Econometrics–2

1k MMAE 2010–2011

Seminar 22
Nonlinear Least Squares

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Nonlinear Least Squares: "nl" in Stata

$$y = g(x,\beta) + \epsilon$$
$$x - n \times k,\ y - n \times 1,\ \beta - l \times 1.$$
$$RSS = \sum_{i=1}^{n} \left(y_i - g(x_i,\hat{\beta})\right)^2$$

F.O.C.

$$\frac{1}{n}\sum_{i=1}^{n} \left(y_i - g(x_i,\hat{\beta})\right) g_\beta(x_i,\hat{\beta}) = 0$$

parameter estimates computed using numerical optimization method. $g_\beta(x_i,\hat{\beta})$ – quasi-regressor

The estimated covariance matrix:

$$\widehat{Var(\hat{\beta})} = s^2 \left(g_\beta(x,\hat{\beta})' g_\beta(x,\hat{\beta})\right)^{-1}$$

Concentration method

Suppose $g(x,\beta) = z(c_2)c_1$. $z(c_2)$ depends only on x and c_2 (c_2 should be low dimensional: 1 or 2 parameters).

Algorithm:

1. Choose a grid $\left[\underline{c_2}, \overline{c_2}\right]$ for c_2.
2. For each c_2 on this grid compute $\hat{c}_1(c_2)$

$$\hat{\varepsilon}_i = y_i - z_i(c_2)\hat{c}_1(c_2)$$
$$\hat{\sigma}^2(c_2) = \frac{1}{n}\sum_{i=1}^{n} \hat{\varepsilon}_i^2(c_2)$$

3. Choose $\hat{c}_2$ that minimizes $\hat{\sigma}^2(c_2)$. Restore $\hat{c}_1 = \hat{c}_1(\hat{c}_2)$.
4. If needed, repeat with finer grid.

Probem 1: Cobb-Douglas PF

Suppose we have a random sample of n firms with data on output Q; capital K and labor L; and want to estimate the Cobb–Douglas production function

$$Q = \alpha Q^{\theta} L^{1-\theta} \varepsilon \tag{1}$$

where ε has the property $E[\varepsilon|K;L] = 1$. Evaluate the following suggestions of estimation of θ:

(a) Run a linear regression of $ln(Q) - ln(L)$ on a constant and $ln(K) - ln(L)$.

(b) For various values of θ on a grid, run a linear regression of Q on $K^{\theta}L^{1-\theta}$ without a constant, and select the value of θ that minimizes a sum of squared OLS errors.

Problem 2: CES PF

Suppose we want to estimate and test the CES production function

$$Q = \gamma \left(\alpha K^{-\rho} + (1-\alpha) L^{-\rho}\right)^{-\tau/\rho} exp(u) \tag{2}$$

where $\gamma > 0$, $0 \leq \alpha \leq 1$, $\rho \geq 1$, $\tau \geq 0$, and the error u has the property $E[u|K;L] = 0$. Here Q is output, K is capital, L is labor. The random sample $(Q_i; K_i; L_i)_{i=1}^{n}$ is available. Describe in detail how you will run the NLLS estimation employing the concentration method, including construction of standard errors for all parameters.

Problem 3: PF – "smooth transition" model

Consider the data for modeling the cost function $C = f(Q; P_w; P_f; P_k)$ for 145 American electric companies (see "nerlove.dta" and "nerlove.wf1"). The variables (in order) are:

n: The number of the observation
C: Total production cost, in \$millions
Q: Kilowatt-hours of output in billions
P_w: Wage rate per hour
P_f: Price of fuels in cents per million BTUs
P_k: The rental price of capital

(a) Estimate an unrestricted Cobb-Douglas specification

$$E\left[lnC|Q;P_w;P_f;P_k\right] = \alpha_1 + \alpha_2 lnQ + \alpha_3 lnP_w + \alpha_4 lnP_f + \alpha_5 lnP_k \tag{3}$$

by OLS

(b) Test the hypothesis H_0: $\alpha_3 + \alpha_4 + \alpha_5 = 1$ against H_0: $\alpha_3 + \alpha_4 + \alpha_5 < 1$ at the 5% using the asymptotic theory.

(c) Reestimate the model imposing the restriction H_0, following the same steps as in Part 1.

For the rest of this problem, keep the restriction H_0 imposed.

(d) Now we will try a non-linear specification. Consider the restricted model plus the extra term

$$\alpha_6 z \tag{4}$$

where

$$z = lnQ(1 + exp(-(lnQ - \alpha_7)))^{-1} \tag{5}$$

This model is called the smooth transition model. For values of lnQ well below α_7, the variable lnQ has a regression slope of α_2. For values well above α_7, the regression slope is $\alpha_2 + \alpha_6$, and the model imposes a smooth transition between these regimes.

The model is non-linear because of the parameter α_7. Estimate the enhanced model following the same scheme as in Part 1, by using the concentration method. [Hint 1: allow at least 10% of values of lnQ to lie below and at least 10% above potential α_7, so examine the data first and pick an appropriate range for α_7. Hint 2: always keep in mind that α_7 is a parameter and the model is nonlinear.]