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**Graph Neural Networks for Stock Portfolio Optimization**

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## Introduction

From the very beginning, stock market analysis has been focused on two complementary issues: stock forecasting and stock portfolio allocation. If you can correctly predict the future value of stocks, then the problem of portfolio allocation becomes trivial, and vice versa, if you abandon the prediction task and succumb to expectations based on past performance, then the task of stock allocation under uncertainty becomes paramount.

For most of the time, researchers preferred to focus on one of these two problems, limiting the scale of the problem and the possible number of degrees of freedom. In this paper I will try to combine both types of analysis using current developments of Machine Learning, Graph Theory and Neural Networks. In the first part, I will overview current state of research in applying graph models to the tasks of stocks forecasting and portfolio allocation.

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## Literature Overview

The question of wealth allocation preoccupied people minds long before the invention of stock exchanges and can be traced to the beginning of the history. «*Let every man divide his money into three parts, and invest a third in land, a third in business, and a third let him keep by him in reserve*» [Talmud]. The development of international trade starting from the Renaissance provoked people to think about risks and calculate them on par with promising returns. «*Antonio: Believe me, no: I thank my fortune for it, My ventures are not in one bottom trusted, Nor to one place; nor is my whole estate Upon the fortune of this present year: Therefore my merchandise makes me not sad*» [Merchant of Venice, Act I, Scene 1]. However, the academic acclaim of the question can be attributed to Harry Markowitz's famous article (Markowitz 1952). Markowitz's theory gave birth to Modern Portfolio Theory (MPT) with emphasis on mean-variance optimization. Later, the bridge between portfolio optimization and asset pricing was built by establishing Capital Asset Pricing Model (CAPM) developed independently by Jack Trainor (Treynor 1961), William Sharp (Sharpe 1964), John Lintner (Lintner 1965) and Jan Mossin (Mossin 1966). It relies on the assumption that all investors conduct the same exercise of calculating mean-variance optimization (MVO). Further relaxation of strict CAPM assumptions lead to the development of Black-Litterman model (Black and Litterman 1992), which provided space for heterogeneous market expectations.

Simultaneously with the problem of portfolio optimization, the researchers turned their attention to the problem of stock forecasting. There are two main approaches: one based on fundamental factors and the other based on technical ones. One of the most well-known models explaining the relation between fundamental factors and stock performance is the 3-factor model (Eugene F. Fama 1993), which tried to reduce the gap between CAPM forecasts and observed stock performance. Later researchers added two additional factors (profitability and investments) establishing 5-factor model (Fama and French 2015).

The development of neural networks has opened up a new arsenal of tools. The main problem of portfolio optimization is the change in the relationships between individual stocks over time. These relations are often measured by the covariance matrix. Hierarchical Risk Parity (HRP) (Prado 2016) was one of the attempts to apply clustering algorithms to stocks. According to this approach, stocks competed for inclusion in the portfolio not with each other, but with a subset of closely correlated neighbors, which made the diversification less concentrated and stable. Another proposed approach (Jaeger and Marinelli 2022) is to use network graph statistics derived from covariance matrix. In that case graph statistics are considered to be a remedy to covariance matrix instability. At first, using graph filtering methods as minimum spanning tree, authors leave only the connections that are essential to the preservation of graph connectivity. Further, each node can be assigned its own metrics, for example, degree centrality and eigenvector centrality. They characterize the “importance" of the node for the graph. But since we are talking about diversification, the authors take the inverse metrics: inverse degree centrality and inverse eigenvector centrality. By doing so, the authors try to maximize the number of "uncorrelated bets", i.e. to achieve some kind of risk parity. Another important idea from the authors: we can build several graphs based on different correlation matrices, and then combine the coefficients from different graphs into one to get achieve greater weights stability. In another recent article (Taylor and Cerbo 2019), it was proposed to deduce the equity market centrality - an indicator obtained from chart statistics. The idea is that by filtering the chart to the minimum spanning tree, you can find a stock with most connections, and it can represent the "center of the market". The researchers found that some stocks remain strongly correlated with the market. This strong relation can be used to exclude some stocks from the portfolio in order to decrease the correlation with the market. From this incomplete review, it becomes clear that graphs currently attract more and more attention from researchers. One of the advantages of graphs is the ability to succinctly summarize various types of information.

Considering another problem, we find a lot of works devoted to forecasting the paths of single stocks. One of the notable attempts to predict stocks using graph representation of shareholders ownership was made in 2018 (Chen and Wei 2018). The idea was to encode using recurrent neural networks features of singular stocks, then to encode target company based on neighbors’ node representations and use this representation as well as a features from most related companies in Convolutional Graph Network to forecast stock price performance. In another work (Matsunaga, Suzumura and Takahashi 2019), authors extend the universe of relations including not only shareholder-type relations, but also “supplier”, “customer” and “partner” types relations. They also add a dot-product of node embeddings to the convolution layers’ computations to take into account the changing nature of stock co-dependence in time (Temporal Graph Convolution).

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