Markets with Search Frictions and Partially Informed Intermediary

Andrey Minaev*

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Abstract

The paper discusses markets with consumer's search frictions and partial information. The main finding is the better information the platform provides can decrease the average quality of the product consumers purchase and decline in the total economic welfare and consumer surplus. The mechanism is if the platform makes better advise to consumers in average what product to explore first, all consumers have lower expectations about the next products and explore them less often, which decreases the quality of purchased good for consumers who got the wrong recommendation and might lead to reduction of the average quality of purchased products. The effect appears in the case of low search cost, which makes it especially important in the analysis of online search platforms.

JEL classification: D43, D83, L11, L13, L15.

Keywords: Consumer search, Information, Product heterogeneity, Intermediaries, Platforms, Welfare.

*Department of Economics, University of North Carolina at Chapel Hill. E-mail: andrey@unc.edu.
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Introduction

In many markets, consumers are uninformed about the quality of the available goods before they search it, and they rely on advice from information intermediaries. For example, financial advice on suitable insurance and financial products; physician advises on appropriate treatments and drugs; online platform ranks sellers for consumers; search engine ranks ads based on search queries, etc.

As economic activities are increasingly connected through the Internet, consumers can have access to more products at lower search costs, but they also face a much larger set of sellers to choose from. Consumers are thus increasingly dependent on intermediaries to guide their search (in some deliberate order) for sellers and products. This has led to enormous commercial successes of Internet companies such as Google, Amazon, and Expedia. In such circumstances, the importance of understanding how intermediaries influence the market increases. This paper addresses the question of how the market outcomes change if the platform has better information about consumers' preferences and can make better advice what products to explore first.

Intermediary's advice provides consumers additional information about product suitability which supposed to help to make better purchasing decisions in terms of the quality of product consumer leaves the market with. If intermediary perfectly knows consumers' preferences, it can make perfect advice on which product to buy. Therefore the average quality of consumed goods increases, and consumers spend less time searching.

However, the case when the platform does not know for sure which product is the best for the consumer but knows which one is probabilistically better and encourages the consumer to explore this product first provides the unstudied effect. On the one hand, it increases the expected quality of the consumed product because of the increased probability that consumer explores the best product. On the other hand, this also leads to lowered incentives to search further, which decreases chances the consumer explores the best good if the platform made the wrong recommendation. Hence, the effect of better information on the quality of the consumed product is ambiguous without additional analysis. In this paper, I demonstrate that better information always leads to a decline in the quality of purchased goods, consumer and total economic welfare in the case of low enough search cost, and leads to increased quality of goods and welfare vice versa.

Related literature

The paper is related to ordered search literature. Arbatskaya (2007) started this branch with a discussion of homogeneous goods. Later Armstrong et al. (2009) and Zhou (2011) investigate ordered search with heterogenous product, in the framework of Wolinsky (1986) and Anderson and Renault (1999). See recent development by Parakhonyak and Titova (2018) and Ding and Zhang (2018). In the aforementioned papers, search ordered as either exogenous or identical to all consumers. The high search order capture a firm's prominence. The current paper contributes to the literature by allowing the search order to be consumer-specific (or based on consumer's taste). It better captures the business feature in the age of the Internet. Search engines and intermediaries often use "big data" to make an individual recommendation based on consumers' browsing and search history.

Armstrong (2017), Haan et al. (2018) and Choi et al. (2018) also study models with endogenous ordered search. Prior to search, a consumer receives information about the match quality of the products being sold by every firm. In particular, Choi et al. (2018) show that better information leads to higher equilibrium price, which is consistent with the finding in my model. However, my paper adds to the literature by showing the effect of the information on match quality and welfare, especially showing that the product's quality purchased by a consumer might be hurt by better information.

The paper is also related to the literature of information intermediary, which is started by Biglaiser (1993) and Lizzeri (1999). I contribute to them by introducing heterogeneous products and the ordered search settings. The large literature discusses the intermediary as a marketplace that sets fees for firms and ranks them by these fees, but the marketplace itself does not have any information about the quality of the good. Bright representatives of such approach are Athey and Ellison (2011), Chen and He (2011) and Teh and Wright (2018). In these models, firms are sorted by paid fees, which in equilibria determines by the heterogeneous quality of the good, known by the firms only, where only in the latter paper the order of firms is consumer specific. As a result, consumers explore products in a given order, determined by the platform's ranking mechanism. I contribute to this branch of literature by discussing the case of a strategic partially informed platform, which determines by itself the order of search based on incomplete information about consumers' preferences.

There is literature on the relation between information and pricing. See Lewis and Sappington (1994), Anderson and Renault (2006) and Anderson and Renault (2000). Recently, Roesler and Szentes (2017) use the newly developed information design technique a la Kamenica and Gentzkow (2011) and Gentzkow and Kamenica (2016) to further explore this topic. The main message is that more information can be bad because the monopolist can better price discriminate against the consumer. See Boleslavsky et al. (2018) and Armstrong and Zhou (2019) to the effect of information in competition model, and Dogan and Hu (2018) in a context of consumer search. I contribute to this literature by showing another channel on how more information can hurt welfare. Richer information helps the platform to make a better recommendation for consumers, which reduces consumers' incentives to search. As a result, consumers' consideration set shrinks, which might lead to a reduction in the quality purchased product.

The Model

The economy consists of a monopolistic platform, two firms labeled A and B, and a continuum of consumers with a measure of one. The platform is the only way for firms and consumers to meet. Firms produce horizontally differentiated products incurring a constant marginal cost normalized to zero.

Platform charges firms ad valorem fee proportional to the transaction price and maximizes only its revenue by determining the order of firms consumers visit for each consumer individually. Firms maximize their revenue by setting the price conditional on the platform's ranking mechanism.

As in Wolinsky (1986), each consumer j has tastes described by a conditional utility (net of any search cost) of the form

$$u_{j,i} = \epsilon_{j,i} - p_i$$

if he buys the product i at price p_i, and $\epsilon_{j,i}$ is the realization of a random variable with twice differentiable cdf $F(\epsilon)$ whose support is an interval $[\underline{\epsilon}, \bar{\epsilon}]$. The term $\epsilon_{i,j}$ can be interpreted as a matching value between consumer j and product of firm i, and these match values are assumed to be independent across consumers and firms. Each consumer can buy at most one unit of product. The consumer's outside option is low enough to encourage him to purchase the good for any price, which leads to full market coverage.

A consumer must incur a search cost s to learn the price charged by any particular firm, as well as his match value for the product sold by that firm. Consumers search sequentially with costless recall. The utility of consumer j is given by

$$u_{j,i} - k \cdot s = \epsilon_{j,i} - p_i - k \cdot s$$

if she buys product i at price p_i after visiting k firms.

The market interaction proceeds as follows. First, all market participants learn the probability that the platform can make perfect advice for each given consumer (q). Each firm simultaneously sets a price p conditional on q. For every consumer with the probability q, the platform observes the qualities $\epsilon_{j,A}$ and $\epsilon_{j,B}$ of both firms' goods and make an individual order for each consumer. To these consumers, the platform recommends visiting first the firm with a higher quality of the good. For the rest of the consumers, the platform provides an order at random. In equilibrium, it is optimal for the revenue-maximizing platform to propose to consumers such order of the search and for utility optimizing consumers to follow this recommendation. Consumers form the expectations about the prices and follow an ordered sequential search process with search cost s and costless recall. Thereafter the consumer buys the better quality product among ones he explored and pays the price p.

Analysis

In this section, I derive the perfect Bayesian equilibrium by means of backward induction. Because of the symmetry of consumers and firms, I concentrate on the analysis of symmetric perfect Bayesian equilibrium when firms charge equal prices and get equal demand. I derive the consumer's optimal search rule and use it to analyze how better information the platform has changes the quality of the product consumer leaves the market with. After that I derive the demand functions and optimal pricing on the market. Thereafter I show that the price and the revenue of each firm increase if the platform knows better information about consumers' preferences. This verifies it is always optimal for the revenue-maximizing platform to use the entire information in case the platform charges firms ad valorem fee proportional to the price. Lastly, using these result I examine the effects of information on the welfare of market participants.

The search and quality of the purchased product

For each consumer, the platform observes the quality of goods with a probability q. Hence, each consumer holds rational expectations that the first firm in the proposed searching order provides the best product with a probability $\frac{1+q}{2}$. Due to low enough outside option, the consumer always explores the first firm. In the symmetric equilibrium, when both firms charge the same price, if the first product has quality z, the benefit from sampling the second one is described in Equation 1 as $h(z)$.

$$h(z) = \int_{z}^{\bar{\epsilon}} (\epsilon - z) f_{\epsilon|z} d\epsilon \tag{1}$$

where $f_{\epsilon|z}$ is the predicted distribution of the second visited firm's product quality, conditional on the observed quality of the first good.

Lemma 1. *In the region $\epsilon > z$, $f_{\epsilon|z}$ can be expressed as:*

$$f_{\epsilon|z} = Pr(\epsilon > z|z) \cdot f(\epsilon|z, \epsilon > z) = \frac{\frac{1-q}{2}(1 - F(z))}{\frac{1-q}{2}(1 - F(z)) + \frac{1+q}{2}F(z)} \cdot \frac{f(\epsilon)}{1 - F(z)}$$

Lemma 1 describes the consumer beliefs about the second product conditional on the observed quality of the first one. In the special case of $q = 0$, when the model is reduced to Wolinsky (1986) model, the platform does not know anything about consumers' preferences and shows products at random. Hence the consumer simply expects $f(\epsilon)$ to be a distribution of the second good's quality. If $q > 0$, then the higher q is, and the higher z consumer observes, firmer his beliefs that the second product is the worst one, which decreases expectation of the second product's quality and incentives to search further.

Consumer's reservation value x, which solves the equation $s = h(x)$, is such product quality that the benefit from sampling one more product equals the search cost if there is no difference in goods prices. Then the consumer's optimal search strategy without the presence price difference is to stop searching and buy the first product if its quality is higher than x. Otherwise, the consumer should explore the second product and purchase the better one. The next two lemmas discuss how x depends on market parameters.

Lemma 2. *The reservation value x is a decreasing function of the search cost s.*

In accordance with the classic result of Wolinsky (1986), if the search cost increases, it becomes less profitable to explore the next product. As a result, x decreases.[1]

Lemma 3. *The reservation value x is a decreasing function of platform information q.*

Lemma 3 highlights the idea that as the platform has better information (q increases), consumers have lower incentives to search because the first explored product is the better one with higher probability. As a result, consumers less often choose the better product and simply purchase the first investigated product, which decreases the expected quality of the consumed product. On the other hand, as the platform has better information, the first explored product is the better one with a higher probability, which increases the expected quality of the consumed product. The subsequent analysis is designed to resolve ambiguity in the combined effect.

For the rest of the paper, I presume that Assumption 1 is satisfied.

Assumption 1. *$f(\cdot)$, $F(\cdot)$ and $1 - F(\cdot)$ are log-concave and continuously differentiable functions.*

The probability that consumer explores the better product can be expressed as:

$$\begin{aligned} Pr(q) &= \frac{1+q}{2} + \frac{1-q}{2} \cdot F_{min\{\epsilon_1,\epsilon_2\}}(x) = \\ &= \frac{1+q}{2} + \frac{1-q}{2}\Big(1 - (1 - F(x)^2)\Big) \end{aligned} \tag{2}$$

The first term in Equation 2 stands for the probability that the first explored product is the better one. Hence the consumer buys this product for sure even if he decided to explore the second one because there is no difference in products' prices. The second term is the probability that the first product is the worse one, and its quality is below x. In such a situation, consumer explores the second product as well, and as a result, will buy the better one in the symmetric equilibrium when both firms charge equal prices.

As a result, the expected quality of the product consumer leaves the market with is:

$$\begin{aligned} V(q) = Pr(q) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \, dF_{max\{\epsilon_1,\epsilon_2\}}(\epsilon) + (1 - Pr(q)) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \, dF_{min\{\epsilon_1,\epsilon_2\}}(x) = \\ = Pr(q) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \, d\left[F(\epsilon)^2\right] + (1 - Pr(q)) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \, d\big[1 - (1 - F(\epsilon))^2\big] \end{aligned} \tag{3}$$

[1]For richer discussion see Wolinsky (1986)

Notice that the expected quality of the consumed good is an increasing function of $Pr(q)$ due to both integrals in Equation 2 are constants and stand for the quality of the better and the worse product among two respectively.

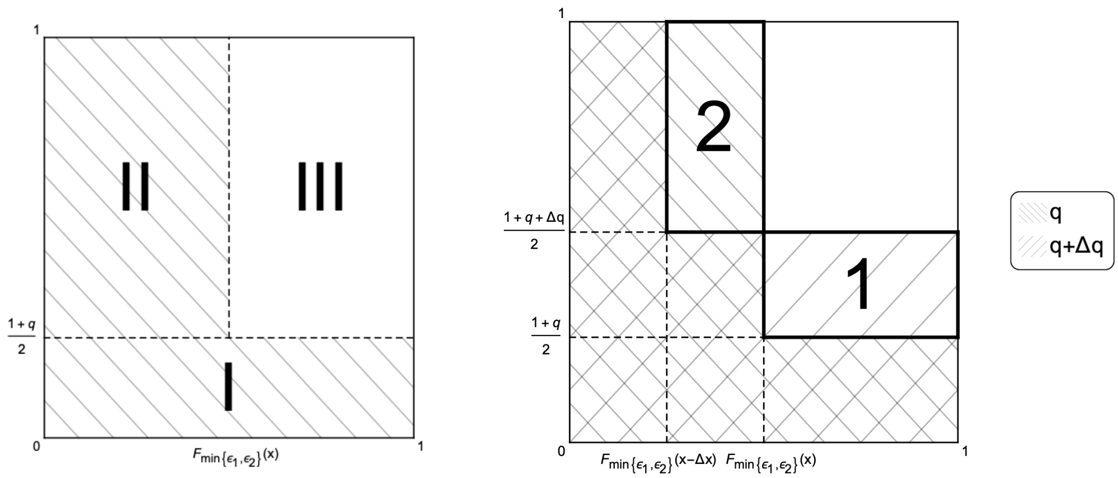
The shaded area in Figure 1a graphically illustrates $Pr(q)$. The vertical axis is the probability the platform makes a correct recommendation for consumer, and hence he explores better product first. The horizontal axis is the probability consumer decided to explore the second product because the quality of the first one was below the reservation value x, conditional on the fact the first product was the worse among two. As a result, region I is the probability the consumer explores a better product first. Region II is the probability the consumer explores a worse product first but then explores a better one as well because the quality of the first product was below the reservation value. And the region III is the probability the consumer does not explore better product at all, leaving the market with the worse one.

The Figure 1b illustrates how $Pr(q)$ changes when q increases. The regions "q" and "$q + \Delta q$" show the probability the consumer purchased the better product when the platform knows consumer's preferences with probabilities q and $q + \Delta q$, respectively. If the platform has better information (increase from q to $q+\Delta q$), then, according to Lemma 3, x decreases to $x - \Delta x$. On the one hand, the consumer now has a better chance to find a better product in the first shop (region 1), but on the other hand, the lower probability of visiting the second shop and find the best product there (region 2).

Figure 1: The probability to explore the better good.

(a) The probability to explore the better good.

(b) The response of probability to explore the better good on change in q.



If search cost s is close to zero, the reservation value x is close to $\bar{\epsilon}$, which makes the region 1 on the Figure 1b be thin and its area be close to zero. Hence the indirect effect of decreased search incentives might outweigh the direct effect of better recommendation, hurting the realized quality of a purchased product. The next proposition summarizes the above discussion in a general statement.

Proposition 1. *Under Assumption 1, for any $\hat{q} \in [0,1]$ exists search cost $\hat{s}$ s.t.*

1. *for any s s.t. $0 < s < \hat{s}$, the expected quality of consumed good decreases in q.*

2. *for any s s.t. $s > \hat{s}$, the expected quality of consumed good increases in q.*

The Demand

Due to the symmetry of firms, them both will have similar demand functions. WLOG I derive the demand function for firm A. Suppose firm A sets price p_A, and firm B sets the equilibrium price p^*. Define $\Delta = p_A - p^*$. It is optimal for consumer, when sampling firm A, to use the search rule, described in the previous section.

The Figure 2 summarizes the demand of firm A. The Figure 2a shows the demand of firm A if it is sampled first. If so, it gets all consumers for whom $\epsilon_A > x + \Delta$ (region I) because these consumers buy from firm A immediately and don't search further. Among the consumers who sample both firms, firm A gets consumers for whom $\epsilon_A - p_A > \epsilon_B - p_B$ (region II). The Figure 2b shows the demand of firm A if it is sampled second. If so, it gets all consumers for whom $\epsilon_A > x + \Delta$ and $\epsilon_B < x$ (region I) because these consumers, at first, don't stop on firm B and search further and, at second, value the firm A product higher. Among the consumers who sample both firms, firm A gets consumers for whom $\epsilon_A - p_A > \epsilon_B - p^*$ (region II). According to the fact that each firm is shown first to the half of consumers because $\epsilon_A \perp\!\!\!\perp \epsilon_B \perp\!\!\!\perp q$, we can express the firm A demand as:

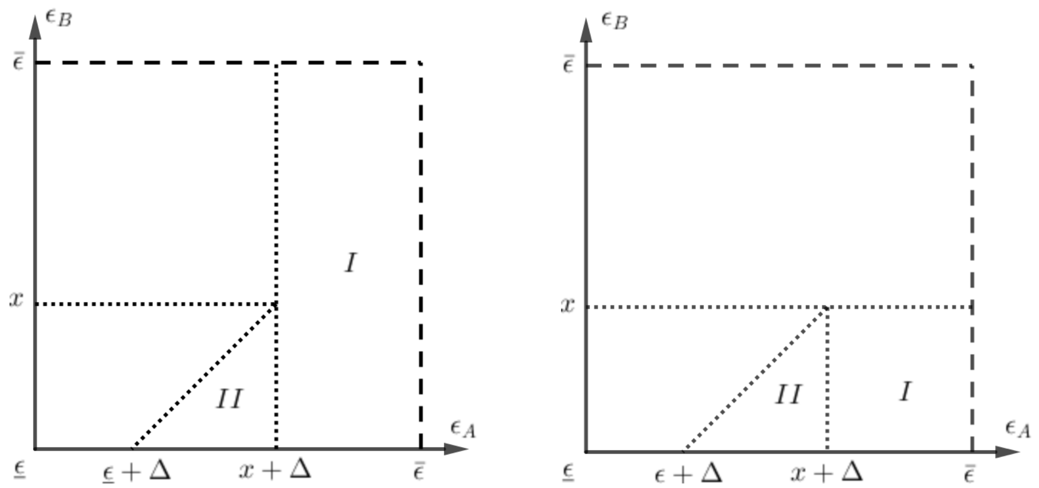
$$D(p_A, p^*) = \frac{1 + F(x)}{2}(1 - F(x + \Delta)) + \int_{\underline{\epsilon}}^{x+\Delta} f(\epsilon)F(\epsilon - \Delta)d\epsilon \tag{4}$$

It's important to note that demand depends on q only through x.

Assuming that the search cost s such that $x \in [\underline{\epsilon}, \bar{\epsilon}]$ $\forall q \in [0,1]$, I now turn to the analysis of the equilibrium. Note that in equilibrium $D(p^*, p^*) = \frac{1}{2}$, i.e. every consumer buys exactly one good.

Figure 2: Firm's demand

(a) Demand if sampled first *(b) Demand if sampled second*



Lemma 4. *Under Assumption 1, there exists a unique symmetric equilibrium:*

$$p_A = p_B = p^* = -\frac{D(p^*,p^*)}{\frac{\partial D}{\partial p}(p^*,p^*)} = \frac{1}{(1-F(x))f(x) + 2\int\limits_{\underline{\epsilon}}^{x} f^2(\epsilon)d\epsilon} \tag{5}$$

As Quint (2014) showed, log-concavity of $f(\epsilon), F(\epsilon)$ and $1-F(\epsilon)$ guarantees that the demand is log-concave in price, hence $-\frac{D(p_A,p^*)}{\frac{\partial D}{\partial p_A}(p_A,p^*)}$ is decreasing in p_A, what guarantees the unique solution of Equation 5.

Lemma 5. *Under Assumption 1, p^* is an increasing function of the search cost s and an increasing function of q.*

Both results of the latter lemma are based on the fact that if consumers have lower incentives to search, the market competitiveness declines and the firms can raise prices. If q increases, rational consumers expect that the platform makes better ranking of firms, hence the consumers has lower expectations of the second firm good's quality, which decreases the incentives to search further.

The equilibrium price is an increasing function of q, while the firms' demand is price non sensitive and constant, which leads to increasing of the firms' profits if the platform uses better information for ranking. As the platform charges firms ad valorem fee proportional to the price, the platform's revenue is an increasing function of q, which guarantees it is optimal for the platform to use for ranking the entire information it has.

The effect of the platform information on welfare of market participants

The level of consumer's search expenditures can be expressed as shown in Equation 6. Due to low enough outside option, all consumers search at least once. $\frac{1+q}{2}$ portion of them will get the best product on the first position and explore the second one only if quality of the first product is below x. The rest $\frac{1-q}{2}$ consumers get the worst off two products on the first position and also explore the second one only if the realized quality of the first product is below x.

$$\begin{aligned} SE(q) &= s \cdot \left(1 + \frac{1+q}{2} \cdot F_{max\{\epsilon_1, \epsilon_2\}}(x) + \frac{1-q}{2} \cdot F_{min\{\epsilon_1, \epsilon_2\}}(x)\right) \\ &= s \cdot \left(1 + \frac{1+q}{2} \cdot F(x)^2 + \frac{1-q}{2} \cdot \left(1 - (1 - F(x))^2\right)\right) \end{aligned} \tag{6}$$

Lemma 6. *Search expenditures is a decreasing function of q.*

As the platform knows better information about consumer's preferences and can make better recommendation, consumers firstly expects lower quality of the second good and secondly get the best good in the first visited firm with higher probability. Both effects lower consumers' incentives to search.

As Proposition 1 states, the product quality can decrease as the platform has better information. On the other hand Lemma 6 shows that as platform makes better recommendation, consumer's search expenditures declines because of the lower search intensity. The next proposition highlights that, according to two these two results, the effect of increased q on the total economy surplus, defined in Equation 7 is ambiguous.

$$TS(q) = V(q) - SE(q) \tag{7}$$

Proposition 2. *Under Assumption 1, for any $\check{q} \in [0, 1]$, exists $\check{s}(\check{q})$ s.t.*

1. *for any s s.t. $0 \leq s < \check{s}$, the total economic welfare decreases in q.*

2. *for any s s.t. $s > \hat{s}$, the total economic welfare increases in q.*

where $\hat{s}$ is defined in Proposition 1

For low enough search cost ($s < \check{s}$), as q increases, the savings in search expenditures are comparatively small and outweighed by the reduction in the quality of the good consumers leave the market with. If search cost is above $\hat{s}$, defined in Proposition 1, the quality of purchased good increases with q, while searching expenditures decreases. In such case both effects increase the total economic welfare. In the intermediate case $\check{s} < s < \hat{s}$ the effect might be non-monotonic and requires extra analysis.

Therefore, the total economic welfare might decrease as the platform has better information even in the case of price non-sensitive demand. The full market coverage for any price level is a notable restriction. In more realistic case consumers have the outside option normalized to zero. In such settings if the equilibrium price rises, more consumers will prefer do not buy any good and leave the market with the outside option. The market shrinking results in decreasing of the total economic welfare due to the marginal cost of production is normalized to zero, and the domain of $f(\epsilon)$ is non-negative. In case of low enough search cost s the latter effect outweighs savings from the reduction of the search intensity. As a result, in such setting the total economic welfare decreases for wider set of parameters.

According to Proposition 2 and Lemma 5, the Total Economy welfare and equilibrium price both increases in q for high enough search cost. Therefore, the effect of increased q on the Consumer Surplus, defined in Equation 8, is ambiguous in general case and depends on the levels of s and q.

$$CS(q) = V(q) - SE(q) - p^*(q) \tag{8}$$

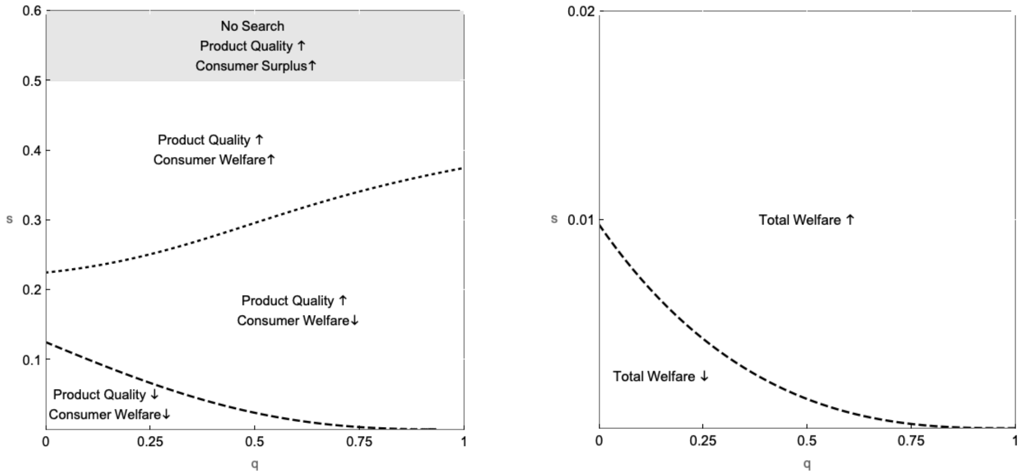
Figure 3 illustrates the compositions of s and q which lead to different dynamics of market outcomes in case of uniform distribution ϵ. If $s > \frac{1}{2}$ then $x < 0 = \underline{\epsilon}$, hence consumers never explore the second product. Therefore, if q increases, price does not change and Consumer Surplus increases the same as the Total Economy Surplus. If $0 < s < \frac{1}{2}$ and $0 < q < 1$, then $0 < x < 1$ and consumers who explored the product with quality below x, search the second good as well. In consistency with Proposition 1 for low enough search cost s, the expected quality of consumed good is decreasing with q. Also, according to the Figure 3, the Consumer Surplus increases with q for high enough search cost because the effects of increased quality of consumed good and decreased search expenditures outweigh the increase in price.

Concluding remark

The paper discusses markets with consumer's search frictions and a partially informed intermediary. The main finding is that the better information the platform has about

Figure 3: The effect of levels of search costs s and platform's information q on market outcomes. The case of uniform distribution.

(a) The effect on product's quality and consumers' welfare *(b) The effect on Total economic welfare*



consumers' preferences can lead to decreasing in the average quality of the product consumers leave the market with. The intuition behind the mechanism is if the platform has better information and makes better advice to consumers what product to explore first, consumers have lower expectations about the next products and explore them less often, which might lead to shrinking of the consideration set and the quality of a chosen product.

Also, in this paper, I argue that consumers and the whole economy can be benefited or hurt if the intermediary knows more information about their preferences and can better manipulate the order in which consumers explore products. The actual effect depends on the search costs and distribution of the qualities of the products and requires additional analysis in each case.

One of the possible extensions of the model for further research is a decreasing platform's monopoly power, for example, by introducing the competition between platforms. That will force the platform to incorporate the consumers' surplus in the platform's objective function and could change the platform's equilibrium strategy. Especially crucial that, if so, the platform can prefer not to use all the information it has about consumers' preferences and steer the search order to do not let the price increase too much.

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Appendices

Proof. of lemma 1

Suppose that ϵ_1, ϵ_2 and ξ are mutually independent, where $Pr(\xi = 1) = 1 - Pr(\xi = 0) = \frac{1+q}{2}$ and ϵ_1, ϵ_2 are identically distributed with density $f(\epsilon)$. Let $V = min\{\epsilon_1, \epsilon_2\}$ and $W = max\{\epsilon_1, \epsilon_2\}$ and define $X = \xi \cdot V + (1-\xi) \cdot W$, $Y = (1-\xi) \cdot V + \xi \cdot W$. I seek for $f_{Y|X}(y|x)$ on the region where $y > x$.

Step 1. Joint density for (V, W)

The joint cdf for (V, W) for $w \geq v$ given by:

$$\begin{aligned} F_{V,W}(v,w) &= Pr(V \leq v, W \leq w) \\ &= Pr(\epsilon_1 \leq v, v < \epsilon_2 \leq w) + Pr(v < \epsilon_1 \leq w, \epsilon_2 \leq v) + Pr(\epsilon_1 \leq v, \epsilon_2 \leq v) = \\ &= 2F(v)[F(u) - F(v)] + F(v)^2 \end{aligned}$$

So the density is given by:

$$f_{V,W}(v,w) = \frac{d^2 F_{V,W}(v,w)}{dvdw} = 2f(v)f(w), \quad for\ v \leq w$$

Step 2. Joint density for (X, Y)

X and Y are functions of (V, W, ξ), so I first derive the density for $(X, Y, \xi) = g(V, W, \xi)$. It follows that $g^{-1}(x, y, \xi) = (\xi x + (1-\xi)y, (1-\xi)x + \xi y, q)$ with Jacobian $J(x, y, \xi) = \begin{pmatrix} \xi & 1-\xi & x-y \\ 1-\xi & \xi & y-x \\ 0 & 0 & 1 \end{pmatrix}$,

so $|detJ(x, y, \xi)| = |\xi^2 - (1-\xi)^2| \cdot 1 = |\xi - (1-\xi)| = |1 - 2\xi| = 1$

Hence the density for (X, Y, ξ) is given by

$f_{X,Y,\xi}(x, y, \xi) = f_{V,W}(\xi x + (1-\xi)y, (1-\xi)x + \xi y) \cdot f_\xi(\xi)$

where I used that ξ is independent of (ϵ_1, ϵ_2) and therefore also independent of (V, W). Next I obtain the density for (X, Y) by integrating out ξ.

$f_{X,Y}(x, y) = \frac{1+q}{2} f_{V,W}(x, y) + \frac{1-q}{2} f_{V,W}(y, x)$.

It follows that $f_{X,Y}(x, y)\mathbb{I}_{x<y} = \frac{1+q}{2} f_{V,W}(x, y)\mathbb{I}_{x<y}$ because $f_{V,W}(y, x) = 0$ for $x < y$.

Step 3. Marginal density for X.

The density for X is given by:

$$f_X(x) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} f_{X,Y}(x,y) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\frac{1+q}{2} f_{V,W}(x,y) + \frac{1-q}{2} f_{V,W}(y,x) \right) dy =$$

$$= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\frac{1+q}{2} 2f(x)f(y)\mathbb{I}_{x\leq y} + \frac{1-q}{2} 2f(y)f(x)\mathbb{I}_{x>y} \right) dy =$$

$$= 2f(x) \left(\frac{1+q}{2} \int_{x}^{\bar{\epsilon}} f(y)dy + \frac{1-q}{2} \int_{\underline{\epsilon}}^{x} f(y)dy \right) =$$

$$= 2f(x) \left(\frac{1+q}{2} + qF(x) \right)$$

Step 4. Conditional density of Y given X.

Finally for $y > x$ we have

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{\frac{1+q}{2} f_{V,W}(x,y)}{2f(x)\left(\frac{1+q}{2} + qF(x)\right)} =$$

$$= \frac{\frac{1+q}{2} 2f(x)f(y)}{2f(x)\left(\frac{1+q}{2} + qF(x)\right)} = \frac{\frac{1+q}{2}(1-F(x))}{\frac{1+q}{2}(1-F(x)) + \frac{1-q}{2}F(x)} \cdot \frac{f(y)}{1-F(x)}$$

Q.E.D.

Proof. of lemma 2

From Equation 1 and definition of x we have:

$$s = \int_{x}^{\bar{\epsilon}} (\epsilon - x) f_{\epsilon|x}(\epsilon) d\epsilon \Rightarrow s = \frac{\frac{1-q}{2}(1-F(x))}{\frac{1-q}{2}(1-F(x)) + \frac{1+q}{2}F(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon) d\epsilon \Rightarrow$$

$$s = \frac{1-q}{1-q+2qF(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon) d\epsilon \Rightarrow$$

$$1 = \frac{\partial}{\partial s} \left(\frac{1-q}{1-q+2qF(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon) d\epsilon \right) \Rightarrow$$

$$1 = -\frac{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int_x^{\bar{\epsilon}} f(y)(y-x)\,dy\right)}{(1-q+2qF(x))^2} \cdot \frac{\partial x}{\partial s} \Rightarrow$$

$$\frac{\partial x}{\partial s} = -\frac{(1-q+2qF(x))^2}{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)\,d\epsilon\right)} < 0 \ \forall \ (q,x,s) \in [0,1] \times [\underline{\epsilon}, \bar{\epsilon}]$$

The alternative expression is $\frac{\partial x}{\partial s} = -\frac{1-q+2qF(x)}{2qsf(x)+(1-q)(1-F(x))}$ Q.E.D.

Proof. *of Lemma* 3

From Equation 1 and definition of x we have:

$$s = \int\limits_x^{\bar{\epsilon}} (\epsilon - x) f_{\epsilon|x}(\epsilon) d\epsilon \Rightarrow s = \frac{\frac{1-q}{2}(1-F(x))}{\frac{1-q}{2}(1-F(x)) + \frac{1+q}{2}F(x)} \cdot \int\limits_x^{\bar{\epsilon}} (\epsilon - x) f(\epsilon) d\epsilon \Rightarrow$$

$$s = \frac{1-q}{1-q+2qF(x)} \cdot \int\limits_x^{\bar{\epsilon}} (\epsilon - x) f(\epsilon) d\epsilon \Rightarrow$$

$$0 = \frac{\partial}{\partial q}\left(\frac{1-q}{1-q+2qF(x)} \cdot \int\limits_x^{\bar{\epsilon}} (\epsilon - x) f(\epsilon) d\epsilon\right) \Rightarrow$$

$$0 = -\frac{2F(x)\int\limits_x^{\bar{\epsilon}} f(\epsilon)(\epsilon - x)\,d\epsilon}{(1-q+2qF(x))^2} - \frac{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}} f(\epsilon)(\epsilon - x)\,d\epsilon\right)}{(1-q+2qF(x))^2} \cdot \frac{\partial x}{\partial q} \Rightarrow$$

$$\frac{\partial x}{\partial q} = -\frac{2F(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)\,d\epsilon}{(1-q)\left((1-F(x))\,(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon\right)} < 0 \ \forall \ (q,x) \in [0,1] \times [\underline{\epsilon}, \bar{\epsilon}]$$

The alternative expression is $\frac{\partial x}{\partial q} = -\frac{2sF(x)}{(1-q)(2qsf(x)+(1-q)(1-F(x)))}$ Q.E.D.

Proof. of Proposition 1

The proof consists of two steps. On the first step I show that $\frac{\partial PQ}{\partial q}$ is negative in the left neighborhood of $\bar{\epsilon}$ (low search cost case) for any arbitrary distribution $F(\epsilon)$, which proves the first part of the proposition. On the second step I show that under the Assumption of log concave distribution $F(\epsilon)$, $\frac{\partial PQ}{\partial q}$ is positive for $x = \underline{\epsilon}$ (high search cost case) and the equation $\frac{\partial PQ}{\partial q} = 0$ has unique solution. This fact, together with continuity of $\frac{\partial PQ}{\partial q}$ proves the second part of the proposition.

Step 1. After differentiating Equation 2:

$$\frac{\partial Pr(q)}{\partial q} = \frac{1 - F(x)}{2}\Big(1 - F(x) + 2(1-q)f(x) \cdot \frac{\partial x}{\partial q}\Big)$$

Using the expression for $\frac{\partial x}{\partial q}$ derived in the section 4, I conclude that

$$\frac{\partial Pr(q)}{\partial q} = \frac{(1 - F(x))}{4qsf(x) + 2(1-q)(1-F(x))} \cdot \Big((1-q)(1-F(x))^2 - 2sf(x)((2+q)F(x) - q)\Big)$$

The fraction part of above expression is positive $\forall\ (q, x, s) \in [0, 1] \times [\underline{\epsilon}, \bar{\epsilon}] \times [0, \infty)$. Hence the sign of $\frac{\partial Pr(q)}{\partial q}$ depends only on the sign of the second part of expression, which can be expressed as a Taylor series approximation in the left neighborhood of $\bar{\epsilon}$ as:

$$(1-q)(1-F(x))^2 - 2sf(x)((2+q)F(x) - q) = -\frac{2(1-q)^2 f\left(\bar{\epsilon}\right)^2}{1+q} \cdot (x - \bar{\epsilon})^2 < 0$$

As a result, $\frac{\partial Pr(q)}{\partial q}$ is negative in the left neighborhood of $\bar{\epsilon}$, and due to continuity of $\frac{\partial Pr(q)}{\partial q}$, there exists $\hat{s}(\hat{q})$ s.t. for any s s.t. $0 < s < \hat{s}(\hat{q})$, $\frac{\partial Pr(q)}{\partial q}$ is negative and, therefore, the expected quality of consumed good decreases in q.

Step 2. Using the expression for $\frac{\partial x}{\partial q}$ derived in the proof of Lemma 3, I conclude that $\frac{\partial x}{\partial q}\Big|_{x=\underline{\epsilon}} = 0$. Hence, $\frac{\partial Pr(q)}{\partial q}\Big|_{x=\underline{\epsilon}} = \frac{1-0}{2}\Big(1 - 0 + 2(1-q)f(x) \cdot 0\Big) = \frac{1}{2} > 0$.

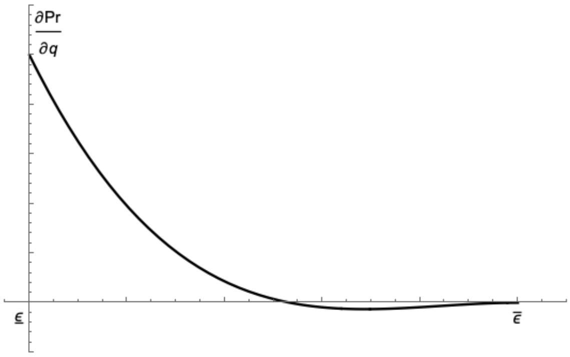
Next I show that $\frac{\partial}{\partial x}\frac{\partial Pr(q)}{\partial q}$ is negative everywhere $\frac{\partial Pr(q)}{\partial q} = 0$ if $1 - F(\cdot)$ is strictly log-concave.

$$\begin{aligned}\frac{\partial}{\partial x}\frac{\partial Pr(q)}{\partial q} &= -\frac{(1-F(x))\left(f(x)^2 + (1-F(x))f'(x)\right)}{2f(x)} \\ &= -\frac{(1-F(x))\left((1-F(x)')^2 - (1-F(x))(1-F(x))''\right)}{2f(x)} < 0\ \forall x \in [\underline{\epsilon}, \bar{\epsilon}]\end{aligned}$$

As a result, $\frac{\partial Pr(q)}{\partial q}$ is positive at $\underline{\epsilon}$, negative at left neighborhood of $\bar{\epsilon}$ and has a negative derivative at any point of where it equals zero. Based on that I conclude $\frac{\partial Pr(q)}{\partial q} = 0$ has a unique solution. The example of $\frac{\partial Pr(q)}{\partial q}$ for $q = \frac{1}{8}$ and $\epsilon \sim U[0, 1]$ is represented on the figure below.

Q.E.D.

Proof. of Proposition 2



Differentiating Equation 7 with accounting to Equation 3 and Equation 6 gives:

$$\frac{\partial TotalSurplus}{\partial q} = s(1-F(z))F(z)\left((1-F(z))\int\limits_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon f(\epsilon)(2F(\epsilon)-1)\,d\epsilon + 1\right) -$$
$$- f(z)\left(2qsF(z) + (1-q)s + 2(1-q)(1-F(z))\int\limits_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon f(\epsilon)(2F(\epsilon)-1)\,d\epsilon\right)x'(q)$$

Q.E.D.

Proof. of Lemma **??**

After differentiating Equation 2 we have:

$$\frac{\partial Pr(q)}{\partial q} = \frac{1-F(x)}{2}\Big(1-F(x)+2(1-q)f(x)\cdot\frac{\partial x}{\partial q}\Big)$$

Hence, $\frac{\partial Pr(q)}{\partial q} < 0$ iff $\frac{\partial x}{\partial q} < -\frac{1-F(x)}{2(1-q)f(x)}$ Q.E.D.

Proof. of Lemma 5

After differentiating Equation 5 and accounting for log-concavity of $1-F(x)$, we have:

$$\frac{\partial p^*}{\partial q} = -\frac{f(x)^2+(1-F(x))f'(x)}{\left((1-F(x))f(x)+2\int\limits_{\underline{\epsilon}}^{x} f(\epsilon)^2 d\epsilon\right)^2} = -\frac{\big((1-F(x))'\big)^2+\big(1-F(x)\big)\big(1-F(x)\big)''}{\left((1-F(x))f(x)+2\int\limits_{\underline{\epsilon}}^{x} f(\epsilon)^2 d\epsilon\right)^2} < 0$$

Q.E.D.

Proof. of Lemma 6

After differentiating Equation 6 we have:

$$\frac{\partial SE}{\partial q} = \underbrace{-s(1-F(x))F(x)}_{<0} + \underbrace{f(x)(1-q+2qF(x))}_{>0}\cdot\underbrace{\frac{\partial x}{\partial q}}_{<0} < 0$$

Q.E.D.

Proof. of Proposition 2

If $s > \hat{s}$, then, according to Proposition 1, the quality of the purchased good is increasing in q. Also, according to Lemma 6, search expenditures is decreasing in q. Hence, the total economic welfare, defined in Equation 7 is increasing in q.

To show that Total economic welfare is decreasing for low enough search cost (or, same, high enough x), I show that the derivative of the total welfare with respect to x is negative in the left neighborhood of $x = \bar{\epsilon}$ by expanding it in Taylor series.

$$\frac{\partial TS}{\partial q} = \left(12(1-q^2)f\left(\bar{\epsilon}\right)^4 \int\limits_{\underline{\epsilon}}^{\bar{\epsilon}} 2\epsilon f(\epsilon)(2F(\epsilon)-1)d\epsilon\right) \cdot \left(x-\bar{\epsilon}\right)^3 + \bar{o}(\left(x-\bar{\epsilon}\right)^3) \leq 0 \quad \forall x < \bar{\epsilon}, 0 \leq q \leq 1$$

Q.E.D.