Two-sender two-dimension cheap talk with impossibility to commit do not collude

Abstract

In this paper, I have studied the information transmission of two-dimensional and two-sender cheap talk model where the set of receiver's policies are exogenously restricted. I consider the case when two senders cannot commit do not collude. It is clear that in such assumptions there is no full information revealed equilibrium (FRE) because senders can collude to report any fake state. Then I adopt this model to the case of two sellers and one buyer who is not sure which the quality of product she should buy. The main results of the paper are constructing of partial information revealed equilibrium (PRE) and proving this equilibrium still worse for all agents than FRE. Also I provide the estimation strategy to evaluate the seller's biases. I use information of only sellers' offers to the buyer and actual buying decision (what quality buyer bought) to evaluate the lower and the upper bounds of both sellers' biases.

Introduction

In (Crawford and Sobel, 1982) authors described canonical cheap-talk model with one sender and one receiver. In this paper, they introduced the main aspects of their partial information revealed equilibria. The idea is when there are two agents – the receiver and the sender. The sender knows the true state of the world but the receiver does not. The receiver learns some information about state of the world from the sender (due to sender send the signal to the receiver about state of the world) and based on this information makes some action, that effects on him and the sender as well. The preferences of the sender are not perfectly aligned with receiver's one ("biased expert"). In addition, sending of message is costless, and both sender and receiver cannot commit themselves how truthfully state will be represented and how this information will be used. In such assumptions, full revealing of the information is impossible and Crawford and Sobel constructed cheap-talk equilibrium concept.

Later, Battaglini in his paper "Multiple referrals and multidimensional cheap talk" (Battaglini, 2002) provided necessary and sufficient condition of existing fully information revealed equilibrium in multi-sender case of model, when the senders cannot collude and receiver knows magnitude and direction of both sender's biases. This idea is very simple – the receiver should punish the senders in case of different messages. Due to the senders cannot collude which fake state to declare and space of possible states is infinite, the probability of declaring the same state by all senders is zero. And with probability one they would be punished if they tried to declare fake state. Hence, they have to declare true state of the world. Punishment should be higher than displeasure of the sender with true state of the world to make sufficient incentives to the senders. But Attila and Takahashi in their paper "Multi-sender cheap talk with restricted state spaces" (Ambrus and Takahashi, 2008) later showed that for restricted state space such equilibria not always exists, because sometimes feasible policy space is too small related to senders' biases and receiver cannot find appropriate punishment. Later (Meyer et al., 2013) studied the multi-dimensional and multi-sender problem in case senders observe the true state of the world with errors. They constructed the fully revealing equilibria that are robust to small mistakes by the senders.

In this paper, I would like to consider the case when the senders cannot commit against collusion. This case seems to be much more plausible in real life. Different employees or company departments likely communicate with each other and can transmit information among themselves and make collusions. The same applies to experts. Generally, there are no so many good experts in narrow field, so they communicate with each other very often. If some of these agents promise the receiver do not collude, she will not have any reasons to believe.

In such assumptions and assumption that utility is quadratic, I derive partial information revealed equilibria. Notice that the receiver still knows magnitude and direction of all expert's biases.

The main idea that the senders strategically combine their preferences (biases) in one bias. Then both of them report one state (not the same) which is close to the real state of the world and both these states and real state lies on the one straight line. Thereby senders report the receiver the line where the real state is, allowing to reduce the dimensionality of the problem to one. By this procedure we get one-dimension one-sender cheap-talk model with new combined bias described by (Crawford and Sobel, 1982). Notice that on the first stage senders sent states consistent to the equilibrium on the second stage (one-dimensional stage).

Due to the initial two-dimension policy space is restricted, it's one-dimension cross-sections can have different length. Moreover, how we know from (Crawford and Sobel, 1982), utility of the sender is higher than smaller his bias related to the length of one-dimension policy space. Due to the senders by their choice of combined bias can handle this fraction, they will choose optimal one (notice that the direction of new combined bias determines which cross-section would be new one-dimension policy space, hence change of combined bias would effect on the length of this new policy space).

As I already mentioned, that the case when senders can't commit do not collude is much more plausible in real life. As a good example we can consider the case of one buyer who is not sure which the quality of product she really needs. There are two senders who can advise to the buyer. But sellers have incentives to "advise" higher quality product due to mark-up for higher quality products usually higher. Hence buyer has no incentives to believe in such advices, due to make an advice is costless for seller. We have standard cheap talk game. In vast majority of cases the biases of sellers are unobservable. And that would be very interesting to estimate how much these advices are really biased.

For this purpose I provided the estimation strategy to evaluate the seller's biases. I use information of only sellers' offers to the buyer and actual buying decision (what quality buyer bought) to evaluate the lower and the upper bounds of both sellers' biases.

Initial Model

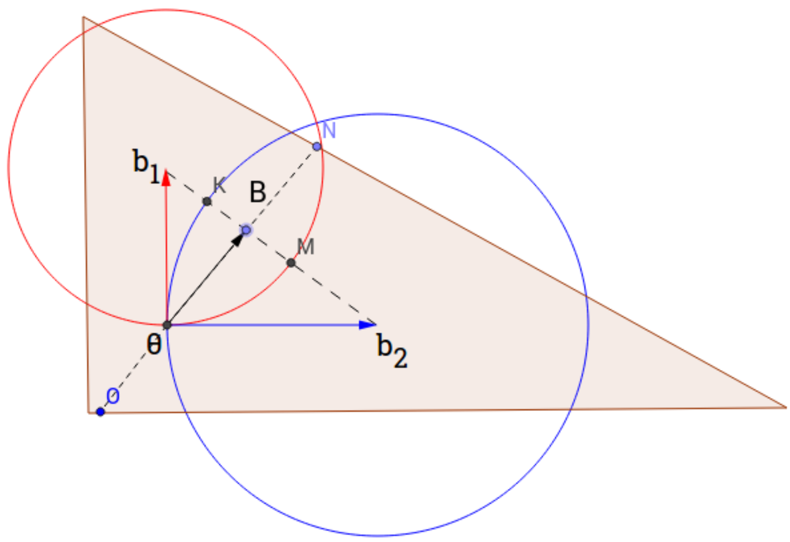
There is two-dimension policy space. Each sender i has bias $\vec{b_i}$. Utility of receiver and senders are:

$$U_R = -|\vec{\theta} - \vec{a}|^2 \tag{1}$$
$$U_{S_i} = -|\vec{\theta} + \vec{b_i} - \vec{a}|^2 \tag{2}$$

where $\vec{\theta}$ – true state of world. Here $|\cdot|^2$ is Euclidean distance for $\mathbb{R}^2$. Hence the indifferent curves are concentric circles. There is exogenous restricted feasible policy space Ω (red triangle on the Figure 1). Receiver apriori believe that θ distributed uniformly in Ω. Receiver makes action $a \in \Omega$ that effects him and all senders. The best action for receiver is θ but for senders best actions is different from θ. Hence the senders have incentives to present fake state of world. Due to they can collude, the solutions represented in (Ambrus and Takahashi, 2008), (Battaglini, 2002) and (Meyer et al., 2013) don't work here. In addition, receiver doesn't have any incentives to believe in state represented by senders.

On the Figure 1 sender 1 and sender 2 prefer any action inside red and blue circles respectively to the real state of the world. Clear that the action they both prefer to real state is inside intersection of these circles. And their best choice of desired action determine new combined bias $\vec{B}$. It might seem that the $\vec{B}$ should be a product of bargaining between two senders, but, as we will see, their incentives about selection of $\vec{B}$ is the same due to the choice of $\vec{B}$ determines new policy state. And even more surprising that their incentives coincide with incentives of the receiver.

Figure 1: Illustration of the model



Let's provide an equilibria. Generally, logic the same like in (Crawford and Sobel, 1982), but in the addition of interval where true state of the world locates, the senders should notice the receiver the line (new policy space) where true state is.

Proposition 1. *There is exist partial information revealed equilibrium:*

1. *Senders send their signals θ' and θ'' ($\theta' \neq \theta''$) s.t.*
   - *θ', θ'' and true state of the world lie on the one line.*
   - *Intersection of this line and feasible policy space determines new restricted policy space Δ.*
   - *New policy space divided to N intervals Δ_i ($\cup_{i=1}^{N}\Delta_i = \Delta$) and sender send any θ' from the same interval where true state θ is.*
2. *Receiver takes θ' and θ'', determine in which Δ_i they are (before that determining Δ) and makes action $a_i = \mathbb{E}[\theta|\theta \in \Delta_i]$*

Let's provide the expressions for senders and receiver utilities in such equilibrium.First of all let's determine borders of Δ_i like θ_{i-1} and θ_i. Due to θ uniformly distributed on Ω and, hence uniformly distributed of Δ too, we have:

$$a_i = \mathbb{E}[\theta|\theta \in [\theta_{i-1}, \theta_i]] = \frac{\theta_{i-1} + \theta_i}{2} \tag{3}$$

In $\vec{\theta_i}$ sender should be indifferent between actions $\vec{a_i}$ and $\vec{a_{i+1}}$, hence

$$(\vec{\theta_i} + \vec{B} - \vec{a_i})^2 = (\vec{\theta_i} + \vec{B} - \vec{a_{i+1}})^2 \tag{4}$$

$$\tag{5}$$

that gives us (since $\vec{a_i} \neq \vec{a_{i+1}}$ and $a_{i-1}, a_i, \theta_i, B$ are collinear)

$$\theta_i + B = \frac{a_i + a_{i+1}}{2} = \frac{\frac{\theta_{i-1}+\theta_i}{2} + \frac{\theta_i+\theta_{i+1}}{2}}{2} \tag{6}$$

$$\theta_{i+1} - \theta_i = \theta_i - \theta_{i-1} + 4B \tag{7}$$

Standard Cheap Talk model result – intervals increase.
Using obtained result and determining length of Δ like k we can find θ_1

$$\theta_1 + (\theta_2 - \theta_1) + \ldots (\theta_{i+1} - \theta_i) + \ldots + (k - \theta_{N-1}) = k \tag{8}$$

$$\theta_1 + (\theta_1 + 4B) + \ldots + (\theta_1 + 4B(N-1)) = k \tag{9}$$

$$\theta_1 = \frac{k - 2BN(N-1)}{N} \tag{10}$$

Finally, let's find expected utility of senders and receiver.
Expected utility of receiver:

$$\mathbb{E}U_R = -\sum_{i=1}^{N} \int_{\theta_{i-1}}^{\theta_i} (\frac{\theta_{i-1} + \theta_i}{2} - \theta)^2 \frac{1}{k} d\theta = \tag{11}$$

$$= - - \sum_{i=1}^{N} \frac{1}{3k} (\frac{\theta_{i-1} + \theta_i}{2} - \theta)^3 \Big|_{\theta_{i-1}}^{\theta_i} = \tag{12}$$

$$= -\frac{k^2}{12N^2} - \frac{B^2(N^2-1)}{3} \tag{13}$$

Expected utility of sender i:

$$\mathbb{E}U_{S_j} = -\sum_{i=1}^{N}\int_{\theta_{i-1}}^{\theta_i}(\overrightarrow{\frac{\theta_{i-1}+\theta_i}{2}} - \vec{\theta} + \vec{b_j})^2\frac{1}{k}d\theta = \tag{14}$$

$$= -\sum_{i=1}^{N}\int_{\theta_{i-1}}^{\theta_i}((\frac{\theta_{i-1}+\theta_i}{2} - (\theta + b_j cos\alpha))^2 + b_j^2 sin^2\alpha)\frac{1}{k}d\theta = \tag{15}$$

$$= -\sum_{i=1}^{N}\int_{\theta_{i-1}}^{\theta_i}((\frac{\theta_{i-1}+\theta_i}{2})^2 - 2\frac{\theta_{i-1}+\theta_i}{2}(\theta + b_j cos\alpha) + (\theta + b_j cos\alpha)^2 + b_j^2 sin^2\alpha = \tag{16}$$

$$= \mathbb{E}U_R - b_j^2 cos^2\alpha - b_j^2 sin^2\alpha - 2b_j cos\alpha \sum_{i=1}^{N}\cancelto{0}{\int_{\theta_{i-1}}^{\theta_i}(\left[\frac{\theta_{i-1}+\theta_i}{2} - \theta\right])}\frac{1}{k}d\theta = \tag{17}$$

$$= \mathbb{E}U_R - b_j^2 cos^2\alpha - b_j^2 sin^2\alpha \tag{18}$$

$$= -\frac{k^2}{12N^2} - \frac{B^2(N^2-1)}{3} - b_j^2 \tag{19}$$

$$\tag{20}$$

As we see, incentives of both senders and receiver are the same – to reduce B and k. Senders will chose B to minimize $\frac{k^2}{12N^2} + \frac{B^2(N^2-1)}{3}$. That is consequent of the quadratic form of loss-function. For another functional form results would be different.

Adaptation of the model to two firms and one buyer problem

Let's assume that one buyer wants to buy two products but doesn't sure how much each product he really need (true state of world θ on the Figure 2), but each of two sellers know what exact amount of each product he need. Not surprising, that both of the sellers want to sell for buyer more products that he really need, hence they have biasses b_1 and b_2 respectively. Having defined the necessary quantity of goods buyer buy both goods from only one seller. He randomly decides from which seller to buy – with probability $\frac{1}{2}$ for each seller. Hence both sellers have incentives to influence on the buyer decision.

Let's assume that both sellers have quadratic cost functions for each of the products. Hence, the profit maximization problem for firm j is (unconditional on the structure of the cheap talk):

$$\pi_j = \max_{q_1^j, q_2^j, b_1^j, b_2^j}\{(q_1^j + b_1^j)p_1 - c_1^j(q_1^j)^2 - \gamma_1^j(b_1^j)^2 + (q_2^j + b_2^j)p_2 - c_2^j(q_2^j)^2 - \gamma_2^j(b_2^j)^2\} \tag{21}$$

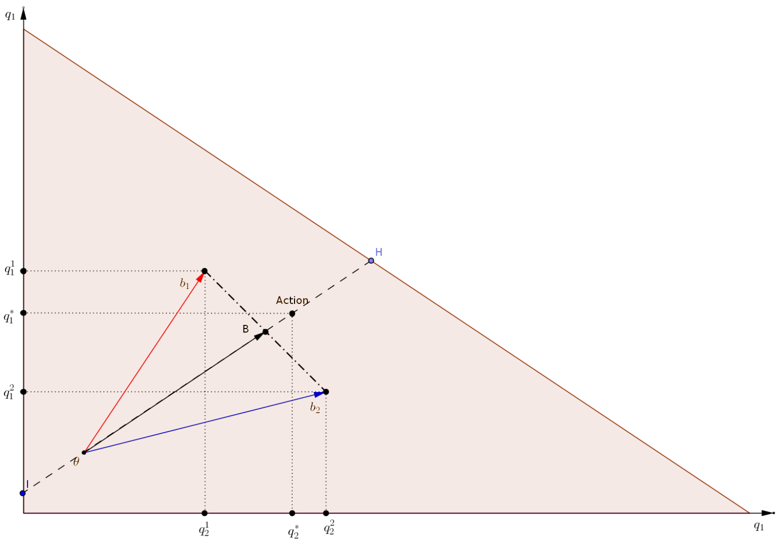
where c_i^j and γ_i^j are constants. Hence each firm j has the biases $b_i^{j\star} = \frac{p_i}{2\gamma_i^j}$ that doesn't depend on q. Later, each firm j has the maximization amount of each good i $q_i^j = \frac{p_i}{2c_i^j}$ (see Figure 2). In addition let's assume that $\frac{\gamma_1^1}{\gamma_2^1} \neq \frac{\gamma_1^2}{\gamma_2^2}$ that guarantee $b_1 \nparallel b_2$.

If buyer buy the amount (q_1^*, q_2^*), then losses from the optimal profit of each firm j for each product i is proportional to quadratic deviation q_i^* from q_i^j. Hence, the objective function of firm j is:

$$Loss_j = -(q_1^j - q_1^*)^2 - (q_2^j - q_2^*)^2 = |\vec{q^j} - \vec{q^*}|^2$$

What is exactly the objective function from my model. Hence results about fully information revealed equilibrium are valid for such interpretation of model as well.

Figure 2: Illustration of the model



Conclusions

In this paper I discussed novel version of two-dimensional and two-senders cheap talk game in which senders can't commit do not collude. The main results is I constructed partial information revealing equilibrium and showed that this model can be used for study of behaviour of agents in two seller one buyer and two goods problem when the buyer doesn't know what exactly quality of each product she really needs but sellers know. This model can be used for study the markets where products are to difficult to buyer could understand which quality she needs but withal the sellers are good informed about buyers' needs. That could be non-retail either B2B markets, for example market of corporative software or B2C markets, for example cosmetics market.

Extensions and further research

Seems like the results are valid for multidimensional and multi-sender case (the key point is the number of sellers should be not less that dimensionality) but I didn't have enough time to prove it accurately. It is in my future plans.

The second extension is using of models that are proposed by (Morgan and Stocken, 2003) and (Li and Madarász, 2008). In these models the receiver does not know the bias of the sender and the message is used to infer both the state and the bias of the sender. Such assumptions seems much more relevant to real live in cases when buyer is choosing from large number of sellers and surely cannot evaluate their incentives to lie and, hence, biases.

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