

Solution. As usual, we denote the directed angle between the lines a and b by $\leq (a, b)$.

Claim 1. Lines AA_0 , BB_0 , CC_0 are concurrent at T, which lies on the circle (ABC).

Proof. Let $T=BB_0\cap CC_0$. Since quadruples of points P,Q,B,B_0 ; Q,P,C,C_0 ; Q,P,A,A_0 are concyclic, we have $\sphericalangle(PB,BT)=\sphericalangle(PB,BB_0)=\sphericalangle(PQ,QB_0)=\sphericalangle(PQ,QC_0)=\sphericalangle(PC,CC_0)=$ = $\sphericalangle(PC,CT)$, which means, that poins P,B,C,T are concyclic. It follows, that $T\in(ABC)$ and $T\in BB_0$. Now we define $T'=BB_0\cap AA_0$. Similarly, points P,B,A,T' are concyclic. That gives us, that $T'\in(ABC)$ and $T'\in BB_0$, which means, that T=T' and we are done. □

Let A', B', C' denote the intersection points of lines BB_0 and CC_0 , AA_0 and CC_0 , BB_0 and AA_0 respectively.

Claim 2. Let $S = (ABC) \cap (A'B'C')$. Then S is symmetric to P with respect to line OQ.

Proof. \square

Claim 3. Lines AA', BB', CC' are comcurrent at S.

Proof. \square