Problem 1. (Concurrent circles) *Consider a triangle ABC which has inscibed circle ω with center I. Let H_a, H_b and H_c denote the feet of altitudes from A,B and C respectively. Let line AI meet ω at зщште A_1, line BI meet ω at point B_1, line CI meet ω at point C_1. Let Ω_a be a circumcircle of triangle AH_aA_1. Similarly, we define Ω_b and Ω_c. Prove, that Ω_a, Ω_b and Ω_c are coaxial.*

Problem 2. (Ellipse's property) *Consider an ellipse $\boldsymbol{\mathcal{P}}$ with foci A and B and an arbitrary point S outside of $\boldsymbol{\mathcal{P}}$. Let C be an arbitrary point on $\boldsymbol{\mathcal{P}}$. Let D be point on $\boldsymbol{\mathcal{P}}$, such that $\angle ASC = \angle BSD$ and C, D belong the same half-plane of the line AB. Then tangent lines at C and D to $\boldsymbol{\mathcal{P}}$ intersect on the angle bisecor of $\angle ASB$.*

Problem 3. (Poncelet porism property) *Hypothesis. Consider a n-gon $\mathcal{A} = A_1A_2...A_n$, inscribed in and circumscribed about two conics $\mathcal{C}_1$ and $\mathcal{C}_2$ respectively. Consider now all intersection points of the diangonals A_iA_{i+2} and let $A_{n+j} = A_j$ for $j \geqslant 1$. They form a n-gon $\mathcal{B} = B_1B_2...B_n$ for $n \geqslant 5$. By the Poncelet theorem, we can «rotate» polygon $\mathcal{A}$ between conics $\mathcal{C}_1$ and $\mathcal{C}_2$. Then, as it turns out, polygon $\mathcal{B}$ rotates between some fixed conics $\mathcal{V}_1$ and $\mathcal{V}_2$.*

Note*. If $n = 4$, then polygon $\mathcal{B}$ degenerates into a point $\mathcal{B}$. By this theorem we can conclude, that as long as quadrilateral $\mathcal{A}$ rotates, point $\mathcal{B}$ is fixed. We will get well-known fact, if we let conics $\mathcal{C}_1$ and $\mathcal{C}_2$ be a circles.*

P.S. It's checked for cases $n = 4, 5, 6$.

Problem 4. (Property of Pascal Theorem for 4 points) *Consider a cyclic quadrilateral $ABCD$, which is inscribed in circle ω. Let $\ell_a, \ell_b, \ell_c, \ell_d$ be the tangent lines to the ω at points A, B, C, D respectively. Denote the intersection point of the lines ℓ_b and ℓ_c by F, the intersection point of the lines ℓ_a and ℓ_d by H. Let E be the common point of the lines AB and CD, G the common point of the lines BD and AC. Let I be an arbitrary point on ω. Lines IE, IF, IG, IH intersect ω at points M, L, K, J.*

(a) Prove, that the points E, F, G, H are collinear. Moreover, $(E, G; F, H)$ is harmonic. (Show that)

(b) Let ℓ be the line, passing through the points E, F, G, H. Prove, that the tangent lines at K, M – ℓ_k, ℓ_m to ω, the line ℓ and the line JL are concurrent.