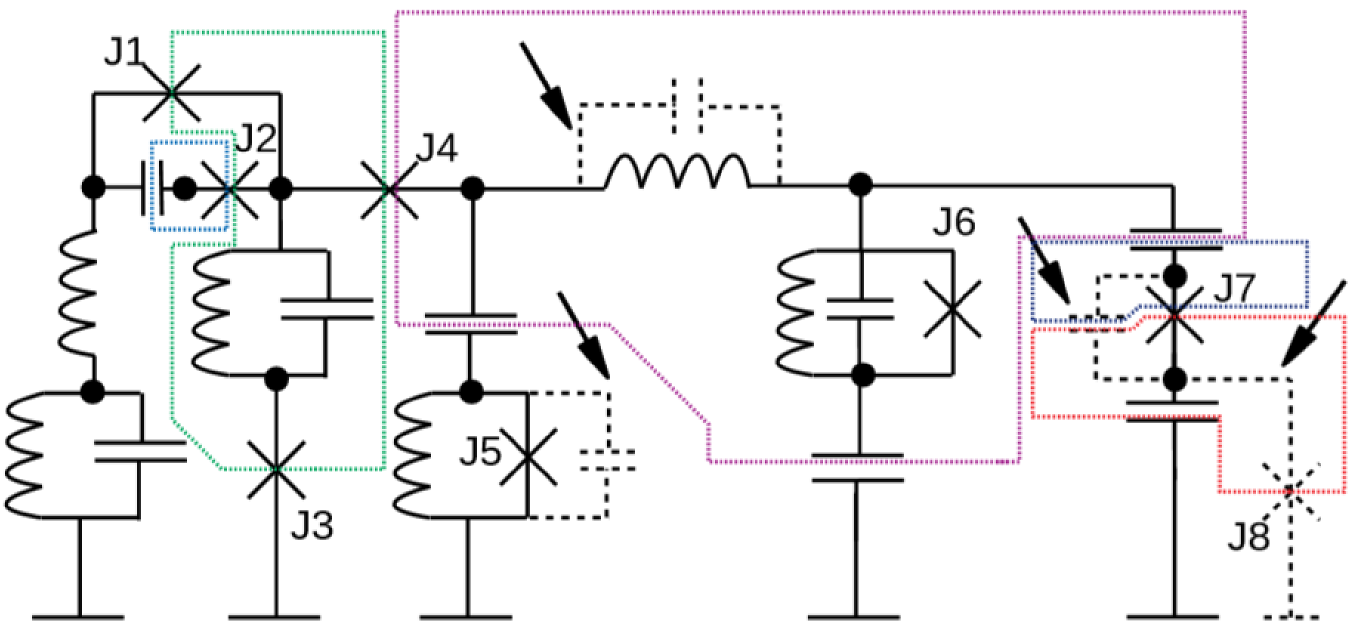
SuperQuant Model

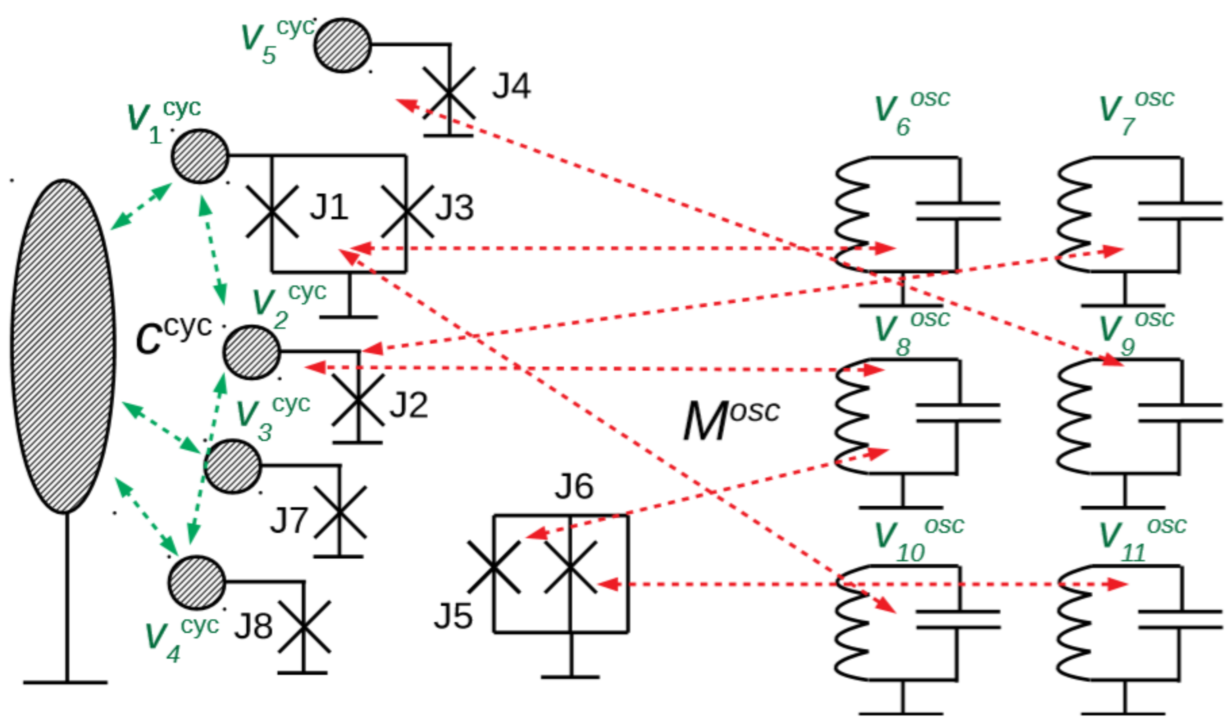
```
   __________________________________________________
 _|                                         |       |
/                                           |       |
\_     S U P E R C O N D U C T I N G        X     __|__
/                                           |     _____
\_            Q U A N T U M                 X       |
/                                           |    ___|___
\_            C I R C U I T                 X   |       |
/                                           |   X       x
\_              M O D E L                   X   |_______|
  |     | |                                 |       |
  |_____| |________\/_______________________|_______|
        | |        /\   (c) Andrey Klots
        | |
```

0. Basic idea.

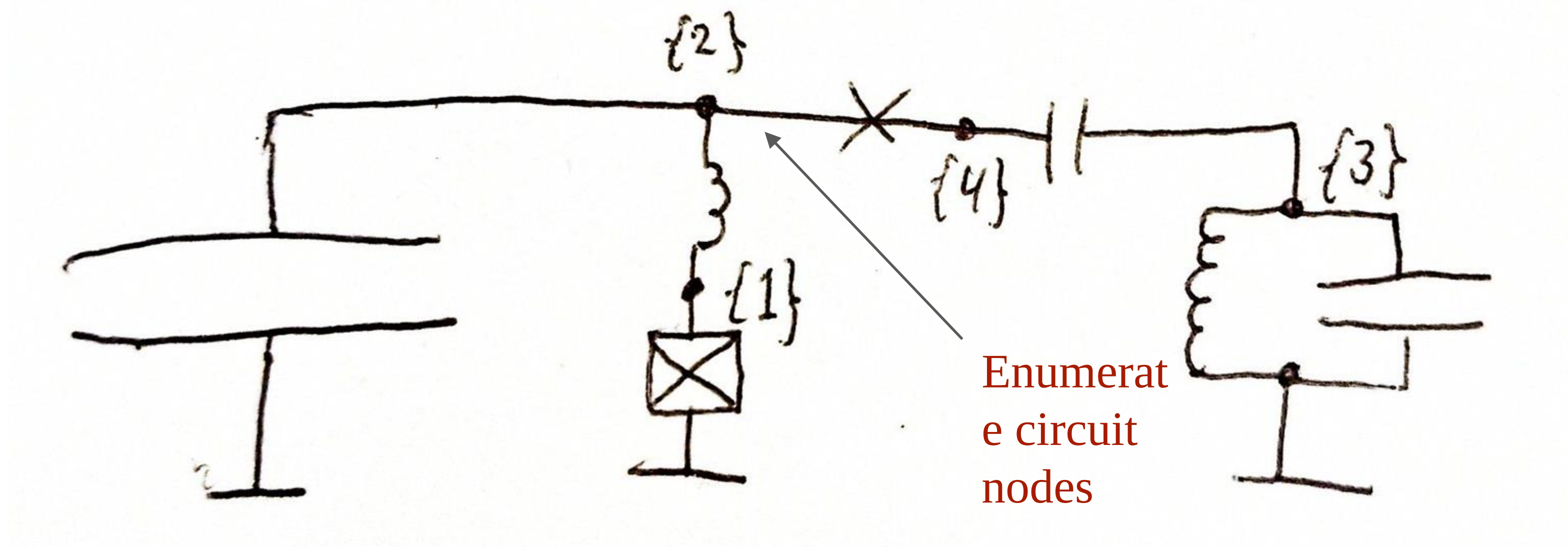
The code is based on a formalism that allows to start with an arbitrary complicated circuit (below)



and separates charge states associated with Josephson junctions from oscillator states associated with inductors. This effectively “pulls” the inductors out of the circuit and allows to represent the circuit and its Hamiltonian in a universal optimized form that is easy for further analysis and allows for efficient numerical simulation. Circuit below is derived from circuit above using the automatized formalism.



1. Sketch Circuit Diagram (arbitrary circuit example)

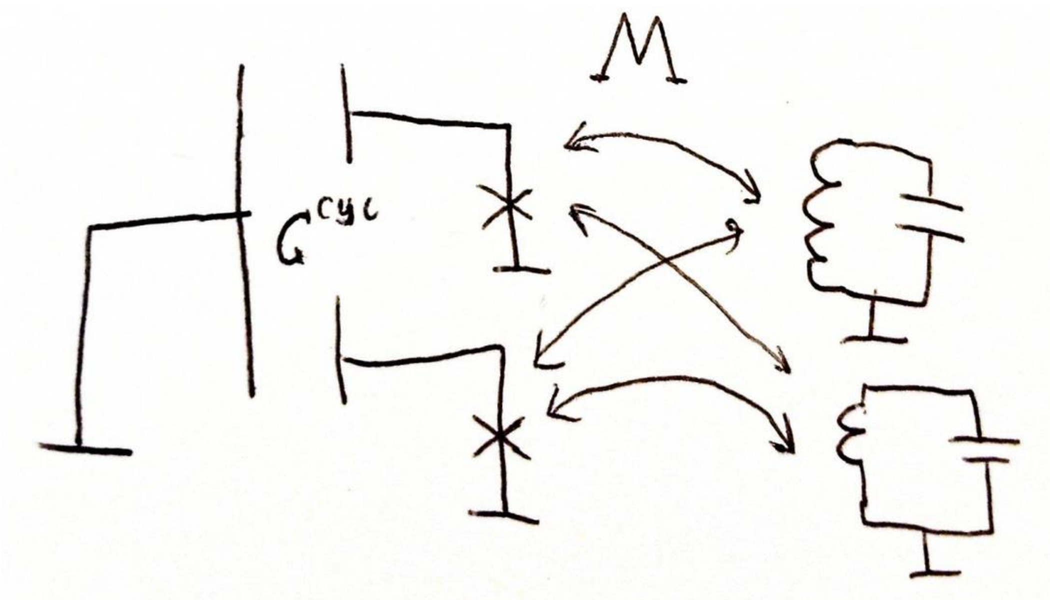


2. Input Circuit Diagram as a Code

```
MyCircuitCode = [\
#           ______
#             |
#             |{Terminal 0: Ground}
#             |
#             |
#  ___________|___________ Large transmon capacitor
   [0,2, "C", C_shunt],         [2,4, "J", E_J_coupling],
#  _______________________
#             |_____________________\/____________
#             |  {Terminal 2}       /\            |
#             C                                   |
#    Small    C                                   |  Coupling
# inductor:   C  wire                             |  capacitor
#   Z~1E-3    C                              _____|_____
    [1,2, "L", L_wire],                 [3,4, "C", C_coupling],
    [1,2, "C", C_wire], #
#                                            ___________
#             |                                   | {Terminal 3}
#             | {Terminal 1}    ..................|.........................
#             |                :           _______|_________               :
#             |                :          C                |  Resonator    :
#             |                :          C                |               :
#             |                :          C                |               :
#            _|_               :          C                |               :
#           |\|/|              :          C                |               :
#           |/|\| Jos. junct   :          C          ______|______         :
     [0,1, "J", E_J],             [0,3, "L", L_res],     [0,3, "C", C_res],#:
     [0,1, "C", C_J]       #    :          |          ____________          :
#             |                :          |________________|               :
#             | {Terminal 0}   :                   | {Terminal 0}          :
#             |                :...................|........................
#           __|__                                __|__
]
```

3. Software performs coordinate transformation.

Software performs linear coordinate transformation that decouples oscillatory degrees of freedom located in inductors from cyclic coordinates, located at charge islands and obeying charge quantization. The Hamiltonian then transforms into an effective Hamiltonian corresponding to a circuit having same number of charge islands and same number of loops:



Here c^{cyc} is capacitance matrix in transformed coordinates. It exists only for cyclic coordinates. Now inductors are effectively “pulled out” of from the circuit and interact with Josephson junctions via mutual inductance matrix *M*.

4. Software Displays New Hamiltonian and Coordinate transformation (result example).

Write down the Hamiltonian:

$$H = H_{cyc} + H_{osc} + H_{jos}$$

Hamiltonian is a sum of
1. cyclic part H_{cyc} representing variables obeying charge quantization.
2. oscillatory part H_{osc} representing oscillator-like coordinates associated with inductors.
3. Josephson part H_{jos} representing interactions between the modes via Josephson junctions.

Circuit has $N^{tot} = 4$ terminals.

Cyclic part of Hamiltonian:

$$H_{cyc} = \sum_{j,k=1}^{N^{cyc}} \frac{1}{2} \left(c^{cyc}\right)^{-1}_{jk} q_j q_k$$

Indexes j,k go over cyclic coordinates.

Circuit has $N^{cyc} = 2$ cyclic coordinates that follow charge quantization.
q-variables represent charge operators.
Inverse capacitance matrix for cyclic coordinates is

$\left(c^{cyc}\right)^{-1} =$

```
[[ 1.6101 -0.1229]
 [-0.1229  0.1229]]
```

Inverse capacitance matrix for cyclic coordinates

Oscillatory part of Hamiltonian:

$$H_{osc} = \sum_{j=N^{tot}-N^{cyc}}^{N^{tot}} \frac{1}{2} w_j \left(q_j^2 + v_j^2\right)$$

Index j goes over oscillatory coordinates of the circuit. The circuit has

$N^{osc} = 2$ oscillator-like coordinates.
Here v- and q- variables are phase and charge operators respectively. Oscillator frequency modes are

$w_j =$

```
[2.399000e-01 7.455139e+02]
```

Oscillator mode frequencies

Josephson part of the Hamiltonian:

$$H_{jos} = -\sum_{J=1}^{N^{jos}} E_J \cos\left[\sum_{j=1}^{N^{tot}} M^{(III)}_{Jj} v_j\right]$$

Index J runs over 2 Josephson junctions. Josephson junction energies are

$E_J =$

```
[[2.  0.3]]
```

Josephson energies

Index j runs over all 4 coordinates.
Indexes in front of transformed (after transformation III) phases v are

$M^{(III)}_{Jj} =$

```
[[1.000e+00 0.000e+00 2.206e-01 3.000e-04]
 [0.000e+00 1.000e+00 0.000e+00 3.060e-02]]
```

Coefficients inside Josephson junction cosines

Note that first 2 cols are indexes in front of cyclic coordinates and are integer.
Remaining 2 indexes correspond to oscillator-like coordinates and do not obey charge quantization and hence do not need to be integer.

Example of Displayed Coordinate Transformation.

```
Coordinate transformation:

Transformed coordinates v  are related
                         j
to original coordinates phi  as
                           j


 _    _     _                               _     _    _
|  cyc |   |  cyc    cyc      cyc    cyc     |   |      |
| v    |   | T       T    ... T      T    ... |   |phi   |
|  1   |   |  1,1    1,2      1,j    1,j+1    |   |   1  |
|      |   |                                  |   |      |
|  cyc |   |   cyc   cyc      cyc    cyc      |   |      |
| v    |   |  T      T    ... T      T    ... |   |phi   |
|  2   |   |   2,1   2,2      2,j    2,j+1    |   |   2  |
| .    |   |  .      .    .   .      .        |   | .    |
| .    |   |  .      .     .  .      .        |   | .    |
| .    |   |  .      .      . .      .        |   | .    |
|  osc | = |   osc   osc      osc    osc      | x |      |
| v    |   |  T      T    ... T      T    ... |   |phi   |
|  1   |   |   1,1   1,2      1,j    1,j+1    |   |   j  |
|      |   |                                  |   |      |
|  osc |   |   osc   osc      osc    osc      |   |      |
| v    |   |  T      T    ... T      T    ... |   |phi   |
|  2   |   |   2,1   2,2      2,j    2,j+1    |   |   j+1|
| .    |   |  .      .    .   .      .     .  |   | .    |
| .    |   |  .      .     .  .      .      . |   | .    |
| .    |   |  .      .      . .      .       .|   | .    |
|_    _|   |_                               _|   |_    _|
```

Matrix that transforms original phase coordinates (*phi*) into new coordinates (*v*). New coordinates are classified in cyclic (first) and oscillatory (last)

```
Coordinate transformation matrix

 (III)
T      =
 jk

[[ 8.3000e-03  9.9170e-01  1.0000e+00 -1.0000e+00]
 [-8.3000e-03 -9.9170e-01  0.0000e+00  0.0000e+00]
 [ 0.0000e+00  0.0000e+00 -4.5322e+00  0.0000e+00]
 [-3.2439e+01  3.2439e+01  0.0000e+00  0.0000e+00]]

  First 2 rows provide extract cyclic
coordinates and remaining 2 rows extract
oscillator-like coordinates from original phase
coordinates \phi .
                j
```

Example of coordinate transformation matrix

5. Other Example. Transmon. Numerical eigenenergy calculation.

Circuit code inputted into the software:

```
MyCircuitCode = [\
#                              ____________________________
#                             |          {Terminal 2}      |
#                            C                             |
#           Small            C                             |   Large
#        inductor:           C                             |   capacitor
#          Z~1E-3            C                    _________|_________
            [1,2, "L", 0.01*ut.pH],        [0,2, "C", 70.*ut.fF],
            [1,2, "C", 10*ut.fF], #       ____________________
#                            |                           |
#                            |  {Terminal 1}             |  {Terminal 0}
#                           _|_                        __|__
#                          |\|/|
#                          |/|\|  Jos. junct
            [0,1, "J", 0.3*ut.K],
            [0,1, "C", 0.5*ut.fF]
#                            |
#                            |  {Terminal 0}
#                          __|__
                ]
```

Use module "MicrowaveUnits" to set units.

Calculated spectrum: