Conditional Approach Gibbs Sampler

$$\underline{Z} = \underline{D}\,\underline{Y} + \underline{\varepsilon} \quad ; \quad \underline{\varepsilon} \sim N(0, \Sigma_\varepsilon(\rho_\varepsilon, \sigma_\varepsilon^2))$$

$$\underline{Y} \sim N(\underline{0}, \underline{\Sigma}) \quad ; \quad \underline{\Sigma} := \cancel{\underline{\Sigma}(\underline{B}(A, \sigma_{11}^2, \sigma_{2|1}^2, K}$$

$$:= \underline{\Sigma}\big(\underline{B}(A, r, \Delta_1, \Delta_2),\ \underline{\Sigma}_1(\sigma_{11}^2, K_{11}),\ \underline{\Sigma}_2(\sigma_{2|1}^2, K_{2|1})\big)$$

$$\therefore\ p(\underline{Y} \mid \underline{Z}, \underline{\theta}) \propto p(\underline{Z} \mid \underline{Y}, \underline{\theta})\, p(\underline{Y} \mid \underline{\theta})$$

$$\propto \exp\Big(-\tfrac{1}{2}(\underline{Z} - \underline{D}\,\underline{Y})^T \underline{\Sigma}_\varepsilon^{-1}(\underline{\theta})(\underline{Z} - \underline{D}\,\underline{Y}) - \tfrac{1}{2}\underline{Y}^T \underline{\Sigma}(\underline{\theta})^{-1}\underline{Y}\Big)$$

$$\propto \exp\Big(-\tfrac{1}{2}\underline{Y}^T\big(\underline{\Sigma}(\underline{\theta})^{-1} + \underline{D}^T\underline{\Sigma}_\varepsilon^{-1}(\underline{\theta})\underline{D}\big)\underline{Y} + \underline{Y}^T\big(\underline{D}^T\underline{\Sigma}_\varepsilon^{-1}(\underline{\theta})\underline{Z}\big)\Big)$$

$$\underline{\Sigma}_Y^* : \big(\underline{\Sigma}(\underline{\theta})^{-1} + \underline{D}^T\underline{\Sigma}_\varepsilon^{-1}(\underline{\theta})\underline{D}\big)^{-1}$$

$$\mu_Y^* = \underline{\Sigma}_Y^*\,\underline{D}^T\underline{\Sigma}_\varepsilon^{-1}(\underline{\theta})\underline{Z}$$

$$p(\sigma_\varepsilon^{-2} \mid \underline{Y}, \underline{Z}, \underline{\theta}^-) \propto p(\underline{Z} \mid \underline{Y}, \sigma_\varepsilon^{-2}, \underline{\theta}^-)\, p(\sigma_\varepsilon^{-2})$$

$$\underline{\theta}^- = \underline{\theta}^{\neq \sigma_\varepsilon^{-2}}$$

$$\propto \exp\Big(-\tfrac{\sigma_\varepsilon^{-2}}{2}(\underline{Z} - \underline{D}\,\underline{Y})^T\underline{\Sigma}_\varepsilon^{-1}(\underline{\theta}^-)(\underline{Z} - \underline{D}\,\underline{Y}) + (\alpha_\varepsilon - 1)\ln\sigma_\varepsilon^{-2} - \beta_\varepsilon\sigma_\varepsilon^{-2}\Big)$$

$$\exp\big(+\tfrac{n}{2}\ln\sigma_\varepsilon^{-2}\big)$$

$$\Rightarrow \alpha^* = \alpha_\varepsilon - 1 + n/2$$

$$\beta^* = \beta_\varepsilon + (\underline{Z} - \underline{D}\,\underline{Y})^T\underline{\Sigma}_\varepsilon^{-1}(\underline{\theta}^-)(\underline{Z} - \underline{D}\,\underline{Y})$$

$$p(\tilde{\rho}_\varepsilon \mid \underline{Y}, \underline{z}, \underline{\theta}) \propto p(\underline{z} \mid \underline{Y}, \underline{\theta})\, p(\tilde{\rho}_\varepsilon)$$

$$\propto \exp\left(-\tfrac{1}{2}(\underline{z} - \underline{D}\,\underline{Y})^T \underline{\Sigma}_\varepsilon(\underline{\theta})^{-1}(\underline{z} - \underline{D}\,\underline{Y})\right)$$

$$\exp\left(-\tfrac{1}{2}\ln|\underline{\Sigma}_\varepsilon(\underline{\theta})|\right)$$

$$\exp\left(-\tfrac{1}{2\sigma^2_{\rho_\varepsilon}}(\tilde{\rho}_\varepsilon - \mu_{\rho_\varepsilon})^2\right) \qquad \rho_\varepsilon = 2\Phi(\tilde{\rho}_\varepsilon) - 1$$

$$p(\sigma^2_{11} \mid \underline{Y}, \underline{z}, \underline{\theta}^-) \propto p(\underline{z} \mid \cancel{\underline{Y}}, \underline{\theta})\, p(\underline{Y} \mid \underline{\theta})\, p(\underline{\theta}^-)$$

$\underline{\theta}^- = \underline{\theta} / \sigma^2_{11}$

$$\propto p(\underline{Y} \mid \underline{\theta})\, p(\underline{\theta}^-)$$

$$\cancel{\propto \exp(} \propto p(\underline{Y}_2 \mid \cancel{\underline{Y}_1}, \underline{\theta})\, p(\underline{Y}_1 \mid \underline{\theta})\, p(\underline{\theta}^-)$$

$$\propto \exp\Big(-\tfrac{\sigma_{11}^{-2}}{2}\underline{Y}_1^T \underline{\Sigma}_{11}(\underline{\theta}^-)^{-1}\underline{Y}_1$$

$$+ \tfrac{n_1}{2}\ln\sigma_{11}^{-2} + (\alpha_{11} - 1)\ln\sigma_{11}^{-2} - \beta_{11}\sigma_{11}^{-2}\Big)$$

$$\Rightarrow \alpha_{11}^* = \alpha_{11} - 1 + n_1/2$$

$$\beta_{11}^* = \beta_{11} + \tfrac{1}{2}\underline{Y}_1^T \underline{\Sigma}_{11}(\underline{\theta}^-)^{-1}\underline{Y}_1$$

$$p(\sigma^2_{2|1} \mid \underline{Y}, \underline{z}, \underline{\theta}^-) \propto p(\underline{Y}_2 \mid \underline{Y}_1, \underline{\theta})\, p(\underline{\theta}^-)$$

$\underline{\theta}^- = \underline{\theta} / \sigma^2_{2|1}$

$$\propto \exp\Big(-\tfrac{\sigma_{2|1}^{-2}}{2}(\underline{Y}_2 - \underline{B}\,\underline{Y}_1)^T \underline{\Sigma}_{2|1}(\underline{\theta}^-)(\underline{Y}_2 - \underline{B}\,\underline{Y}_1)$$

$$+ \tfrac{n_2}{2}\ln\sigma_{2|1}^{-2} + (\alpha_{2|1} - 1)\ln\sigma_{2|1}^{-2} - \beta_{2|1}\sigma_{2|1}^{-2}\Big)$$

$$\Rightarrow \alpha_{2|1}^* = \alpha_{2|1} - 1 + n_2/2$$

$$\beta_{2|1}^* = \beta_{2|1} + \tfrac{1}{2}(\underline{Y}_2 - \underline{B}\,\underline{Y}_1)^T \underline{\Sigma}_{2|1}(\underline{\theta}^-)(\underline{Y}_2 - \underline{B}\,\underline{Y}_1)$$

$$p(A \mid \underline{Y}, \underline{z}, \underline{\theta}^-) \propto p(\underline{z} \mid \cancel{\underline{Y}_1}, \underline{\theta})\, p(\underline{Y}_2 \mid \underline{Y}_1, \underline{\theta})\, p(A)$$

$$\underline{\theta}^- \equiv \underline{\theta}^{/A}$$

$$\propto \exp\Big(-\tfrac{1}{2}\big(\underline{Y}_2 - A\underline{B}^-\underline{Y}_1\big)^T \underline{\Sigma}_{2|1}^{-1}(\underline{\theta})\big(\underline{Y}_2 - A\underline{B}^-\underline{Y}_1\big) - \frac{\sigma_A^{-2}}{2}(A - \mu_A)^2\Big)$$

$$\propto \exp\Big(-\tfrac{1}{2}A^2\big(\sigma_A^{-2} + \underline{Y}_1^T \underline{B}^{-T}\underline{\Sigma}_{2|1}^{-1}(\underline{\theta})\underline{B}^-\underline{Y}_1\big) + A\big(\underline{Y}_1^T\underline{B}^{-T}\underline{\Sigma}_{2|1}(\underline{\theta})\underline{Y}_2 + \sigma_A^{-2}\mu_A\big)\Big)$$

$$\Rightarrow \sigma_A^{-2*} = \sigma_A^{-2} + \underline{Y}_1^T\underline{B}^{-T}\underline{\Sigma}_{2|1}^{-1}(\underline{\theta})\underline{B}^-\underline{Y}_1$$

$$\mu_A^* = \sigma_A^{2*}\big(\underline{Y}_1^T\underline{B}^{-T}\underline{\Sigma}_{2|1}^{-1}(\underline{\theta})\underline{Y}_2 + \sigma_A^{-2}\mu_A\big)$$

$$p(\Delta, r \mid \underline{Y}, \underline{z}, \underline{\theta}^-) \propto \exp\Big(-\tfrac{1}{2}\big(\underline{Y}_2 - AB(\underline{\theta})\big)^T\underline{\Sigma}_{2|1}^{-1}(\underline{\theta})\big(\underline{Y}_2 - AB\underline{Y}_1\big) - \frac{1}{2\sigma_r^2}(\tilde{r} - \mu_r) - \frac{1}{2\Delta_1^2}(\Delta_1 - \mu_{\Delta_1}) - \frac{1}{2\Delta_2}(\Delta_2 - \mu_{\Delta_2})\Big) \quad ; r = e^{\tilde{r}}$$

$$\underline{\theta}^- \equiv \underline{\theta}^{/(\Delta, \tilde{r})}$$

$$p(K_{2|1} \mid \underline{Y}, \underline{z}, \underline{\theta}^-) \propto \exp\Big(-\tfrac{1}{2}(\underline{Y}_2 - A\underline{B})^T\underline{\Sigma}_{2|1}(\underline{\theta})(\underline{Y}_2 - A\underline{B}) - \frac{1}{2\sigma_{2|1}^2}(\tilde{K}_{2|1} - \mu_{2|1})^2 \quad ; K_{2|1} = e^{\tilde{K}_{2|1}}$$

$$\underline{\theta}^- \equiv \underline{\theta}^{/\tilde{K}_{2|1}}$$

$$p(K_{11} \mid \underline{Y}, \underline{z}, \underline{\theta}^-) \propto \exp\Big(-\tfrac{1}{2}\underline{Y}_1^T\underline{\Sigma}_{11}(\underline{\theta})^{-1}\underline{Y}_1 - \frac{1}{2\sigma_{K_{11}}^2}(\tilde{K}_{11} - \mu_{11})^2\Big)$$

$$\underline{\theta}^- \equiv \underline{\theta}^{/\tilde{K}_{11}}$$

$$K_{11} = e^{\tilde{K}_{11}}$$