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Pattern Recognition on Oriented Matroids

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To the memory of

<i>Alexander Semënovich</i>	<i>Grigory Alexeevich</i>
<i>Antonov</i>	<i>Matveev</i>
1866–1919	1865–1920
<i>Elizaveta Vasilyevna</i>	<i>Yulia Nikiforovna</i>
<i>Antonova–Gryaznykh</i>	<i>Matveeva</i>
⟨ <i>Chistyakova</i> ⟩	⟨ <i>Batmanova</i> ⟩
1880–1948	1872–1953

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