Problems 1: Random problems

1. There is a parallelogram $ACDE$ externally constructed on the side AC of the triangle ABC. Let O be the intersection point of the diagonals of $ACDE$; N and K are midpoints of the sides BC and BA respectively. Prove, that lines DK, NE and BO are concurrent.

2. In the right triangle ABC the side AC is the hypotenuse and BH is the altitude. Points D, E, F are chosen on the side BC, segment BH and segment CH respectively, such that $\angle BAD = \angle CAE$ and $\angle AFE = \angle CFD$. Prove, that $\angle AEF = 90^\circ$.

3. In acute triangle ABC, BH is the altitude. Points M and N are midpoints of the segments of AH and CH respectively. In the circumcircle Ω of BMN, BB' is the diameter. Prove that $AB' = CB'$.

4. Triangle ABC is inscribed in the circle ω. Let M be the point on the arc BC (not containing the point A) and point M_1 is symmetric to point M with respect to the side BC. Prove, that segment AM_1 is divided in half by the circumcircle of the medial triangle.

5. Consider the triangle ABC, where BL is the bisector. Bisector of the segment BL intersects external bisectors of angles A and C at points P and Q respectively. Prove, that the circumcircle of PBQ touches the circumcircle of ABC.