Эконометрика-2 ММАЭ
Семинар 26

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Бутстрап
Bootstrap Approach

Задача 1. © *S. Anatolyev, NES, 2007*

Suppose we have the following linear mean regression:

$$y = \beta_1 x_1 + \beta_2 x_2 + \varepsilon,\ E(\varepsilon \mid x_1, x_2) = 0.$$

There is random sample $\{x_{1i}, x_{2i}, y_i\}_{i=1}^{n}$. You will evaluate the quality of approximation of the exact distributions of the estimator $\hat{\theta} = \dfrac{\hat{\beta}_1}{\hat{\beta}_2}$ of the parameter $\theta = \dfrac{\beta_1}{\beta_2}$, and its t-ratio $t_{\hat{\theta}} = \dfrac{\hat{\theta} - \theta}{s.e.(\hat{\theta})}$, where $(\hat{\beta}_1, \hat{\beta}_2)'$ is the OLS estimator of $(\beta_1, \beta_2)'$, by corresponding asymptotic distributions.

When generating artificial data, use the following information: $\varepsilon \sim N(0,3)$ and independent of $(x_1, x_2)'$, $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim N(0, I_2)$, $y = x_1 + 0.5x_2 + \varepsilon$. Of course, this information cannot be used when finding the approximations or constructing statistics of interest, because a potential researcher does not know it.
What is the asymptotic distribution of $\hat{\theta}$?

Evaluate the quality of approximation of the exact distributions of the estimator $\hat{\theta} = \dfrac{\hat{\beta}_1}{\hat{\beta}_2}$ of the parameter $\theta = \dfrac{\beta_1}{\beta_2}$, where $(\hat{\beta}_1, \hat{\beta}_2)'$ is the OLS estimator of $(\beta_1, \beta_2)'$, by a corresponding bootstrap distribution.

1. Describe the bootstrap algorithm that you will be using.
2. Plot the bootstrap and asymptotic distributions on the same graph. Which approximation, bootstrap or asymptotic, seems more precise?
3. Repeat (1)-(2) for various n (sample size). Discuss the role of sample size in the quality of bootstrap approximations.