```
#############################################################################################
#################################### Problem 2 ##############################################
#############################################################################################
Converting the metric to sp.Matrix object...
Inverting the matrix...
Succesfully inverted the matrix.
Calculating Christoffel symbols...
Done.
Calculating Riemann tensor...
Done.
Calculating Ricci tensor...
Done.
Calculating Ricci scalar...
Done.
Calculating Einstein tensor...
Done.
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
Gamma ANALYSIS
	Gamma[ttr] = \frac{0.5 R}{r \left(- R + r\right)}
	Gamma[trt] = \frac{0.5 R}{r \left(- R + r\right)}
	Gamma[rtt] = \frac{0.5 R \left(R - r\right)}{r^{3}}
	Gamma[rrr] = \frac{0.5 R}{r \left(R - r\right)}
	Gamma[rthetatheta] = 1.0 R - 1.0 r
	Gamma[rphiphi] = \left(R - 1.0 r\right) \sin^{2}{\left (\theta \right )}
	Gamma[thetartheta] = \frac{1.0}{r}
	Gamma[thetathetar] = \frac{1.0}{r}
	Gamma[thetaphiphi] = - 0.5 \sin{\left (2 \theta \right )}
	Gamma[phirphi] = \frac{1.0}{r}
	Gamma[phithetaphi] = \frac{1.0}{\tan{\left (\theta \right )}}
	Gamma[phiphir] = \frac{1.0}{r}
	Gamma[phiphitheta] = \frac{1.0}{\tan{\left (\theta \right )}}
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
riemann ANALYSIS
	riemann[trtr] = \frac{1.0 R}{r^{2} \left(- R + r\right)}
	riemann[trrt] = \frac{1.0 R}{r^{2} \left(R - r\right)}
	riemann[tthetattheta] = - \frac{0.5 R}{r}
	riemann[tthetathetat] = \frac{0.5 R}{r}
	riemann[tphitphi] = - \frac{0.5 R \sin^{2}{\left (\theta \right )}}{r}
	riemann[tphiphit] = \frac{0.5 R \sin^{2}{\left (\theta \right )}}{r}
	riemann[rttr] = \frac{1.0 R \left(R - r\right)}{r^{4}}
	riemann[rtrt] = \frac{1.0 R \left(- R + r\right)}{r^{4}}
	riemann[rthetartheta] = - \frac{0.5 R}{r}
	riemann[rthetathetar] = \frac{0.5 R}{r}
	riemann[rphirphi] = - \frac{0.5 R \sin^{2}{\left (\theta \right )}}{r}
	riemann[rphiphir] = \frac{0.5 R \sin^{2}{\left (\theta \right )}}{r}
	riemann[thetatttheta] = \frac{0.5 R \left(- R + r\right)}{r^{4}}
	riemann[thetatthetat] = \frac{0.5 R \left(R - r\right)}{r^{4}}
	riemann[thetarrtheta] = \frac{0.5 R}{r^{2} \left(- R + r\right)}
	riemann[thetarthetar] = \frac{0.5 R}{r^{2} \left(R - r\right)}
	riemann[thetaphithetaphi] = \frac{1.0 R \sin^{2}{\left (\theta \right )}}{r}
	riemann[thetaphiphitheta] = - \frac{1.0 R \sin^{2}{\left (\theta \right )}}{r}
	riemann[phittphi] = \frac{0.5 R \left(- R + r\right)}{r^{4}}
	riemann[phitphit] = \frac{0.5 R \left(R - r\right)}{r^{4}}
	riemann[phirrphi] = \frac{0.5 R}{r^{2} \left(- R + r\right)}
	riemann[phirphir] = \frac{0.5 R}{r^{2} \left(R - r\right)}
	riemann[phithetathetaphi] = - \frac{1.0 R}{r}
	riemann[phithetaphitheta] = \frac{1.0 R}{r}
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
ricci_tensor ANALYSIS
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
ricci_scalar ANALYSIS
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
einstein ANALYSIS
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
Converting the metric to sp.Matrix object...
Inverting the matrix...
Succesfully inverted the matrix.
Calculating Christoffel symbols...
```

```
Done.
Calculating Riemann tensor...
Done.
Calculating Ricci tensor...
Done.
Calculating Ricci scalar...
Done.
Calculating Einstein tensor...
Done.
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
Gamma ANALYSIS
	Gamma[txx] = 1.0 H e^{2 H t}
	Gamma[tyy] = 1.0 H e^{2 H t}
	Gamma[tzz] = 1.0 H e^{2 H t}
	Gamma[xtx] = 1.0 H
	Gamma[xxt] = 1.0 H
	Gamma[yty] = 1.0 H
	Gamma[yyt] = 1.0 H
	Gamma[ztz] = 1.0 H
	Gamma[zzt] = 1.0 H
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
riemann ANALYSIS
	riemann[txtx] = 1.0 H^{2} e^{2 H t}
	riemann[txxt] = - 1.0 H^{2} e^{2 H t}
	riemann[tyty] = 1.0 H^{2} e^{2 H t}
	riemann[tyyt] = - 1.0 H^{2} e^{2 H t}
	riemann[tztz] = 1.0 H^{2} e^{2 H t}
	riemann[tzzt] = - 1.0 H^{2} e^{2 H t}
	riemann[xttx] = 1.0 H^{2}
	riemann[xtxt] = - 1.0 H^{2}
	riemann[xyxy] = 1.0 H^{2} e^{2 H t}
	riemann[xyyx] = - 1.0 H^{2} e^{2 H t}
	riemann[xzxz] = 1.0 H^{2} e^{2 H t}
	riemann[xzzx] = - 1.0 H^{2} e^{2 H t}
	riemann[ytty] = 1.0 H^{2}
	riemann[ytyt] = - 1.0 H^{2}
	riemann[yxxy] = - 1.0 H^{2} e^{2 H t}
	riemann[yxyx] = 1.0 H^{2} e^{2 H t}
	riemann[yzyz] = 1.0 H^{2} e^{2 H t}
	riemann[yzzy] = - 1.0 H^{2} e^{2 H t}
	riemann[zttz] = 1.0 H^{2}
	riemann[ztzt] = - 1.0 H^{2}
	riemann[zxxz] = - 1.0 H^{2} e^{2 H t}
	riemann[zxzx] = 1.0 H^{2} e^{2 H t}
	riemann[zyyz] = - 1.0 H^{2} e^{2 H t}
	riemann[zyzy] = 1.0 H^{2} e^{2 H t}
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
ricci_tensor ANALYSIS
	ricci_tensor[tt] = - 3.0 H^{2}
	ricci_tensor[xx] = 3.0 H^{2} e^{2 H t}
	ricci_tensor[yy] = 3.0 H^{2} e^{2 H t}
	ricci_tensor[zz] = 3.0 H^{2} e^{2 H t}
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
ricci_scalar ANALYSIS
	ricci_scalar[] = 12.0 H^{2}
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
-----------------------------------------------------
SHOWING ONLY NONZERO COMPONENTS...
einstein ANALYSIS
	einstein[tt] = 3.0 H^{2}
	einstein[xx] = - 3.0 H^{2} e^{2 H t}
	einstein[yy] = - 3.0 H^{2} e^{2 H t}
	einstein[zz] = - 3.0 H^{2} e^{2 H t}
ALL OTHER COMPONENTS ARE ZERO.
-----------------------------------------------------
For de Sitter space, the product (g^mn)*(G_mn) is -12.0*H**2.
############################################################################################
###################################### Problem 3 ###########################################
############################################################################################
TO WRITE THE INVERTED KERR METRIC TO A FILE, UNCOMMENT THE LINES BELOW HERE.
Calculating Christoffel symbols...
```

```
Done.
Calculating Riemann tensor...
Done.
Calculating Ricci tensor...
Done.
Calculating Ricci scalar...
Done.
Calculating Einstein tensor...
Done.
SUPPRESSING PRINTING FOR THIS PROBLEM AS THE KERR METRIC LEADS TO VERY LARGE EXPRESSIONS.
UNCOMMENT `analysis.print_results(kerr_analysis)` IF REQUIRED.
Evaluating the Einstein tensor at this point:
	t = 0
	r = 0.5
	theta = 1.0471975511965976
	phi = 0.7853981633974483
	a = 100
	M = 0.1
	G = 1
The result is...
	G[tt] = -3.66086978058819E-19
	G[tr] = 0
	G[ttheta] = 0
	G[tphi] = -3.59989002583078E-21
	G[rt] = 0
	G[rr] = 1.82977728170711E-19
	G[rtheta] = 6.36968776335234E-19
	G[rphi] = 0
	G[thetat] = 0
	G[thetar] = 6.86951992029363E-19
	G[thetatheta] = -1.67490939737350E-15
	G[thetaphi] = 0
	G[phit] = -3.59989002583078E-21
	G[phir] = 0
	G[phitheta] = 0
	G[phiphi] = 1.53700565729692E-15
#############################################################################################
###################################### Problem 4 ############################################
#############################################################################################
Converting the metric to sp.Matrix object...
Inverting the matrix...
Succesfully inverted the matrix.
Calculating Christoffel symbols...
Done.
Calculating Riemann tensor...
Done.
Calculating Ricci tensor...
Done.
Calculating Ricci scalar...
Done.
Calculating Einstein tensor...
Done.
Converting the metric to sp.Matrix object...
Inverting the matrix...
Succesfully inverted the matrix.
\frac{d^{2}}{d y^{2}} r{\left (y \right )} = 0
\left(- \frac{R}{r{\left (y \right )}} + 1.0\right) \left(\frac{d}{d y} t{\left (y \right )}\right)^{2} + 1.0
r^{2}{\left (y \right )} \left(\frac{d}{d y} \phi{\left (y \right )}\right)^{2} + \frac{\left(\frac{d}{d y}
r{\left (y \right )}\right)^{2}}{- \frac{R}{r{\left (y \right )}} + 1.0} = -1.0
The solution for $\Big{(}\frac{dr}{dy}\Big{)}^2$ is:
\left [ - E^{2} + \frac{L^{2} R}{r^{3}{\left (y \right )}} - \frac{L^{2}}{r^{2}{\left (y \right )}} + \frac{R}
{r{\left (y \right )}} - 1.0\right ]
```