CSC 284/484 - homework 3 (MCMC, Geometric Algorithms)

`http://www.cs.rochester.edu/~stefanko/Teaching/17CS484`

Students that take the course as 484 are required to do **both** 284/484 and 484 parts of the homework. Students that take the course as 284 are only required to do 284/484 part of the homework (of course you are welcome to solve/turn-in the 484 part as well).

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1.1 Theoretical part

Exercise 1.1 (**due 4/11/2017**) Consider the following Markov chain. The states are $\Omega = \{-n, -(n-1), \dots, -1, 0, 1, \dots, n-1, n\}$. The transition probabilities are

$$P(x,x) = 1/2 \quad \text{for all } x \in \Omega,$$

$$P(-n, -(n-1)) = P(n, n-1) = 1/2,$$
$$P(0,1) = P(0,-1) = 1/4,$$

For $x \in \{1, \dots, n-1\}$

$$P(-x, -(x+1)) = P(x, x+1) = 1/3,$$
$$P(-x, -(x-1)) = P(x, x-1) = 1/6.$$

Give a bound on the mixing time of the chain.

Exercise 1.2 (**due 4/11/2017**) Consider the following Markov chain. The states are $\Omega = \{-n, -(n-1), \dots, -1, 0, 1, \dots, n-1, n\}$. The transition probabilities are

$$P(x,x) = 1/2 \quad \text{for all } x \in \Omega,$$

$$P(-n, -(n-1)) = P(n, n-1) = 1/2,$$
$$P(0,1) = P(0,-1) = 1/4,$$

For $x \in \{1, \dots, n-1\}$

$$P(-x, -(x+1)) = P(x, x+1) = 1/6,$$
$$P(-x, -(x-1)) = P(x, x-1) = 1/3.$$

Give a bound on the mixing time of the chain.

Exercise 1.3 (**due 4/11/2017**) We are given a set S of n points $(x_1, y_1), \dots, (x_n, y_n)$ in the plane. We want to preprocess the points and output a data structure that supports answering the following queries. Given a point $p = (p_x, p_y)$ output a triangle containing p whose vertices are in S (or report that no such triangle exists). (Point p is contained in a triangle if it is inside or on the boundary of the triangle.) Clearly state: 1) preprocessing algorithm, 2) the preprocessing time, 3) query algorithm, 4) query processing time. Faster query processing $\Longrightarrow$ more points.

INPUT: n, $(x_1, y_1), \dots, (x_n, y_n)$ in the plane

QUERY: (p_x, p_y)

OUTPUT: $i, j, k \in \{1, \ldots, n\}$ such that (p_x, p_y) is in the triangle with vertices $(x_i, y_i), (x_j, y_j), (x_k, y_k)$, or NO if no such triangle exists.

1.2 Applied part

Exercise 1.4 (**due 4/11/2017**) Implement coupling from the past algorithm on a problem of your choice.

Exercise 1.5 (**due 4/11/2017**) Implement Graham Scan and Jarvis' March algorithms for computing convex hulls. Your program should read the input from the `stdin` in the following format: first line contains n, the number of points; the next n lines contain coordinates of the points (two numbers (integers) per line). Write the output to `stdout` in the following format: first line contains h, the number of points on the convex hull; the next h lines contain coordinates of points on the convex hull in clockwise direction starting from the point with the lexicographically smallest coordinates.

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2.1 Theoretical/applied part

Exercise 2.1 (**due 4/11/2017**) We are given n points in the plane. We want to find a line ℓ that maximizes the number of pairs of points a, b that are reflections of each other with respect to ℓ (two points a, b are reflections of each other with respect to ℓ if the segment ab is perpendicular to ℓ and the midpoint of the segment ab lies on ℓ). Give an $O(n^2 \log n)$ algorithm for the problem.

2.2 Applied part

Exercise 2.2 (**due 4/11/2017**) Implement the $O(n \log n)$ closest pair of points algorithm. Your program should read the input from the `stdin` in the following format: first line contains n, the number of points; the next n lines contain coordinates of the points (two numbers (integers) per line). Write the output to `stdout` in the following format: coordinates of the two closest points on two separate lines.