An overview of active learning methods for insurance with fairness appreciation

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Abstract

This paper addresses and solves some challenges in the adoption of machine learning in insurance with the democratization of model deployment. The first challenge is reducing the labelling effort (hence focusing on the data quality) with the help of *active learning*, a feedback loop between the model inference and an oracle: as in insurance the unlabeled data is usually abundant, active learning can become a significant asset in reducing the labelling cost. For that purpose, this paper sketches out various classical active learning methodologies before studying their empirical impact on both synthetic and real datasets. Another key challenge in insurance is the *fairness* issue in model inferences. We will introduce and integrate a *post-processing* fairness for multi-class tasks in this active learning framework to solve these two issues. Finally numerical experiments on unfair datasets highlight that the proposed setup presents a good compromise between model precision and fairness.

Keywords: Active learning, algorithmic fairness, risk classification, insurance datasets.

1 Introduction

Over the last decade, technological advances have allowed the emergence of Artificial Intelligence (AI) solutions for many applications. This emergence has been accompanied by an amplification of AI research, for which major tech companies have largely participated in its democratization. This democratization has allowed many technological advances (autonomous cars, translation...) and facilitates the implementation and deployment of AI solutions. New functionalities are regularly made possible thanks to the availability of new neural network structures that are pre-trained and rapidly integrated into high-level frameworks such as keras¹ in python. However, this appearance of simplicity hides in some cases a relatively complex prerequisite: having a large volume of labelled training data. In order to understand this need, it may be useful to distinguish between generic and specific AI solutions:

• A cognitive service can be said to be generic as soon as it is not dedicated to a sector of activity². They are built, made available and updated by the Big Tech and do not require

¹We can cite for example the language model BERT[DCLT19] which has significantly improved performances in automatic language processing.

²For example, the Vision API of Azure (OCR technology) has been trained to extract printed text in several languages, handwritten text in English as well as numbers and monetary symbols from images or PDFs. Whatever

any adaptation before reuse. So when a company pays to use this service, it is paying for three main things: the mathematical performance of the underlying model, the technical performance of the infrastructure hosting the solution, and the sheer volume of data and machine power that was required to train the model.

• At the opposite end of the spectrum from these solutions are the so-called specific cognitive services. By analogy, these services are so called because they are totally or partially dedicated to a specific sector of activity³. But unlike generic services, these solutions are rarely considered by the big tech companies. Not because their specific character makes them more complex but rather because their niche status leads to a low Return On Investment (ROI), mainly due to the reduced scope of their use and because they require the intervention of rare and expensive business experts. As a result, it is difficult or even impossible to find pre-trained solutions that meet the company's needs, either free of charge or for a fee.

If the separation between specific and generic services seems relatively simple, it is illusory to think that the main AI business projects are based on a single category of service. The reality in the field is that the two approaches are mixed. As a result, most AI projects require large amounts of data to train specific algorithms, which in practice are often based on deep learning models and are extremely data-intensive. For example, for text classification (i.e. topic detection), the training set must contains at least 100 documents for each topic and the building of an insurance document parser will need a training set of at least 5000 documents. The labellisation step is obviously an important part of the AI process (and perhaps the most important). Indeed qualitative labeled data helps in calibrating the learning model to correctly map instances and labels and the lack of it can badly impact the performance of the model and may sometimes introduce biases and ethical problems (e.g. the fairness in AI solutions like bias problems in recruitment). This time consuming (and expensive) task has to be performed by experts in order to ensure the quality of labels. Labeling campaigns are therefore often carried out under time (and therefore volume) constraints. Having algorithms to prioritize the data to be labeled instead of randomly selecting these examples is in some cases as important as the deep learning algorithm that will be trained with these data. These algorithms refer to the field of active learning. This field, which is a sub-section of machine learning, enables the learning algorithm to interactively query a human expert (a.k.a. oracle) to label new data points with the desired outputs.

Both on theoretical and practical ends, the field of active learning has received a lot of attention in the recent years, such as [BGNK18, SED19, SNL+18] among others. The literature already contains many active learning surveys [Set09, FZL13, RXC+20]. However few works tackle the special challenges in active learning related to actuarial problems such as fairness and transparency. It is well-known that enforcing fairness in the model drops the model accuracy as shown by several experimental results [ZVGRG19, ABD+18, DOBD+18, CDH+19, BHN18]. Nevertheless, fairness in AI is one of the key challenges of insurers (notably for EU insurers with the development of regulations such as GDPR) and the fairness evaluation of active learning methods are under-studied. Up to our knowledge only three papers investigate the issue of fairness in active learning. [AAT20] and [SDI20] developed each an algorithm for fair active learning that sample data points to be labeled considering a balance between model accuracy and fairness. [BCAR+21] studies whether models trained with uncertainty-based (deep) active learning is fairer in their decisions with respect to a sensitive feature (e.g. gender) than those trained with passive learning. It appears that, with neural network architecture, active learning can preemptively mitigate fairness issues. The objective of this paper is to give a review and comparison of active learning algorithms for data labeling together with its fairness analysis on real actuarial datasets. For that purpose, this paper is focused on two areas of study: (1) address the fairness issue in Artificial Intelligence and (2) enhance the data quality in labeling datasets with active learning for insurance companies.

Section 2 provides a quick overview of AI governance and the fairness issue, specifically in insurance. Section 3 formally and mathematically defines both fairness and labeling problems. Section 4 gives an overview of classical active learning methods and numerically illustrates them on a simple synthetic

the field of activity and under the assumption of not using a specific language, it is possible to use this service without retraining

³For example, let us consider an entity wishing to measure compliance with GDPR rules in the context of calls made on its platforms. If the model to be implemented is based on generic speech-to-text algorithms, the notion of "compliant" or "non-compliant" verbatims in the GDPR and insurance sense is specific to the sector of activity (or even to the company in the case where its "jargon" is strong)

dataset (cf. Section 4.7). In Section 5 we empirically study the impact of active learning methods together with a novel filtered fairness algorithm on real datasets (both fair and unfair datasets).

2 Fairness issue in Artificial Intelligence

Following the increase of available data (democratization of the datalakes to stock the data) and of the computing power, Artificial Intelligence (AI) constitutes a well established motivating force for the development and the transformation of the insurance sector. Indeed, the insurance use cases integrating machine learning are numerous, while the competition with new actors (GAFAM, Insurtechs) creates pressure on margins and a risk of adverse selection. Therefore the actuary must seize these new and efficient methodologies to keep and reinforce their expertise of the risk. The precision of machine learning algorithm to provide a better segmentation of risk, to achieve large scale automation (e.g. using IoT technologies, extracting information from business records), or to design a decision-making process (e.g. automation of document processing with NLP and computer vision, development of voice bot for call centers with Speech-to-Text and NLP) can both improve the risk assessment and the operational efficiency, and reduce the costs of a company. Therefore a race to deploy these AI systems has progressively taken place in insurance companies.

However, with this ever-evolving AI technology and application, one major drawback is the lack of interpretability. This may explain why adoption of AI into actuarial sciences has been slower than in other fields. This black-box algorithms make it difficult to ensure that the model does not discriminate nor induce unintentional bias, that may expose the company to operational and reputational risks.

Notably for EU's insurers, the fairness principle is an obligation. Indeed, it is recognised in the *Insurance Distribution Directive* (IDD) where it states that insurance distributors shall "always act honestly, fairly and professionally in accordance with best interests of their customers" (Art. 17(1) - IDD). The fairness principle is also stated in Article 5 of *General Data Protection Regulation* (GDPR): personal data shall be "processed lawfully, fairly and in transparent manner in relation to the data subject" (Art. 5(1) - GDPR). For example, since the decision of the European Court of Justice of December 21, 2012, EU's insurers must no longer use gender criteria in in the computation of insurance premiums.

Thus the development of AI governance framework becomes essential to mitigate (or, if possible, to remove) some ethical issues. The idea behind such governance is to render the adoption of AI systems fair in terms of ethics in technological advancement. One of the biggest challenge in establishing governance is the Machine Learning (ML) model fairness assessment. AI systems learn to make decisions based on labeled data, which can include biased human decisions or reflect social inequities even if the sensitive variables (like gender or age) are removed. The unfairness (e.g. gender and race) lies in the data as either the data reflects biased human decisions or the data is simply too imbalanced to be representative and leads to important errors for minority classes. Furthermore the unfairness issue becomes more significant where specific judgements of a black-blox algorithm cannot be specifically explained in a meaningful way (transparency issue).

Unfairness reduction algorithms can be categorized into (1) pre-processing in enforcing fairness directly in the data, (2) in-processing which enforces fairness in the training step of the learning model and (3) post-processing which reduces unfairness in the model inferences. As we will see in the section 4, a key component of active learning is to sample from a pool of unlabelled data. It seems natural to enforce fairness during sampling strategies, as such, falling in the category (1). Contrary to the interpretability for which there does not exist yet a clear metric (cf. [MCB20, Mai21]), one can define formally an unfairness criteria as in 3.5. This criteria will be taken into account with respect to the machine learning algorithms and will be empirically studied in Section 5 both with and without a post-processing unfairness reduction algorithm.

3 Problem formulation

Let us provide the theoretical framework and some mathematical notations. First we introduce different measures to quantity the quality of a classification and then, we present a fairness criterion.

3.1 Theoretical and empirical misclassification risk

Let us denote $\mathcal X$ the space of instances and $\mathcal Y$ the space of labels (or classes). We also consider the adequate probability measures for these spaces: $\mathcal P$ the distribution over $\mathcal X$ and $\mathcal P_{\mathcal X}$ the marginal distribution of $\mathcal P$ over $\mathcal X$. We denote $\mathcal H: \mathcal X \to \mathcal Y$ the space of hypotheses (also called the set of predictors). For a given labeled instance $(x,y) \in \mathcal X \times \mathcal Y$ and a given predictor $h \in \mathcal H$, $\hat y = h(x)$ is the prediction of the label of x.

A prediction is evaluated by a loss function denoted $l: \mathcal{Y} \times \mathcal{Y} \to [0, +\infty[$. For instance, a classical way for a classification problem (i.e. a task restricted to a finite set of classes \mathcal{Y}) is to choose the *misclassification loss* defined by $l_{0-1}(y,y') = \mathbb{1}(y \neq y')$ and the *square loss* for regression tasks (i.e. our task is to predict a continuous output) defined by $l_2(y,y') = (y-y')^2$. Note that a loss function often satisfies the following properties: for $a,y \in \mathcal{Y}$, l(y,y) = 0 and l(a,y) is an increasing function of |a-y|. Interested readers can consult the appendix for more examples on loss functions.

We note that the loss function evaluates the performance of a predictor on a single observation. To evaluate the predictors on a set \mathcal{X} we define the following (theoretical) risk function: for any loss function l we have

$$R(h) := \mathbb{E}[l(h(x), y)] = \int_{\mathcal{X} \times \mathcal{Y}} l(h(x), y) \mathcal{P}(x, y) dx dy \tag{1}$$

In practice, the distribution \mathcal{P} is often unknown, therefore it is an analytically intractable. A (natural) estimator of this risk is its *empirical risk*: If we denote $(x_i, y_i)_{i=1}^N \sim \mathcal{P}^N$ the observations then

$$\hat{R}(h) := \frac{1}{N} \sum_{i=1}^{N} l(h(x_i), y_i)$$
(2)

If we consider the misclassification loss then we have the following misclassification risk:

Theoretical risk: it represents a probability that the predictor h predicts a different answer than the oracle

$$R(h) = \mathbb{E}[\mathbb{1}(h(x) \neq y)] = \mathbb{P}(h(x) \neq y) \tag{3}$$

Empirical risk: it represents the average of times the predictor misclassify on the data

$$\hat{R}(h) = \frac{1}{N} \sum_{i=1}^{N} \mathbb{1}(h(x_i) \neq y_i)$$

3.2 Active sampling and Empirical Risk Minimization

Let $\mathcal{D}^{(train)} = \{(x_i^{(train)}, y_i^{(train)})\}_{i=1}^L$ be the training set and $\mathcal{D}^{(test)} = \{(x_i^{(test)}, y_i^{(test)})\}_{i=1}^T$ the test set where (x_i, y_i) are drawn i.i.d. according to the distribution \mathcal{P} . If we assume that we have access to a large pool of unlabelled dataset denoted $D_X^{(pool)} = \{x_1^{(pool)}, \dots, x_U^{(pool)}\}$ (we will also call it pool-set) and one or several oracles to label the data then the statistical learning process consists in the following steps:

- 1. Querying step. This step consists of labeling the "raw" data allowing to set up labeled data (for instance both training data $\mathcal{D}^{(train)}$ and test data $\mathcal{D}^{(test)}$). We consider two types of labeling: a passive labeling which consists in randomly querying the data and an active labeling which queries the data according to an importance criterion. We note that passive labeling allows generating an i.i.d. database whereas active labeling generates a database that does not check the independence condition. The test data (also called the hold-out set) should be generated with passive labeling in order to empirically replicate $\mathcal{X} \times \mathcal{Y}$.
- 2. **Training step.** Given the training set $D^{(train)}$, this step consists in finding an estimator $\hat{h} \in \mathcal{H}$ such that, for any labeled point $(x^{(train)}, y^{(train)})$, $h(x^{(train)})$ is "as close as possible" to $y^{(train)}$ while avoiding its overfitting⁴. More formally, in the training step, the

⁴too much capture of random fluctuations and variations in the training data resulting to a poor generalization of the prediction of an unlearned data

objective is to find out the optimal predictor $h^* \in \mathcal{H}$ i.e. a minimizer of the theoretical risk Eq. (2). The theoretical risk being intractable in practice, we focus on \hat{h} a function that minimizes its empirical form Eq. (3) instead. We call this minimization the *Empirical Risk Minimization* (ERM):

$$\hat{h} = \underset{h \in \mathcal{H}}{\operatorname{argmin}} \hat{R}_{train}(h)$$

where

$$\hat{R}_{train}(h) := \frac{1}{L} \sum_{i=1}^{L} l\left(h(x_i^{(train)}), y_i^{(train)}\right)$$

After the learning phase we expect that minimizing the empirical risk is approximately equivalent to minimizing the true risk:

$$\hat{R}_{train}(\hat{h}) \approx R(h^*)$$

3. **Testing step.** This step studies the performance of our estimator h on a hold-out set $D^{(test)}$. Note that this step helps detecting whether the model overfit or underfit on the training set. In a more formal way, this amounts to checking that \hat{h} verifies the condition:

$$\hat{R}_{train}(\hat{h}) \approx R_{test}(\hat{h})$$

where

$$\hat{R}_{test}(h) := \frac{1}{T} \sum_{i=1}^{T} l\left(h(x_i^{(test)}), y_i^{(test)}\right).$$

The notion of overfitting (or underfitting) is closely related to the estimation (a.k.a. variance) and approximation (a.k.a. bias) errors.

3.3 Bias and variance trade-off

Given the set \mathcal{H} , let us denote \hat{h} an estimator (e.g. ERM estimator), h^* the optimal estimator w.r.t. Eq. (3) with $R^* = R(h^*)$ its error. Then the *excess error* can be viewed as a bias/variance decomposition:

$$\underbrace{R(\hat{h}) - R(h^*)}_{\text{Excess error}} = \underbrace{\left\{R(\hat{h}) - \inf_{h \in \mathcal{H}} R(h)\right\}}_{\text{Error of estimation or variance}} + \underbrace{\left\{\inf_{h \in \mathcal{H}} R(h) - R(h^*)\right\}}_{\text{Error of approximation or bias}}.$$

The first term is called the estimation error (or variance) and the second term the approximation error (or bias). This bias-variance dilemma is a conflict that exists when we try to simultaneously minimize these two sources of error which prevent supervised learning algorithms from generalizing beyond their training set.

Bias of an estimator. It measures how well the optimal estimator can be approximated within a hypothesis space \mathcal{H} . The bias is an error coming from wrong assumptions in the learning algorithm. A high bias (a too simple learning model) can lead to an algorithm taking little account of the relevant relationships between the features and the target outputs (underfitting). However a too low bias can impact and lead to a higher variance.

Variance of an estimator. The variance depends on the chosen hypothesis h. The error of estimation measures the error of h relative to the lower bound of the errors realized by the assumptions in \mathcal{H} . In other words, the variance is an error arising from being too sensitive to small fluctuations in the learning base. A high variance (overly complex learning model) may lead to an algorithm that models random noise in the training data, rather than the target outputs (overfitting). A too low variance can lead to a higher bias.

Thus, the model overfit (resp. underfit) if it has a too high variance (resp. bias) whereas a too low value can lead to a higher bias (resp. variance) and therefore to a poorer model's performance.

Hence the goal in supervised machine learning is to choose a hypothesis space \mathcal{H} that verifies a good trade-off between approximation and estimation errors.

In this study the model performance is not only tied to the model metric but also to the model unfairness. Hence the evaluation of a model is defined by an adequate model metric and an adequate model unfairness.

3.4 Precision evaluation

The choice of the metric is essential for measuring the precision of the model. The well-known accuracy (e.g. empirical form of the complement of the misclassification risk) will not be used considering its misinterpretation for imbalanced datasets: a very high accuracy might not represent a good model (e.g. a constant classifier always predicting the majority class). For that purpose in binary classification we consider simultaneously two metrics:

- precision: the number of true positive results divided by the number of all positive results;
- **recall:** the number of true positive results divided by the number of all samples that should have been identified as positive.

As we want the same importance of these two metrics we consider their harmonic mean **F1-score** which presents a good balance between precision and recall therefore giving good results on imbalanced classification problems. For a multi-class classification task we can compute a F1-score per class in a one-vs-rest manner and then average the results. For a more precise definition, refer to Eq. (6). Empirically, this metric will be further studied in Section 5.

3.5 Unfairness evaluation

As mentioned in Section 2, the unfairness is one of the biggest challenge in insurance (and, more broadly, in many AI governance) therefore it needs to be rigorously defined alongside with its evaluation metric. Let us first define the most widely used definitions of fairness of an estimator.

We consider $S = \{-1, +1\}$ the sensitive feature (e.g. gender or sex) which is included in \mathcal{X} . We also consider $\mathcal{X}_{-\mathcal{S}} := \mathcal{X} \setminus \mathcal{S}$ the space of instances without the sensitive feature, K the total number of classes and Y the true response of the task.

Definition 3.1 (Equalized Odds). In Equalized Odds (a.k.a. Positive Rate Parity) (see [HPS16]), we say that a classifier $h \in \mathcal{H}$ is fair with respect to the distribution \mathbb{P} on $\mathcal{X}_{-S} \times \mathcal{S} \times [K]$ if h(X) and S are independent conditional on Y. For binary classification, this definition states that protected and unprotected group should have equal true positive rates and false positive rates.

A relaxation of equalized odds is possible in binary case: we can require to have non-discrimination group only within the "advantaged" outcome (e.g. university admissions or employee recruitment). This unfairness is called *Equal Opportunity*.

Definition 3.2 (Equal Opportunity). *In binary setting, we say that a classifier* $h \in \mathcal{H}$ *satisfies Equal Opportunity (a.k.a. True Positive Parity)(see [HPS16]) with respect to the distribution* \mathbb{P} *on* $\mathcal{X}_{-S} \times \mathcal{S} \times \{0,1\}$ *if*

$$\mathbb{P}(h(X) = 1 | S = 1, Y = 1) = \mathbb{P}(h(X) = 1 | S = -1, Y = 1)$$

A relaxation of the above fair definitions is possible in the multi-class case: the *Demographic Parity* which require to have non-discrimination group in all (predicted) outcomes.

Definition 3.3 (Demographic Parity). *In Demographic Parity (a.k.a. Statistical Parity) see [CKP09], we say that a classifier* $h \in \mathcal{H}$ *is fair with respect to the distribution* \mathbb{P} *on* $\mathcal{X}_{-S} \times \mathcal{S} \times [K]$ *if*

$$\mathbb{P}(h(X) = k|S = 1) = \mathbb{P}(h(X) = k|S = -1), \quad \forall k \in [K].$$

This definition states that protected and unprotected group should have equal likelihood. In an approximate version we want: for a given small $\epsilon > 0$,

$$|\mathbb{P}(h(X) = k|S = 1) - \mathbb{P}(h(X) = k|S = -1)| \le \epsilon, \quad \forall k \in [K].$$

In this study, we consider multi-class classification problems under **Demographic Parity** fairness constraint [CKP09], that requires the independence of the prediction function from the sensitive feature S for all classes.

Evaluation in Demographic Parity. Given a classifier $h \in \mathcal{H}$, when fairness is required, two important aspects of the classifier need to be controlled: its misclassification risk R defined in Eq. (1) and its unfairness that will be evaluated by

$$\mathcal{U}(h) := \frac{1}{K} \sum_{k=1}^{K} \left| \mathbb{P}\left(h(X) = k | S = 1\right) - \mathbb{P}\left(h(X) = k | S = -1\right) \right|.$$

Naturally, taking into account the definition above, a classifier h is more fair as $\mathcal{U}(h)$ becomes small. This quantity being intractable, its empirical form (see Eq. (7)) will be used in practice.

4 Active Learning methods

For the present study let us introduce the active learning setting before studying its sampling methods.

4.1 Definitions and framework

Given a set of hypothesis \mathcal{H} , active learning consists in iteratively querying oracle to label instances $x \in D_X^{(pool)}$ that provides the most information for learning an hypothesis $h \in \mathcal{H}$. Informally, active learning enables a learning model to perform better with fewer labeled data if we can query from a pool of unlabelled data. Following each query the model is trained and updated w.r.t. the new labelled data. There are several ways to access unlabeled data, among them there are:

- 1. the offline scenario where the raw data is directly accessible in large quantities, for example, product reviews left by Internet users on a website. The query strategy associated to this scenario is called pool-based sampling (or offline sampling). In this scenario, we assume that the unlabeled data $D_X^{(pool)} \subset \mathcal{X}$ is completely available. Given a trained model $h \in \mathcal{H}$ the goal of an active learner would be querying the most informative instance according to a well-defined importance score $I(\cdot,h)$. We call batch-mode sampling the procedure of querying more than one instance per active learning iteration. A natural approach for a batch-mode sampling would be querying the top instances according to the importance score.
- 2. the *online scenario* where the data is collected one by one, for example, e-mails received in an e-mail box. The query strategy associated to this scenario is called *stream-based sampling* (or *online sampling*). In this scenario, the active learner decides whether each new instance $x^{(stream)}$ should be queried or not. For each accepted query, we obtain the label $y^{(stream)}$ of $x^{(stream)}$ (via the oracle). Given an importance score function, a natural approach for online sampling would be querying instances that have their score above a given threshold.

The goal of active learning in both scenarios is to retain the optimal set of queries that maximizes the model's performance (e.g. minimizing the empirical risk on test-set). In contrast, the traditional model of supervised learning is trained on a dataset queried randomly (passive sampling) from the pool-set. This last process is called *passive learning*.

In active learning, the objective is to build actively a model more accurate than the passive one. In other words, if we denote $\hat{h}_a, \hat{h}_p \in \mathcal{H}$ the models trained respectively by active and passive learning process with the same cardinal of training set, then we expect to have

$$\hat{R}_{test}(\hat{h}_a) < \hat{R}_{test}(\hat{h}_p).$$

About the training step. Let $h_t \in \mathcal{H}$ be the previous trained model. We assume that (x^*, y^*) is the current queried data. During the training step of the active learner, we can either (1) construct h_{t+1} in re-training h_t on the whole labeled set including (x^*, y^*) (called *batch learning* approach)

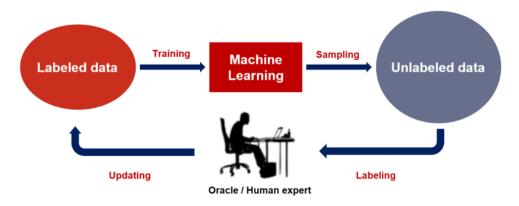


Figure 1: Active learning in an offline scenario.

or (2) construct h_{t+1} in updating the weight of h_t based only on (x^*, y^*) (called *online learning* approach. See [SS⁺11] for more details).

About the learning task. In the current literature, each query strategy often depends on the learning task we are studying: sampling strategies for a classification problem often differ from those for a regression. This is mainly due to the nature of the model responses: unlike regression, a probabilistic classification model often gives a directly interpretable response (for instance a posterior probability that a given class is correct) therefore leading to some natural heuristic choices for the importance score function.

In practice, although some tabular data require the intervention of an additional tool for labeling, active learning sees its usefulness in the labeling of unstructured data such as texts, images or sounds. Indeed unstructured data often suffer from a lack of labeling due to its difficulty and its high complexity in time and cost. Labeling unstructured data involves the intervention of oracles / human experts (hence the *cost*) that may require some expertise⁵ (hence the *difficulty*) to label the data one by one (hence the *time*), especially when the data is in sound, video or text format.

Settings. Motivated by the above practical insights we study the active learning for (both binary and multi-class) **classification tasks**. Furthermore, we assume that unlabeled data is inexpensive and abundant but labeling them is difficult, expensive and time consuming. Therefore we are in an **offline scenario**. Given an importance score function I, we present in Algorithm 1 the active learning process.

⁵For instance some textual data in insurance need to be studied by legal experts, e.g. the GDPR-compliancy.

Algorithm 1 Outline of an active learner process in an offline scenario

Input: h a base estimator, $\mathcal{D}^{(train)}$ the initial training set and $\mathcal{D}_{X}^{(pool)}$ the initial pool-set.

Step 0. Fit h on the training set $\mathcal{D}^{(train)}$.

Step 1. The active learner query the instance $x^* \in \mathcal{D}_X^{(pool)}$ that maximizes the importance score function I(x,h) i.e.

$$x^* = \underset{x \in \mathcal{D}_X^{(pool)}}{\operatorname{argmax}} \{I(x, h)\}$$

Step 2. Update the training set and the pool-set: if we denote y^* its label then

$$\mathcal{D}^{(train)} = \mathcal{D}^{(train)} \cup \{(x^*, y^*)\}$$

$$\mathcal{D}_X^{(pool)} = \mathcal{D}_X^{(pool)} - \{x^*\}$$

Step 3. As long as we do not reach a stopping criterion (e.g. exhaustion of the labeling budget or convergence of the performance), we repeat this process (**return to step 0**).

Output: the final estimator h

Some statistical guarantees. Let us denote $h^* \in \mathcal{H}$ the Bayes classifier with d its Vapnik-Chervonenkis (VC) dimension (see [VC15]), $\hat{h}_a \in \mathcal{H}$ the active learner and $\hat{h}_p \in \mathcal{H}$ the passive learner. The VC dimension is a measure of a learning algorithm's "complexity": it is defined by the cardinality of the largest set of points that the algorithm can shatter (given any labeled data points the algorithm can always learn a perfect classifier). We also denote classification error excesses:

$$\varepsilon_p = R(\hat{h}_p) - R(h^*)$$
 and $\varepsilon_a = R(\hat{h}_a) - R(h^*)$.

Then, it is shown by [BBL09] that under good conditions on the label distribution like the *Massart noise*⁶ we have the following convergence rates (n being the number of labeled data):

$$\varepsilon_p \sim \frac{d}{n} \quad \text{and} \quad \varepsilon_a \sim \exp\left(-\text{constant} \times \frac{d}{n}\right).$$
 (4)

This means that (under this condition) active learning outperforms passive learning and therefore reduces the labeling costs. Note that these results are achieved by a disagreement-based active learning algorithm called A^2 (Agnostic active learning) algorithm, as detailed in [BBL09]. The notion of disagreement-based sampling will be defined and studied in Section 4.3.

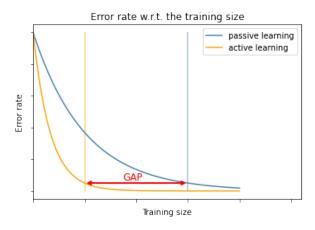


Figure 2: The shape of the curve (we are supposed to have) with active learning process.

⁶The *Massart noise* verifies: $\exists \beta < 1/2$ such that $\mathbb{P}(Y \neq h^*(X)|X) \leq \beta$ almost everywhere with $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$.

In the next sections we will introduce the various classical approaches in active learning to define *I* before studying them on both synthetic and real datasets.

4.2 Sampling based on uncertainty

This approach consider that the most informative instances are the ones located in the region of uncertainty of the learning model. Therefore, the idea is to query and learn the most uncertain sample according to the estimator. Intuitively this approach tends to avoid querying redundant instances since the model learned on an uncertain data point will probably be more certain. Let us formally define the uncertainty of an instance.

Given the current predictor $h_t \in \mathcal{H}$ (t being the t-th update of the model) and an instance $x \in \mathcal{X}$, we denote $p_{t,x}(y) := \mathbb{P}(h_t(x) = y|x)$ the probabilistic response of classifying x as y according to h_t . This approach is studied under both binary and multi-class classification task.

Binary classification. In a binary task, a natural method is to sample the least confident instances (LC) for our learning model. In a more formal way we query the instance

$$\hat{x}_{\text{LC}} = \underset{x \in \mathcal{D}_{x}^{(pool)}}{\operatorname{argmin}} \left\{ \left| p_{t,x}(y) - \frac{1}{2} \right| \right\}$$

where the unimportance criterion is defined by

$$I_{\mathrm{LC}}(x, h_t) = \left| p_{t,x}(y) - \frac{1}{2} \right|$$

Thus defined, the queried instance \hat{x}_{LC} is in the region of uncertainty of h_t . We therefore hope, via this approach, that the updated model h_{t+1} will either perform better (section 3) than the unupdated model h_t or perform better than passive learning.

Multi-class classification. Let K>2 be the number of classes in a multi-class classification setting. A naive approach would be to generalize the approach presented in the binary case by querying the most uncertain instance according to

$$\hat{x}_{LC} = \underset{x \in \mathcal{D}_X^{(pool)}}{\operatorname{argmin}} \left\{ p_{t,x}(y^{(K)}) \right\}$$

Where we denote $y^{(k)}$ the argument of the k-th highest value of $p_{t,x}$. It follows $p_{t,x}(y^{(1)}) \le p_{t,x}(y^{(2)}) \le \cdots \le p_{t,x}(y^{(K)})$ and therefore $y^{(K)}$ is highest probable class for the instance x under the model h_t ,

$$y^{(K)} = \underset{y \in [K]}{\operatorname{argmax}} \ p_{t,x}(y)$$

 \hat{x}_{LC} thus is the queried instance with the smallest probability (in the pool-set) of the most probable class. Note this approach focus only on $y^{(K)}$ and ignores the distribution of the other classes $y^{(k)}$, $k \neq K$ and therefore it raises the following issue: for a given instance x if the margin between $p_{t,x}(y^{(K)})$ and $p_{t,x}(y^{(K-1)})$ is small then x can be considered as an *uncertain* data point *undetected* by the the LC criteria. In contrast, a given instance can be, naturally, considered as a *certain* data point if there is a large margin between class probabilities. The Shannon entropy (defined in [Sha48]) can overcome this problem.

The Shannon entropy is often used in statistical learning as a measure of uncertainty (as for example in the calculation of decision trees with the entropy measure). In this case, the Shannon entropy assigns to a given instance x an uncertainty score according to the distribution of $p_{t,x}=(p_{t,x}(1),p_{t,x}(2),\cdots,p_{t,x}(K))$. This measure classifies instances according to whether the posterior probability distribution of the labels is uniform. More precisely a distribution close to the uniform distribution underlines the difficulty for the model to decide on the right class while a distribution far from the uniform distribution implies a more reliable prediction. For any instance x and fixed time t we note the uncertainty score $H_{t,x}:=H_{t,x}(Y)$ with Y the discrete random variable on $\mathcal Y$ where each event $\{Y=y\}$ occurs with probability $p_{t,x}(y)$.

The entropy-based uncertainty score is defined by $H_{t,x} = -\sum_{k=1}^{K} p_{t,x}(k) \log p_{t,x}(k)$ and we thus query the instance that verifies the equation

$$\hat{x}_H = \underset{x \in \mathcal{D}_X^{(pool)}}{\operatorname{argmax}} \left\{ H_{t,x} \right\}.$$

The corresponding sampling is called uncertainty sampling. In the definition of $H_{t,x}$ we use the convention $0 \log 0 = 0$.

Let us denote $I_Y(y) = -\log p_{t,x}(y)$ the information (or *self-information*) that the random event $\{Y=y\}$ contains. We remark that more uncertain an event is, the more informative it is (thus interesting) and a certain event contains no information. We note that the entropy can also be written as

$$H_{t,x}(Y) = \mathbb{E}[-\log p_{t,x}(Y)] = \mathbb{E}[I_Y].$$

The Shannon entropy of a random variable Y is therefore the expectation of the information contained in the variable Y.

Although these uncertainty-based methods can strengthen the learning model, especially on regions of uncertainty, they remain difficult to use when the model is not enough reliable for prediction. Indeed, the current training set may not contain sufficient information for the model: we call this problem, the *cold-start problem*⁷. Moreover, this sampling method works only for probabilistic model and therefore is not adequate for deterministic or non-probabilistic models (e.g. SVM). Alternative methods exist to overcome (at least partially) these problems such as disagreement-based sampling.

4.3 Sampling based on disagreement

For an active learning process the choice of the initial labelled data is crucial so as to get an accurate uncertainty score. If the dataset is insufficiently representative of \mathcal{X} then too many plausible model parameters can be suggested for fitting such dataset leading therefore to a *model robustness* issue. We call this phenomenon a *high epistemic uncertainty*. By contrast a *low epistemic uncertainty* indicates a robust model. Usually epistemic uncertainty arises in areas where there are fewer samples for training.

Instead of relying on the uncertainty measure based on a single model, disagreement-based sampling proposes a more robust method by combining the result of several learning models all different from each other (so-called *ensembling* methods). The idea described in [SOS92] is to rely on a set of models to "vote" the informativeness of each instances. This set of models is named a *Committee*. For a committee an informative instance is characterized by the highest voting disagreement among the models. For a given instance, a model's vote can be characterized either by its label prediction (a.k.a. "hard" vote) or by its label posterior probability (a.k.a. "soft" vote). This approach is called *Query-By-Committee* (a.k.a. *QBC*). QBC requires two main components, namely the construction of the **committee** and the definition of the **disagreement measure** answering respectively the following questions: (1) how can we define a set of models close enough to sample the adapted regions of uncertainty but different enough to ensure the informativeness of the queries? (2) How to measure the degree of disagreement of the Committee?

Committee. Consider a committee of models $h_t^{\text{committee}} = \{h_t^1, \dots, h_t^C\}$ where each $h_t^i \in \mathcal{H}$ is trained on the current training-set $\mathcal{D}^{(train)}$. QBC is a query strategy based on the maximum disagreement of $h_t^{\text{committee}}$ constructed by a randomized copies of a learned model. At each iteration, the algorithm generates a new committee of classifiers based on the updated training-set.

Initially in his simulations [SOS92] had implemented a Gibbs training to learn two perceptrons: each perceptron is consistent with the training-set $\mathcal{D}^{(train)}$ with slightly different parameters due to the randomness of the Gibbs algorithm. These generated models query the instance where their predictions are the most dispersed. Note that [FST97] showed that for a given performance (see Eq. (4)) and given some assumptions this process leads to an exponential decrease in the number of labeled instances required compared to passive learning. However, in spite of these theoretical

⁷The term comes from the analogy with starting very cold engines such as cars. Cold starting a car can be difficult to deal with but as soon as the engine is running the car starts to move forward and the performance increases.

guarantees, Gibbs algorithms has a high computational complexity and is hard to implement for more complex learning models. We overcome this problem by using other approaches to generate $h_{\text{committee}}$:

- For generative models such as linear discriminant analysis (LDA) or naive Bayesian classification, we can randomly sample an arbitrary number of models from a $h \to \mathbb{P}(h|\mathcal{D}^{(train)})$ distribution with $h \in \mathcal{H}$. As an example, [DE95] samples Hidden Markov Models (HMM) using normal distribution;
- For other classes of models, such as discriminative (e.g. logistic regression) or non-probabilistic (e.g. SVM) models, Ensemble methods such as Bagging or Boosting are used for constructing the committee. These query strategies were first proposed by [AM98] and are defined as follows:

Query-by-bagging. Let $h \in \mathcal{H}$ be a base estimator and $\mathcal{D}^{(train)}$ our current training-set. The first step of query-by-bagging method consists in bootstrapping $\mathcal{D}^{(train)}$ into C sets of the same size as the original set denoted by $\mathcal{D}^{(train),1},\ldots,\mathcal{D}^{(train),C}$. Then we construct the committee of models $h^{\text{committee}} = \{h^1,\ldots,h^C\}$ such that each member h^i corresponds to the base model h trained on the bootstrapped training-set $\mathcal{D}^{(train),i}$.

Query-by-boosting. Let $h \in \mathcal{H}$ be a base estimator and $\mathcal{D}^{(train)}$ our current training-set. The method *query-by-boosting* consists in building the committee $h^{\text{committee}} = \{h^1, \dots, h^C\}$ via *Adaboost* (*Adaptive Boosting* [FS97]) on the base model h where (h^1, \dots, h^C) is a sequence of copies of h trained on an iteratively modified dataset: the weights of incorrectly classified instances are adjusted such that following classifiers focus more on challenging examples. Query-by-boosting approach chooses an instance for which the weighted vote obtained by boosting is the most dispersed.

Disagreement measure. Given a committee $h_t^{\text{committee}} = \{h_t^1, \dots, h_t^C\}$ with t the number of updates (or active learning iterations), there exists various disagreement measures for evaluating the dispersion of votes for multi-class classification problems. In the active learning litterature we have the following known methods:

 vote by entropy proposed by [DE95] is an entropy-based method combined with a hard committee vote:

$$\hat{x}_{VE} = \underset{x \in \mathcal{D}_{X}^{(pool)}}{\operatorname{argmax}} \left\{ -\sum_{k=1}^{K} \frac{v_{t,x}^{\text{committee}}(k)}{C} \log \frac{v_{t,x}^{\text{committee}}(k)}{C} \right\}$$

where $v_{t,x}^{\text{committee}}(k) := \sum_{c=1}^{C} \mathbb{1}\{h_t^c(x) = k\}$ is the number of *hard* votes of the committee for the label k given the instance x.

• the mean Kullback-Leibler (KL) divergence [MN98], is a method based on the KL divergence combined with a weak committee vote:

$$\hat{x}_{KL} = \underset{x \in \mathcal{D}_{v}^{(pool)}}{\operatorname{argmax}} \left\{ \frac{1}{C} \sum_{c=1}^{C} D(p_{t,x}^{c} || p_{t,x}^{committee}) \right\}$$

where for all $c \in [C]$, $D(|p_{t,x}^c|||p_{t,x}^{committee}|)$ is the KL divergence defined by

$$D(\;p_{t,x}^c \mid\mid p_{t,x}^{committee}\;) = \sum_{k=1}^K p_{t,x}^c(k) \log \left\{ \frac{p_{t,x}^c(k)}{p_{t,x}^{committee}(k)} \right\}$$

with $p_{t,x}^c(k) = \mathbb{P}(h_t^c(x) = k|x)$ the probability that the model h_t^c predicts that the class of x is k and $p_{t,x}^{committee}(k) = \frac{1}{C}\sum_{c=1}^{C}\mathbb{P}(h_t^c(x) = k|x) = \frac{1}{C}\sum_{c=1}^{C}p_{t,x}^c(k)$ is an averaged (over all committee member) probability that k is the correct class.

So far, we have presented techniques for querying instances that lie on an uncertain regions of classification. These uncertain areas are characterized either by the uncertainty of a classifier (iterative update of a *single set* of parameters) or by the disagreement of a model committee on its classification (iterative update of *several sets* of parameters). However, these approaches intend to improve only a local (and not a global) prediction quality of an estimator. Indeed, these methods do not take into account the influence of a queried input instance \hat{x} on the other parts of the input space: implicitly they try to increase the quality of the model near \hat{x} . On the other hand the following algorithms propose to query the instances that "impact" the most learning models.

4.4 Sampling based on model change

The idea of these approaches is to choose the instance that gives the most change (or impact) on our learning model if we know its label. The word "impact" may seem vague but it highlights a class of possibilities in terms of sampling criteria. In the current literature, when we know its label a candidate instance can impact the model mainly in two manners: (1) *impact on the model parameters* and (2) *impact on the model performance*.

4.4.1 Impact on the model parameters

The idea is to query instances that can change the learning model $h_t \in \mathcal{H}$ as much as possible. This can be done by evaluating the change of the model parameters between the updated model h_{t+1} and the current model h_t . Intuitively, if an instance is able to modify considerably the parameters of a model, then this instance contains information on the underlying distribution \mathcal{X} which is not (or rarely) found in the training-set. In the following, we call this set of strategies the *Expected Model Change* (EMC). An example of a change measure is the *Expected Gradient Length* (EGL).

Expected Gradient Length. The EGL strategy applies to all learning models that require the computation of the gradient of a loss function during training (e.g. training via gradient descent). Consider l_t a loss function with respect to the model $h_t \in \mathcal{H}$ and ∇l_t its gradient. The degree of change of the model can be measured by the Euclidean norm of the training gradient $\|\nabla l_t(\mathcal{D}^{(train)})\|$: if we denote $\mathcal{D}_t^{(train)}$ and $\mathcal{D}_{t+1}^{(train)}$ the data labeled at iteration t and t+1 respectively then a "small" impact results in a norm $\|\nabla l_t(\mathcal{D}_{t+1}^{(train)})\| \approx \|\nabla l_t(\mathcal{D}_t^{(train)})\| \approx 0$ while a "large" impact results in a large margin between these two norms. Thus, the instance to query is the instance x which, if labeled and added to $\mathcal{D}^{(train)}$, results (on average over the set of possible labels) in a larger gradient size. Formally this amounts to querying x which maximizes the term

$$\mathbb{E}\left[\left\|\nabla l_t\left(\mathcal{D}^{(train)}\cup(x,Y)\right)\right\|\right]$$

with $Y \sim p_{t,x}$ a discrete random variable on \mathcal{Y} . That is

$$\hat{x}_{EGL} = \underset{x \in \mathcal{D}_{X}^{(pool)}}{\operatorname{argmax}} \left\{ \sum_{k=1}^{K} p_{t,x}(k) \left\| \nabla l_{t} \left(\mathcal{D}^{(train)} \cup (x,k) \right) \right\| \right\}.$$

For the sake of time complexity we approximate \hat{x}_{EGL} by

$$\tilde{x}_{EGL} = \underset{x \in \mathcal{D}_{X}^{(pool)}}{\operatorname{argmax}} \left\{ \sum_{k=1}^{K} p_{t,x}(k) \left\| \nabla l_{t} \left((x,k) \right) \right\| \right\}.$$

Indeed after training the model h_t on $\mathcal{D}^{(train)}$, $||\nabla l_t(\mathcal{D}^{(train)})|| \approx 0$ (l_t reaches a local minimum) and we often assume that the training-set is independent.

4.4.2 Impact on the model performance

The idea is to query instances that can reduce the generalization error. We can reduce the forecast error by estimating this error directly empirically (e.g. *Expected Error Reduction*) or indirectly by reducing the variance present in the risk of a learning model.

Expected Error Reduction. The Expected Error Reduction (EER), proposed by [RM01], is a strategy consists in choosing the instance that minimizes the *expected* of generalization error since the class of the instance is currently unknown. Let us note

- h_t a predictor of \mathcal{H} trained on $\mathcal{D}_t^{(train)}$ at time t;
- $h_{t+1}^{(x,k)}$ the updated predictor re-trained on $\mathcal{D}^{(train)} \cup (x,k)$ at time t+1;
- $p_{t+1,u}^{(x,k)}(v)=\mathbb{P}(h_{t+1}^{(x,k)}(u)=v|u)$ the probability that the class of u is v under $h_{t+1}^{(x,k)}$;
- $\bullet \ y^{(K)} = \underset{y \in \mathcal{Y}}{\operatorname{argmax}} \ p_{t+1,u}^{(x,k)}(y).$

[RM01] proposed the following approach for minimizing the expected error (based on 0-1 loss):

$$\hat{x}_{EER} = \underset{x \in \mathcal{D}_X^{(pool)}}{\operatorname{argmin}} \left\{ \sum_{u \in \mathcal{D}_X^{(pool)}} \mathbb{E}_Y \left[1 - p_{t+1,u}^{(x,Y)}(y^{(K)}) \right] \right\}.$$

The main drawback of this method is its time complexity. Indeed this sampling method involves $\left|\mathcal{D}_{X}^{(pool)}\right| \times K$ re-training of h_{t} .

Thus far we have presented querying techniques that are based either on the impact of the instances on the learning model, either on the overall predictive quality of the model or on the degree of expected model change. In the next section we study a complementary sampling strategy based on the representativeness of the instances.

4.5 Sampling based on representativeness

All the sampling methods presented so far aim at choosing the instance that gives the highest informativeness to the learning model: an informativeness based on the quality of the local prediction (cf. Section 4.2 and 4.3) or global prediction (cf. Section 4.4). However, some of these instances may not be representative of the distribution of \mathcal{X} leading to a possible decrease of the model performance. As an example, it is possible that in our pool-set $\mathcal{D}_X^{(pool)}$ we have anomalies such as *outliers* which are not representative of \mathcal{X} distribution but which are possibly considered informative in the sense of the sampling approaches presented above⁸. Thus, this subsection introduces methods allowing to take into account the representativeness of the queried data: an informativeness component alone is not enough and the approaches presented here propose to add a representativeness component (an instance must thus verify a good compromise between informativeness and representativeness). An example of this methodology, called *Information Density*, is presented by [SC08] and formally defined in Eq. (5).

We denote $I_A: \mathcal{X} \times \mathcal{H} \to \mathbb{R}$ an importance criterion (for the query) according to an informativeness measure A (e.g. entropy sampling). Let I_R be a representativeness (or density) measure. Let us choose the following measure: for all $x \in \mathcal{X}$

$$I_R(x) = \frac{1}{|\mathcal{D}_X^{(pool)}|} \sum_{\mathbf{u} \in \mathcal{D}_X^{(pool)}} \operatorname{sim}(\mathbf{x}, \mathbf{u})$$
 (5)

with sim a similarity measure between two instances (e.g. the cosine similarity). $I_R(x)$ measures the average similarity between the instance x and the instances of the set $\mathcal{D}_X^{(pool)}$.

Information Density. A sampling strategy that relies on this I_R measure is the density weighted methods proposed by [SC08]:

$$\hat{x}_{ID} = \left(\underset{x \in \mathcal{D}_X^{(pool)}}{\operatorname{argmax}} I_A(x, h)\right) \cdot I_R(x)^{\beta}$$

⁸An outlier may be in the area of model uncertainty or may result in a significant impact on the model parameters after it is labeled.

with β a parameter tuning the importance given to the representativeness measure. Taking into account both informativeness and representativeness, the instances thus selected have a low predictive quality and are much requested.

There are many other active learning methods that take into account the representativeness and the density of the instances. [SC08] proposed a variant of the weighted density method by integrating a clustering method: first they cluster the set $\mathcal{D}_X^{(pool)}$ before computing the average similarity with the instances of the same cluster.

In this section, we have cited the most common query strategies in the current active learning literature. In the next subsection, we present some recent active learning methods based on deep learning models.

4.6 Sampling based on neural nets architecture

In a "classical" active learning setup, Deep Learning methods raise additional challenges:

- 1. Active learning methods rely on training models on a small amount of labeled data whereas recent Deep Learning algorithms are increasingly greedy in terms of labeled data due to the explosion of the number of parameters. Therefore a too complex neural network architecture can overfit to a "simple" data. Moreover, the *cold-start* problem stated in section 4.2 may intensify this issue and consequently may render the previous sampling methods unusable.
- 2. Most active queries are based on the uncertainty given by the model. However, standard Deep Learning algorithms for classification (or regression) problems do not capture well uncertainties. Indeed, in multi-class classification, the probabilities obtained with a *softmax* output layer (which is often the case in recent architectures) are often misinterpreted as the confidence in the model. [GG16] shows that the *softmax* function results in extrapolations with unjustifiably high confidence for points far from the training data.

Thus, this section presents existing approaches in active learning literature to overcome these difficulties.

Contrary to the previous sections, it is natural to present approaches that are not agnostic to learning models. Indeed the following methods are designed for neural network architectures and can be divided into two categories: *uncertainty-based* sampling and *representativeness-based* sampling approaches.

Uncertainty of instances. We recall that our goal is to estimate an uncertainty score for each unlabeled instance (see section 4.2 and 4.3). However, as mentioned above neural networks do not capture uncertainty (or only to a limited extent). Nevertheless, recent studies have shown that uncertainties can be estimated by the introduction of Bayesian methods in neural networks. We call them *Bayesian Deep Learning* methods. Applying Bayesian methods on neural networks to approximate as closely as possible the true distribution of the posterior probability of the parameters $p(w|\mathcal{D}^{(train)})$ (with w the parameters of the neural networks) have been extensively studied in the literature. Recently, [GG15b] proposes to rely on recent advances in *Bayesian Deep Learning* for the estimation of the uncertainty of the learning model. The principle is as follows: if the probabilistic model of neural networks is defined by h(x, w) with $x \in \mathcal{X}$ and w the parameters (or weights) then p(w) is the a priori law on the space of the parameters (often the a priori law is Gaussian $p(w) = \mathcal{N}(w|\mu,\sigma^2)$) and the likelihood is often defined by a $softmax\ p(y=c|x,w) = softmax(h(x,w))$. Our goal is therefore to find (or approximate) the following posterior distribution on w:

$$p(w|X,Y) = \frac{p(Y|X,w) \cdot p(w)}{p(Y|X)}$$

[GG15a] proposes to approximate this value by variational inference. Later, [GG16] proposed to use the regularizer *dropout* in deep networks as a Bayesian approximation of a Gaussian process. In their paper, they showed that any model trained with *dropout* is an approximation of a Bayesian model, and the estimated uncertainty is the variance of multiple predictions (output of the *softmax* layer) with different *dropout* filters: this method is also known as *Monte Carlo Dropout* (MC-Dropout). We note that for this methodology the dropout regularizer is used both in the learning phase ("classical

regularizer") and in the inference phase (the predictions are therefore no longer deterministic). [LPB17] on the other hand proposes a non-Bayesian methodology by considering a *ensembling* approach to estimate this uncertainty. Specifically, the authors of the paper propose to train a set of the same neural network architecture (but with different randomly initialized initialization weights) and take the average of the *softmax* vectors.

Representativeness of instances. The objective is to estimate a representativeness score of the instances: the selected instances must fairly represent the underlying distribution \mathcal{X} . [SS18] proposes density-based methods. To this end, the authors propose to define the active learning problem as a selection core-set: finding a small subset of a large labeled dataset such that a model learned on the small dataset is competitive on the entire labeled dataset. The idea is to choose c centroids such that the largest distance between these crentroids and the rest of the unlabeled data is as small as possible. However, the core-set approaches require to compute a large distance matrix on the unlabeled data which leads to a high computational complexity.

4.7 Numerical illustrations

In this section we discuss several numerical aspects of the studied procedure on synthetic datasets. passive learning is used as a benchmark. In a nutshell we illustrate the efficiency and behaviour of different active learning processes to build a ML model.

Toy examples. All studies in this section are realized on Gaussian datasets called *Two Gaussians*. We have generated 2000 Gaussian instances of dimension 2: 1000 *red* examples and 1000 *blue* examples with same variance but different means. These examples will then constitute the set of labeled data (red or blue points) and unlabeled data (gray points). At initialization (iteration 0 of AL) 10 instances are randomly queried for constructing the first training set (see Figure 3). Note that since the dataset is balanced the metric accuracy (*acc*) is used for comparing the models performances.

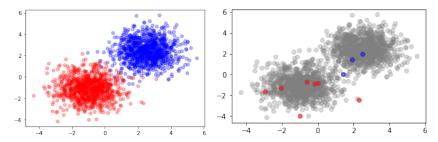


Figure 3: **Synthetic dataset: Two Gaussian.**Left: fully labeled dataset. Right: pool-set (gray points) and train-set (red or blue points).

Results. Figure 4 displays the behaviour of passive learning on the synthetic dataset. This figure is used as a benchmark. After 3 iterations, passive learning updates slowly the estimator keeping roughly the same accuracy whereas Figures 5, 6, 7 and 8 illustrate how active learning outperforms the classical querying way. Uncertainty and disagreement based sampling (Figures 5 and 6) focus on the model uncertainty to classify. The loss-based sampling (Figure 7) focus more on the *non-dense* tails of the two normal distributions (near model uncertainty regions but far from dense regions). At last, the density-based sampling (Figure 8) query both uncertain and representative (near dense regions) instances.

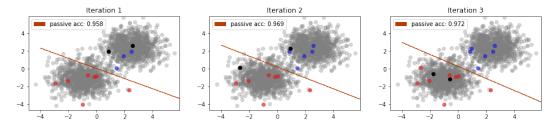


Figure 4: **passive learning on synthetic data.** Here we illustrate the first 3 iterations of passive learning with, for each iteration, a random querying of two instances (in black). The green line illustrates the decision threshold of the logistic model after training on the labeled data.

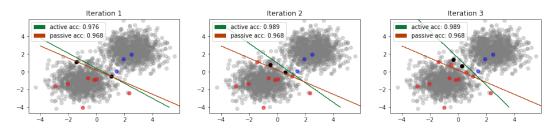


Figure 5: **Sampling by Shannon Entropy.** We illustrate here the first 3 iterations of active learning with, for each iteration, two instances (in black) queried according to their uncertainty entropy-based score. The *green line* illustrates the decision threshold of the logistic model after training on the labeled data and the *red line* the last updated passive model in Figure 4.

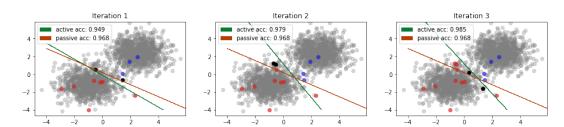


Figure 6: **Sampling by Query by Bagging.** We illustrate here the first 3 iterations of active learning with, for each iteration, two instances queried according to their disagreement score. Here, the disagreement measure used is the **vote entropy** and the model committee was built by the **query by bagging** method by replicating 8 logistic regression models.

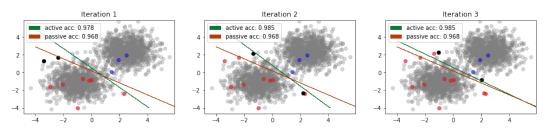


Figure 7: **Sampling by Expected Gradient Length.** We illustrate here the first 3 iterations of active learning with, for each iteration, two instances queried according to their Expected Gradient Length score used on a binary logistic model.

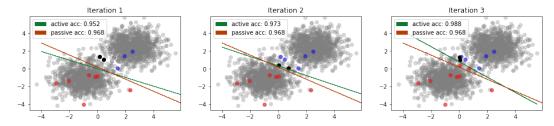


Figure 8: **Sampling by Information Density.** We illustrate here the first 3 iterations of active learning with, for each iteration, two instances queried according to their representativeness (and uncertainty) score. The representativeness measure used is the Information Density on a logistic model.

5 Application on real datasets

In this section, we will present the empirical effectiveness of active learning methods on real data. In particular, we will apply them on textual data and evaluate their performance.

5.1 Practical considerations

For several years now, research in active learning has been progressing but the proposed methodologies remain a challenge in terms of feasibility in practice. Indeed, experimental active learning is often difficult for researchers because they do not have access to a labeling oracle. Thus, in order to evaluate the effectiveness of the proposed methodologies, a widely used trick is to take a labeled database and transform it into a database suitable for active learning methods by "masking" the label of the data that is not queried by active querying. To build up unlabeled data, we need to "hide" their label. This naturally raises to some important issues in practice that must be considered when implementing active learning methodologies:

- How to start labelling? The problem of cold-start remains a challenge: we want to have enough representative data for our learning model before starting active querying. Indeed, if the learning model is poorly calibrated, the selected instances may be less informative than a passive query. For the initialization step, [ST11] and [RS13] propose for example to query representative instances first in order to have a large decision scope and uncertainty-based queries are then used to frame the decision scope and improve learning performance.
- How to evaluate active learning methodologies? In practice it is difficult to compare them when we work with limited budget: active learning approaches must be executed only once and therefore it will be a waste to exhaustively implement all methodologies. Moreover, part of the budget might be used for setting up of a hold-out set. We note that in practice, it is advisable to first label the test base randomly in order to be as close as possible to the underlying distribution before labeling the training set.
- When to stop the active learning process? In addition to business aspects (e.g. limited budget or labeling time) there are many other possible stopping criteria: for example [ZH07] proposes the *Max-confidence* stopping criterion which comes to analyze the entropy measure of the pool-set. More specifically we stop the active learning process when the entropy of each unlabeled example becomes smaller than a given threshold (e.g. 0.001). The authors also propose the *Min-error* which consists in analyzing the performance of the learning model: we stop the active learning iterations when the models have reached a given performance.
- Which learning models to use? In active learning, most of the queried samples are adjusted to a given learning model. Since we do not know the labels in advance, the choice of the learning model remains delicate because at initialization the labeled data are not complex and abundant enough to choose the right learning model (a good fitted learning model at the beginning doesn't mean a good fit at the end of the active learning process).
- What happens when labels are not reliable (mislabeling)? The most common method is to re-label some (uncertain) data because of the possibility of errors in labelling the oracles.

5.2 Metrics and datasets

In order to evaluate the model performance we have to consider two components: an adequate model metric and a model unfairness evaluation.

Model precision. We use F1-score for evaluating the performance of a binary model on a hold-out set. F1-score measures the harmonic mean of *precision* (i.e. number of true positives divided by the total number of predicted positives) and *recall* (i.e. number of true positives divided by the total number of actual positives):

$$F_1 = 2 \times \frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}}$$
 (6)

Model unfairness. The fairness of an estimator $h \in \mathcal{H}$ is measured on a test set $\mathcal{D}^{(test)}$ via the empirical counterpart of the unfairness measure $\mathcal{U}(h)$ given in Section 3.5. For simplicity's sake, we note $\hat{\nu}_{h|s}(k) = \frac{1}{|\mathcal{T}^s|} \sum_{(X,Y) \in \mathcal{T}^s} \mathbb{1}_{\{h(X)=k\}}$ the empirical distribution of h(X)|S=s on $\mathcal{T}^s = \{(X,Y) \in \mathcal{D}^{(test)}|S=s\}$ the conditional test set. The unfairness is formally defined by:

$$\hat{\mathcal{U}}(h) = \frac{1}{K} \sum_{k=1}^{K} \left| \hat{\nu}_{h|-1}(k) - \hat{\nu}_{h|1}(k) \right|. \tag{7}$$

Let us now present the datasets.

Dataset. The textual data that we study is the NPS (Net Promoter Score) verbatims of Société Générale Insurance. The NPS is an indicator that measures the level of customer satisfaction. This indicator is measured through a survey carried out at regular intervals with a sample of customers. Within the framework of the present study, the NPS analyzed concerns in particular the so-called "hot NPS"; this term expresses the fact that the survey is sent immediately to the customer following a call with a customer relations centers. The question asked to the customer to measure the level of satisfaction is the following:

"Would you recommend our insurance company to a friend or your family?"

The answer to this question is a score ranging from 1 (detractor) to 10 (promoter). These scores constitute our label space $\{1,\cdots,K\}$ with K a value between 2 (binary classification) and 10 (multiclass classification) depending on the split. We decided to study a binary classification framework $\mathcal{Y} = \{0,1\} = \{\text{score} \leq c, \text{score} > c\}$ with $c \in [K-1]$ being the *imbalanced* parameter. We can choose c such that we can range from a 10% (*imbalanced* case) to 50% (*balanced* case) imbalanced rate. In addition to the question the following question is also asked on surveys:

"Why did you give this rating?"

The answer to this question is a free text comment allowing clients to justify the rating given by explaining the main reasons for their satisfaction or dissatisfaction. This text comment is named "verbatim nps". It should be noted that a construction of a numerical representation model of verbatims in a "semantic space" allowing to measure the similarity of the verbatims has been implemented: for this purpose the algorithm Doc2vec is used [LM14]. Unlike embedding algorithms of type Word2vec [MSC+13], Doc2vec has the advantage of providing in a "native" way a digital representation of an entire textual document (here a verbatim for example) without having to resort to an aggregation of the embeddings of each of the words composing the verbatim. Each numerical representation of the textual documents represents our sample from \mathcal{X} . The sample made available for the active learning study consists of around 5000 verbatims that correspond to a collection period of approximately one year (late April 2018 to late May 2019).

Unfair datasets. The fairness is studied on two unfair datasets:

• Unfair NPS (UNPS) dataset where we render the NPS dataset randomly unfair by introducing a sensitive feature S such that the distribution of S, given that Y = k, is

$$(S|Y = k) \sim 2 \cdot \mathcal{B}(p) - 1, \quad \text{if} \quad k \le \lfloor K/2 \rfloor$$

and

$$(S|Y=k) \sim 2 \cdot \mathcal{B}(1-p) - 1, \quad \text{if} \quad k > \lfloor K/2 \rfloor$$

with p the parameter that measures the historical bias in the dataset. For instance, for $k \le \lfloor K/2 \rfloor$, S takes the values 1 and -1 with probability p and 1-p respectively. Specifically, the model becomes fair when p=1/2 and completely unfair when $p \in \{0,1\}$. We set p=0.7.

• Law School Admissions (LAW) dataset [WR98] presents national longitudinal bar passage data and has 20649 examples. The task is to predict a students GPA divided into K=2 classes such that the dataset is balanced. The sensitive attribute is the race (white versus non-white).

5.3 Methods and settings

We will study the efficiency of active learning methods. The learning model used is XGBoost (*Extreme Gradient Boosting* proposed by [CG16]). This algorithm has the advantage of being flexible and is faster to train than Gradient Boosting. Thanks to a regularization term it adapts rather well to small or medium size data. We consider an uncertainty sampling with entropy measure (*EntropySampling*), a disagreement-based sampling with Query-by-bagging (*QbagSampling*), a model change sampling strategy with EGL (*EGLSampling*) and a density-based sampling (*DensityWeightedSampling*). The benchmark used here is a passive sampling (*RandomSampling*). In a nutshell, we will study active learning with respect to:

- the metric evaluation of the model to both balanced and imbalanced datasets;
- the unfairness evaluation of the *crude* model (without fairness filter) to both fair and unfair dataset;
- the unfairness evaluation of the *fair* model (with post-processed fairness filter) to both fair and unfair dataset. The fairness method is detailed in [DEHH21] and will be briefly presented below.

We split the data into three sets: 100 initial training set, 1000 test set and the remainder constitutes the pool set.

Fair multi-class classification. [DEHH21] provides an optimal fair classifiers with respect to the misclassification risk under Demographic Parity constraint. This method is called *argmax-fair*. The difficulty of obtaining an optimal fair classifier consists in finding a good equilibrium between misclassification risk and fairness criterion. Hence, given a hypothesis h, [DEHH21] introduces and calibrates the parameter $\lambda = (\lambda_1, \dots, \lambda_K) \in \mathbb{R}^K$ such that the so-called *fair-risk*

$$R_{\lambda}(h) := R(h) + \sum_{k=1}^{K} \lambda_{k} \left[\mathbb{P}\left(h(X) = k | S = 1 \right) - \mathbb{P}\left(h(X) = k | S = -1 \right) \right]$$

is minimized with respect to λ . The authors provide a plug in estimator for the optimal fair classifier h^*_{fair} with strong theoretical guarantees both in terms of fairness and risk. In particular, the fairness guarantee is distribution-free. Let us denote $(p^s_{t,x}(k))_k$ the conditional probabilities (e.g., Random Forest, SVM, etc.) at time t for the instance x with the associated sensitive feature s calibrated by the training set $\mathcal{D}^{(train)}$. We also need the pool-set $\mathcal{D}^{(pool)}_X$ to compute:

- $(\hat{\pi}_s)_{s \in \mathcal{S}}$ the empirical frequencies for estimating the distribution of the sensitive feature \mathcal{S} ;
- N_s the number of observations corresponding to S = s. Therefore $N_{-1} + N_1 = \left| D_X^{(pool)} \right|$;
- and the feature vector in $D_X^{(pool)}$ denoted $X_1^s,\dots,X_{N_s}^s$ composed of *i.i.d.* data from \mathbb{P}_{X^s} , the distribution of $X^s:=\{X|S=s\}$.

Given small perturbations $(\zeta_k)_{k \in [K]}$ and $(\zeta_{k,i}^s)$ as independent copies of a uniform distribution on [0, u] (e.g. $u = 10^{-5}$), the randomized fair classifier \hat{h}^s by plug-in is

$$\hat{h}^s(x) = \underset{k \in [K]}{\operatorname{argmax}} \left(\hat{\pi}_s(p^s_{t,x}(k) + \zeta_k) - s\hat{\lambda}_k \right), \text{ for all } (x,s) \in \mathcal{X} \times \mathcal{S}$$

with $\hat{\lambda} \in \mathbb{R}^K$ given as

$$\hat{\lambda} \in \underset{\lambda}{\operatorname{argmin}} \sum_{s \in \mathcal{S}} \frac{1}{N_s} \sum_{i=1}^{N_s} \left[\max_{k \in [K]} \left(\hat{\pi}_s(p_{t,X_i^s}^s(k) + \zeta_{k,i}^s) - s\lambda_k \right) \right] .$$

[DEHH21] proposes to solve this optimization problem by smoothing the problem by soft-max (a.k.a. LogSumExp) and then use a gradient-based optimization method, such as accelerated gradient descent [Nes83, Nes13].

Note that as for all fairness-awareness algorithms, the downside of this method is that the model accuracy in predictions is poorer.

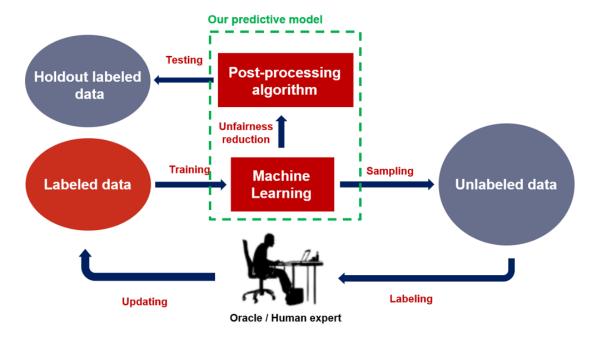


Figure 9: Fair active learning process: fairness-unawareness in queries.

5.4 Results

All the graphs presented in this section are averaged over 15 simulations and colored area corresponds to the standard deviation.

Model precision analysis. Numerical studies in Figures 10 and 11 show that active learning (AL) strategies outperform passive learning (PL) by sampling better quality data for the studied machine learning model (i.e. XGBoost). Indeed, most of the AL strategies converge to 0.84 with 600 labeled data instead of 2000 with PL. As shown by Figure 12 AL seems is robust to the imbalanced NPS. Figure 13 highlights it by evaluating the gap in performance between AL and PL:

$$GAP = 1 - \frac{f1_score_passive}{f1_score_active}$$

where $f1_score_passive$ (resp. $f1_score_active$) is the F_1 -score of the passive learning (resp. active learning) process. The gap in performance between AL and PL shown in Figure 13 indicates the efficiency of AL. However, the more unbalanced the data, the closer the performance of AL is to the performance of PL (Figure 14). Precisely [EHBG07] shows that AL performs well to a slightly imbalanced case but can be inefficient to a heavily imbalanced one.

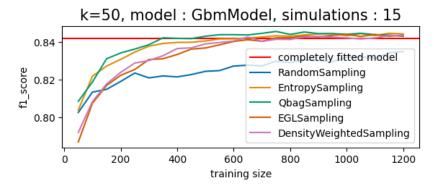


Figure 10: Performance of the classification procedures in terms of **F1-score** for the **XGBoost** estimator w.r.t the training set iteratively constructed by PL and AL methods mentioned above. In each iteration we query k = 50 instances on a **balanced** dataset (30%).

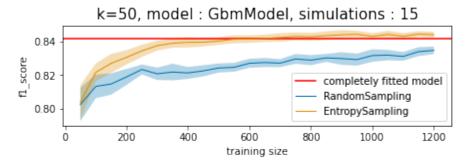


Figure 11: Performance of the classification procedures in terms of **F1-score** for the **XGBoost** estimator w.r.t the training set constructed by RandomSampling and EntropySampling. In each iteration we query k = 50 instances on a **balanced** dataset (30%).

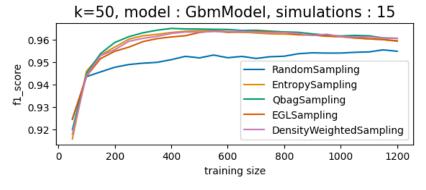


Figure 12: Performance of the classification procedures in terms of **F1-score** for the **XGBoost** estimator w.r.t the training set iteratively constructed by *RandomSampling* and AL methods. We query k = 50 instances on a **imbalanced** dataset (10%).

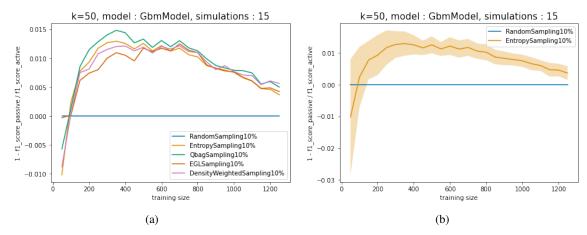


Figure 13: Performance of the classification procedures in terms of **F1-score** for the **XGBoost** estimator w.r.t the training set iteratively set by PL and AL methods on a **imbalanced** dataset (10%) where each iteration we query k = 50 instances. Each line corresponds to the mean over 15 simulations and the colored area the standard deviation (Figure (b)).

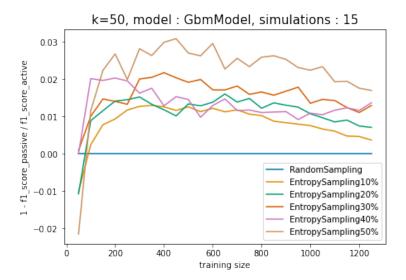


Figure 14: Performance of the classification procedures in terms of **F1-score** for the **XGBoost** estimator w.r.t the training set iteratively set by passive learning and uncertainty-based methods. Each line corresponds to the mean over 15 simulations and in each iteration we query k=50 instances w.r.t the dataset with different **rates of imbalance** (10% to 50%).

Model unfairness analysis. Figure 15 displays the metric (Figure 15a) and the fairness of both AL and PL w.r.t. training size. Figure 15b illustrates that PL is better than query-by-bagging method in terms of unfairness. However, the unfairness reduction algorithm proposed in this paper works well for both learning processes. We note that active learning processes work well for both crude and fair model. Thus active learning with post-processing fairness reduction algorithm seems a good trade-off between model accuracy and model unfairness.

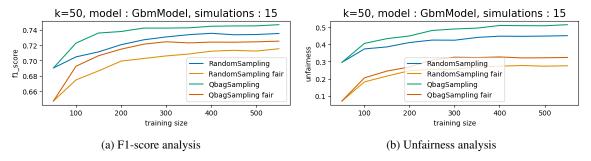


Figure 15: UNPS Dataset. Model performance (accuracy and unfairness) w.r.t. the training size. We study both fair and unfair model. Each line corresponds to the mean over 15 simulations and in each iteration we query k=50 instances.

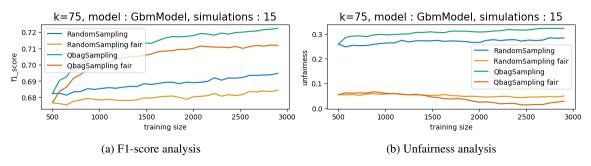


Figure 16: LAW Dataset. Model performance (accuracy and unfairness) w.r.t. the training size. We study both unfair and fair model. Each line corresponds to the mean over 15 simulations and in each iteration we query k=75 instances.

6 Conclusion

With the adoption of Artificial Intelligence, insurance companies are tied to the following objectives: (1) need of quality data alongside with an efficient learning model for deploying their AI solutions and (2) need of a good AI governance (interpretation and fairness) for managing and trusting their model. The present study introduces an overview of active learning for resolving these concerns: numerical analysis and application on real datasets show that active learning strategies can reduce considerably the amount of labelled data needed for calibrating an efficient machine learning model. In our study, the downside of unfiltered active learning seems that the model becomes more unfair. However it can be mitigated by adding a post-processing fairness such as *argmax-fair* leading to a good trade-off between model precision and unfairness. With its wide range of applications, fairness in multi-class classification is a rapidly expanding domain and we believe that considering both active learning and fairness will lead to enhance AI performance and mitigate operational and reputational risks in insurance companies.

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