Markets with Search Frictions and Partially Informed Intermediary

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Abstract

The paper discusses markets with consumer's search frictions and partially informed intermediary. The main finding is the increase in the intermediary information might decrease the average quality of the product consumers purchase and decline in the total economic welfare and consumer surplus. The mechanism is if the intermediary makes better advise to consumers in average what product to explore first, all consumers have lower expectations about the next products and explore them less often, which decreases the quality of purchased good for consumers who got the wrong recommendation and might lead to reduction of the average quality of purchased products. The effect appears in the case of low search cost, which makes it especially important in the analysis of online search platforms.

JEL classification: D43, D83, L11, L13, L15.

Keywords: Consumer search, Information, Product heterogeneity, Intermediaries, Platforms, Welfare.

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Introduction

In many markets, consumers are initially uninformed about the quality of the available products. They may conduct a search to learn about product qualities, and in many cases these searches are facilitated by information intermediaries. For example, financial advisors provide information about insurance and investment products, physicians provide advice about appropriate treatments and drugs, and online platforms give consumers a ranking of sellers. As economic activities are increasingly connected through the Internet, consumers can have access to more products at lower search costs, but they also face a much larger set of sellers to choose from. Consumers are thus increasingly dependent on intermediaries to guide their search (in some deliberate order) for sellers and products. This has led to enormous commercial successes of Internet platforms such as Google, Amazon, and Expedia. In such circumstances, the importance of understanding how intermediaries influence the market increases.

This paper analyzes how platform's information about consumer preferences changes market outcomes. There are two main features of the model: first, with some probability, the platform observes the preferences of each consumer individually, and second, it proposes to each consumer personal order to visit firms. The key variable in the analysis is the probability that the platform knows the quality of the various goods to each consumer. In the special case in which this probability is one, the platform always recommends to the consumer to visit first the firm with the highest quality good. The main focus of this paper is more general case in which this probability is not one. As the probability increases (the platform has better information about consumers preferences), there are two countervailing forces. The direct effect is the platform recommends to higher mass of consumers to visit first the firm with the highest product quality, which increases the mean quality of the products, purchased by consumers. The indirect effect is consumers expect lower quality of next products, which decreases their incentives to search further and leads to decrease in the quality of purchased product for those consumers, preferences of which the platform does not know.

I demonstrate that better information leads to a decline in the quality of purchased products, consumer and total economic welfare in the case of low search cost. Though, in the case of high search cost, the result is opposite, and better information increases the mean quality of purchased products and welfare.

Related literature

The paper is related to ordered search literature. Arbatskaya (2007) started this branch with a discussion of homogeneous goods. Later Armstrong et al. (2009) and Zhou (2011) investigate ordered search with heterogenous product, in the framework of Wolinsky (1986) and Anderson and Renault (1999). See recent development by Parakhonyak and Titova (2018) and Ding and Zhang (2018). In the aforementioned papers, search ordered as either exogenous or identical to all consumers. The high search order capture a firm's prominence. The current paper contributes to the literature by allowing the search order to be consumer-specific (or based on consumer's taste). It better captures the business feature in the age of the Internet. Search engines and intermediaries often use "big data" to make an individual recommendation based on consumers' browsing and search history.

Armstrong (2017), Haan et al. (2018) and Choi et al. (2018) also study models with endogenous ordered search. Prior to search, a consumer receives information about the match quality of the products being sold by every firm. In particular, Choi et al. (2018) show that better information leads to higher equilibrium price, which is consistent with the finding in my model. However, my paper adds to the literature by showing the effect of the information on match quality and welfare, especially showing that the product's quality purchased by a consumer might be hurt by better information.

The paper is also related to the literature of information intermediary, which is started by Biglaiser (1993) and Lizzeri (1999). I contribute to them by introducing heterogeneous products and the ordered search settings. The large literature discusses the intermediary as a marketplace that sets fees for firms and ranks them by these fees, but the marketplace itself does not have any information about the quality of the good. Bright representatives of such approach are Athey and Ellison (2011), Chen and He (2011) and Teh and Wright (2018). In these models, firms are sorted by paid fees, which in equilibria determines by the heterogeneous quality of the good, known by the firms only, where only in the latter paper the order of firms is consumer specific. As a result, consumers explore products in a given order, determined by the platform's ranking mechanism. I contribute to this branch of literature by discussing the case of a strategic partially informed platform, which determines by itself the order of search based on incomplete information about consumers' preferences.

There is literature on the relation between information and pricing. See Lewis and Sappington (1994), Anderson and Renault (2006) and Anderson and Renault (2000). Recently, Roesler and Szentes (2017) use the newly developed information design technique a la Kamenica and Gentzkow (2011) and Gentzkow and Kamenica (2016) to further explore this topic. The main message is that more information can be bad because the monopolist can better price discriminate against the consumer. See Boleslavsky et al. (2018) and Armstrong and Zhou (2019) to the effect of information in competition model, and Dogan and Hu (2018) in a context of consumer search. I contribute to this literature by showing another channel on how more information can hurt welfare. Richer information helps the platform to make a better recommendation for consumers, which reduces consumers' incentives to search. As a result, consumers' consideration set shrinks, which might lead to a reduction in the quality purchased product.

The Model

The economy consists of a monopolistic platform, two firms labeled A and B, and a continuum of consumers with a measure of one. The platform is the only way for firms and consumers to meet. Firms produce horizontally differentiated products incurring a constant marginal cost normalized to zero. The platform steers consumers' search process by providing each consumer individual order to visit firms, based on the consumers' preferences to firms' products. For each consumer the platform observes his preferences with probability q. The platform charges firms ad valorem fee proportional to the transaction price. Firms maximize their revenue by setting the price conditional on the platform's ranking mechanism.

As in Wolinsky (1986), a consumer must incur a search cost s to learn the price charged by any particular firm and its product quality. Consumers search sequentially with costless recall. If consumer j buys product of firm i at price p_i after visiting k firms, he receives utility

$$U_{j,i} = u_{j,i} - k \cdot s = \epsilon_{j,i} - p_i - k \cdot s$$

where $\epsilon_{j,i}$ is the realization of a random variable with twice differentiable cdf $F(\epsilon)$, pdf $f(\epsilon)$ and support $[\underline{\epsilon}, \bar{\epsilon}]$. The term $\epsilon_{i,j}$ can be interpreted as firm i product quality for consumer j, and is assumed to be independent across consumers and firms. Each consumer can buy at most one unit of product. The consumer's outside option is low enough to encourage him to purchase the good for any price, which leads to full market coverage.

The market interaction proceeds as follows. First, all participants learn q, the probability that the platform observes the quality $\epsilon_{j,A}$ and $\epsilon_{j,B}$ for a given consumer j. Then firms simultaneously set prices p_A and p_B, conditional on q. After that the platform provides individual search order for each consumer as follows: if the platform observes the quality of products for a given consumer, it recommends this consumer visit the firm with higher quality first; if the platform does not observe the quality, it makes the recommendation at random with equal probabilities. In later constructed equilibrium, it is optimal for a revenue-maximizing platform to propose to consumers such order of the search, and for utility optimizing consumers to follow this recommendation. After that consumers form the expectations about the prices and follow an ordered sequential search process with search cost s and costless recall. Thereafter the consumer buys utility maximizing product among ones he explored and pays the price.

Analysis

In this section, I derive the perfect Bayesian equilibrium by means of backward induction. Because of ex-ante symmetry of consumers and firms, I concentrate on the analysis of symmetric equilibrium when firms charge equal prices and get equal demand. First, I derive the consumer's optimal search rule and use it to obtain the demand functions and optimal pricing on the market. Thereafter I show that the price and the revenue of each firm increase with better information (higher q). This verifies, that for the revenue-maximizing platform, charging firms ad valorem fee proportional to the price, it is always optimal to use the entire information it has, i.e. if the platform observes products qualities for a given consumer, the platform always recommends him to visit first the firm with better product. Lastly, in the section Information effect on the market I use these result to analyze the effects of information on the quality of the product consumer purchases and welfare of market participants.

Consumers' search

For each consumer, the platform observes the quality of goods with a probability q. The consumer does not know whether the platform observes his preferences. Hence each consumer uses the law of total probability, and expects that the first firm in the proposed searching order provides the better product with a probability $q + \frac{1-q}{2} = \frac{1+q}{2}$. Due to low enough outside option, the consumer always explores the first firm. If both firms charge the same price and the product of first visited firm has quality z, net benefit from visiting the second one is derived in Equation 1 as $h(z)$.

$$h(z) = \int_{z}^{\bar{\epsilon}} (\epsilon - z) f_{\epsilon|z}(\epsilon) d\epsilon \ , \tag{1}$$

where $f_{\epsilon|z}(\epsilon)$, described in Lemma 1, is the predicted distribution of the second visited firm's product quality, conditional on the observed quality of the good provided by the first firm. In the special case of $q = 0$, the platform does not know anything about consumers' preferences and shows products at random. Hence the firms' goods qualities are uncorrelated, and the consumer simply expects $f_{\epsilon}(\epsilon)$ to be a distribution of the second good's quality. In such a case the model is reduced to Wolinsky (1986) model. If $q > 0$, the platform observes the quality of products for some consumers, and steers them to visit first the firm with a better product. That makes the quality of the second visited firm's product to be correlated with the quality of the first visited one. The higher q is, and the higher z the consumer observes in the first firm, then firmer his beliefs that the second product is the worse one, which decreases expectation of the second product's quality and the incentive to search further.

Lemma 1. *In the region $\epsilon > z$, $f_{\epsilon|z}$ can be expressed as:*

$$f_{\epsilon|z}(\epsilon) = Pr(\epsilon > z|z) \cdot f(\epsilon|z, \epsilon > z) = \frac{\frac{1-q}{2}(1 - F(z))}{\frac{1-q}{2}(1 - F(z)) + \frac{1+q}{2}F(z)} \cdot \frac{f(\epsilon)}{1 - F(z)}$$

The consumer's reservation value x, which solves the equation $s = h(x)$, is the product quality such that the benefit from sampling one more product equals the search cost. If there is no difference in goods prices, then the consumer's optimal strategy is to stop searching if good's quality is higher than x. Otherwise, the consumer should explore the second product and purchase the better one. The next two lemmas discuss how x depends on market parameters.

Lemma 2 shows, that in accordance with the classic result of Wolinsky (1986), if the search cost increases, it becomes less profitable to explore the next product. As a result, x decreases.

Lemma 2. *The reservation value x is a decreasing function of the search cost s.*

Lemma 3 highlights the idea that if the platform has better information (q increased), consumers have lower incentives to search because the first explored product is the better one with higher probability.

Lemma 3. *The reservation value x is a decreasing function of q.*

The Demand

For the rest of the paper, I presume that Assumption 1 is satisfied.

Assumption 1. *$f(\cdot)$, $F(\cdot)$ and $1 - F(\cdot)$ are log-concave and continuously differentiable functions.*

Due to ex-ante symmetry of firms, they have similar demand functions. Without loss of generality I derive the demand function for firm A. Suppose firm A sets price p_A, and firm B sets the equilibrium price p^*. Define $\Delta = p_A - p^*$. If the consumer visits firm A first, observe the price p_A, and expects that the firm B charges the price p^*, he will stop his search and buy if and only if $\epsilon_A - p_A > x - p^*$, or, same, $\epsilon_A > x + \Delta$.

The Figure 1 summarizes the demand of firm A. The shaded area on Figure 1a shows the demand of firm A if it is visited first. If so, it gets all consumers for whom $\epsilon_A > x + \Delta$ (region I) because these consumers buy from firm A immediately and don't search further. Among the consumers who sample both firms, firm A gets consumers for whom $\epsilon_A > \epsilon_B + \Delta$ (region II). The shaded area on Figure 1b shows the demand of firm A if it is visited second. If so, it gets all consumers for whom $\epsilon_B < x$ and $\epsilon_A > \epsilon_B + \Delta$ because these consumers, at first, don't stop on firm B and search further and, at second, value the firm A product higher than firm B product. According to the fact that each firm is shown first to the half of consumers because ϵ_A, ϵ_B and q are independent, we can express the firm A demand as:

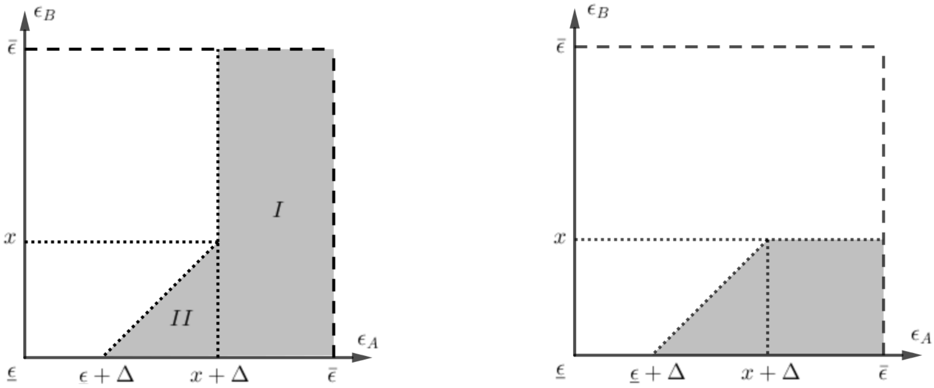
$$D(p_A, p^*) = \frac{1 + F(x)}{2}(1 - F(x + \Delta)) + \int_{\underline{\epsilon}}^{x+\Delta} f(\epsilon)F(\epsilon - \Delta)d\epsilon \tag{2}$$

It's important to note that demand depends on q only through x.

Assuming that the search cost s such that $x \in [\underline{\epsilon}, \bar{\epsilon}]$ $\forall q \in [0, 1]$, I now turn to the analysis of the equilibrium. Note that in equilibrium $D(p^*, p^*) = \frac{1}{2}$, i.e. every consumer buys exactly one good.

Figure 1: Firm's demand

(a) Demand if visited first *(b) Demand if visited second*



Lemma 4. *Under Assumption 1, there exists a unique symmetric price equilibrium:*

$$p_A = p_B = p^* = -\frac{D(p,p^*)}{\frac{\partial D}{\partial p}(p,p^*)}\Big|_{p=p^*} = \frac{1}{(1-F(x))f(x) + 2\int\limits_{\underline{\epsilon}}^{x} f^2(\epsilon)d\epsilon} \tag{3}$$

As Quint (2014) showed, log-concavity of $f(\epsilon), F(\epsilon)$ and $1 - F(\epsilon)$ guarantees that the demand is log-concave in price, hence $-\frac{D(p,p^*)}{\frac{\partial D}{\partial p}(p,p^*)}$ is decreasing in p, what guarantees the unique solution of Equation 3.

As shown in lemma 4, the price is the function of x, which, in turn, is a function of search cost and information the platform has. If s increases, then, in accordance with the classic result of Wolinsky (1986), consumers search less often. If q increases, rational consumers expect that the platform makes better ranking of firms, and they have lower expectations of the second firm product's quality, which decreases the incentives to search further. If consumers search less, the market competitiveness declines and the firms can raise prices. These results are summarized in the next lemma.

Lemma 5. *Under Assumption 1, p^* is an increasing function of s and q.*

The equilibrium price is an increasing function of q, while the firms' demand is price non sensitive and constant due to low enough outside option and full market coverage. That leads to increase in the firms' profits if the platform uses better information for ranking. Hence, as the platform charges firms ad valorem fee proportional to the price, the platform's revenue is an increasing function of q, which guarantees it is optimal for the platform to use for ranking the entire information it has i.e. always recommend the consumer to visit first the firm with better product if the platform observes products qualities for this consumer.

Information effect on the market

Information effect on the quality of purchased product

In this section I address the main question of the paper – how the quality of the product that consumer purchases varies with information the platform has. On the one hand, as platform has better information and q increases, the product that consumer explores first, is the better one with a higher probability, which increases the expected quality of the consumed product. On the other hand, according to lemma 3, as q increases, consumers have lower incentive to search, and more often do not visit the second firm and simply purchase the first investigated product. Hence, consumers, have smaller chances to choose the better product. That decreases the expected quality of the consumed product. The analysis in this chapter is designed to resolve ambiguity in the combined effect.

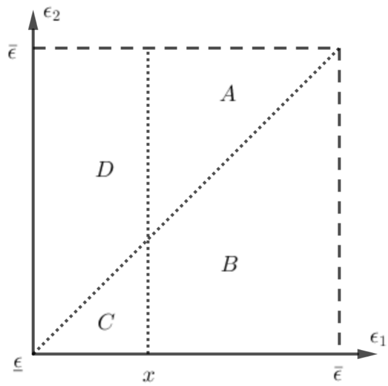
The expected quality of the product consumer leaves the market with can be found as described in Equation 4, where $P(q)$ is the probability that consumer purchases the worse product. The first term in the sum stands for the expected quality of the better product, multiplied by the probability the consumer purchases the better product. The second term stands for the expected quality of the worse product, multiplied by the probability the consumer purchases the worse product.

$$\begin{aligned} V(q) &= (1 - P(q)) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \; dF_{max\{\epsilon_1,\epsilon_2\}}(\epsilon) + P(q) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \; dF_{min\{\epsilon_1,\epsilon_2\}}(x) = \\ &= (1 - P(q)) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \; d\left[F(\epsilon)^2\right] + P(q) \cdot \int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon \; d\big[1 - (1 - F(\epsilon))^2\big] \end{aligned} \tag{4}$$

Notice that the expected quality of the consumed good is a decreasing function of $P(q)$ due to both integrals in Equation 5 are constants and stand for the quality of the better and the worse product among two respectively. In the subsequent analysis I discuss how $P(q)$, the probability that the consumer purchases the worse product, varies with information the platform has.

Figure 2 illustrates probabilities of different search outcomes. ϵ_1 and ϵ_2 stand for the qualities of products of the first and the second firm in consumers' search order respectively. Regions B and C have combined area $\frac{1+q}{2}$, and illustrate the mass of consumers who find the better product in the first visited firm ($\epsilon_1 > \epsilon_2$). Accordingly, the regions A and D with combined area $\frac{1-q}{2}$ depict the mass of consumers who visit first the firm with the worse product ($\epsilon_1 < \epsilon_2$). In regions B and C the product of the first visited firm is the better one, and consumer purchases this product. In region D the product quality of the first visited firm is below x, hence consumer visits the second firm and purchases the better product. In region A the product quality of the first visited firm is above x, hence the consumer decides do not search further. But this product is the worse one among two. As a result, the only region where consumer leaves the market with the worse product is the region A. As a result, the probability that consumer purchases the worse product is represented as the region A in Figure 2 and is indicated in Equation 5

Figure 2: Probabilities of the search outcomes



$$P(q) = (1-q) \cdot \frac{\left(1-F(x)\right)^2}{2} \tag{5}$$

Differentiating the Equation 5 we get:

$$\frac{\partial P(q)}{\partial q} = -\frac{1-F(x)}{2} \cdot (1 - F(x) + 2(1-q)f(x)\frac{\partial x}{\partial q}) \tag{6}$$

Hence the indirect effect of decreased search incentives might outweigh the direct effect of better recommendation, hurting the realized quality of a purchased product if consumers decrease the search intensity a lot enough in response to increased q:

$$\frac{\partial P(q)}{\partial q} > 0 \Leftrightarrow \frac{\partial x}{\partial q} < -\frac{1-F(x)}{2(1-q)f(x)} \tag{7}$$

Proposition 1. *Under Assumption 1, for any $\hat{q} \in [0,1]$ exists search cost $\hat{s}$ s.t.*

1. *for any s s.t. $0 < s < \hat{s}$, the expected quality of consumed good decreases in q.*

2. *for any s s.t. $s > \hat{s}$, the expected quality of consumed good increases in q.*

Information effect on welfare

Due to low enough outside option, all consumers search at least once, and search the second time only if the quality of the first product is below x. The first explored product is the better one for $\frac{1+q}{2}$ portion of consumers, while for the rest mass of consumers $\frac{1-q}{2}$, the first product is the worse one. As a result, the level of consumer's search expenditures can be expressed as shown in Equation 8.

$$\begin{aligned} SE(q) &= s \cdot \Big(1 + \frac{1+q}{2} \cdot F_{max\{\epsilon_1,\epsilon_2\}}(x) + \frac{1-q}{2} \cdot F_{min\{\epsilon_1,\epsilon_2\}}(x)\Big) \\ &= s \cdot \Big(1 + \frac{1+q}{2} \cdot F(x)^2 + \frac{1-q}{2} \cdot \big(1 - (1 - F(x))^2\big)\Big) \end{aligned} \tag{8}$$

When the platform has better information about consumers' preferences, there are two effects. First, consumers expect lower quality of the second product. Second, there is higher probability that consumers get the better product at the first visited firm. Both effects lower consumers' incentives to search. This result is summarized in the next lemma.

Lemma 6. *Search expenditures is a decreasing function of q.*

Define the Total Surplus in the Equation 9 as

$$TS(q) = V(q) - SE(q) \tag{9}$$

Lemma 6 shows that $SE(q)$ is decreasing in q. Proposition 1 gives condition under which $V(q)$ is decreasing and decreasing in q. Hence the net effect may be ambiguous. Proposition 2 provides conditions under which Total Surplus is decreasing and increasing in q.

Proposition 2. *Under Assumption 1, for any $\check{q} \in [0,1]$, exists $\check{s}(\check{q})$ s.t.*

1. *for any s s.t. $0 \leq s < \check{s}$, the total economic welfare decreases in q.*

2. *for any s s.t. $s > \hat{s}$, the total economic welfare increases in q.*

where $\hat{s}$ is defined in Proposition 1

For low enough search cost $(s < \check{s})$, as q increases, the savings in search expenditures are comparatively small and outweighed by the reduction in the quality of the good consumers purchase. If search cost is above $\hat{s}$, defined in Proposition 1, the quality of purchased good increases with q, while searching expenditures decreases. In such case both effects increase the total economic welfare. In the intermediate case $\check{s} < s < \hat{s}$ the effect might be non-monotonic and requires extra analysis.

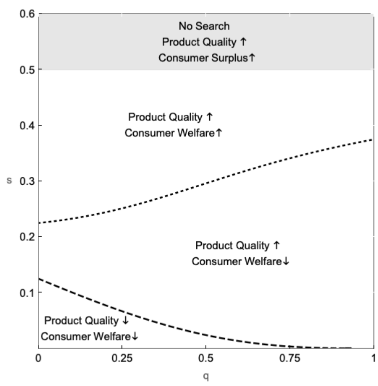
According to Proposition 2 and Lemma 5, for high enough search cost, the Total Surplus and equilibrium price both increases in q. Therefore, the effect of increased q on the Consumer Surplus, defined in Equation 10, is ambiguous in the general case and depends on the levels of s and q.

$$CS(q) = V(q) - SE(q) - p^*(q) \tag{10}$$

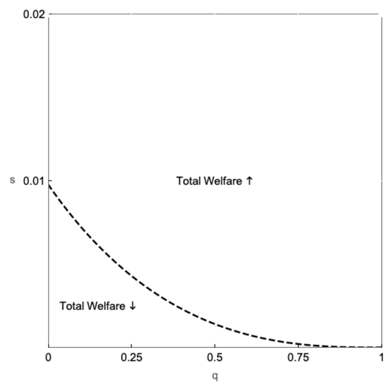
Figure 3 illustrates the compositions of s and q which lead to different dynamics of market outcomes in case of uniform distribution ϵ. If $s > \frac{1}{2}$ then $x < 0 = \underline{\epsilon}$, hence consumers never explore the second product. Therefore, if q increases, price does not change and Consumer Surplus increases the same as the Total Surplus. If $0 < s < \frac{1}{2}$ and $0 < q < 1$, then $0 < x < 1$ and consumers who explored the product with quality below x, search the second good as well. Consistent with Proposition 1, for low enough search cost s, the expected quality of the consumed product is decreasing with q. Also, according to the Figure 3, the Consumer Surplus increases with q for high enough search cost because the effects of increased quality of consumed good and decreased search expenditures outweigh the increase in price.

Figure 3: The effect of levels of search costs s and platform's information q on market outcomes. The case of uniform distribution.

(a) The effect on product's quality and consumers' welfare



(b) The effect on Total economic welfare



Concluding remark

The paper discusses markets with consumer's search frictions and a partially informed intermediary. The main finding is when the intermediary has better information about consumers' preferences, the average quality of the product consumers purchase can decrease. The intuition behind the mechanism is if the intermediary has better information and makes better advice to consumers about which product to explore first, consumers have lower expectations about the next products quality and explore them less often, which decreases the number of explored products and the quality of a chosen one. Also, I show that consumers and the whole economy can be benefited or hurt if the intermediary knows more information about the preferences and can better manipulate the order in which consumers explore products. The actual effect depends on the search costs.

One of the possible extensions of the model for further research is a decreasing intermediary's monopoly power, for example, by introducing the competition between intermediaries. That will force the intermediary to incorporate the consumers' surplus in the its objective function . Especially important that, in that case, the intermediary may prefer not to use all the information it has about consumers' preferences and steers the search order to keep the firm's incentives to not increase prices too much.

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Appendices

Proof. of lemma 1

Suppose that ϵ_1, ϵ_2 and ξ are mutually independent, where $Pr(\xi = 1) = 1 - Pr(\xi = 0) = \frac{1+q}{2}$ and ϵ_1, ϵ_2 are identically distributed with density $f(\epsilon)$. Let $V = min\{\epsilon_1, \epsilon_2\}$ and $W = max\{\epsilon_1, \epsilon_2\}$ and define $X = \xi \cdot V + (1-\xi) \cdot W$, $Y = (1-\xi) \cdot V + \xi \cdot W$. I seek for $f_{Y|X}(y|x)$ on the region where $y > x$.

Step 1. Joint density for (V, W)

The joint cdf for (V, W) for $w \geq v$ given by:

$$
\begin{aligned}
F_{V,W}(v, w) &= Pr(V \leq v, W \leq w) \\
&= Pr(\epsilon_1 \leq v, v < \epsilon_2 \leq w) + Pr(v < \epsilon_1 \leq w, \epsilon_2 \leq v) + Pr(\epsilon_1 \leq v, \epsilon_2 \leq v) = \\
&= 2F(v)[F(u) - F(v)] + F(v)^2
\end{aligned}
$$

So the density is given by:

$$
f_{V,W}(v, w) = \frac{d^2 F_{V,W}(v, w)}{dvdw} = 2f(v)f(w), \quad for\ v \leq w
$$

Step 2. Joint density for (X, Y)

X and Y are functions of (V, W, ξ), so I first derive the density for $(X, Y, \xi) = g(V, W, \xi)$. It follows that $g^{-1}(x, y, \xi) = (\xi x + (1-\xi)y, (1-\xi)x + \xi y, q)$ with Jacobian $J(x, y, \xi) = \begin{pmatrix} \xi & 1-\xi & x-y \\ 1-\xi & \xi & y-x \\ 0 & 0 & 1 \end{pmatrix}$,

so $|detJ(x, y, \xi)| = |\xi^2 - (1-\xi)^2| \cdot 1 = |\xi - (1-\xi)| = |1 - 2\xi| = 1$

Hence the density for (X, Y, ξ) is given by

$f_{X,Y,\xi}(x, y, \xi) = f_{V,W}(\xi x + (1-\xi)y, (1-\xi)x + \xi y) \cdot f_\xi(\xi)$

where I used that ξ is independent of (ϵ_1, ϵ_2) and therefore also independent of (V, W). Next I obtain the density for (X, Y) by integrating out ξ.

$f_{X,Y}(x, y) = \frac{1+q}{2} f_{V,W}(x, y) + \frac{1-q}{2} f_{V,W}(y, x)$.

It follows that $f_{X,Y}(x, y)\mathbb{I}_{x<y} = \frac{1+q}{2} f_{V,W}(x, y)\mathbb{I}_{x<y}$ because $f_{V,W}(y, x) = 0$ for $x < y$.

Step 3. Marginal density for X.

The density for X is given by:

$$
\begin{aligned}
f_X(x) &= \int_{\underline{\epsilon}}^{\bar{\epsilon}} f_{X,Y}(x,y) = \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\frac{1+q}{2} f_{V,W}(x,y) + \frac{1-q}{2} f_{V,W}(y,x) \right) dy = \\
&= \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left(\frac{1+q}{2} 2f(x)f(y)\mathbb{I}_{x\leq y} + \frac{1-q}{2} 2f(y)f(x)\mathbb{I}_{x>y} \right) dy = \\
&= 2f(x) \left(\frac{1+q}{2} \int_{x}^{\bar{\epsilon}} f(y)dy + \frac{1-q}{2} \int_{\underline{\epsilon}}^{x} f(y)dy \right) = \\
&= 2f(x) \left(\frac{1+q}{2} + qF(x) \right)
\end{aligned}
$$

Step 4. Conditional density of Y given X.

Finally for $y > x$ we have

$$
\begin{aligned}
f_{Y|X}(y|x) &= \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{\frac{1+q}{2} f_{V,W}(x,y)}{2f(x)\left(\frac{1+q}{2} + qF(x)\right)} = \\
&= \frac{\frac{1+q}{2} 2f(x)f(y)}{2f(x)\left(\frac{1+q}{2} + qF(x)\right)} = \frac{\frac{1+q}{2}(1-F(x))}{\frac{1+q}{2}(1-F(x)) + \frac{1-q}{2}F(x)} \cdot \frac{f(y)}{1-F(x)}
\end{aligned}
$$

Q.E.D.

Proof. of lemma 2

From Equation 1 and definition of x we have:

$$
\begin{aligned}
s &= \int_{x}^{\bar{\epsilon}} (\epsilon - x) f_{\epsilon|x}(\epsilon)d\epsilon \Rightarrow s = \frac{\frac{1-q}{2}(1-F(x))}{\frac{1-q}{2}(1-F(x)) + \frac{1+q}{2}F(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon)d\epsilon \Rightarrow \\
s &= \frac{1-q}{1-q+2qF(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon)d\epsilon \Rightarrow \\
1 &= \frac{\partial}{\partial s} \left(\frac{1-q}{1-q+2qF(x)} \cdot \int_{x}^{\bar{\epsilon}} (\epsilon - x) f(\epsilon)d\epsilon \right) \Rightarrow \\
1 &= -\frac{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int_x^{\bar{\epsilon}} f(y)(y-x)\,dy\right)}{(1-q+2qF(x))^2} \cdot \frac{\partial x}{\partial s} \Rightarrow
\end{aligned}
$$

$$\frac{\partial x}{\partial s} = -\frac{(1-q+2qF(x))^2}{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)\,d\epsilon\right)} < 0 \ \forall \ (q,x,s) \in [0,1] \times [\underline{\epsilon}, \bar{\epsilon}]$$

The alternative expression is $\frac{\partial x}{\partial s} = -\frac{1-q+2qF(x)}{2qsf(x)+(1-q)(1-F(x))}$ Q.E.D.

Proof. *of Lemma* 3

From Equation 1 and definition of x we have:

$$s = \int\limits_x^{\bar{\epsilon}}(\epsilon - x)f_{\epsilon|x}(\epsilon)d\epsilon \Rightarrow s = \frac{\frac{1-q}{2}(1-F(x))}{\frac{1-q}{2}(1-F(x)) + \frac{1+q}{2}F(x)} \cdot \int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon \Rightarrow$$

$$s = \frac{1-q}{1-q+2qF(x)} \cdot \int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon \Rightarrow$$

$$0 = \frac{\partial}{\partial q}\left(\frac{1-q}{1-q+2qF(x)} \cdot \int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon\right) \Rightarrow$$

$$0 = -\frac{2F(x)\int\limits_x^{\bar{\epsilon}} f(\epsilon)(\epsilon - x)\,d\epsilon}{(1-q+2qF(x))^2}$$

$$-\frac{(1-q)\left((1-F(x))(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}} f(\epsilon)(\epsilon - x)\,d\epsilon\right)}{(1-q+2qF(x))^2} \cdot \frac{\partial x}{\partial q} \Rightarrow$$

$$\frac{\partial x}{\partial q} = -\frac{2F(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)\,d\epsilon}{(1-q)\left((1-F(x))\,(1-q+2qF(x)) + 2qf(x)\int\limits_x^{\bar{\epsilon}}(\epsilon - x)f(\epsilon)d\epsilon\right)} < 0 \ \forall \ (q,x) \in [0,1] \times [\underline{\epsilon}, \bar{\epsilon}]$$

The alternative expression is $\frac{\partial x}{\partial q} = -\frac{2sF(x)}{(1-q)(2qsf(x)+(1-q)(1-F(x)))}$ Q.E.D.

Proof. of Proposition 1

The proof consists of two steps. On the first step I show that $\frac{\partial PQ}{\partial q}$ is negative in the left neighborhood of $\bar{\epsilon}$ (low search cost case) for any arbitrary distribution $F(\epsilon)$, which proves the first part of the proposition. On the second step I show that under the Assumption of log concave distribution $F(\epsilon)$, $\frac{\partial PQ}{\partial q}$ is positive for $x = \underline{\epsilon}$ (high search cost case) and the equation $\frac{\partial PQ}{\partial q} = 0$ has unique solution. This fact, together with continuity of $\frac{\partial PQ}{\partial q}$ proves the second part of the proposition.

Step 1. After differentiating Equation 5:

$$\frac{\partial Pr(q)}{\partial q} = \frac{1-F(x)}{2}\Big(1 - F(x) + 2(1-q)f(x)\cdot\frac{\partial x}{\partial q}\Big)$$

Using the expression for $\frac{\partial x}{\partial q}$ derived in the section 5, I conclude that

$$\frac{\partial Pr(q)}{\partial q} = \frac{(1-F(x))}{4qsf(x) + 2(1-q)(1-F(x))}\cdot\Big((1-q)(1-F(x))^2 - 2sf(x)((2+q)F(x) - q)\Big)$$

The fraction part of above expression is positive $\forall\ (q,x,s) \in [0,1]\times[\underline{\epsilon},\bar{\epsilon}]\times[0,\infty)$. Hence the sign of $\frac{\partial Pr(q)}{\partial q}$ depends only on the sign of the second part of expression, which can be expressed as a Taylor series approximation in the left neighborhood of $\bar{\epsilon}$ as:

$$(1-q)(1-F(x))^2 - 2sf(x)((2+q)F(x) - q) = -\frac{2(1-q)^2 f(\bar{\epsilon})^2}{1+q}\cdot(x-\bar{\epsilon})^2 < 0$$

As a result, $\frac{\partial Pr(q)}{\partial q}$ is negative in the left neighborhood of $\bar{\epsilon}$, and due to continuity of $\frac{\partial Pr(q)}{\partial q}$, there exists $\hat{s}(\hat{q})$ s.t. for any s s.t. $0 < s < \hat{s}(\hat{q})$, $\frac{\partial Pr(q)}{\partial q}$ is negative and, therefore, the expected quality of consumed good decreases in q.

Step 2. Using the expression for $\frac{\partial x}{\partial q}$ derived in the proof of Lemma 3, I conclude that $\frac{\partial x}{\partial q}\Big|_{x=\underline{\epsilon}} = 0$. Hence, $\frac{\partial Pr(q)}{\partial q}\Big|_{x=\underline{\epsilon}} = \frac{1-0}{2}\Big(1 - 0 + 2(1-q)f(x)\cdot 0\Big) = \frac{1}{2} > 0$.

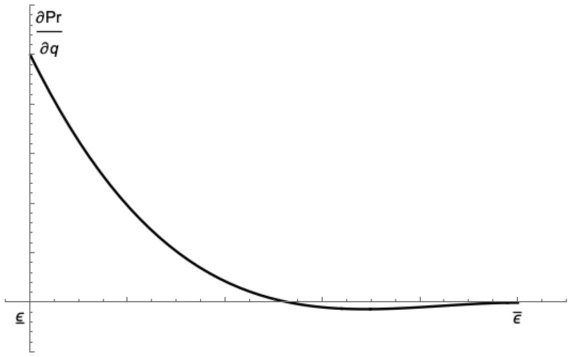
Next I show that $\frac{\partial}{\partial x}\frac{\partial Pr(q)}{\partial q}$ is negative everywhere $\frac{\partial Pr(q)}{\partial q} = 0$ if $1 - F(\cdot)$ is strictly log-concave.

$$\begin{aligned}\frac{\partial}{\partial x}\frac{\partial Pr(q)}{\partial q} &= -\frac{(1-F(x))\left(f(x)^2 + (1-F(x))f'(x)\right)}{2f(x)} \\ &= -\frac{(1-F(x))\left((1-F(x)')^2 - (1-F(x))(1-F(x))''\right)}{2f(x)} < 0\ \forall x \in [\underline{\epsilon},\bar{\epsilon}]\end{aligned}$$

As a result, $\frac{\partial Pr(q)}{\partial q}$ is positive at $\underline{\epsilon}$, negative at left neighborhood of $\bar{\epsilon}$ and has a negative derivative at any point of where it equals zero. Based on that I conclude $\frac{\partial Pr(q)}{\partial q} = 0$ has a unique solution. The example of $\frac{\partial Pr(q)}{\partial q}$ for $q = \frac{1}{8}$ and $\epsilon \sim U[0,1]$ is represented on the figure below.

Q.E.D.

Proof. of Proposition 2



Differentiating Equation 9 with accounting to Equation 4 and Equation 8 gives:

$$\frac{\partial TotalSurplus}{\partial q} = s(1-F(z))F(z)\left((1-F(z))\int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon f(\epsilon)(2F(\epsilon)-1)\,d\epsilon + 1\right) -$$

$$- f(z)\left(2qsF(z) + (1-q)s + 2(1-q)(1-F(z))\int_{\underline{\epsilon}}^{\bar{\epsilon}} \epsilon f(\epsilon)(2F(\epsilon)-1)\,d\epsilon\right) x'(q)$$

Q.E.D.

Proof. of Lemma **??**

After differentiating Equation 5 we have:

$$\frac{\partial Pr(q)}{\partial q} = \frac{1-F(x)}{2}\Big(1 - F(x) + 2(1-q)f(x)\cdot\frac{\partial x}{\partial q}\Big)$$

Hence, $\frac{\partial Pr(q)}{\partial q} < 0$ iff $\frac{\partial x}{\partial q} < -\frac{1-F(x)}{2(1-q)f(x)}$ Q.E.D.

Proof. of Lemma 5

After differentiating Equation 3 and accounting for log-concavity of $1-F(x)$, we have:

$$\frac{\partial p^*}{\partial q} = -\frac{f(x)^2 + (1-F(x))f'(x)}{\left((1-F(x))f(x) + 2\int_{\underline{\epsilon}}^{x} f(\epsilon)^2 d\epsilon\right)^2} = -\frac{\big((1-F(x))'\big)^2 + \big(1-F(x)\big)\big(1-F(x)\big)''}{\left((1-F(x))f(x) + 2\int_{\underline{\epsilon}}^{x} f(\epsilon)^2 d\epsilon\right)^2} < 0$$

Q.E.D.

Proof. of Lemma 6

After differentiating Equation 8 we have:

$$\frac{\partial SE}{\partial q} = \underbrace{-s(1-F(x))F(x)}_{<0} + \underbrace{f(x)(1-q+2qF(x))}_{>0}\cdot\underbrace{\frac{\partial x}{\partial q}}_{<0} < 0$$

Q.E.D.

Proof. of Proposition 2

If $s > \hat{s}$, then, according to Proposition 1, the quality of the purchased good is increasing in q. Also, according to Lemma 6, search expenditures is decreasing in q. Hence, the total economic welfare, defined in Equation 9 is increasing in q.

To show that Total economic welfare is decreasing for low enough search cost (or, same, high enough x), I show that the derivative of the total welfare with respect to x is negative in the left neighborhood of $x = \bar{\epsilon}$ by expanding it in Taylor series.

$$\frac{\partial TS}{\partial q} = \left(12(1-q^2)f\left(\bar{\epsilon}\right)^4 \int\limits_{\underline{\epsilon}}^{\bar{\epsilon}} 2\epsilon f(\epsilon)(2F(\epsilon)-1)d\epsilon\right) \cdot \left(x-\bar{\epsilon}\right)^3 + \bar{o}(\left(x-\bar{\epsilon}\right)^3) \leq 0 \quad \forall x < \bar{\epsilon}, 0 \leq q \leq 1$$

Q.E.D.