

Solution. As usual, we denote the directed angle between the lines a and b by $\angle(a,b)$.

Claim 1. Lines AA_0 , BB_0 , CC_0 are concurrent at T, which lies on the circle (ABC).

Proof. Let $T = BB_0 \cap CC_0$. Since quadruples of points $P, Q, B, B_0; Q, P, C, C_0; Q, P, A, A_0$ are concyclic, we have

$$\angle(PB,BT) = \angle(PB,BB_0) = \angle(PQ,QB_0) = \angle(PQ,QC_0) = \angle(PC,CC_0) = \angle(PC,CT).$$

which means, that points P, B, C, T are concyclic. It follows, that $T \in (ABC)$ and $T \in BB_0$. Now we define $T' = BB_0 \cap AA_0$. Similarly, points P, B, A, T' are concyclic. That gives us, that $T' \in (ABC)$ and $T' \in BB_0$, which means, that T = T' and we are done. \square

Let A', B', C' denote the intersection points of lines BB_0 and CC_0 , AA_0 and CC_0 , BB_0 and AA_0 respectively.

Claim 2. Let $S = (ABC) \cap (A'B'C')$. Then S is symmetric to P with respect to line OQ.

Proof. \square

Claim 3. Lines AA', BB', CC' are concurrent at S.

Proof. \square