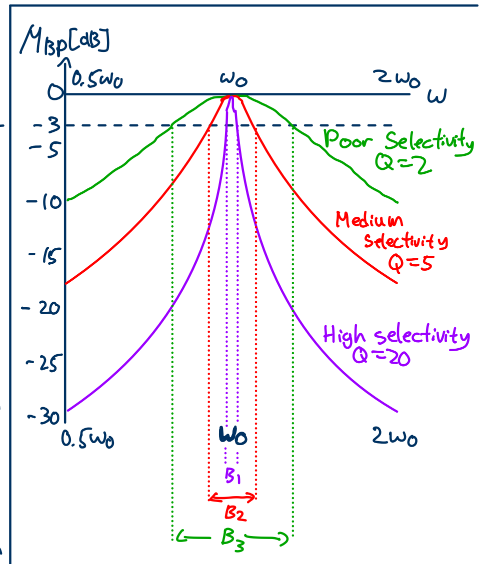
$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \left(\frac{1}{LC}\right)}$$

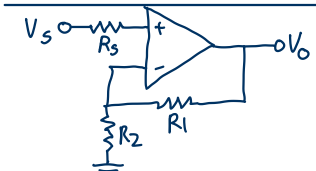
$$B = \omega_{c2} - \omega_{c1} = \frac{R}{L} \qquad \omega_0 = \sqrt{\omega_{c1}\omega_{c2}}$$

$$Q = \frac{\omega_0 L}{R} = \frac{\omega_0}{B}$$

$$Q = 2\pi\left(\frac{W_{stor}}{W_{diss}}\right) \text{ @ } \omega = \omega_0$$

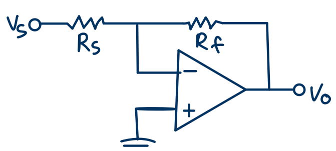
High Q = high selectivity & narrow bandwidth





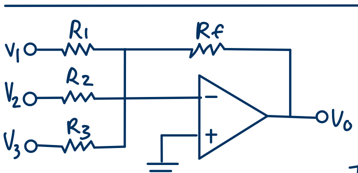
$V_s \longrightarrow \boxed{G = \frac{R_1 + R_2}{R_2}} \longrightarrow V_0 = GV_s$

Noninverting Amp
(V_0 independent of R_s)



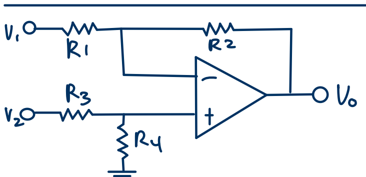
$V_s \longrightarrow \boxed{G = -\frac{R_f}{R_s}} \longrightarrow V_0 = GV_s$

Inverting Amp



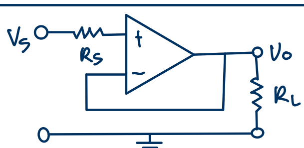
$V_1 \longrightarrow \boxed{G_1 = -R_f/R_1}$
$V_2 \longrightarrow \boxed{G_2 = -R_f/R_2}$
$V_3 \longrightarrow \boxed{G_3 = -R_f/R_3}$
$\longrightarrow V_0 = G_1V_1 + G_2V_2 + G_3V_3$

Inverting Summing Amp



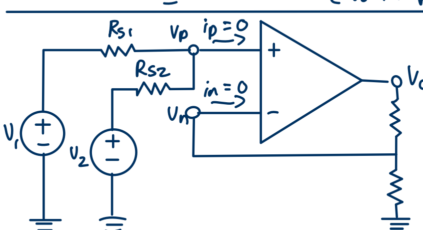
$V_1 \longrightarrow \boxed{G_1 = -\frac{R_2}{R_1}}$
$V_2 \longrightarrow \boxed{G_2 = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right)}$
$\longrightarrow V_0 = G_1V_1 + G_2V_2$

Subtracting Amp



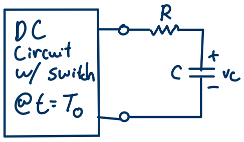
$\longrightarrow \boxed{G = 1} \longrightarrow V_0 = V_s$

Voltage Follower/Buffer
(V_0 independent of R_s & R_L)



$V_1 \longrightarrow \boxed{G_1 = \left(\frac{R_1 + R_2}{R_2}\right)\left(\frac{R_{s2}}{R_{s1} + R_{s2}}\right)}$
$V_2 \longrightarrow \boxed{G_2 = \left(\frac{R_1 + R_2}{R_2}\right)\left(\frac{R_{s1}}{R_{s1} + R_{s2}}\right)}$
$\longrightarrow V_0 = G_1V_1 + G_2V_2$

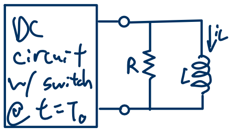
Noninverting Summing Amp



$$v_C(t) = V_C(\infty) + [V_C(T_0) - V_C(\infty)]e^{-(t-T_0)/\tau}$$

$$\tau = R_{eq}C$$

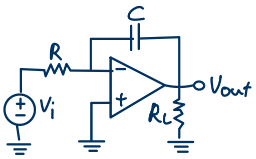
$$i_C(t) = -\frac{1}{R}[V_C(T_0) - V_C(\infty)]e^{-(t-T_0)/\tau}$$



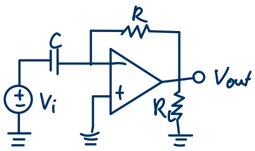
$$i_L(t) = i_L(\infty) + [i_L(T_0) - i_L(\infty)]e^{-(t-T_0)/\tau}$$

$$\tau = L/R_{eq}$$

$$V_L(t) = -R\,[i_L(T_0) - i_L(\infty)]\,e^{-(t-T_0)/\tau}$$

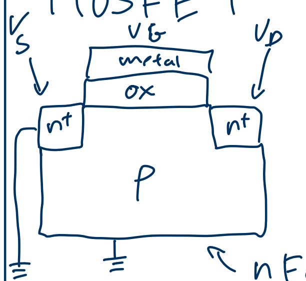


$$V_{out}(t) = -\frac{1}{RC}\int_{t_0}^{t} v_i\,dt + V_{out}(t_0)$$



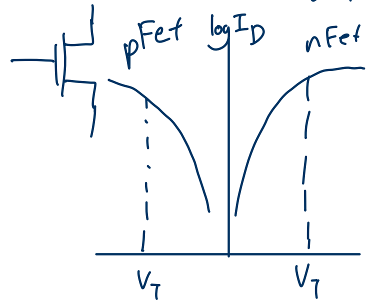
$$V_{out}(t) = -RC\frac{dV_i}{dt}$$

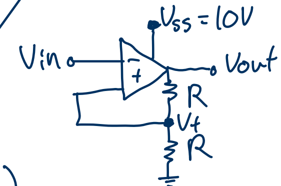
MOSFET

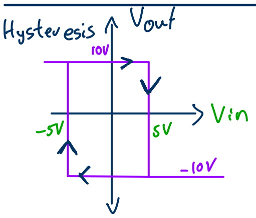


+/− in n/p indicates extent of doping. N+ means heavily doped

nFet; pFet → swap







① $V_- > V_+$

$V_{out} = A(V_+ - V_-) < 0$

↓

V_{out} and V_+ flips sign

↓

As long as $V_- > -5$; $V_+ - V_- < 0$

if V_{in} goes under −5V.

② $V_+ - V_- > 0 \rightarrow V_{out}$ & V_+ flip sign again

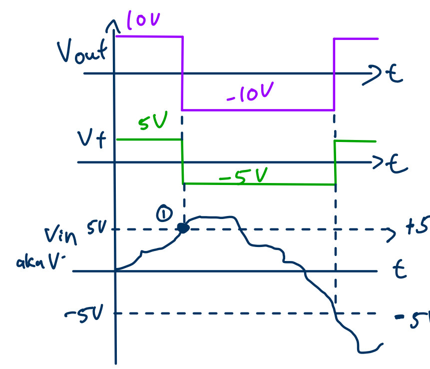
$$I_D = Ae^{\frac{BV_G}{KT}} \quad \text{(exp before } V_T\text{)}$$

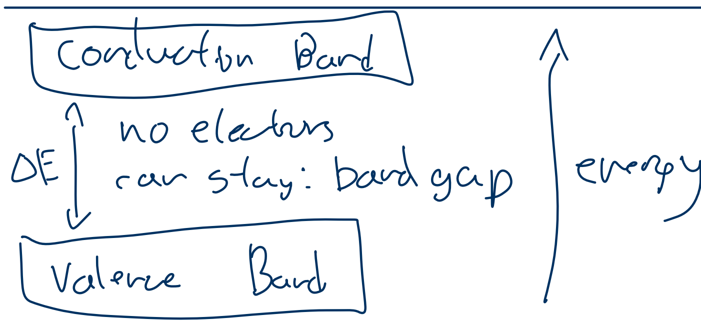
$$I_D = C(V_g - V_T)^2$$

(quad. after V_T)

ideal op amp characteristics

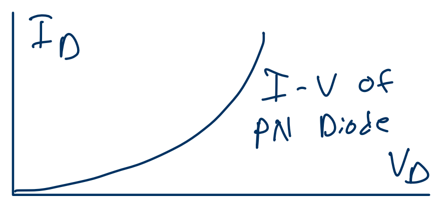
- $i_p = i_n = 0$
- $v_p = v_n$
- $A = \infty$
- $R_i = \infty$
- $R_o = 0$





smaller band gap → more conductive

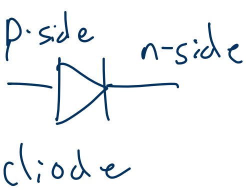
P(electron at C band due to thermal energy) $\sim e^{-\Delta E/K_B T}$

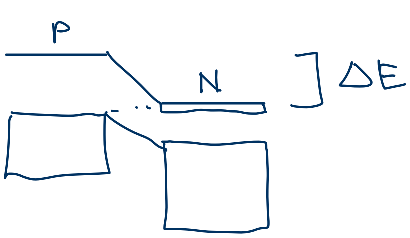


K_B: boltzmann constant

n-type material: extra electrons added to conduction band from donor material (phosphorus for silicon)

p-type material: atoms with one fewer valence electron used to dope the silicon, creates holes in conduction band





forward bias

$V_D > 0$

$\rightarrow \Delta E' < \Delta E$

reverse bias

$V_D < 0$

$\Delta E' > \Delta E$

Quality factor

According to the foregoing discussion, the choice of values we make for R, L, and C will specify the overall shape of the transfer function completely, as well as its center frequency ω_0 and bandwidth B.

> ► The ***quality factor*** of a circuit Q is an attribute commonly used to characterize the ***degree of selectivity*** of the circuit. A high Q circuit has a narrow bandwidth (relative to the center frequency) and high selectivity. ◄

Figure 9-16 displays frequency responses for three circuits, all with the same ω_0. The high-Q circuit exhibits a sharp response with a narrow bandwidth (relative to ω_0), the medium-Q circuit has a broader pattern, and the low-Q circuit has a pattern with limited selectivity.

For the bandpass-filter response, Q obviously is related to the ratio ω_0/B, but the formal definition of Q applies to any resonant circuit and is based on energy considerations, namely

$$Q = 2\pi \left(\frac{W_{\text{stor}}}{W_{\text{diss}}} \right) \bigg|_{\omega=\omega_0}, \tag{9.53}$$

The current $\mathbf{I}$ flowing through the loop in **Fig. 9-15(a)** is given by

$$\mathbf{I} = \frac{\mathbf{V}_s}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)} = \frac{j\omega C \mathbf{V}_s}{(1-\omega^2 LC) + j\omega RC}, \tag{9.44}$$

where we multiplied the numerator and denominator by $j\omega C$ to simplify the form of the expression. The transfer function corresponding to $\mathbf{V}_R$ is

$$\mathbf{H}_{\text{BP}}(\omega) = \frac{\mathbf{V}_R}{\mathbf{V}_s} = \frac{R\mathbf{I}}{\mathbf{V}_s} = \frac{j\omega RC}{(1-\omega^2 LC) + j\omega RC}, \tag{9.45}$$

where we added the subscript "BP" in anticipation of the fact that $\mathbf{H}_{\text{BP}}(\omega)$ is the transfer function of a bandpass filter. Its magnitude and phase angle are given by

$$M_{\text{BP}}(\omega) = |\mathbf{H}_{\text{BP}}(\omega)| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 R^2 C^2}}, \tag{9.46}$$

and

$$\phi_{\text{BP}}(\omega) = 90^\circ - \tan^{-1}\left[\frac{\omega RC}{1-\omega^2 LC}\right]. \tag{9.47}$$

$$M^2_{\text{BP}}(\omega) = \frac{1}{2} \qquad @\ \omega_{c_1} \text{ and } \omega_{c_2}. \tag{9.49}$$

Upon inserting the expression for $M_{\text{BP}}(\omega)$ given by Eq. (9.46) and carrying out several steps of algebra, we obtain the solutions

$$\omega_{c_1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \tag{9.50a}$$

and

$$\omega_{c_2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}. \tag{9.50b}$$

The bandwidth then is given by

$$B = \omega_{c_2} - \omega_{c_1} = \frac{R}{L}. \tag{9.51}$$

It is worth noting that ω_0 is equal to the geometric mean of ω_{c_1} and ω_{c_2}:

$$\omega_0 = \sqrt{\omega_{c_1}\omega_{c_2}}. \tag{9.52}$$

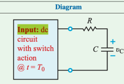
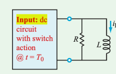
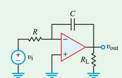
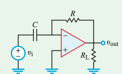
Upon substituting Eqs. (9.59) and (9.60) into Eq. (9.53), we obtain the result

$$Q = \frac{\omega_0 L}{R} \qquad \textbf{(bandpass filter)}. \qquad (9$$

Using the relation given by Eq. (9.51), the expression fo quality factor becomes

$$Q = \frac{\omega_0}{B} \qquad \textbf{(bandpass filter)}, \tag{9.62}$$

Table 5-5: Response forms of basic first-order circuits.

Circuit	Diagram	Response
RC		$v_C(t) = \left\{ v_C(\infty) + [v_C(T_0) - v_C(\infty)] e^{-(t-T_0)/\tau} \right\} u(t - T_0)$ $(\tau = RC)$
RL		$i_L(t) = \left\{ i_L(\infty) + [i_L(T_0) - i_L(\infty)] e^{-(t-T_0)/\tau} \right\} u(t - T_0)$ $(\tau = L/R)$
Ideal integrator		$v_{out}(t) = -\frac{1}{RC}\int_{t_0}^{t} v_i \, dt' + v_{out}(t_0)$
Ideal differentiator		$v_{out}(t) = -RC \frac{dv_i}{dt}$

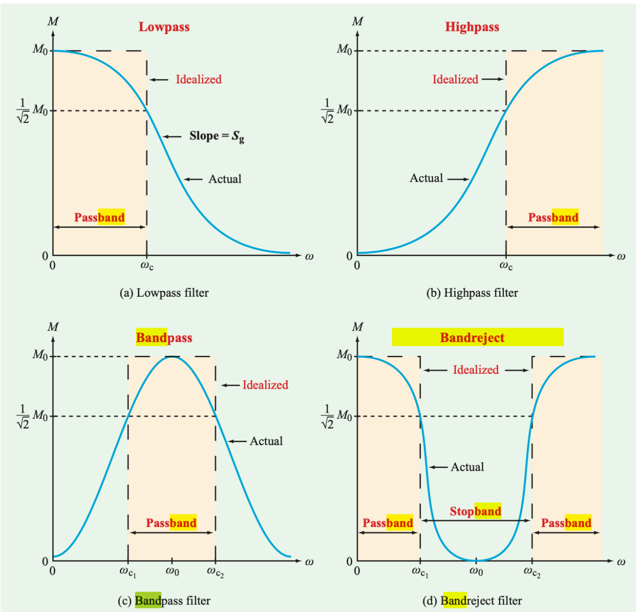
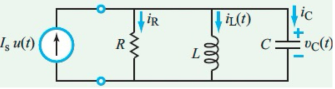


Figure 9-2: Typical magnitude spectral responses for the four types of filters.



finding diff e in terms of $i_L(t)$

$$i_s = i_R + i_L + i_c$$
$$= \frac{V_R}{R} + i_L + C\frac{dV_c}{dt}$$
$$= \frac{V_L}{R} + i_L + C\frac{dV_L}{dt}$$
$$i_s = \frac{L}{R}\frac{di_L}{dt} + i_L + CL\frac{d^2 i_L}{dt^2}$$

to standard form

$$\boxed{i_L'' + \frac{1}{RC} i_L' + \frac{1}{LC} i_L = \frac{1}{LC} i_s}$$

determining initial conditions

$i_L(0^+)$: $i_L(0^+) = i_L(0^-) = 0$

$i_L'(0^+)$: $i_L'(0^+) = 0$ since all current flows through the capacitor, which acts as short

$i_L(\infty)$: I_s since all current will flow through inductor, which acts as short

solving for $i_L(t)$

solve for homogeneous soln

$$i_L'' + \frac{1}{RC} i_L' + \frac{1}{LC} i_L = 0$$

$$s^2 + 2\alpha s + w_0^2 = 0$$

$2\alpha = \frac{1}{RC}$ $\quad$ $w_0^2 = \frac{1}{LC}$

$\alpha = \frac{1}{2RC}$ $\quad$ $w_0 = \frac{1}{\sqrt{LC}}$

assuming $R = 5$
$C = 1mF$
$L = 1pH$

$\alpha = 10^5$ $\quad$ $w_0 = 10^9$

$\alpha < w_0 \Rightarrow$ underdamped

$$X_h(t) = e^{-\alpha t}\left(C\cos(w_d t) + D\sin(w_d t)\right)$$

solve for C & D

$$i_L(0) = e^0(C + 0)$$
$$= C = 0$$

$$X_h'(t) = -\alpha e^{-\alpha t} C\cos(w_d t) + e^{-\alpha t} C w_d(-\sin(w_d t))$$
$$-\alpha e^{-\alpha t} D\sin(w_d t) + e^{-\alpha t} D w_d\cos(w_d t)$$

$$X_h'(0) = -\alpha(1)C(1) + \cancel{(1)C w_d(0)}$$
$$\cancel{-\alpha(1)D(0)} + (1)D w_d(1)$$
$$= -\alpha C + w_d D$$

now soln is homogeneous soln + particular soln

$X = X_h + X_p$ $\quad$ $X_p = X(\infty) = i_L(\infty) = I_s$

$$i_h(t) = Ae^{-(\alpha - \sqrt{\alpha^2 - w_0^2})t} + Be^{-(\alpha + \sqrt{\alpha^2 - w_0^2})t}$$

$$i'_h(t) = -(\alpha - w_d)\, Ae^{-(\alpha - w_d)t} - (\alpha + w_d) Be^{-(\alpha + w_d)t}$$

$$i'_h(0) = -(\alpha - w_d)A - (\alpha + w_d)B$$

$$= (w_d - \alpha)A - (w_d + \alpha)B$$