asymptotics

- $f(n) = O(g(n))$
- $f(n) = \Theta(g(n))$
- $f(n) = \Omega(g(n))$

$\lim_{n\to\infty} \frac{f(n)}{g(n)} \begin{cases} < \infty \\ = c \\ > 0 \end{cases}$

exponential > polynomial > logarithm

$3^n > 2^n$ > $n^5 > n^4 > n^{0.1}$ > $(\log n)^3 > (\log n)^2 > \log\log n$

- try substitution! or L'Hopital's!

maths

- $\log_b(M^k) = k \cdot \log_b M$
- $\log_b(b^k) = k$
- $a^{\log_b n} = n^{\log_b a}$
- $\log_b n = \frac{\ln n}{\ln b}$
- $\log_b(M \cdot N) = \log_b M + \log_b N$
- $\log_b(M/N) = \log_b M - \log_b N$
- $1/\log_a b = \log_b a$
- $\sum_{i=1}^{k} \frac{1}{i} = \log k$
- $\ln n \to 1/n$
- $a^n \to a^n \ln n$
- $(\ln n)^k \to k(\ln n)^{k-1}/n$
- $n \log n \to \log n$
- $1 + 2 + \dots + n = O(n^2)$
- $1 + \frac{1}{2} + \dots + \frac{1}{n} = O(\log N)$
- $a + ar + \dots ar^n = \frac{a(1-r^n)}{1-r}$
- $1 + p + p^2 + \dots p^k = \frac{p^{k+1}-1}{p-1}$
- inf geometric $\to \frac{a}{1-r}$

master's theorem

$T(n) = aT(n/b) + O(n^d)$

$d > \log_b a$	$O(n^d)$
$d = \log_b a$	$O(n^d \log n)$
$d < \log_b a$	$O(n^{\log_b a})$

BFS/DFS: $O(|V| + |E|)$

Kosaraju's: $O(|V| + |E|)$

- run DFS
- run DFS on G_{rev} w/ decreasing post-order

Dijkstra's: $O((|E| + |V|)\log|V|)$

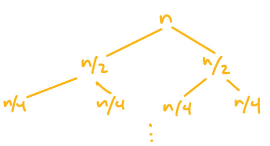
- init dist $= \infty$, par = None
- dist(s) = 0, add to minH
- update all neighbors of first

Bellman-Ford: $O(|V||E|)$

- works w/ neg edges!
- repeat $|V|-1$ times:
  - for each e in E:
    - update dist(v)

tree method

$T(n) = 2T(\frac{n}{2}) + O(\frac{n}{\log n})$ w/ $T(1) = 1$



$n/\log n$

$\frac{n/2}{\log \frac{n}{2}} \cdot 2 = \frac{n}{\log n/2}$

$n/\log\frac{n}{4}$

$n/n \mid n/\log 1$

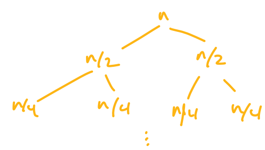
$n \sum_{j=0}^{\log n} \frac{1}{\log \frac{n}{2^j}} = n \sum_{j=0}^{\log n} \frac{1}{\log n - \log 2^j}$

$= n \sum_{j=0}^{\log n} \frac{1}{\log n - j}$

$= n \left(\sum_{i=1}^{\log n} \frac{1}{i} \right)$

$= n \log\log n$

$T(n) = 2T(n/2) + O(n \log n)$



$n \log n$

$n \log n/2$

$n \log n/4$

$n \log n/n$

$n \sum_{i=0}^{\log n} \log \frac{n}{2^i} = n \sum_{i=0}^{\log n} \log n - i$

$= n \log^2 n$

squeeze + guess & check

- guess U/L bounds

$T(n) = T(3n/5) + T(4n/5)$

guess an^b, $a=1$ since $T(1)=1$

$T(n) = \quad n^b = \quad (3n/5)^b + (\frac{4n}{5})^b$

$\Rightarrow b = 2$

change of variables

$T(n) = 2T(\sqrt{n}) + O(\log n)$

$m = \log n$

$T(2^m) = 2T(2^{m/2}) + O(m)$

$S(m) = T(2^m)$

$S(m) = 2S(m/2) + O(m)$

$\Rightarrow S(m) = m \log m$

$T(2^m) = m \log m$

$T(2^{\log n}) = T(n) = \log n \log\log n$

$T(n) = T(\sqrt{n}) + 1$

$m = \log n$

$T(2^m) = T(2^{m/2}) + 1$

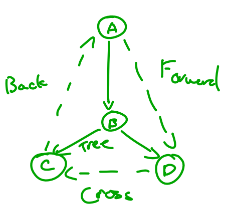
$S(m) = T(2^m)$

$S(m) = S(m/2) + 1$

$\Rightarrow S(m) = \log m$

$T(2^m) = \log m$

$T(2^{\log n}) = T(n) = \log\log n$



Tree/Forward if $\underset{u}{[}\underset{v}{[}\underset{v}{]}\underset{u}{]}$

pre(u) < pre(v) < post(v) < post(u)

Back if $\underset{v}{[}\underset{u}{[}\underset{u}{]}\underset{v}{]}$

pre(v) < pre(u) < post(u) < post(v)

cross if: $\underset{v}{[}\underset{v}{]}\underset{u}{[}\underset{u}{]}$

pre(v) < post(v) < pre(u) < post(u)