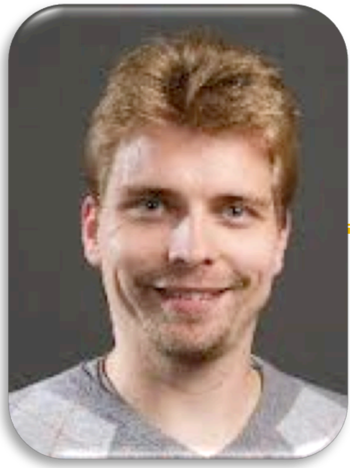

Computational Structures in Data Science

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Lecture 6: Environment Diagrams, Recursion Review, Midterm Review

March 4, 2019

http://inst.eecs.berkeley.edu/~cs88

On Computer Science Exams

In computer science exams, we try to assess the student's understanding of concepts and his or her ability to practically apply these.

- In CS, we do not:
  - require extensive memorization (e.g. we allow cheat sheet)
  - require a lot of reading
  - require essay writing skills

In CS, we do:
- require the ability to translate a given textual problem into programming code
- require you to be able to read other people's code
- value solutions that are almost right over no solution
- accept solutions we did not think about if they work
- prioritize math (logic) and science (experiment) over opinion or authority

How to prepare for a CS exam

- Explain the content of the computational concepts toolbox to somebody else
  - Describe the concept
  - What is an example of using it?
  - When does it not work? Corner cases?
  - Why does it exist?
- Practice programming:
  - Play around with the examples from lecture, lab, homework
  - Think about your own similar examples
- In the exam:
  - Make sure you understand the question: What is the given input? What is the required output?
  - Think of easy cases first (e.g. n=1?).
  - What is the iteration/recursion doing (e.g. i=i+1)?
  - What are corner cases that need explicit handling (e.g. division by zero, negative numbers, empty list)?

Computational Concepts Toolbox

- Data type: values, literals, operations,
  - e.g., int, float, string
- Expressions, Call expression
- Variables
- Assignment Statement
- Sequences: tuple, list
  - indexing
- Data structures
- Tuple assignment
- Call Expressions
- Function Definition Statement
- Conditional Statement
- Iteration:
  - data-driven (list comprehension)
  - control-driven (for statement)
  - while statement
- Higher Order Functions
  - Functions as Values
  - Functions with functions as argument
  - Assignment of function values
- Recursion
- Environment Diagrams

Recursion Key concepts – by example

1. Test for simple "base" case

2. Solution in simple "base" case

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

3. Assume recusive solution to simpler problem

4. Transform soln of simpler problem into full soln

In words

- The sum of no numbers is zero
- The sum of 1^2 through n^2 is the
  - sum of 1^2 through $(n-1)^2$
  - plus n^2

```
def sum_of_squares(n):
    if n < 1:
        return 0
    else:
        return sum_of_squares(n-1) + n**2
```

How does it work?

- Each recursive call gets its own local variables
  - Just like any other function call
- Computes its result (possibly using additional calls)
  - Just like any other function call
- Returns its result and returns control to its caller
  - Just like any other function call
- The function that is called happens to be itself
  - Called on a simpler problem
  - Eventually bottoms out on the simple base case

- Reason about correctness “by induction”
  - Solve a base case
  - Assuming a solution to a smaller problem, extend it

Local variables

```
def sum_of_squares(n):
    n_squared = n**2
    if n < 1:
        return 0
    else:
        return n_squared + sum_of_squares(n-1)
```

- Each call has its own “frame” of local variables
- What about globals?
- Let’s see the environment diagrams

https://goo.gl/CiFaUJ

Environments Example

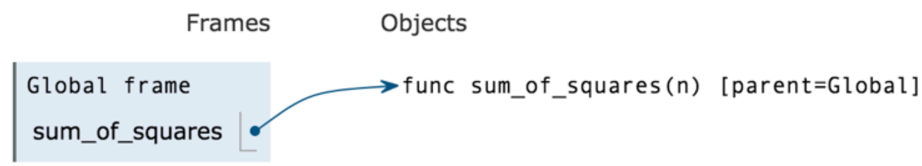
Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First   < Back   Step 2 of 17   Forward >   Last >>



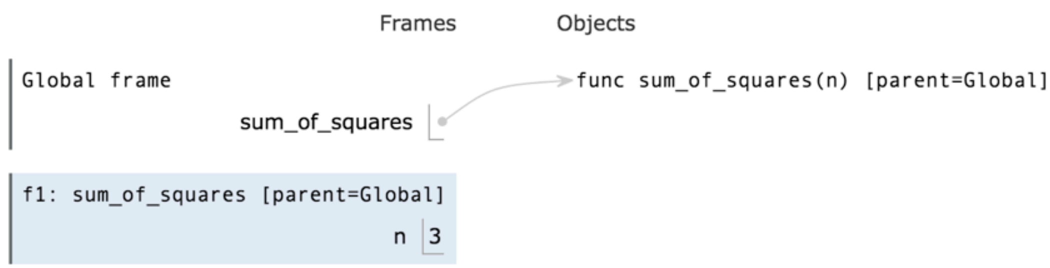
Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First   < Back   Step 3 of 17   Forward >   Last >>



pythontutor.com

Environments Example

Python 3.3

```python
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

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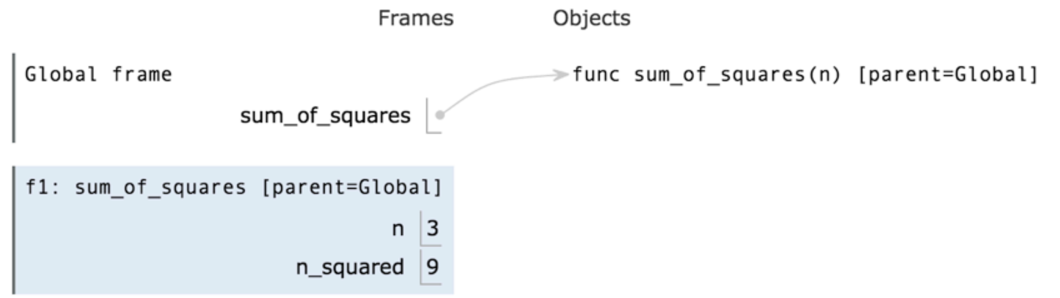
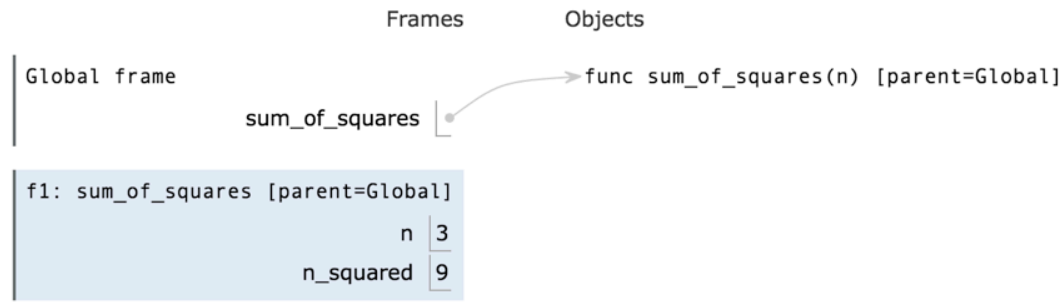


Python 3.3

```python
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```



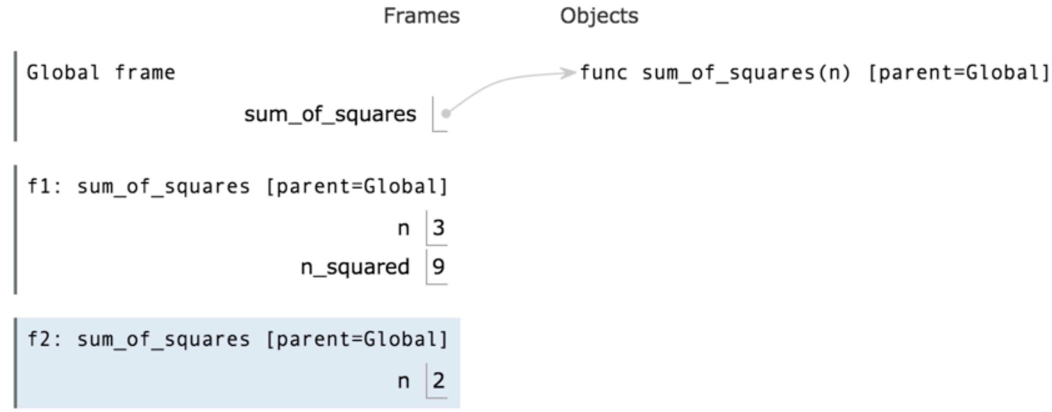
Environments Example

Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code



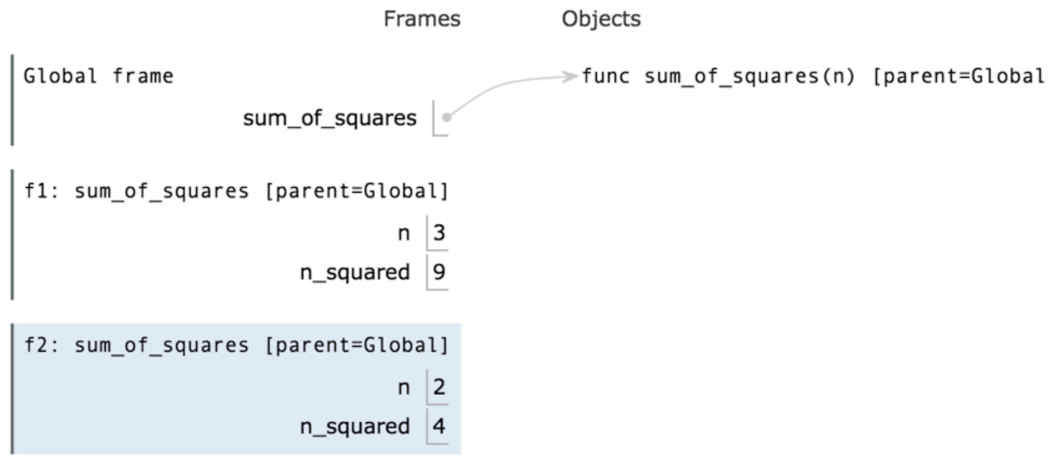
Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First | < Back | Step 9 of 17 | Forward > | Last >>



Environments Example

Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First | < Back | Step 10 of 17 | Forward > | Last >>

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n | 3
n_squared | 9

f2: sum_of_squares [parent=Global]
n | 2
n_squared | 4

Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First | < Back | Step 11 of 17 | Forward > | Last >>

that has just executed
: line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]
n | 3
n_squared | 9

f2: sum_of_squares [parent=Global]
n | 2
n_squared | 4

f3: sum_of_squares [parent=Global]
n | 1

Environments Example

Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First | < Back | Step 13 of 17 | Forward > | Last >>

that has just executed
t line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]

n	3
n_squared	9

f2: sum_of_squares [parent=Global]

n	2
n_squared	4

f3: sum_of_squares [parent=Global]

n	1
n_squared	1

Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First | < Back | Step 14 of 17 | Forward > | Last >>

that has just executed
t line to execute

Frames | Objects

Global frame
sum_of_squares → func sum_of_squares(n) [parent=Global]

f1: sum_of_squares [parent=Global]

n	3
n_squared	9

f2: sum_of_squares [parent=Global]

n	2
n_squared	4

f3: sum_of_squares [parent=Global]

n	1
n_squared	1

Environments Example

Python 3.3

```python
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

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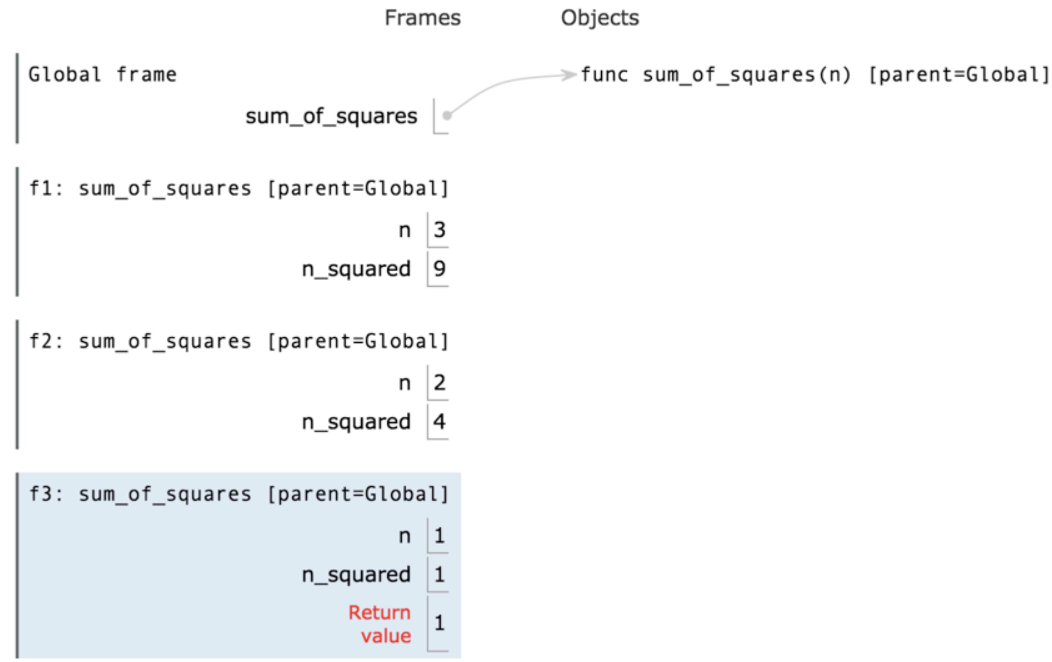
e that has just executed

xt line to execute



Environments Example

Python 3.3

```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First < Back Step 16 of 17 Forward > Last >>

: that has just executed

:t line to execute

Environments Example

Python 3.3

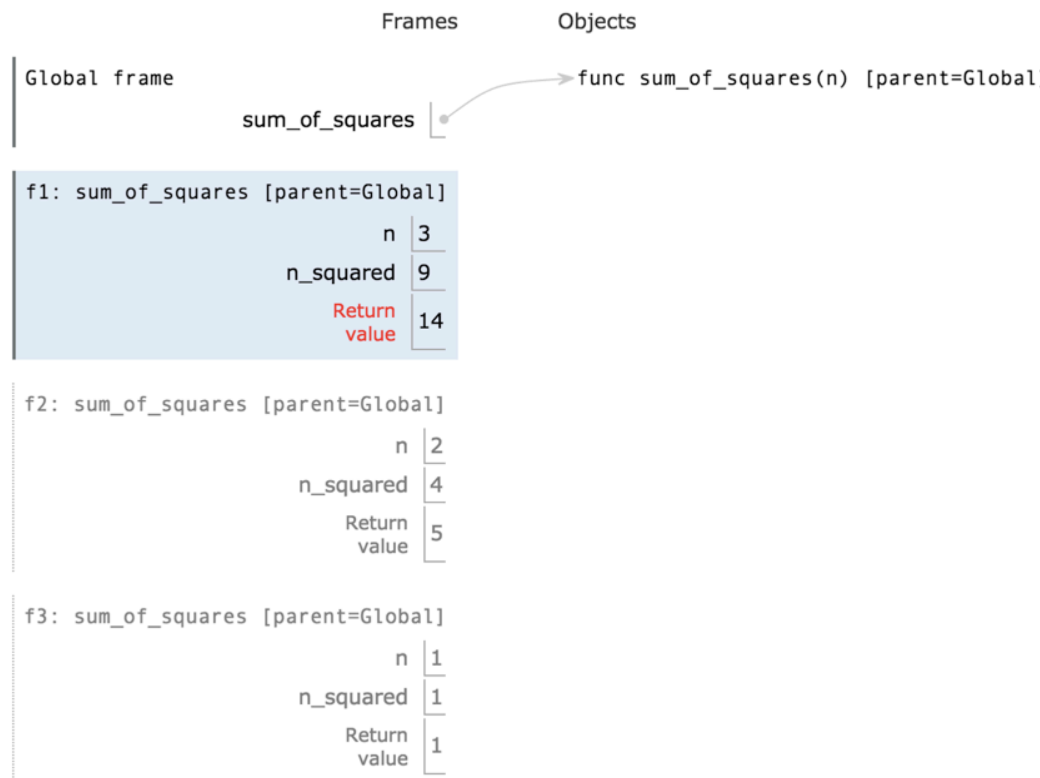
```
def sum_of_squares(n):
    n_squared = n**2
    if n == 1:
        return 1
    else:
        return n_squared + sum_of_squares(n-1)

sum_of_squares(3)
```

Edit code

<< First < Back Step 17 of 17 Forward > Last >>

e that has just executed
xt line to execute



How much ???

- Time is required to compute `sum_of_squares(n)`?
  - Recursively?
  - Iteratively ?
- Space is required to compute `sum_of_squares(n)`?
  - Recursively?
  - Iteratively ?

Linear
proportional to n
cn for some c

- Count the frames…
- Recursive is linear, iterative is constant!

Tail Recursion

- All the work happens on the way down the recursion
- On the way back up, just return

```
def sum_up_squares(i, n, accum):
    """Sum the squares from i to n in incr. order"""
    if i > n:
        Base Case
    else:
        Tail Recursive Case


>>> sum_up_squares(1,3,0)
14
```

Tree Recursion

- **Break the problem into multiple smaller sub-problems, and Solve them recursively**

```
def split(x, s):
    return [i for i in s if i <= x], [i for i in s if i > x]

def qsort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""
    if not s:
        return []
    else:
        pivot = first(s)
        lessor, more = split(pivot, rest(s))
        return qsort(lessor) + [pivot] + qsort(more)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

QuickSort Example

```
[3, 3, 1, 4, 5, 4, 3, 2, 1, 17]
[3, 1, 3, 2, 1]        [4, 5, 4, 17]
[1, 3, 2, 1]   []      [4]   [5, 17]
[1]   [3, 2]           [] []   []   [17]
[] []   [2]   []       [4]     [] []
[1]   [] []            [5, 17]
[2, 3]                 [4, 4, 5, 17]
[1, 1, 2, 3]
[1, 1, 2, 3, 3]
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

Tree Recursion with HOF

```
def qsort(s):
    """Sort a sequence - split it by the first element,
    sort both parts and put them back together."""

    if not s:
        return []
    else:
        pivot = first(s)
        lessor, more = split_fun(leq_maker(pivot), rest(s))
        return qsort(lessor) + [pivot] + qsort(more)

>>> qsort([3,3,1,4,5,4,3,2,1,17])
[1, 1, 2, 3, 3, 3, 4, 4, 5, 17]
```

Computational Concepts Toolbox

- Data type: values, literals, operations,
  - e.g., int, float, string
- Expressions, Call expression
- Variables
- Assignment Statement
- Sequences: tuple, list
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Answers for the Wandering Mind

The computer choses a random element *x* of the list generated by `range(0,n)`. What is the <u>smallest amount</u> of iteration/recursion steps the best algorithm needs to guess *x?*

$\log_2 n$

How would the algorithm look like?

Guess the binary digits of x starting with the highest significant digit. This is, ask questions of the form
"smaller than 2^{n-1}?" (yes => 0…),
"smaller than 2^{n-2}?" (no   => 0 1…),
"smaller than $2^{n-2}+2^{n-3}$?", …

This method is also called: binary search

Quantum physics: Allow less than $\log_2 n$ guesses.