Bootstrap and flavour spectroscopy

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Introduction

- Physics motivation:
  - Heavy quark meson sector contains a lot of interesting physics
  - Potentially enrich our knowledge of flavour physics: Nature
  - Experiments are dedicated to measure these effects: Belle(II), LHCb
  - Development of phenomenological models to describe mass spectra measurements
- Results motivation:
  - Improve the error treatment with statistical methods
  - The correct error treatment could guide future research
  - These errors could help to improve the phenomenological model itself
- Strategy:
  - Perform a parameter fitting using least minimum squares
  - Perform a statistical sampling to determine the statistics of the parameters

Changes w.r.t. last presentation

- Computation of the masses and errors directly from the bootstrap sample
- Formal statistic test were performed to ensure the normality of the simulated masses
- Cross-check of the parameter computation using linear algebra
- Fits were performed in several states groups: all, omegas, cascades, sigmas and lambdas
- Inclusion of asymmetric errors, using the quantile technique

Goal: determine the mass splitting parameters A, B, E, G by fitting the model Hamiltonian (plus the ω from the harmonic oscillator):

$$H = H_{\mathrm{h.o.}} + A\mathbf{S}^2 + B\mathbf{S}\cdot\mathbf{L} + E\mathbf{I}^2 + G\mathbf{C_2}(\mathrm{SU}(3)_{\mathrm{f}}) \tag{1}$$

Mass eigenstates $|ssc, S_\rho, S_{tot}, I_\rho, I_\lambda, J\rangle$ follow:

$$H|ssc, S_\rho, S_{tot}, I_\rho, I_\lambda, J\rangle = m|ssc, S_\rho, S_{tot}, I_\rho, I_\lambda, J\rangle \tag{2}$$

where m is the measured (predicted) meson masses.
`https://doi.org/10.1140/epjc/s10052-019-7527-4`

Strategy: bootstrap

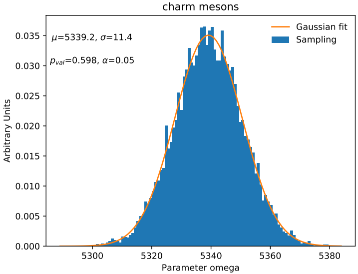
- Use experimental masses and errors to build a Gaussian distribution, with $\mu =$mass and $\sigma =$Error$_{exp}$ for each observed mass
- Randomly sample the Gaussian distribution to obtained a re-sampled mass spectrum
- With the re-sampled experimental masses perform a least minimum squares fit to fit the predicted/modelled masses:

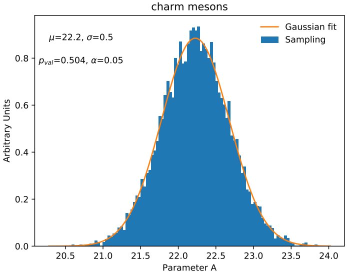
$$d^2 = \sum_i (m_{sampled} - m_{predicted})^2 \tag{3}$$

- Iterate the previous steps to produce a bootstrap sample and obtain smooth Gaussian statistics (μ, σ)
- Inspired on Molina *et.al*. `https://arxiv.org/abs/2001.05408`

Results: sampling distributions

A p_{value} was obtained to formally test the null hypothesis of data being Gaussian distributed, with a $\alpha = 0.05$





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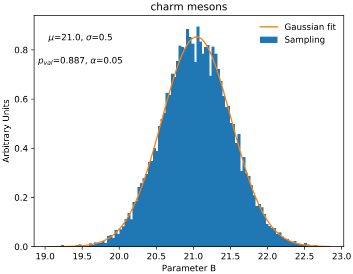

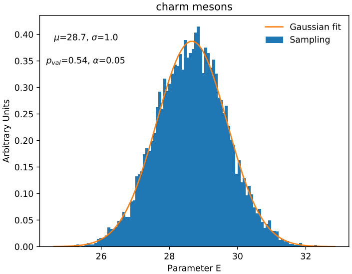

Results: sampling distributions

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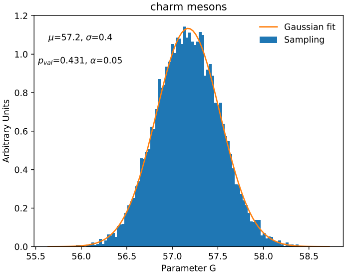
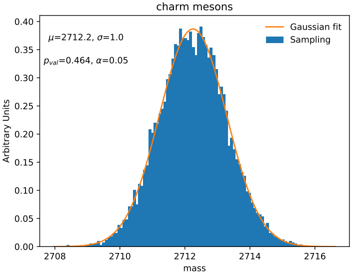




Results: parameters

	K	A	B	E	G
Paper	5727.12$\pm x.xx$	21.54$\pm$0.37	23.91$\pm$0.31	30.34 $\pm$0.23	54.37$\pm$0.58
Sampled	5339.2 $\pm$11.38	22.26 $\pm$0.46	21.07 $\pm$0.47	28.62 $\pm$1.02	57.16 $\pm$0.36
L.Algebra	5339.1	22.24	21.05	28.63	57.17

Table: Model pararameters in MeV, for states: *All*

	K	A	B	E	G
Paper	5727.12$\pm x.xx$	21.54$\pm$0.37	23.91$\pm$0.31	30.34 $\pm$0.23	54.37$\pm$0.58
Sampled	5995.1 $\pm$26.74	26.79 $\pm$0.48	31.92 $\pm$0.52	0.0 $\pm$0.0	49.56 $\pm$0.54
L.Algebra	5994.5	26.8	31.93	0.0	49.56

Table: Model pararameters in MeV, for states: *omega*

Results: parameters

	K	A	B	E	G
Paper	5727.12$\pm x.xx$	21.54$\pm$0.37	23.91$\pm$0.31	30.34 $\pm$0.23	54.37$\pm$0.58
Sampled	5463.8 $\pm$30.43	20.99 $\pm$0.7	23.82 $\pm$0.73	34.87 $\pm$4.75	56.76 $\pm$1.11
L.Algebra	5462.3	21.0	23.82	35.04	56.73

Table: Model pararameters in MeV, for states: *cascades*

	K	A	B	E	G
Paper	5727.12$\pm x.xx$	21.54$\pm$0.37	23.91$\pm$0.31	30.34 $\pm$0.23	54.37$\pm$0.58
Sampled	5013.3 $\pm$30.7	17.93 $\pm$0.92	14.93 $\pm$1.06	43.19 $\pm$2.5	50.81 $\pm$0.99
L.Algebra	5013.6	17.93	14.92	43.23	50.8

Table: Model pararameters in MeV, for states: $sigma_lamb$

Masses, asymmetric errors calculated via 68% quantile method

Mass State	Experiment (MeV)	Predicted mass old (MeV)	Predicted mass sampled (MeV)	diff pred (%)	diff sampl (%)
$\vert ssc, 1/2, 1/2, 0, 0, 10/3\rangle$	2695.0 ± 2.0	2702.4 $\pm xx$	2712.2 $^{+1.0}_{-1.0}$	-7.4 (0.3)	-17.2 (0.6)
$\vert ssc, 3/2, 3/2, 0, 0, 10/3\rangle$	2766.0 ± 2.0	2767.0 $\pm xx$	2779.0 $^{+1.4}_{-1.4}$	-1.0 (0.0)	-13.0 (0.5)
$\vert ssc, 1/2, 1/2, 1, 0, 10/3\rangle$	3000.4 ± 0.4	3015.8 $\pm xx$	3005.6 $^{+1.0}_{-1.0}$	-15.4 (0.5)	-5.2 (0.2)
$\vert ssc, 1/2, 3/2, 1, 0, 10/3\rangle$	3050.2 ± 0.3	3044.5 $\pm xx$	3040.8 $^{+0.4}_{-0.4}$	5.7 (0.2)	9.4 (0.3)
$\vert ssc, 3/2, 1/2, 1, 0, 10/3\rangle$	3065.6 ± 0.4	3051.6 $\pm xx$	3037.2 $^{+0.6}_{-0.5}$	14.0 (0.5)	28.4 (0.9)
$\vert ssc, 1/2, 1/2, 0, 0, 10/3\rangle$	3090.2 ± 0.7	3080.4 $\pm xx$	3072.4 $^{+0.6}_{-0.6}$	9.8 (0.3)	17.8 (0.6)
$\vert suc, 1/2, 1/2, 0, 1/2, 10/3\rangle$	2578.0 ± 2.9	2570.1 $\pm xx$	2578.7 $^{+1.0}_{-1.1}$	7.9 (0.3)	-0.7 (0.0)
$\vert suc, 3/2, 3/2, 0, 1/2, 10/3\rangle$	2645.9 ± 0.6	2634.8 $\pm xx$	2645.5 $^{+0.9}_{-0.9}$	11.1 (0.4)	0.4 (0.0)
$\vert suc, 1/2, 3/2, 1, 1/2, 10/3\rangle$	2923.0 ± 0.4	2934.1 $\pm xx$	2927.5 $^{+1.0}_{-1.0}$	-11.1 (0.4)	-4.5 (0.2)
$\vert suc, 3/2, 1/2, 1, 1/2, 10/3\rangle$	2938.5 ± 0.3	2941.2 $\pm xx$	2924.0 $^{+0.7}_{-0.8}$	-2.7 (0.1)	14.5 (0.5)
$\vert suc, 3/2, 3/2, 1, 1/2, 10/3\rangle$	2964.9 ± 0.3	2969.9 $\pm xx$	2959.1 $^{+0.6}_{-0.6}$	-5.0 (0.2)	5.8 (0.2)
$\vert uuc, 1/2, 1/2, 0, 1, 10/3\rangle$	2453.9 ± 0.1	2453.1 $\pm xx$	2459.5 $^{+1.9}_{-2.0}$	0.8 (0.0)	-5.6 (0.2)
$\vert uuc, 3/2, 3/2, 0, 1, 10/3\rangle$	2518.0 ± 2.3	2517.7 $\pm xx$	2526.3 $^{+1.3}_{-1.4}$	0.3 (0.0)	-8.3 (0.3)
$\vert uuc, 1/2, 1/2, 1, 1, 10/3\rangle$	2801.0 ± 6.0	2819.0 $\pm xx$	2802.0 $^{+2.5}_{-2.6}$	-18.0 (0.6)	-1.0 (0.0)
$\vert udc, 1/2, 1/2, 0, 0, 4/3\rangle$	2286.5 ± 0.1	2283.7 $\pm xx$	2287.9 $^{+0.4}_{-0.4}$	2.8 (0.1)	-1.4 (0.1)
$\vert udc, 1/2, 1/2, 1, 0, 10/3\rangle$	2592.3 ± 0.4	2649.7 $\pm xx$	2630.4 $^{+0.8}_{-0.7}$	-57.4 (2.2)	-38.1 (1.5)
$\vert udc, 3/2, 1/2, 1, 0, 4/3\rangle$	2625.0 ± 0.2	2685.6 $\pm xx$	2662.0 $^{+0.4}_{-0.4}$	-60.6 (2.3)	-37.0 (1.4)
$\vert ssc, 1/2, 1/2, 0, 1/2, 4/3\rangle$	2469.0 ± 4.0	2461.2 $\pm xx$	2464.4 $^{+0.6}_{-0.6}$	7.8 (0.3)	4.6 (0.2)
$\vert ssc, 1/2, 1/2, 1, 1/2, 4/3\rangle$	2792.0 ± 3.3	2796.5 $\pm xx$	2778.0 $^{+1.3}_{-1.3}$	-4.5 (0.2)	14.0 (0.5)
$\vert ssc, 3/2, 1/2, 1, 1/2, 10/3\rangle$	2815.0 ± 0.2	2832.4 $\pm xx$	2809.7 $^{+0.8}_{-0.8}$	-17.4 (0.6)	5.3 (0.2)
			Total diff	261.0	232.2

Masses, asymmetric errors calculated via 68% quantile method

Mass State	Experiment (MeV)	Predicted mass old (MeV)	Predicted mass sampled (MeV)	diff pred (%)	diff sampl (%)
$\lvert ssc, 1/2, 1/2, 0, 0, 10/3\rangle$	2695.0 ± 2.0	2702.4 $\pm xx$	2690.3 $^{+1.6}_{-1.6}$	-7.4 (0.3)	4.7 (0.2)
$\lvert ssc, 3/2, 3/2, 0, 0, 10/3\rangle$	2766.0 ± 2.0	2767.0 $\pm xx$	2770.6 $^{+1.6}_{-1.6}$	-1.0 (0.0)	-4.6 (0.2)
$\lvert ssc, 1/2, 1/2, 1, 0, 10/3\rangle$	3000.4 ± 0.4	3015.8 $\pm xx$	3011.4 $^{+0.7}_{-0.8}$	-15.4 (0.5)	-11.0 (0.4)
$\lvert ssc, 1/2, 3/2, 1, 0, 10/3\rangle$	3050.2 ± 0.3	3044.5 $\pm xx$	3043.9 $^{+0.3}_{-0.3}$	5.7 (0.2)	6.3 (0.2)
$\lvert ssc, 3/2, 1/2, 1, 0, 10/3\rangle$	3065.6 ± 0.4	3051.6 $\pm xx$	3059.3 $^{+0.4}_{-0.4}$	14.0 (0.5)	6.3 (0.2)
$\lvert ssc, 1/2, 1/2, 0, 0, 10/3\rangle$	3090.2 ± 0.7	3080.4 $\pm xx$	3091.8 $^{+0.8}_{-0.8}$	9.8 (0.3)	-1.6 (0.1)
			Total diff	53.0	34.6

Table: Every quantity is in MeV, except for percentage differences. States: omega

Masses, asymmetric errors calculated via 68% quantile method

Mass State	Experiment (MeV)	Predicted mass old (MeV)	Predicted mass sampled (MeV)	diff pred (%)	diff sampl (%)
$\vert suc, 1/2, 1/2, 0, 1/2, 10/3\rangle$	2578.0 $\pm$2.9	2570.1 $\pm xx$	2581.1 $^{+2.2}_{-2.2}$	7.9 (0.3)	-3.1 (0.1)
$\vert suc, 3/2, 3/2, 0, 1/2, 10/3\rangle$	2645.9 $\pm$0.6	2634.8 $\pm xx$	2644.0 $^{+1.3}_{-1.2}$	11.1 (0.4)	1.9 (0.1)
$\vert suc, 1/2, 3/2, 1, 1/2, 10/3\rangle$	2923.0 $\pm$0.4	2934.1 $\pm xx$	2927.0 $^{+0.9}_{-0.9}$	-11.1 (0.4)	-4.0 (0.1)
$\vert suc, 3/2, 1/2, 1, 1/2, 10/3\rangle$	2938.5 $\pm$0.3	2941.2 $\pm xx$	2935.5 $^{+1.2}_{-1.2}$	-2.7 (0.1)	3.0 (0.1)
$\vert suc, 3/2, 3/2, 1, 1/2, 10/3\rangle$	2964.9 $\pm$0.3	2969.9 $\pm xx$	2962.8 $^{+0.7}_{-0.7}$	-5.0 (0.2)	2.1 (0.1)
$\vert ssc, 1/2, 1/2, 0, 1/2, 4/3\rangle$	2469.0 $\pm$4.0	2461.2 $\pm xx$	2467.6 $^{+2.3}_{-2.5}$	7.8 (0.3)	1.4 (0.1)
$\vert ssc, 1/2, 1/2, 1, 1/2, 4/3\rangle$	2792.0 $\pm$3.3	2796.5 $\pm xx$	2786.3 $^{+1.8}_{-1.8}$	-4.5 (0.2)	5.7 (0.2)
$\vert ssc, 3/2, 1/2, 1, 1/2, 10/3\rangle$	2815.0 $\pm$0.2	2832.4 $\pm xx$	2822.0 $^{+1.1}_{-1.2}$	-17.4 (0.6)	-7.0 (0.2)
			Total diff	68.0	28.2

Table: Every quantity is in MeV, except for percentage differences. States: cascades

Masses, asymmetric errors calculated via 68% quantile method

Mass State	Experiment (MeV)	Predicted mass old (MeV)	Predicted mass sampled (MeV)	diff pred (%)	diff sampl (%)
$\|uuc, 1/2, 1/2, 0, 1, 10/3\rangle$	2453.9 ± 0.1	2453.1 $\pm xx$	2464.2 $^{+1.6}_{-1.6}$	0.8 (0.0)	-10.3 (0.4)
$\|uuc, 3/2, 3/2, 0, 1, 10/3\rangle$	2518.0 ± 2.3	2517.7 $\pm xx$	2518.0 $^{+2.2}_{-2.3}$	0.3 (0.0)	0.0 (0.0)
$\|uuc, 1/2, 1/2, 1, 1, 10/3\rangle$	2801.0 ± 6.0	2819.0 $\pm xx$	2790.6 $^{+4.8}_{-4.8}$	-18.0 (0.6)	10.4 (0.4)
$\|udc, 1/2, 1/2, 0, 0, 4/3\rangle$	2286.5 ± 0.1	2283.7 $\pm xx$	2276.2 $^{+1.6}_{-1.6}$	2.8 (0.1)	10.3 (0.5)
$\|udc, 1/2, 1/2, 1, 0, 10/3\rangle$	2592.3 ± 0.4	2649.7 $\pm xx$	2602.6 $^{+1.6}_{-1.6}$	-57.4 (2.2)	-10.3 (0.4)
$\|udc, 3/2, 1/2, 1, 0, 4/3\rangle$	2625.0 ± 0.2	2685.6 $\pm xx$	2625.0 $^{+0.2}_{-0.2}$	-60.6 (2.3)	-0.0 (0.0)
			Total diff	140.0	41.3

Table: Every quantity is in MeV, except for percentage differences. States: sigma$_{l}amb$

Results: summary

	K	A	B	E	G
K	1				
A	0.24	1			
B	0.43	0.46	1		
E	0.1	0.1	-0.03	1	
G	-0.53	-0.71	-0.23	-0.49	1

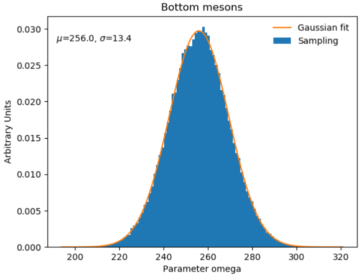
Table: Correlation matrix for the parameters

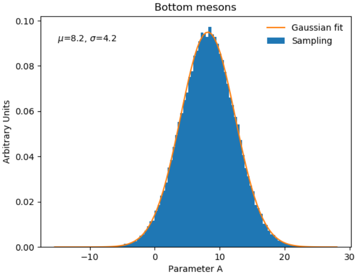
Summary and future work

- The bootstrap method was implemented successfully
- Results look promising
- Compute asymmetric uncertainties
- More sampling statistical methods could be applied for comparison
- Code is found on GitHub:
  `https://github.com/andrex-naranjas/flavor_phys`

Thanks for listening!

Results: sampling distributions





Results: sampling distributions