# Learning Certified Individually Fair Representations

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#### Abstract

To effectively enforce fairness constraints one needs to define an appropriate notion of fairness and employ representation learning in order to impose this notion without compromising downstream utility for the data consumer. A desirable notion is individual fairness as it guarantees similar treatment for similar individuals. In this work, we introduce the first method which generalizes individual fairness to rich similarity notions via logical constraints while also enabling data consumers to obtain fairness certificates for their models. The key idea is to learn a representation that provably maps similar individuals to latent representations at most  $\epsilon$  apart in  $\ell_{\infty}$ -distance, enabling data consumers to certify individual fairness by proving  $\epsilon$ -robustness of their classifier. Our experimental evaluation on six real-world datasets and a wide range of fairness constraints demonstrates that our approach is expressive enough to capture similarity notions beyond existing distance metrics while scaling to realistic use cases.

## 1 Introduction

The increasing use of machine learning in sensitive applications (e.g., crime risk assessment (Brennan et al., 2009)) has raised concerns that methods based on learning from data can reinforce human biases, discriminate, and lack fairness (Sweeney, 2013). To formalize these issues, McNamara et al. (2017) proposed a framework which partitions the landscape into three parties: a data regulator who defines fairness for the particular task at hand, a data producer who processes sensitive user data and transforms it into another representation, and a data consumer who performs predictions based on the new representation.

In this setting, a machine learning model M is composed of two models: a model  $f_{\theta}$  provided by the data producer and a model  $h_{\psi}$  provided by the data consumer. The data regulator then selects a definition of fairness the model M should satisfy. Most work so far has explored two main families of fairness definitions (Chouldechova & Roth, 2018): statistical and individual. Statistical notions define specific groups in the population and require that particular statistics, computed based on model decisions, should be equal for all groups. Popular notions of this kind are, for example, demographic parity (Dwork et al., 2012) and equalized odds (Hardt et al., 2016). While these notions do not require any assumptions on the data and are easy to certify, they offer no guarantees for individuals or other subgroups in the population (Kearns et al., 2017). In contrast, individual notions of fairness (Dwork et al., 2012) explicitly require that similar individuals in the population are treated similarly.

Learning individually fair models is a hard problem which requires solving two key challenges: (i) the definition of similarity is typically done via a fixed distance metric which can be hard to come up with in real world tasks, and (ii) providing a certificate that a model M indeed does satisfy individual fairness (harder than providing only empirical evidence). In this work we propose methods that address both of these challenges. At a high level, our approach is based on the observation that recent advances in training machine learning models with logical constraints (Fischer et al., 2019) together with new methods for proving that constraints are satisfied (Tjeng et al., 2019) open the possibility for tackling these challenges.

Concretely, we make individual fairness more practical by enabling data regulators to express flexible definitions via an interpretable logical constraint  $\phi$  between two individuals (instead of

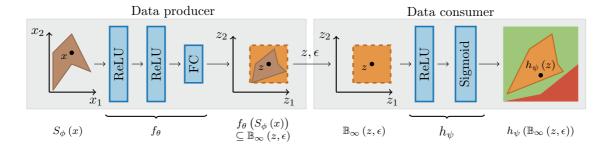


Figure 1: Conceptual view of our framework. The left side shows the component corresponding to the data producer who learns a representation  $f_{\theta}$  which maps the entire set of individuals  $S_{\phi}(x)$  that are similar to individual x, according to the similarity notion  $\phi$ , to points near  $f_{\theta}(x)$  in the latent space. The data producer then computes the worst-case distance  $\epsilon$  in the latent space and passes it to the data consumer, along with  $z = f_{\theta}(x)$ . The data consumer receives the latent representation z and radius  $\epsilon$ , trains a classifier  $h_{\psi}$ , and certifies that the entire  $\ell_{\infty}$ -ball centered around z with radius  $\epsilon$  is classified correctly (green color shows fair output region).

fixed distance functions). For example,  $\phi$  could be defined as true for a pair of individuals if either of the following holds: (i) they both have age below some threshold and numerical attributes (e.g., salary, age) are in a range  $\Delta_{Young}$ , (ii) they both have age above some threshold and their numerical attributes are in the range  $\Delta_{Old}$ .

For the certification challenge, a key requirement is that the data consumer should be able to provide a certificate for the model M, without having to reason about the fairness constraint  $\phi$ . This modular view is beneficial as it enables data consumers to employ standard training procedures (independent of  $\phi$ ). To realize this modular view, our solution consists of two parts. First, we enable the data producer to learn a representation of user data with the property that two individuals which satisfy  $\phi$  should be mapped at most  $\delta$  apart in  $\ell_{\infty}$ -distance in latent space. Second, given a user dataset D, for each point  $x \in D$ , we compute a box centered at  $z = f_{\theta}(x)$  with radius  $\epsilon$ , which captures the latent representations of all individuals similar to x according to  $\phi$ . Then, to perform certification of end-to-end individual fairness on M, for each point  $x \in D$ , it is enough to prove local robustness of  $h_{\psi}$  at a point z with radius  $\epsilon$ . Importantly, certification now can be accomplished without knowing the fairness constraint  $\phi$  (training of  $h_{\psi}$  also does not require special procedures).

#### Main contributions Our key contributions are:

- A generalization of similarity notions in individual fairness via interpretable logical constraints.
- A method to learn individually fair representations (defined using expressive logical constraints) which comes with provable certificates.
- An end-to-end implementation of our method in a tool called LCIFR, together with an extensive evaluation on several datasets, constraints and architectures. We make LCIFR publicly available at https://github.com/eth-sri/lcifr.

### 2 Overview

In this section we provide a high-level overview of our approach, with the overall flow shown in Figure 1. As introduced earlier, our setting consists of three parties. The first party is a data regulator who defines an individual fairness property  $\phi$ . The property  $\phi$  is problem-specific and can be expressed in a rich logical fragment which we describe later in Section 3. A deterministic classifier M satisfies individual fairness with respect to  $\phi$  if it is the case that for all inputs

 $x, x' \in \mathbb{R}^n$ , from  $\phi(x, x')$  it follows that M(x) = M(x'). Here, the classifier M is a composition of two functions: an encoder  $f_{\theta} : \mathbb{R}^n \to \mathbb{R}^k$ , provided by the data producer, and a classifier  $h_{\psi} : \mathbb{R}^k \to \mathbb{R}^o$ , provided by the data consumer.

In this work, we propose a method for training both the encoder  $f_{\theta}$  and the classifier  $h_{\psi}$  so that the end-to-end model  $M = h_{\psi} \circ f_{\theta}$  satisfies the above definition of individual fairness (given a notion of similarity  $\phi$ ). We define the set of all points similar to x as  $S_{\phi}(x) = \{x' \in \mathbb{R}^n \mid \phi(x, x')\}$ . In Figure 1,  $S_{\phi}(x)$  is represented as a brown shape and x is shown as a single point inside of  $S_{\phi}(x)$ .

The key idea of our approach is to train an encoder  $f_{\theta}$  to map point x and all points  $x' \in S_{\phi}(x)$  close to one another in the latent space, specified as

$$\phi(x, x') \implies ||f_{\theta}(x') - f_{\theta}(x)||_{\infty} \le \delta, \tag{1}$$

where  $\delta$  is a tunable parameter of the method. If the encoder indeed satisfies Equation (1), the data consumer can train a classifier  $h_{\psi}$  in a way which is in essence independent of the particular similarity notion  $\phi$ . Namely, the data consumer only has to train  $h_{\psi}$  to be robust to perturbations up to  $\delta$  in the  $\ell_{\infty}$ -norm, which can be solved via a standard min-max optimization procedure, discussed in Section 3.

We now explain our end-to-end inference with provable guarantees using a trained encoder  $f_{\theta}$  and classifier  $h_{\psi}$ .

**Processing the producer model** Given a data point x, we first propagate both x and its set of similar points  $S_{\phi}(x)$  through the encoder, as shown in Figure 1, and obtain  $z = f_{\theta}(x)$  (the latent representation of x) and  $f_{\theta}(S_{\phi}(x))$  (the latent representations of all data points similar to x). Due to the stochastic nature of training, Equation (1) may not hold for the particular x, so we compute the smallest  $\epsilon$  such that  $f_{\theta}(S_{\phi}(x)) \subseteq \mathbb{B}_{\infty}(z, \epsilon)$ . This  $\ell_{\infty}$ -bounding box with center z and radius  $\epsilon$  is shown as orange in Figure 1.

Processing the consumer model Next, we provide both the latent representation z and the radius  $\epsilon$  to the data consumer (that is, a bounding box). The data consumer then knows that all points similar to x are in the  $\ell_{\infty}$ -ball of radius  $\epsilon$ , but does not need to know similarity constraint  $\phi$  nor the particular convex shape  $f_{\theta}(S_{\phi}(x))$ . The key observation is the following: if the data consumer can prove its classifier  $h_{\psi}$  is robust to  $\ell_{\infty}$ -perturbations up to  $\epsilon$  around  $f_{\theta}(x)$ , then the end-to-end classifier  $M = h_{\psi} \circ f_{\theta}$  satisfies individual fairness with respect to the similarity rule  $\phi$  imposed by the data regulator.

There are two key technical challenges we need to address here. The first challenge is how to train an encoder to satisfy Equation (1), while not making any domain-specific assumptions about the point x or the similarity constraint  $\phi$ . The second challenge is how to provide a certificate of individual fairness, which requires both computing the smallest radius  $\epsilon$  such that  $f_{\theta}(S_{\phi}(x)) \subseteq \mathbb{B}_{\infty}(z, \epsilon)$  as well as certifying  $\ell_{\infty}$ -robustness of the classifier  $h_{\psi}$ .

To train an encoder, we build on Fischer et al. (2019) which provides a translation from a logical constraint  $\phi$  to a differentiable loss function. The training of the encoder network can then be formulated as a min-max optimization problem which alternates between (i) searching for counterexamples  $x' \in S_{\phi}(x)$  that violate Equation (1), and (ii) training  $f_{\theta}$  on the counterexamples. We utilize gradient descent to minimize a joint objective composed of a classification loss and the constraint loss obtained from translating Equation (1). Once no more counterexamples are found, we can conclude the encoder empirically satisfies Equation (1). We discuss the detailed procedure in Section 3.

To obtain a proof of individual fairness, we compute two certificates. First, to provide guarantees on the latent representations obtained using an encoder  $f_{\theta}$ , we solve the following optimization problem:

$$\epsilon = \max_{x' \in S_{\phi}(x)} ||z - f_{\theta}(x')||_{\infty}.$$

Note that the set  $S_{\phi}(x)$  generally contains an infinite number of individuals x' and thus the above optimization problem cannot be solved by simple enumeration. In Section 4 we show how this

optimization problem is encoded as a mixed-integer linear program (MILP) and solved using offthe-shelf MILP solvers. After obtaining  $\epsilon$ , a second certificate (using MILP) proves local robustness of the classifier  $h_{\psi}$  around  $z = f_{\theta}(x)$ : here we prove that for each z' where  $||z' - z|| \leq \epsilon$ , the classification results of  $h_{\psi}(z')$  and  $h_{\psi}(z)$  are the same. This implies the overall model  $M = h_{\psi} \circ f_{\theta}$ satisfies individual fairness.

## 3 Learning Individually Fair Representations

We now present our method for learning representations which are individually fair with respect to the property  $\phi$ . To illustrate our method we consider a scenario where the data regulator proposes the following similarity constraint:

$$\phi(x, x') = \bigwedge_{i \in \text{Cat} \setminus \{\text{race}\}} x_i = x'_i \bigwedge_{j \in \text{Num}} |x_j - x'_j| \le \alpha.$$

According to  $\phi$ , individual x' is considered similar to x if: (i) all categorical attributes except for race are equal to those of x, and (ii) all numerical attributes (e.g., income) of x and x' differ by at most  $\alpha$ . Under the constraint  $\phi$ , similarity of individuals x and x' does not depend on their races.

### 3.1 Enforcing Individual Fairness

To learn a representation that satisfies  $\phi$ , we build on the recent work DL2 (Fischer et al., 2019). Concretely, we aim to enforce the following constraint on the encoder  $f_{\theta}$  used by the data producer:

$$\phi(x, x') \implies \|f_{\theta}(x) - f_{\theta}(x')\|_{\infty} \le \delta, \tag{2}$$

where  $\delta$  is a tunable constant. With DL2, this implication can be translated into a non-negative, differentiable loss  $\mathcal{L}(\phi)$  such that  $\mathcal{L}(\phi)(x,x')=0$  if and only if the implication is satisfied. Here, we denote  $\omega(x,x'):=\|f_{\theta}(x)-f_{\theta}(x')\|_{\infty}\leq \delta$  and translate the constraint in Equation (2) as

$$\mathcal{L}\left(\phi \implies \omega\right) = \mathcal{L}\left(\neg\phi \lor \omega\right) = \mathcal{L}\left(\neg\phi\right) \cdot \mathcal{L}\left(\omega\right),$$

and further

$$\mathcal{L}(\omega)(x, x') = \mathcal{L}(\|f_{\theta}(x) - f_{\theta}(x')\|_{\infty} \le \delta)$$
$$= \max\{\|f_{\theta}(x) - f_{\theta}(x')\|_{\infty} - \delta, 0\}.$$

where  $\mathcal{L}(\phi' \wedge \phi'')$  is translated to  $\mathcal{L}(\phi') + \mathcal{L}(\phi'')$  and negations are propagated through the constraint (via standard logic). We refer interested readers to the original work (Fischer et al., 2019) for further details on the translation.

Using this differentiable loss, the data producer can now approximate the problem of finding an encoder  $f_{\theta}$  such that the probability that the constraint  $\phi \Longrightarrow \omega$  is satisfied for all individuals is as large as possible. This is solved via the following min-max optimization problem, defined as follows (in two steps): First, we find a counterexample

$$x^* = \underset{x' \in S_{\phi}(x)}{\operatorname{arg \, min}} \mathcal{L} \left( \neg \left( \phi \implies \omega \right) \right) \left( x, x' \right),$$

where  $S_{\phi}(x) = \{x' \in \mathbb{R}^n \mid \phi(x, x')\}$  denotes the set of all individuals considered similar to x according to  $\phi$ .

Then, in the second step we find the parameters  $\theta$  which minimize the constraint loss at  $x^*$ :

$$\underset{\theta}{\operatorname{arg\,min}} \mathbb{E} \left[ \mathcal{L} \left( \phi \implies \omega \right) \left( x, x^* \right) \right].$$

Note that in the outer loop we are finding  $\theta$  which minimize the loss of the original constraint from Equation (2), while in the inner loop we are finding  $x^*$  by minimizing the loss corresponding to the

negation of this constraint. We use Adam (Kingma & Ba, 2015) for optimizing the outer problem. For the inner minimization problem, Fischer et al. (2019) further refine the loss by excluding constraints that have closed-form analytical solutions, e.g.,  $\max\{\|x-x'\|_{\infty}-\delta,0\}$  which can be minimized by projecting x' onto the  $\ell_{\infty}$ -ball of radius  $\delta$  around x. The resulting objective is thus

$$x^{*} = \operatorname*{arg\,min}_{x' \in \mathbb{C}} \mathcal{L}\left(\rho\right)\left(x, x'\right),$$

where  $\mathbb{C}$  is the resulting convex set and  $\rho$  is  $\neg (\phi \Longrightarrow \omega)$  without the respective constraints. It has been shown (Madry et al., 2018) that such an objective can be efficiently solved with Projected Gradient Descent (PGD).

Handling categorical constraints A key challenge here is that DL2 does not support constraints  $\phi$  involving categorical constraints, while these are critical to the fairness context. As mentioned above, numerical attribute constraints of the form  $|x_j - x_j'| \leq \alpha$  can be solved efficiently by projecting  $x_j'$  onto  $[x_j - \alpha, x_j + \alpha]$ . Unfortunately, this does not directly extend to categorical constraints. To see this, consider the constraint that two individuals x and x' should be considered similar irrespective of their race. Further, consider an individual x with only one (categorical) attribute, namely  $x = [\text{race}_1]$ , and r distinct races. After a one-hot encoding, the features of x are  $[1,0,\ldots,0]$ . Now, one could try to translate the constraint as  $|x_k - x_k'| \leq \alpha$  for all  $k = 1,\ldots,r$ . However, choosing e.g.,  $\alpha = 0.3$  would only allow for x' with features of the form  $[0.7,0.3,0,\ldots,0]$  which still represent the same race when considering the maximum element. Thus, this translation would not consider individuals with different races as similar. At the same time, choosing a larger  $\alpha$ , e.g.,  $\alpha = 0.9$ , would yield a translation which considers an individual x' with features  $[0.9,0.9,\ldots,0.9]$  similar to x. Clearly, this does not provide a meaningful relaxation of the categorical constraint.

To overcome this problem, we relax the categorical constraint to  $x_k' \in [0,1]$  and normalize the sum over all possible races as  $\sum_k x_k' = 1$  with every projection step, thus ensuring a meaningful feature vector. Moreover, it can be easily seen that our translation allows x' to take on any race value irrespective of the race of x. We note that although our relaxation can produce features with fractional values, e.g.,  $[0, 0.2, 0.3, 0, \dots, 0.5]$ , we found it works well in practice.

#### 3.2 Predictive Utility of the Representation

Recall that our method is modular in the sense the data producer and data consumer models are learned separately. Therefore, to ensure the latent representation remains informative for downstream applications (represented by some data consumer model  $h_{\psi}$ ), we simulate a data consumer model while training the encoder  $f_{\theta}$ . That is, we train a classifier  $q: \mathbb{R}^k \to \mathbb{R}$  that receives latent representation  $z = f_{\theta}(x)$  as an input and tries to predict the target label y. For this purpose, any suitable classification loss  $\mathcal{L}_{C}$  (e.g., cross entropy) can be employed. Thus, the data producer seeks to jointly train the encoder  $f_{\theta}$  and a classifier q to minimize the combined objective

$$\underset{f,k}{\operatorname{arg\,min}} \mathbb{E}_{x,y} \left[ \mathcal{L}_{C} \left( q \left( f_{\theta} \left( x \right) \right), y \right) + \gamma \mathcal{L}_{F} \left( x, f_{\theta}(x) \right) \right],$$

where  $\mathcal{L}_F$  is the loss obtained from DL2 and the hyperparameter  $\gamma$  balances the two objectives.

We also consider the task of fair transfer learning (Madras et al., 2018) which requires the latent representation to be fair on a variety of different tasks: here, the data producer can additionally learn a decoder g(z) which tries to predict the original attributes x from the latent representation, thereby retaining as much information as possible. This amounts to adding a reconstruction loss  $\mathcal{L}_R(x, g(f_\theta(x)))$ , e.g.,  $\ell_2$ , to the objective. We empirically show that our method is compatible with this other fairness notion in Section 5.

### 3.3 Training Robust Classifier $h_{\psi}$

Here we assume the encoder  $f_{\theta}$  was trained to both maintain predictive utility and satisfy Equation (2). Recall that, given this assumption, the data consumer who wants to ensure their classifier

 $h_{\psi}$  is individually fair, only needs to ensure local robustness of the classifier for perturbations up to  $\delta$  in  $l_{\infty}$ -norm. This is a standard problem in robust machine learning (Ben-Tal et al., 2009) and can be solved via min-max optimization, recently found to work well for neural network models (Madry et al., 2018):

$$\min_{\psi} \mathbb{E}_{z \sim \mathcal{D}} \left[ \max_{\pi \in [\pm \delta]} \mathcal{L}_{C} \left( z + \pi, y \right) \right],$$

where  $\mathcal{L}_C$  is a suitable classification loss. The optimization alternates between trying to find  $\pi \in [\pm \delta]$  that maximizes  $\mathcal{L}_C(z + \pi, y)$  and updating  $\psi$  to minimize  $\mathcal{L}_C(z + \pi, y)$  under such worst-case perturbations  $\pi$ .

## 4 Certifying Individual Fairness

In this section we discuss how the data consumer can compute a certificate of individual fairness for its model  $h_{\psi}$  trained on the latent representation (as described in Section 3.3 above). We split this process into two steps: (i) the data producer propagates a data point x through the encoder to obtain  $z = f_{\theta}(x)$  and computes the radius  $\epsilon$  of the smallest  $\ell_{\infty}$ -ball around z that contains the latent representations of all similar individuals  $f_{\theta}(S_{\phi}(x))$ , i.e.,  $f_{\theta}(S_{\phi}(x)) \subseteq \mathbb{B}_{\infty}(z, \epsilon)$ , and (ii) the data consumer checks whether all points in the latent space that differ by at most  $\epsilon$  from z are classified to the same label, i.e.,  $h_{\psi}(z) = h_{\psi}(z')$  for all  $z' \in \mathbb{B}_{\infty}(z, \epsilon)$ . We now discuss both of these steps.

### 4.1 Certifying Latent Similarity

To compute the minimum  $\epsilon$  which ensures that  $f_{\theta}(S_{\phi}(x)) \subseteq \mathbb{B}_{\infty}(z, \epsilon)$ , the data producer models the set of similar individuals  $S_{\phi}(x)$  and the encoder  $f_{\theta}$  as a mixed-integer linear program (MILP).

**Modeling**  $S_{\phi}$  as MILP We use an example to demonstrate the encoding of logical constraints with MILP. Consider an individual x that has two categorical features  $x_1 = [1, 0, \dots, 0]$  and  $x_2 = [0, \dots, 0, 1]$  and one numerical feature  $x_3$ , with the following constraint for similarity:

$$\phi(x, x') := (x_1 = x_1') \wedge (|x_3 - x_3'| \le \alpha).$$

Here x is an individual from the test dataset and can be treated as constant, while x' is encoded using mixed-integer variables. For every categorical feature  $x_i'$  we introduce k binary variables  $v_i^l$  with  $l=1,\ldots,k$ , where k is the number of distinct values this categorical feature can take. For the fixed categorical feature  $x_1'$ , which is equal to  $x_1$ , we add the constraints  $v_1^1=1$  and  $v_1^l=0$  for  $l=2,\ldots,k$ . To model the free categorical feature  $x_2'$  we add the constraint  $\sum_l v_2^l=1$  thereby enforcing it to take on exactly one of k potential values. Finally, the numerical attribute  $x_3'$  can be modeled by adding a corresponding variable  $v_3$  with the two constraints:  $v_3 \geq x_3 - \alpha$  and  $v_3 \leq x_3 + \alpha$ . It can be easily verified that our encoding of  $S_\phi$  is exact.

Consider now a fairness constraint including disjunctions, i.e.,  $\phi := \phi_1 \lor \phi_2$ . To model such a disjunction we introduce two auxiliary binary variables  $v_1$  and  $v_2$  with the constraints  $v_i = 1 \iff \phi_i(x, x')$  for i = 1, 2 and  $v_1 + v_2 \ge 1$ .

Handling general constraints Note that, the encodings demonstrated on these two examples can be applied for general constraints  $\phi$ . A full formalization of our encoding is found in Appendix A.

Modeling  $f_{\theta}$  as MILP To model the encoder we employ the method from Tjeng et al. (2019) which is exact for neural networks with ReLU activations. We recall that a ReLU performs  $\max\{x,0\}$  for some input x. Given an upper and lower bound on x, i.e.,  $x \in [l,u]$  we can encode the output of ReLU exactly via case distinction: (i) if  $u \leq 0$  add a variable with upper and lower

bound 0 to MILP, (ii) if  $l \ge 0$  add a variable with upper and lower bounds u and l respectively to MILP, and (iii) if l < 0 < u, add a variable v and a binary indicator i to MILP in addition to the following constraints:

$$\begin{aligned} &0 \leq v \leq x \cdot i, \\ &x \leq v \leq x - l \cdot (1 - i), \\ &i = 1 \iff 0 \leq x. \end{aligned}$$

Finally, given MILP formulation of  $S_{\phi}$  and  $f_{\theta}$  we can compute  $\epsilon$  by solving the following k MILP instances:

$$\hat{\epsilon}_{j} = \max_{x' \in S_{\phi}(x)} |f_{\theta}^{(j)}(x) - f_{\theta}^{(j)}(x')|.$$

We compute the final result as  $\epsilon = \max\{\hat{\epsilon}_1, \hat{\epsilon}_2, \dots \hat{\epsilon}_k\}$ .

### 4.2 Certifying Local Robustness

The data consumer obtains a point in latent space z and a radius  $\epsilon$ . To obtain a fairness certificate, the data consumer certifies that all points in the latent space at  $\ell_{\infty}$ -distance at most  $\epsilon$  from z are mapped to the same label as z. This amounts to solving the following MILP optimization problem for each label y' different from the true label y:

$$\max_{z' \in \mathbb{B}_{\infty}(z,\epsilon)} h_{\psi}^{(y')}(z') - h_{\psi}^{(y)}(z').$$

If the solution of the above optimization problem is less than zero for each  $y' \neq y$ , then robustness of the classifier  $h_{\psi}$  is provably established. Note that, the data consumer can employ same methods as the data producer to encode the classifier as MILP (Tjeng et al., 2019) and benefit from any corresponding advancements in solving MILP instances in the context of neural network certification, e.g., (Singh et al., 2019c).

## 5 Experimental Evaluation

We implement our method in a tool called LCIFR and present an extensive experimental evaluation of our method. We consider a variety of different datasets (described below) for which we perform the following preprocessing: (i) normalize numerical attributes to zero mean and unit variance, (ii) one-hot encode categorical features, (iii) drop rows/columns with missing values, and (iv) split into train, test and validation sets. Although, we only consider datasets with binary classification tasks, we note that our method straightforwardly extends to the multiclass case. We make all code, datasets and preprocessing pipelines publicly available at https://github.com/eth-sri/lcifr to ensure reproducibility of our results.

Adult The Adult Income dataset (Dua & Graff, 2017) is extracted from the 1994 US Census database. Every sample represents an individual and the goal is to predict whether that person's income is over 50K / year.

Compas The COMPAS Recidivism Risk Score dataset contains data collected on the use of the COMPAS risk assessment tool in Broward County, Florida Angwin (Angwin et al., 2016). The task is to predict recidivism within two years for all individuals.

**Crime** The Communities and Crime dataset (Dua & Graff, 2017) contains socio-economic, law-enforcement, and crime data for communities within the US. We try to predict whether a specific community is above or below the median number of violent crimes per population.

Table 1: Statistics for train, validation, and test datasets. Note that most of the datasets, namely Adult, German, Health, and Law School, have a highly skewed distribution of positive labels.

	Train		VALIDATION		Test	
	Size	Positive	Size	Positive	Size	Positive
Adult	24129	24.9%	6033	24.9%	15060	24.6%
Compas	3377	52.3%	845	52.2%	1056	55.6%
Crime	1276	48.7%	319	55.5%	399	49.6%
GERMAN	640	70.5%	160	66.9%	200	71.0%
Health	139785	68.0%	34947	68.6%	43683	68.0%
Law School	5053	27.3%	13764	26.8%	17205	26.3%

**German** The German Credit dataset (Dua & Graff, 2017) contains 1000 instances describing individuals who are either classified as good or bad credit risks.

**Health** The Heritage Health dataset (https://www.kaggle.com/c/hhp) contains physician records and insurance claims. For every patient we try to predict ten-year mortality by binarizing the Charlson Index, taking the median value as a cutoff.

**Law School** This dataset from the Law School Admission Council's National Longitudinal Bar Passage Study (Wightman, 2017) has application records for 25 different law schools. The task is to predict whether a student passes the bar exam.

We note that for some of these datasets the label distribution is highly unbalanced as displayed in Table 1. For example, for the Law School dataset, learning a representation that maps all individuals to the same point in the latent space and classifying that point as negative would yield 73.7% test set accuracy. Moreover, individual fairness would be trivially satisfied for any constraint  $\phi$  as all individuals are mapped to the same outcome. It is thus important to compare the performance of all models with the base rates from Table 1. Moreover, for every table containing accuracy values we provide an analogous table with balanced accuracy in Appendix B.1.

### 5.1 Learning and Certifying Individual Fairness

In our first experiment, we propose a range of different constraints for which we apply our method.

**Noise (Noise)** Under this constraint, two individuals are similar if their normalized numerical features differ less than a noise level  $\alpha$ . We consider  $\alpha = 0.3$  for all experiments, which means e.g., for Adult: two individuals are similar if their ages difference is smaller than roughly 3.95 years.

Categorical (CAT) We consider two individuals similar if they are identical except for one or multiple categorical attributes. For Adult and German, we choose the binary attribute gender. For Compas, two people are to be treated similarly regardless of race. For Crime, we enforce the constraint that the state should not affect prediction outcome for two neighborhoods. For Health, two identical patients, except for gender and age, should observe the same ten-year mortality at their first insurance claim. For Law School, we consider two individuals similar regardless of their race and gender.

Categorical and noise (CAT + Noise) This constraint combines the two previous constraints and considers two individuals as similar if their numerical features differ no more than  $\alpha$  regardless of their values for certain categorical attributes.

Conditional attributes (ATTRIBUTE) In this case,  $\phi$  is composed of a disjunction of two mutually exclusive cases, one of which has to hold for similarity. For this, we consider a numerical attribute and a certainty threshold  $\tau$ . If two individuals are both below  $\tau$ , then they are similar if their normalized attribute differences are less than  $\alpha_1$ . If both individuals are above  $\tau$ , similarity holds if the attribute differences are less than  $\alpha_2$ . Concretely, consider two applicants from the Law School dataset. If both of their GPAs are below  $\tau = 3.4$  (the median), then they are similar only if their difference in GPA is less than 0.1694 ( $\alpha_1 = 0.4$ ). However, if both their GPAs are above 3.4, then we consider the applicants similar if their GPAs differ less than 0.847 ( $\alpha_2 = 0.2$ ). For Adult, we consider the median age as threshold  $\tau = 37$ , with  $\alpha_1 = 0.2$  and  $\alpha_2 = 0.4$  which corresponds to age differences of 2.63 and 5.26 years respectively. For German, we also consider the median age as threshold  $\tau = 33$ , with  $\alpha_1 = 0.2$  and  $\alpha_2 = 0.4$  which corresponds to age differences of roughly 0.24 and 0.47 years respectively.

Subordination (QUANTILES) We follow Lahoti et al. (2019b) and define a constraint that counters subordination between social groups. We consider the Law School dataset and differentiate two social groups by race, one group containing individuals of white race and the other containing all remaining races. To counter subordination, we compute within-group ranks based on the GPAs and define similarity if the rank difference for two students from different groups is less than 24. Thus, two students are considered similar if their performance relative to their group is similar even though their GPAs may differ significantly.

Applying our method in practice We assume that the data regulator has defined the above constraints. First, we take on the role of the data producer and learn a representation that enforces the individual fairness constraints using our method from Section 3.1. After training, we compute  $\epsilon$  for every individual data point in the test set and pass it to the data consumer along with the latent representation of the entire dataset as described in Section 4.1. Second, we act as data consumer and use our method from Section 3.3 to learn a locally-robust classifier from the latent representation. Finally, to obtain a certificate of individual fairness, we use  $\epsilon$  to certify the classifier via our method from Section 4.2.

**Hyperparameters** We model the encoder  $f_{\theta}$  as a neural network, and we use logistic regression as a classifier  $h_{\psi}$  for all datasets. We perform a grid search over model architectures and loss balancing factors  $\gamma$  which we evaluate on the validation set. As a result, we consider  $f_{\theta}$  with 1 hidden layer of 20 neurons (except for Law School where we do not have a hidden layer) and a latent space of dimension 20. We fix  $\gamma$  to 10 for Adult, Crime, and German, to 1 for Compas and Health, and to 0.1 for Law School.

We show our results in Table 2 where we compare the accuracy and percentage of certified individuals with a baseline encoder and classifier obtained from setting  $\gamma=0$ . It can be observed that LCIFR drastically increases the percentage of certified individuals across all constraints and datasets. We would like to highlight the relatively low (albeit still significantly higher than baseline) certification rate for the Law School dataset. This occurs because we set the loss balancing factor  $\gamma=0.1$ , thereby only weakly enforcing the individual fairness constraint during training. Moreover, comparing with Table 1, we note that all our classifiers achieve accuracies that are well above the accuracy obtained from always predicting the majority class (except for CAT + Noise on German).

Finally, we report the following mean certification runtime per input, averaged over all constraints: 0.29s on Adult, 0.35s on Compas, 1.23s on Crime, 0.28s on German, 0.68s on Health, and 0.02s on Law School, showing that our method is computationally efficient.

#### 5.2 Scaling to Large Networks

We recall that scalability is not an issue for our method on the datasets considered here: the longest average certification runtime across all datasets and constraints is 1.18s for CAT on the Health dataset. Nevertheless, we show that our method can be easily scaled to larger networks.

Table 2: Accuracy and certified individual fairness. For every dataset we fix the neural network architectures of the encoder  $f_{\theta}$ , and we use logistic regression as a classifier on the latent representation. For every dataset and constraint we compare the accuracy and percentage of certified individuals with a baseline obtained from setting the loss balancing factor  $\gamma=0$ . It can be seen that LCIFR produces a drastic increase in certified individuals while only incurring minor decrease in accuracy.

		Accus	RACY (%)	Севти	FIED (%)
Constraint	Dataset	BASE	LCIFR	BASE	LCIFR
	A	02.0	01.4	50.0	
	ADULT	83.0	81.4	59.0	97.8
	Compas	65.8	63.4	32.1	79.0
Noise	Crime	84.4	83.1	7.4	66.9
1,0101	German	76.5	74.0	71.0	97.5
	${ m Health}$	80.8	81.1	75.4	97.8
	Law School	84.4	84.6	57.9	69.2
	Adult	83.3	83.1	79.9	100
	Compas	65.6	66.3	90.9	100
<b>G</b> . –	Crime	84.4	83.9	78.3	100
Сат	German	76.0	75.5	88.5	100
	Неагтн	80.7	80.9	64.1	99.8
	Law School	84.4	84.4	25.6	51.1
	Adult	83.3	81.3	47.5	97.6
	Compas	65.6	63.7	30.9	75.6
O	Crime	84.4	81.5	6.2	63.3
Cat + Noise	German	76.0	70.0	68.0	95.5
	Неагтн	80.7	80.7	24.7	97.3
	Law School	84.4	84.5	11.6	28.9
	Adult	83.0	80.9	49.3	94.6
Attribute	German	76.5	73.5	65.0	96.5
	Law School	84.3	86.9	46.4	62.6
QUANTILES	Law School	84.2	84.2	56.5	76.9

Table 3: Accuracy and percentage of certified individuals for transferable representation learning on Health dataset with CAT + NOISE constraint. The transfer labels are omitted during training and the data producer objective is augmented with a reconstruction loss. This allows the data consumer to achieve high accuracies and certification rates across a variety of (potentially unknown) tasks

Task	Label	Accuracy (%)	Certified (%)
Original	CHARLSON INDEX	73.8	96.9
Transfer	MSC2A3 METAB3 ARTHSPIN NEUMENT RESPR4	73.7 75.4 75.4 73.8 72.4	86.1 93.6 93.7 97.1 98.4

For this we train a larger encoder  $f_{\theta}$  with 200 hidden neurons and latent space dimension 200. For such large models we can relax the MILP encoding to a linear program (Ehlers, 2017) and solve for robustness via overapproximation. Running this relaxation for our large network and the NOISE constraint on Adult we can certify fairness for 91.4% of the individuals with 82.8% accuracy and average certification runtime of 1.13s. In contrast, the complete solver can certify 92.6% of individuals with average runtime of 31.9s. We note that for even larger model architectures, one can use one of the recent state-of-the-art network verifiers (Singh et al., 2019a).

### 5.3 Transfer Learning

We follow Madras et al. (2018) to demonstrate that our method is compatible with transferable representation learning. We also consider the Health dataset, for which the original task is to predict the Charlson Index. To demonstrate transferability, we omit the primary condition group labels from the set of features, and try to predict them from the latent representation without explicitly optimizing for the task. The data producer learns both the encoder  $f_{\theta}$  and the corresponding decoder g from the reduced feature set. Moreover, the original objective is augmented with an  $\ell_2$ -reconstruction loss, thus not only retaining task-specific information on the Charlson Index. Assuming that our representations are in fact transferable, the data consumer is now free to choose any classification objective. We note that our certification method straightfowardly extends to all possible prediction tasks allowing the data consumer to obtain fairness certificates regardless of the objective. Here, we let the data consumer train classifiers for both the original task and to predict the 5 most common primary condition group labels. We display the accuracy and percentage of certified data points on all tasks in Table 3. The table shows that our learned representation transfers well across tasks while additionally providing provable individual fairness guarantees. We note that all accuracies are above the base rate achieved by majority class prediction and we refer to Appendix B.2 for further details.

#### 6 Related Work

We now discuss work most closely related to ours.

Learning fair representations There has been a long line of work on learning fair representations. Zemel et al. (2013) introduced a method to learn fair representations ensuring group fairness and protection of sensitive attribute. Representations which are invariant to sensitive attributes could also be learned using variational autoencoders (Louizos et al., 2016), adversarial learning (Edwards & Storkey, 2016; Madras et al., 2018) or disentanglement (Creager et al.,

2019). Zemel et al. (2013); Madras et al. (2018) also consider the problem of fair transfer learning which we experiment with in our work. Song et al. (2019) used duality to unify some of the mentioned work under the same framework. McNamara et al. (2019) discuss theoretical guarantees for learning fair representations.

Most work so far focuses on learning representations which satisfy statistical notions of fairness, but there has also been some recent work on learning individually fair representations. These works learn fair representations with alternative definitions of individual fairness based on Wasserstein distance (Feng et al., 2019; Yurochkin et al., 2020), fairness graphs (Lahoti et al., 2019b) or distance measures (Lahoti et al., 2019a). In our work, we defined individual fairness via interpretable logical constraints. While here we focus on learning fair representations, there is also another line of work on training fair classifiers end-to-end (Zafar et al., 2015; Kusner et al., 2017; Agarwal et al., 2018).

Certification of Neural Networks Certification of neural networks has become an effective way to prove that these models are robust to adversarial perturbations. Certification approaches are typically based on SMT solving (Katz et al., 2017), abstract interpretation (Gehr et al., 2018), mixed-integer linear programming (Tjeng et al., 2019) or linear relaxations (Zhang et al., 2018; Singh et al., 2019b,a). A recent work (Urban et al., 2019) also leverages certification methods to verify fairness. However, they only consider certifying fairness of existing models, while we also focus on the actual training of models to be fair. Further, their method can only certify counterfactual fairness on very small networks (4x5) while we investigate both, learning flexible representations but also certification of individual fairness of much larger networks within less than a second.

In our work, we investigate modular certification. For the case of the data producer, we need to propagate the input shape through both logical operators (e.g., conjunctions and disjunctions) and the neural network. While in our work we used a MILP encoding, other approaches could also be applied by crafting specialized convex relaxations. For example, if our approach is applied to learn individually fair representations of complex data such as images, where encoder networks are usually larger than in tabular data which we consider in this work, one could leverage the certification framework from Singh et al. (2019a). On the data consumer side, any of the existing approaches from above could be applied as they are all designed to certify  $\ell_{\infty}$ -robustness which we consider in our work.

### 7 Conclusion

We introduced a novel end-to-end framework for learning representations with provable certificates of individual fairness, which we generalized from fixed distance metric to interpretable logical constraints. We also demonstrated that our method is compatible with existing notions of fairness, such as transfer learning. Our evaluation across different datasets and fairness constraints demonstrates the practical effectiveness of our method.

## 8 Acknowledgments

We would like to thank Anna Chmurovič for her help with initial investigations on combining fairness and differentiable logic during ETH Student Summer Research Fellowship.

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## A Full Encoding

Here, we present our fairness constraint language and show how to encode constraints as a mixed-inter linear program (MILP). We closely follow Fischer et al. (2019).

Logical language We recall that our framework allows the data regulator to define notions of similarity via a logical constraint  $\phi$ . Our language of logical constraints consists of boolean combinations of comparisons between terms where each term t is a linear function over a data point x. We note that although Fischer et al. (2019) support terms with real-valued functions, we only consider linear functions since nonlinear constraints, e.g.,  $x^2 < 3$ , cannot be encoded exactly as MILP. Unlike Fischer et al. (2019), our constraint language also supports constraints on categorical features. To form comparison constraints, two terms t and t' can be combined as t = t',  $t \le t'$ ,  $t \ne t'$ , and t < t'. Finally, a logical constraint  $\phi$  is either a comparison constraint, a negation  $\neg \phi'$  of a constraint  $\phi'$ , or a conjunction  $\phi' \wedge \phi''$  or disjunction  $\phi' \vee \phi''$  of two constraints  $\phi'$  and  $\phi''$ .

Encoding as MILP Given an individual x and a logical constraint  $\phi$  capturing some notion of similarity, the data producer needs to compute the radius  $\epsilon$  of the smallest  $\ell_{\infty}$ -ball around the latent representation  $z = f_{\theta}(x)$  that contains the latent representations of all similar individuals  $f_{\theta}(S_{\phi}(x))$ , i.e.,  $\arg\min_{\epsilon} f_{\theta}(S_{\phi}(x)) \subseteq \mathcal{B}_{\infty}(z, \epsilon)$ . To that end, the data producer is required to encode  $S_{\phi}(x)$  as a MILP which can be performed in a recursive manner.

The individual x belongs to the test dataset and can thus be treated as a constant. To model  $S_{\phi}(x)$ , we encode a similar individual x' by considering numerical and categorical features separately. For all numerical features we add a real-valued variable  $v_i$  to the MILP. For all categorical features we add  $k_j$  binary variables  $v_j^l$  for  $l=1,\ldots,k_j$ , where  $k_j$  is the number of distinct values this categorical feature can take, to the MILP. Furthermore, we add the constraint  $\sum_l v_j^l = 1$  for every categorical variable, thereby ensuring that it takes on one and only one of its values

With these variables, each term can be directly encoded as it consists of a linear function. Likewise, the comparison constraints =,  $\leq$ , and < can be directly encoded in the MILP. We encode  $t \neq t'$  as  $(t < t') \lor (t' < t)$  for continuous variables and as  $\bigvee_{l \neq t'} t = l$  for categorical variables

Next, we consider the case where  $\phi$  is a boolean combination of constraints  $\phi' \wedge \phi''$  or  $\phi' \vee \phi''$ . The first case can be encoded straightforwardly in the MILP. To encode the disjunction  $\phi' \vee \phi''$  we add two additional binary variables v' and v'' to the MILP with the constraints

$$v' = 1 \iff \phi',$$
  
 $v'' = 1 \iff \phi'',$   
 $v' + v'' > 1.$ 

Finally, if  $\phi$  is a negation  $\neg \phi'$  of  $\phi'$ , the constraint is preprocessed and rewritten into a logically equivalent constraint before encoding as MILP:

$$\neg (t = t') := t \neq t', 
\neg (t \leq t') := t' < t, 
\neg (t \neq t') := t = t', 
\neg (t < t') := t' \leq t, 
\neg (\phi' \land \phi'') := \neg \phi' \lor \neg \phi'', 
\neg (\phi' \lor \phi'') := \neg \phi' \land \neg \phi'', 
\neg (\neg \phi') := \phi'.$$

Table 4: Balanced accuracy for encoders and classifiers from Table 2. We recall that for every dataset we fix the neural network architectures of the encoder  $f_{\theta}$ , and we use logistic regression as a classifier on the latent representation. It can be observed that LCIFR incurs only minor decreases in terms of balanced accuracy in comparison with the baseline.

		Balance	D Accuracy (%)
Constraint	Dataset	Base	LCIFR
	Adult	74.5	70.9
	Compas	65.1	62.3
Noise	Crime	84.4	83.2
NOISE	German	69.6	60.8
	HEALTH	77.1	76.5
	Law School	74.5 65.1 84.4 69.6 77.1 76.1 74.7 64.9 84.4 69.2 77.2 L 76.1 74.7 64.9 84.4 69.2 77.2 L 76.1 74.5 69.6 L 76.1	75.8
	Adult	74.7	73.9
	Compas	64.9	65.7
Сат	Crime	84.4	83.9
CAI	GERMAN	69.2	68.3
	HEALTH	77.2	77.1
	Law School	76.1	75.5
	Adult	74.7	70.8
	Compas	ATASET BASE LCIFR  ADULT 74.5 70.9  OMPAS 65.1 62.3  CRIME 84.4 83.2  ERMAN 69.6 60.8  EALTH 77.1 76.5  SCHOOL 76.1 75.8  ADULT 74.7 73.9  OMPAS 64.9 65.7  CRIME 84.4 83.9  ERMAN 69.2 68.3  EALTH 77.2 77.1  SCHOOL 76.1 75.5  ADULT 74.7 70.8  OMPAS 64.9 62.5  CRIME 84.4 81.7  ERMAN 69.2 49.8  CRIME 84.4 81.7  CRIME 84.4 81.7  ADULT 74.7 70.8  CRIME 84.4 81.7  ADULT 74.7 70.8  CRIME 84.4 81.7  CR	62.5
Cat + Noise	CRIME 84.4 GERMAN 69.6 HEALTH 77.1 LAW SCHOOL 76.1  ADULT 74.7 COMPAS 64.9 CRIME 84.4 GERMAN 69.2 HEALTH 77.2 LAW SCHOOL 76.1  ADULT 74.7 COMPAS 64.9 CRIME 84.4 GERMAN 69.2 HEALTH 77.2 LAW SCHOOL 76.1  ADULT 74.7 COMPAS 64.9 CRIME 84.4 GERMAN 69.2 HEALTH 77.2 LAW SCHOOL 76.1  ADULT 74.5 GERMAN 69.6 LAW SCHOOL 76.1	81.7	
CAI + NOISE	German	GERMAN       69.2       68.3         HEALTH       77.2       77.1         AW SCHOOL       76.1       75.5         ADULT       74.7       70.8         COMPAS       64.9       62.5         CRIME       84.4       81.7         GERMAN       69.2       49.8         HEALTH       77.2       76.5	
	Health	77.2	76.5
	Law School	76.1	75.5
	Adult	74.5	70.1
Attribute	GERMAN	69.6	61.8
	Law School	76.1	74.3
Quantiles	Law School	76.1	75.8

## B Additional Results

#### B.1 Balanced Accuracy

We recall that some of the datasets considered are highly imbalanced (cf. Table 1). For that reason we evaluate the balanced accuracies for the results displayed in Tables 2 and 3 and display them in Tables 4 and 5 respectively.

In Table 4 we observe that LCIFR performs only slightly worse than the baseline in terms of balanced accuracy across all constraints and datasets. The only exception is CAT + NOISE on German which was already pointed out above.

In Table 5 we observe that LCIFR performs well in terms of balanced accuracy. However, we note that the balanced accuracies are inversely proportional to the label imbalance as can be seen in Table 6.

Table 5: Balanced accuracy for transferable representation learning on Health dataset with CAT + Noise constraint from Table 3. We recall that the transfer labels are omitted during training and the data producer's objective is augmented with a reconstruction loss.

Task	Label	Balanced Accuracy (%)
Original	Charlson Index	63.9
Transfer	MSC2A3 METAB3 ARTHSPIN NEUMENT RESPR4	70.8 68.5 66.0 58.9 56.0

Table 6: Percentage of positive labels for train, validation, and test datasets for transfer learning tasks. Note, that the percentages do not sum to 100% as the labels are aggregated by patient and year.

	TRAIN	Positive (%) Validation	Test
MSC2A3 METAB3 ARTHSPIN NEUMENT RESPR4	62.0 34.9 31.5 28.4 27.5	61.9 34.9 31.7 28.5 27.5	61.9 34.9 32.1 28.6 27.5

## **B.2** Transfer Learning Base Rates

In Table 6 we present the label distribution for the transfer tasks from Section 5.3. Comparing with Table 3 we observe that the transfer accuracies of LCIFR are above the base rates achieved by majority class prediction in all cases except for RESPR4.