

Explainable Decision Making with Lean and Argumentative Explanations

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Abstract

It is widely acknowledged that transparency of automated decision making is crucial for deployability of intelligent systems, and explaining the reasons why some decisions are “good” and some are not is a way to achieving this transparency. We consider two variants of decision making, where “good” decisions amount to alternatives (i) meeting “most” goals, and (ii) meeting “most preferred” goals. We then define, for each variant and notion of “goodness” (corresponding to a number of existing notions in the literature), explanations in two formats, for justifying the selection of an alternative to audiences with differing needs and competences: *lean* explanations, in terms of goals satisfied and, for some notions of “goodness”, alternative decisions, and *argumentative* explanations, reflecting the decision process leading to the selection, while corresponding to the lean explanations. To define argumentative explanations, we use *assumption-based argumentation* (ABA), a well-known form of structured argumentation. Specifically, we define ABA frameworks such that “good” decisions are admissible ABA arguments and draw argumentative explanations from *dispute trees* sanctioning this admissibility. Finally, we instantiate our overall framework for explainable decision-making to accommodate connections between goals and decisions in terms of *decision graphs* incorporating defeasible and non-defeasible information.

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1. Introduction

Decision making can be understood as a process of selecting “good” *decisions* amongst several alternatives, e.g. using *decision criteria*, based on relevant information available to the decision maker and a representation of this information. For systems assisting decision making process, it is important that these systems not only generate “good” selections, but be able to explain their selections to their users. Indeed, it is widely acknowledged that transparency of automated decision making is crucial and explaining the reasons why some decisions are “good” and some are not is a way to achieving this transparency (Goodman & Flaxman, 2017). It is less clear however what these explanations may be but, given that the decisions may need to be understood by different kinds of users, different forms of explanations are desirable, tailored to the needs and abilities of their users (Sokol & Flach, 2020; Sheh, 2017; Phillips, Hahn, Fontana, Yates, Greene, Broniatowski, & Przybocki, 2021).

In this paper we consider two different types of explanations for several forms of decision-making, based on different decision criteria. The first type of explanation is targeted at users who do not need or do not care to understand the mechanisms for selecting a decision as “good” (according to a chosen decision criterion): these explanations thus simply refer to the inputs for the decision-making process, namely the goals satisfied by the “good” decisions, and, in some settings, preferences over these goals and alternative decisions. We refer to explanations of this first type as *lean*, given that they focus solely on the building-blocks of decision making and ignore the intricacies of the underlying decision-making criterion sanctioning the decision as “good”. The second type of explanation is targeted at users who need and want to understand the underpinning mechanisms: these explanations bring up the reasoning behind the identification of the explanations. We refer to explanations of this second type as *argumentative*, because we use *dispute trees* (Dung, Kowalski, & Toni, 2006; Dung, Mancarella, & Toni, 2007; Toni, 2013), as understood in *computational argumentation* (see e.g. (Bench-Capon & Dunne, 2007; Besnard & Hunter, 2008; Rahwan & Simari, 2009; Atkinson, Baroni, Giacomin, Hunter, Prakken, Reed, Simari, Thimm, & Villata, 2017)), to represent the reasoning leading to the identification of the explained decisions as “good”. The two forms of explanations are related, as we show that lean explanations can be obtained from argumentative ones.

For illustration, consider the following scenario. A financial advisor is to make investment suggestions to their client. The decision alternatives are *Shares*, *Bonds*, *Unit Trusts* and *Real Estate*; these have attributes *Regular Income*, *Liquidity*, and *Diversity*; and the goals in consideration are *Stability* and *Low Risk*. The relations amongst these are depicted in Figure 1 as a *decision graph (DG)*. To explain why *Bonds* is a good investment choice, the advisor can either give the reason “*Bonds* meets goals *Stability* and *Low Risk*”, or go the long way saying something along the lines “*Bonds* brings *Regular Income*, so it brings *Stability*; it also has good *Diversity* so it has *Low Risk*”. Both explanations can be seen as lean, as they refer exclusively to the building blocks of the decision problem (how the decisions fulfil the goals in the first case and how they bring intermediate attributes in turn determining the goals). However, neither explanations give any indications about the reasoning of the decision maker towards sanctioning the decisions as “good”. An argumentative explanation in this toy setting, unearthing the underpinning reasoning, may, for

example, amount to a dispute between fictional proponent and opponent players whereby the proponent starts by arguing for the “goodness” of *Bonds*, followed by the opponent questioning its satisfaction of goals *Stability* and *Low Risk*, and ending with the proponent proving (by reasoning on the DG) that both goals are satisfied, which the opponent cannot object against. The underpinning argumentative reasoning here is straightforward (given that the decision problem is simple and one alternative decision is obviously “best”, meeting as it does both goals). For other settings, though, where more sophisticated decision criteria may be required to discriminate amongst alternatives, argumentative explanations introduce more transparency into decision making.

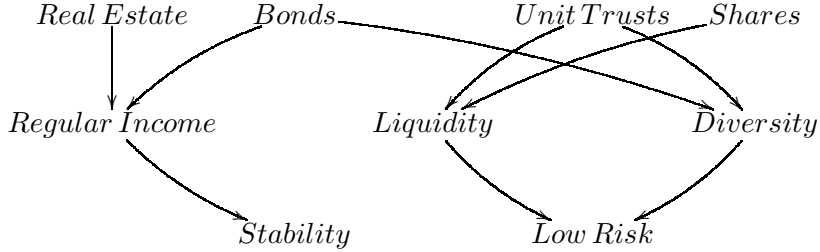


Figure 1: Investment Product Example.

Lean explanations are, in spirit, similar to model-agnostic explanations for machine learning models (e.g. the popular LIME (Ribeiro, Singh, & Guestrin, 2016) and SHAP (Lundberg & Lee, 2017)) and model-specific explanations (e.g. counterfactual explanations as in (Tolomei, Silvestri, Haines, & Lalmas, 2017; Wachter, Mittelstadt, & Russell, 2017)): like these existing explanation methods, our lean explanations focus on explaining the outputs of AI systems solely in terms of their inputs. Instead, argumentative explanations are, in spirit, similar to work on visualisation of deep learning models (Olah, Satyanarayan, Johnson, Carter, Schubert, Ye, & Mordvintsev, 2018) or explanations of their inner workings (Dejl, He, Mangal, Mohsin, Surdu, Voinea, Albini, Lertvittayakumjorn, Rago, & Toni, 2021; Sukpanichnant, Rago, Lertvittayakumjorn, & Toni, 2021): like these existing explanation methods, our argumentative explanations attempt to shed light on the underpinning workings of the AI system towards obtaining outputs from inputs. Our argumentative explanations also share with (Dejl et al., 2021; Sukpanichnant et al., 2021) (as well as with several other works, as overviewed in (Cyras, Rago, Albini, Baroni, & Toni, 2021)) the choice of computational argumentation as the underpinning methodology, and strives towards explanations that are in line with recommendations from the social sciences (e.g. as overviewed in (Miller, 2019)) and in particular with interpretations of human-oriented explanations as argumentative (e.g. see (Antaki & Leudar, 1992; Mercier & Sperber, 2017)).

We consider two variants of decision making. The first variant considers *Abstract Decision Frameworks (ADFs)* consisting of *decisions*, *goals* and a *table* giving a *decisions-meeting-goals* relation. In the context of ADFs, we study three decision criteria, *strong dominance* (the most basic, which we illustrated in the earlier toy example), *dominance*, and *weak dominance*, respectively enforcing, as “good”, decisions meeting all goals, meeting all goals that are met by any decision, and meeting goals not met by any other decision. The second variant considers *Preferential Decision Frameworks (PDFs)*, which add to ADFs *preferences* over goals. In the context of PDFs, we study the decision criterion (that we

call g-preferred) enforcing, as “good”, decisions meeting the most preferred goals. The decision criteria that we consider are strongly connected to other decision making works in the literature. In particular, we show that strong dominance corresponds to selected decisions using the conjunctive method in standard multi-attribute decision making (Yoon & Hwang, 1995), weak dominance corresponds to the well-known decision criterion of Pareto optimality in decision theory (Emmerich & Deutz, 2006) and g-preferred generalizes the lexicographic method in multi-attribute decision making (Yoon & Hwang, 1995). For each of the proposed decision criteria, we give lean and argumentative explanation for a decision candidate, either justifying the selection or identifying conditions that the decision fails to meet. We also show that the lean explanations that we define (of various kinds, corresponding to the various decision criteria and decision framework) can be directly obtained from (the corresponding) argumentative explanations.

In order to provide argumentative explanations, for each of the proposed decision making frameworks (ADFs and PDFs) and respective decision criteria, we give mappings into assumption-based argumentation (ABA) frameworks (Bondarenko, Toni, & Kowalski, 1993; Bondarenko, Dung, Kowalski, & Toni, 1997; Dung, Kowalski, & Toni, 2009; Toni, 2014), a well known form of computational argumentation with provably correct computational mechanisms (dispute trees) with respect to several semantics (Dung et al., 2006, 2007; Toni, 2013), from which argumentative explanations can be readily extracted. Specifically, we prove that “good” decisions correspond to *admissible arguments*, namely arguments belonging to admissible sets (which are conflict-free and can defend themselves against all attacks); admissible arguments can be computed through the construction of *admissible abstract dispute trees*: argumentative explanations are then drawn from variants of these trees.

Finally, we consider an instance of our overall framework for explainable decision making, using DGs (generalising that illustrated in Figure 1) for capturing more complex information about the relations between decisions and goals, in ADFs and PDFs. In a DG, a decision leads to *intermediate goals*, which in turn may lead to additional intermediate goals and eventually to goals. With DGs, we have the ability to model *defeasibility* in decision making, by expressing, essentially, that “a decision normally meets a goal, unless some conditions hold”. In this instantiation, the argumentative counterpart of decision making giving argumentative explanations can also be used to support reasoning leading to “good” decisions.

The paper is organised as follows. We position our paper in the context of related work in Section 2. In Section 3 we define ADFs and PDFs with their decision criteria (and in Appendix A show relations with existing decision-making methods). In Section 4 we define our lean explanations and in Section 5 we define our argumentative explanations, after providing some necessary background on ABA (in Section 5.1.1) and after introducing (in Section 5.1.2) variants of the *abstract dispute trees* used in this work. We introduce DGs and explainable decision making therein, with and without defeasible information, in Section 6. We conclude in Section 7.

This paper builds upon and generalises existing prior work as follows. ADFs, PDFs and DGs introduced in this work are abstractions of the decision frameworks, preferential decisions frameworks and DGs introduced in (Fan & Toni, 2013), (Fan, Craven, Singer, Toni, & Williams, 2013; Fan, Toni, Mocanu, & Williams, 2014) and (Carstens, Fan, Gao, &

Toni, 2015), respectively. In this paper, we have expanded the theoretical presentation of these various decision making frameworks (including relations with existing decision making methods) and, crucially, focused on formulations of explanation, absent from this prior work.

2. Related Work

Even though most of the recent work on explainable AI focuses on explaining black-box models, there is also significant effort in explaining transparent methods, especially when non-expert users need to make sense of the outputs of these methods (Fox, Long, & Magazzeni, 2017). Specifically, (Chakraborti, Sreedharan, & Kambhampati, 2020) provide a survey of existing work on explainable planning as a form of decision making: our argumentative explanations can be seen as forms of their “algorithm-based explanations”. Also, (Cyras, Letsios, Misener, & Toni, 2019; Cyras, Karamlou, Lee, Letsios, Misener, & Toni, 2020) focus on scheduling, another form of decision making: like us, they developing argumentation-based explanations, but based on abstract argumentation (Dung, 1995) rather than ABA.

Our proposal of two types of (lean and argumentative) explanations is in line with the recent advocacy of multiple forms of explanations. Specifically, (Sheh, 2017), in the context of Human-Robot Interaction (HRI) research, introduce an explanation categorization containing five types: *teaching*, *introspective tracing*, *introspective informative*, *post-hoc* and *execution* for the purpose of “matching machine learning capabilities with HRI requirements”. In the context of explaining machine learning predictions, (Sokol & Flach, 2020) have looked at explaining prediction models globally with *model visualisation* and *feature importance* and prediction instances locally with *decision rule*, *counterfactual* and *exemplar*. They suggest that personalised explanations, which present different forms of explanations to different users, are beneficial for achieving interpretable machine learning. Moreover, a recent National Institute of Standards and Technology report published by the U.S. Department of Commerce (Phillips et al., 2021) reports that “the audience will strongly influence the purpose of the explanation” and “the explanation’s purpose will influence its style”.

Our argumentative explanations leverage on the field of computational argumentation, as overviewed, for example, in (Bench-Capon & Dunne, 2007; Besnard & Hunter, 2008; Rahwan & Simari, 2009; Modgil, Toni, Bex, Bratko, Chesñevar, Dvořák, Falappa, Fan, Gaggl, García, González, Gordon, Leite, Možina, Reed, Simari, Szeider, Torroni, & Woltran, 2013; Atkinson et al., 2017). Computational argumentation in many forms and shapes is widely used in explainable AI (e.g. see the recent survey by (Cyras et al., 2021)). Our approach uses ABA (Bondarenko et al., 1993, 1997; Dung et al., 2009; Toni, 2014; Cyras, Fan, Schulz, & Toni, 2017), a specific form of computational argumentation, well-suited for our purposes for a number of reasons. First, it is a general purpose argumentation framework suited in principle to any application (e.g. see (Dung et al., 2009; Toni, 2012, 2014)), that has already proven useful to support decision making (Matt, Toni, & Vaccari, 2009; Dung, Thang, & Toni, 2008; Fan et al., 2013; Fan & Toni, 2013; Fan et al., 2014; Zeng, Fan, Wu, & Miao, 2018; Carstens et al., 2015). ABA has strong theoretical foundations (Bondarenko et al., 1993, 1997), with several formal results readily available characterising the computational complexity of various tasks in ABA (Dimopoulos, Nebel, & Toni, 2002; Dunne, 2009; Cyras, Heinrich, & Toni, 2021). Another essential feature of ABA, for the purposes

of this paper, is that it is equipped with provably correct computational mechanisms (dispute trees) with respect to several semantics (Dung et al., 2006, 2007; Toni, 2013), which includes *admissibility*. We rely upon aspects of these mechanisms, as well as their soundness, to define our argumentative explanations. Finally, ABA is a framework for *structured* argumentation (Besnard, Garcia, Hunter, Modgil, Prakken, Simari, & Toni, 2014), allowing a fine-grained representation of the various components in a decision model. At the same time, the specific form of ABA we need (referred to in the literature as *flat* (Toni, 2014)), is an instance of abstract argumentation (Dung et al., 2007; Toni, 2012, 2014) and it admits abstract argumentation as an instance (Toni, 2012, 2014). Moreover, (flat) ABA is an instance of ASPIC+ (Prakken, 2010), another framework for structured argumentation. Thus, our ABA formulations could serve as a starting point for other explanation styles, different from our argumentative explanations, built using these other related argumentation frameworks.

ABA has already been studied in XAI in the literature. For instance, (Fan & Toni, 2015) introduce *related admissibility* to identify arguments and assumptions that explain argument acceptability in ABA; (Wakaki, 2017) model ABA with preference and experiment with decision making; (Fan, 2018, 2018) study how ABA can be used to generate explainable plans; and (Zeng, Shen, Tan, Chin, Leung, Wang, Chi, & Miao, 2020) use ABA for explainable diagnostics and prognostics of Alzheimer’s Disease. To the best of our knowledge, though, our work is unique for the variety of decision problems it considers and in showing how argumentative explanations relate to lean explanations, of a more conventional spirit, as they are directly defined in terms of the basic components of the decision problems.

(Cyras et al., 2021) categorise work on explainable AI based on computational argumentation according to the relation between the explanations and the method being explained and according to the format of argumentation-based explanations in particular. In their context, our work can be categorised as “post-hoc complete”, in that we provide a one-to-one mapping between decisions and argumentative explanations while also, in addition, formally relating the latter and lean explanations.

We define abstract decision problems (with and without preferences over goals) as well as a specific instance thereof integrating information in the form of decision graphs (possibly integrating defeasible knowledge). This instance can be seen as a form of (explainable) argumentation-based decision making, in the sense that it can be used to directly support the identification of “good” decisions and their explanations. Argumentation-based decision making in general has attracted a considerable amount of research interest over the years (e.g. see (Fox, Krause, & Elvang-Gøransson, 1993; Nawwab, Bench-Capon, & Dunne, 2008; Amgoud & Prade, 2009; Fox, Glasspool, Patkar, Austin, Black, South, Robertson, & Vincent, 2010; Müller & Hunter, 2012)). Argumentation-based decision making is a form of *qualitative decision theory* (Doyle & Thomason, 1999), understood as an alternative to classical *quantitative decision theory* (French, 1986) when a decision problem cannot be easily formulated in standard decision-theoretic terms using decision tables, utility functions and probability distributions. In decision-theoretic terms, argumentation can be used to compute a utility function which is too complex to be given a simple analytic expression in closed form, and/or when a transparent justification of decisions is beneficial. Existing approaches to decision making using argumentation are predominantly *descriptive*, in that

Table 1: Works on Argumentation-based Decision Making.

Works	Argumentation Frameworks	Preference	Explanation Type
Matt et al. 2009	ABA	No	None
Fox et al. 2007	ASPIC	No	Arguments
Marreiros et al. 2007	Logic Programming	Decision	None
Visser et al. 2012	Structured	Decision	None
Dung et al., 2008	ABA	Goals & States	None
Fan & Toni, 2013	ABA	Goals	None
Fan et al., 2013	ABA	Goals	None
Fan et al., 2014	ABA	No	Dialogue
Zeng et al., 2018	ABA	No	Argument & Context
Zeng et al., 2020	ABA	Yes	Dialogue
Kakas and Moraitis, 2003	Logic Programming	No	None
Müller and Hunter, 2012	ASPIC+	Value	None
Amgoud and Prade, 2006	Structured	Decision	Arguments
Amgoud and Prade, 2009	AA	Decisions	Arguments
Teze et al., 2020	DeLP	Arguments	None
Labreuche 2013	Propositional Logic	Attributes	None

they are inspired by what people actually do and aim at justifying decisions by equipping them with a transparent explanation. For example, (Atkinson, Bench-Capon, & McBurney, 2004) focuses on structuring arguments according to well-defined argument schemes in order to justify and explain decisions. In this work, we focus on *normative* decision theory, regulated by decision criteria fulfilling properties of rationality. In other words, we first define the type of decision making problems we want to solve, the decision criteria for solving them and the types of explanations that justify the selection, then we use argumentation as an alternative way to identify the solutions and generate explanations.

A summary of related work on argumentation-based decision making is shown in Table 1. These existing works can use different argumentation frameworks, e.g. ABA (Toni, 2014), ASPIC+ (Prakken, 2010), Logic Programming (Lloyd, 1987). Many of them accommodate preference modelling for which preferences can be expressed over various elements of their decision making models, as indicated in the table. The majority of these works (Kakas & Moraitis, 2003; Matt et al., 2009; Marreiros, Santos, Novais, Machado, Ramos, Neves, & Bulas-Cruz, 2007; Dung et al., 2008; Müller & Hunter, 2012; Visser, Hindriks, & Jonker, 2012; Fan & Toni, 2013; Fan et al., 2013; Labreuche, 2013; Teze, Gottifredi, García, & Simari, 2020) do not explicitly consider explanations but rely on the underlying argumentation formalism to realise decision making interpretability. A few works (Fox, Glasspool, Grecu, Modgil, South, & Patkar, 2007; Amgoud & Prade, 2006, 2009) explicitly construct and label arguments for and against decision candidates as a form of explanations. (Fan et al., 2014) and (Zeng et al., 2020) employ dialogues as a form of explanation. Lastly, (Zeng et al., 2018) consider explanations in the form of arguments and contexts.

3. Decision Making Abstractions

In this section, we introduce Abstract Decision Frameworks (ADF) and Preferential Decision Frameworks (PDF) to model decision making problems involving decisions, goals and preferences. We introduce several decision making criteria to identify “good” decisions in these frameworks. Some of these notions correspond to notions in the literature as shown in Appendix A.

3.1 Abstract Decision Frameworks (ADFs)

Abstract Decision Frameworks describe the relation between *decisions* and *goals* in a decision problem.

Definition 3.1. An *Abstract Decision Framework (ADF)* is a tuple $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$, with

- a (finite) set of *decisions* $\mathbf{D} = \{d_1, \dots, d_n\}, n > 0$;
- a (finite) set of *goals* $\mathbf{G} = \{g_1, \dots, g_l\}, l > 0$;
- a mapping $\gamma : \mathbf{D} \rightarrow 2^{\mathbf{G}}$ such that given a decision $d \in \mathbf{D}$, $\gamma(d) \subseteq \mathbf{G}$ denotes the set of goals *met by* d .

The following simple example, adapted from (Matt et al., 2009), illustrates the notion of ADF.

Example 3.1. An agent is deciding on accommodation in London. The three candidate decisions (\mathbf{D}) are John Howard Hotel (*jh*), Imperial College Halls (*ic*), and Ritz (*ritz*). The agent deems two goals (\mathbf{G}) *cheap* and *near* as important, where *jh* meets the goal *near* ($\gamma(jh) = \{\text{near}\}$), *ic* meets both goals ($\gamma(ic) = \{\text{near}, \text{cheap}\}$), and *ritz* meets no goal ($\gamma(ritz) = \{\}$).

In the remainder of this section, unless otherwise specified, we assume as given a generic ADF $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$.

We define three decision criteria for ADFs, characterising different notions of “good” decisions: *strongly dominant* decisions, which select as “good” decisions meeting *all* goals; *dominant* decisions, meeting all goals that are *ever met* by any decision; and *weakly dominant* decisions, meeting a set of goals which is *not a subset* of the set of goals met by any other decision. Formally,

Definition 3.2. A decision $d \in \mathbf{D}$ (in $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$) is

- *strongly dominant* if and only if $\gamma(d) = \mathbf{G}$;
- *dominant* if and only if there is no $g' \in \mathbf{G} \setminus \gamma(d)$ with $g' \in \gamma(d')$ for some $d' \in \mathbf{D} \setminus \{d\}$;
- *weakly dominant* if and only if there is no $d' \in \mathbf{D} \setminus \{d\}$ with $\gamma(d) \subset \gamma(d')$.

In Example 3.1, *ic* is a strongly dominant decision as it meets both *cheap* and *near*, and there is no other strongly dominant decision. In the same example, *ic* is the only dominant and the only weakly dominant decision. The following examples illustrate, respectively, the cases when dominant decisions exist, but there is no strongly dominant decision, and when weakly dominant decisions exist, but there is not dominant or strongly dominant decision.

Example 3.2. We again consider the problem of an agent deciding accommodation in London, but represented by an ADF $\langle D, G, \gamma \rangle$ with $D = \{jh, ritz\}$, $G = \{cheap, near\}$, $\gamma(jh) = \{near\}$, and $\gamma(ritz) = \{\}$. Now, differently from Example 3.1, no decision meets both goals, *cheap* and *near*. Nevertheless, *jh* is a better decision than *ritz* as it meets *near* whereas *ritz* meets no goal. Thus, although there is no strongly dominant decision, *jh* is a dominant (as well as a weakly dominant) decision.

Example 3.3. We now consider the decision problem represented by the ADF $\langle D, G, \gamma \rangle$ with $D = \{jh, ic, ritz\}$, $G = \{cheap, near, clean\}$, $\gamma(jh) = \{near, clean\}$, $\gamma(ic) = \{cheap, clean\}$, and $\gamma(ritz) = \{clean\}$. Here, differently from Example 3.1, there is a new goal *clean* that is met by all decisions. However, *ic* no longer meets *near*, hence *ic* is not strongly dominant. Moreover, *ic* does not meet *near*, which is met by *jh*, hence *ic* is not dominant. Similarly, *jh* is neither strongly dominant nor dominant. However, since *ic* and *jh* both meet goals that are not met by the other, they are both weakly dominant. Finally, *ritz* meets no goal that is not met by any other decision and is not even weakly dominant.

In the remainder of this section, we identify simple connections amongst properties of the three given decision criteria. First, it is easy to see that all dominant decisions meet the same set of goals:

Proposition 3.1. For any $d, d' \in D$, if d, d' are dominant then $\gamma(d) = \gamma(d')$.

Moreover, dominant decisions are guaranteed to be weakly dominant and strongly dominant decisions are guaranteed to be dominant:

Proposition 3.2. For any $d \in D$:

- if d is strongly dominant then it is dominant;
- if d is dominant then it is weakly dominant.

Further, if there are strongly dominant decisions, then decisions are (weakly) dominant if and only if they are strongly dominant:

Proposition 3.3. Let $S_s = \{d \in D \mid d \text{ is strongly dominant}\}$, $S_d = \{d \in D \mid d \text{ is dominant}\}$, and $S_w = \{d \in D \mid d \text{ is weakly dominant}\}$. If $S_s \neq \{\}$, then $S_s = S_d = S_w$.

Similarly, if there are dominant decisions, then a decision is weakly dominant if and only if it is dominant:

Proposition 3.4. Let $S_d = \{d \in D \mid d \text{ is dominant}\}$ and $S_w = \{d \in D \mid d \text{ is weakly dominant}\}$. If $S_d \neq \{\}$, then $S_d = S_w$.

Proposition 3.1 indicates that, when there are (strongly) dominant decisions for the given ADF, then they are all equally “good” in meeting goals, since $\gamma(d) = G$ for all strongly dominant decisions d , and $\gamma(d) = \gamma(d')$ for all d, d' dominant decisions. Instead, as illustrated by Example 3.3, if there are only weakly dominant decisions, then there are multiple such decisions and they meet different sets of goals. Formally:

Theorem 3.1. Let $S_d = \{d \in \mathcal{D} \mid d \text{ is dominant}\}$ and $S_w = \{d \in \mathcal{D} \mid d \text{ is weakly dominant}\}$. If $S_d = \{\}$ and $S_w \neq \{\}$, then there exists $d, d' \in S_w, d \neq d'$ and $\gamma(d) \neq \gamma(d')$.

Hence, whereas discriminating amongst (strongly) dominant decisions is not an issue for the decision maker, selecting one weakly dominant decision amongst all possible such decisions requires other measures to discriminate amongst goals. We introduce *preferences* next to further identify suitable decisions.

3.2 Abstract Decision Frameworks with Preferences (PDFs)

Preference rankings over sets of goals could help us further rank weakly dominant decisions. We incorporate these rankings into ADFs as follows.

Definition 3.3. A *Preferential Decision Frameworks* (PDF) is a tuple $\langle \mathcal{D}, \mathcal{G}, \gamma, \leq_g \rangle$, in which $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$ is an ADF and \leq_g is a partial order over $2^{\mathcal{G}}$.

In the remainder of this section, when we refer to decision frameworks with preferences, we assume as given a generic PDF $F_e = \langle \mathcal{D}, \mathcal{G}, \gamma, \leq_g \rangle$. Moreover, as conventional, we will use $s <_g s'$ to denote $s \leq_g s'$ and $s' \not\leq_g s$.

To ease presentation, we define the notion of *comparable goal set*, namely the set of all goal sets appearing in the ranking, as follows:

Definition 3.4. The *comparable goal set* (in F_e) is $\mathcal{S} \subseteq 2^{\mathcal{G}}$ such that

- for every $s \in \mathcal{S}$, there is $s' \in \mathcal{S}, s \neq s'$, such that either $s \leq_g s'$ or $s' \leq_g s$;
- for every $s \in 2^{\mathcal{G}} \setminus \mathcal{S}$, there is no $s' \in 2^{\mathcal{G}}$, such that $s \leq_g s'$ or $s' \leq_g s$.

Hence sets of goals in \mathcal{S} are “comparable” (according to \leq_g). We illustrate \mathcal{S} with the following example.

Example 3.4. Let \mathcal{G} be $\{g_1, g_2, g_3, g_4, g_5\}$ and \leq_g be such that¹

$$\{g_5\} <_g \{g_4\} <_g \{g_3\} <_g \{g_4, g_5\} <_g \{g_2\} <_g \{g_1\}.$$

Then the comparable goal set is $\mathcal{S} = \{\{g_1\}, \{g_2\}, \{g_3\}, \{g_4\}, \{g_5\}, \{g_4, g_5\}\}$.

Definition 3.5. A decision $d \in \mathcal{D}$ is *g-preferred* (in $\langle \mathcal{D}, \mathcal{G}, \gamma, \leq_g \rangle$) iff it is weakly dominant in $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$ and for all weakly dominant decisions $d' \in \mathcal{D}$ such that $d \neq d'$, for all $s \in \mathcal{S}$ (the comparable goal set in $\langle \mathcal{D}, \mathcal{G}, \gamma, \leq_g \rangle$):

- if $s \not\subseteq \gamma(d)$ and $s \subseteq \gamma(d')$, then there exists $s' \in \mathcal{S}$, such that

$$s' \leq_g s, \quad s' \subseteq \gamma(d), \quad \text{and} \quad s' \not\subseteq \gamma(d').$$

Definition 3.5 states that to select a decision d , firstly, we check that d is weakly dominant; then, we check against all other weakly dominant decisions d' to ensure that for any member s of the comparable goal set, if d' meets s but d does not, then there exists

1. Here and in the remainder of the paper, we sometimes give $<_g$ instead of \leq_g and assume that \leq_g is any partial order with the strict counterpart $<_g$.

some member s' more preferred than, or as preferred as, s such that d , but not d' , meets (all goals in) s' . Thus, the notion of g-preferred decisions uses the preference ranking only to discriminate amongst weakly dominant decisions; indeed, if a decision d is not weakly dominant in the first place, there exists some other decision meeting all goals met by d and more, and thus “better”.

Example 3.5. Consider $\langle D, G, \gamma, \leq_g \rangle$ with decisions $D = \{d_1, d_2\}$, goals $G = \{g_1, g_2, g_3, g_4, g_5\}$, $\gamma(d_1) = \{g_2, g_4, g_5\}$, $\gamma(d_2) = \{g_2, g_3\}$, and \leq_g as in Example 3.4. Here, both decisions meet g_2 , d_1 meets both g_4 and g_5 whereas d_2 meets g_3 . Also, although g_3 is more preferred than g_4 and g_5 individually, g_4 and g_5 together are more preferred than g_3 . Hence, d_1 is a g-preferred decision and d_2 is not.

4. Lean Explanations

Thus far, we have introduced several decision making criteria for selecting “good” decisions amongst a set of candidates. Since there can be multiple decisions, goals and reference relations involved in a decision making process, it would be useful to *explain* the reason for selecting a decision by identifying decisions and goals that “influence” the selection. In this section we define *lean explanations* for a decision meeting (or not meeting) certain decision criteria as follows.

Definition 4.1. Given $\langle D, G, \gamma \rangle$ and $d \in D$

- $\gamma(d) = G$ is a *lean explanation* for d being *strongly dominant*;
- $G \setminus \gamma(d)$ is a *lean explanation* for d **not** being *strongly dominant*;
- $(\gamma(d), G \setminus \gamma(d))$ is a *lean explanation* for d being *dominant*;
- $\{(d_i, g_j) | d_i \in D, g_j \in G, \text{ and } g_j \notin \gamma(d), g_j \in \gamma(d_i)\}$ is a *lean explanation* for d **not** being *dominant*;
- $(G, \{(g_1, d_1), \dots, (g_n, d_n)\})$ for which $G \subseteq \gamma(d)$ and $g_i \in \gamma(d), g_i \notin \gamma(d_i)$ such that for each $d' \in D, d' \neq d$, either $\gamma(d') \subseteq G$ or $d' \in \{d_1, \dots, d_n\}$ is a *lean explanation* for d being *weakly dominant*;
- $\{d' \in D | \gamma(d) \subset \gamma(d')\}$ is a *lean explanation* for d **not** being *weakly dominant*.

Basically, to explain a decision being strongly dominant (or not), we identify the goals it meets being the set of all goals (or not). To explain a decision being dominant, we identify the goals the decision meets and does not meet. To explain a decision not being dominant, we identify decision-goal pairs such that the goal in each pair is met by the decision in the pair but not by the decision under consideration. To explain a decision being weakly dominant, we identify goals met by this decision and goal-decision pairs such that the goal in a pair is met by the decision under consideration but not by the decision in the pair. To explain a decision not being weakly dominant, we identify the set of decisions that meet more goals than the decision under consideration.

Example 4.1. Given the ADF in Example 3.1:

- $\{cheap, near\}$ is a lean explanation for ic being strongly dominant as these are the two goals to be met in this example;
- $\{cheap\}$ is a lean explanation for jh being not strongly dominant as $cheap$ is the goal not met by jh ;
- $\{cheap, near\}$ is a lean explanation for $ritz$ being not strongly dominant as $cheap$ and $near$ are the goals not met by $ritz$.

Given the ADF in Example 3.2:

- $(\{near\}, \{cheap\})$ is a lean explanation for jh being dominant as $near$ is a goal met by jh but $cheap$ is not;
- $\{(jh, near)\}$ is a lean explanation for $ritz$ being not dominant as $near$ is a goal met by jh but not by $ritz$.

Given the ADF in Example 3.3:

- $(\{clean\}, (near, ic))$ is a lean explanation for jh being weakly dominant as in addition to meeting the goal $clean$, jh also meets the goal $near$, which is not met by the alternative decision ic ;
- $(\{clean\}, (cheap, jh))$ is a lean explanation for ic being weakly dominant as in addition to meeting the goal $clean$, ic also meets the goal $cheap$, which is not met by the alternative decision jh ;
- $\{jh, ic\}$ is a lean explanation for $ritz$ not being weakly dominant as both jh and ic meet all goals met by $ritz$ and more.

Lean explanations fulfil several properties, considered next.

Proposition 4.1. Given $\langle D, G, \gamma \rangle$ and $d \in D$

1. d has at least one lean explanation as given in Definition 4.1;
2. if d is not weakly dominant, let E be a lean explanation for d ; then there is no $E' \neq E$ which is a lean explanation for d such that $E' \neq E$.

Proposition 4.1 sanctions, firstly, that each decision in an ADF always admits at least one lean explanation: this is desirable as it implies that each “good” decision can be somewhat justified. Secondly, unless a decision is weakly dominant, its lean explanation (per Definition 4.1) is unique: this is desirable as it implies consistency across multiple requests for explanation for the same “good” decision (these requests may be by the same explainee at different times or by multiple explainees).

Analogously to the case of ADFs, we would like to explain why decisions in PDFs are g-preferred or not. We can do so as follows.

Definition 4.2. Given $\langle D, G, \gamma, \leq_g \rangle$ and $d \in D$,

- a *lean explanation for d being g -preferred* is a pair $(G, \{(S_1, d'_1), \dots, (S_n, d'_n)\})$ where $G \subseteq \gamma(d)$, $S_i \subseteq \gamma(d)$, $S_i \not\subseteq \gamma(d'_i)$ and there is no $S' \subseteq \gamma(d'_i)$ such that $S' \leq_g S_i$, for $i \in \{1, \dots, n\}$. Moreover, for each $d' \in \mathcal{D}$, $d' \neq d$, $\gamma(d') \subseteq G$ or $d' \in \{d_1, \dots, d_n\}$;
- a *lean explanation for d not being g -preferred* is $\{d' \in \mathcal{D} \mid \text{either } \gamma(d) \subset \gamma(d') \text{ or there exists } S \subseteq \gamma(d'), \text{ such that for all } S' \subseteq \gamma(d), S \leq_g S'\}$.

Definition 4.2 is given in the same spirit as Definition 4.1, where a decision (not) being g -preferred is explained in terms of its relation with other decisions and goals as well as with the preference relations.

Example 4.2. Given the PDF shown in Example 3.5:

- $(\{g_2\}, \{(\{g_4, g_5\}, d_2)\})$ is a lean explanation for d_1 being g -preferred as in addition to d_1 meeting g_2 , d_1 also meets the more preferred goals g_4, g_5 , which are not met by d_2 ;
- $\{d_1\}$ is a lean explanation for d_2 being not g -preferred as d_1 meets more preferred goals than d_2 .

Proposition 4.2. Given $\langle \mathcal{D}, \mathcal{G}, \gamma, \leq_g \rangle$ and $d \in \mathcal{D}$,

1. d has at least one lean explanation as given in Definition 4.2;
2. if d is not g -preferred, let E be a lean explanation for d not being g -preferred; then there is no E' which is a lean explanation for d not being g -preferred such that $E' \neq E$.

The explanations defined in this section are lean in that they provide the basic reasons behind a decision, but ignore any indication of the role the underlying decision criterion has in explaining the decisions. Argumentative explanations, defined next, remedy this issue, while still embedding all information in lean explanations, as we will show.

5. Argumentative Explanations for ADFs and PDFs

Thus far, we have introduced a set of decision criteria and lean explanations for decisions meeting these criteria or not. In this section we define argumentative explanations for these decisions. To do so, given an ADF or PDF and a decision criterion, we construct an ABA framework so that admissible arguments are selected decisions in the ADF with respect to the decision criterion. We can then use special forms of dispute trees as argumentative explanations, explaining the mechanisms by means of which those decisions are inferred as “good”, in such a way that corresponding lean explanations can be extracted from them too. We first give preliminary background on ABA and (our variants of) dispute trees (Section 5.1). Then, we define argumentative explanations for ADFs (Section 5.2) and PDFs (Section 5.3). The proofs of the technical results for this section are in Appendix B.

5.1 Preliminaries

5.1.1 ABA

Material in this section is mostly adapted from (Toni, 2014; Cyras et al., 2017). An *ABA framework* is a tuple $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ where

- $\langle \mathcal{L}, \mathcal{R} \rangle$ is a deductive system, with a *language* \mathcal{L} and a set of inference *rules* \mathcal{R} of the form $\beta_0 \leftarrow \beta_1, \dots, \beta_m (m > 0)$ or $\beta_0 \leftarrow$ with $\beta_i \in \mathcal{L}$, for $i = 0, \dots, m$,
- $\mathcal{A} \subseteq \mathcal{L}$ is a (non-empty) set, whose elements are referred to as *assumptions*,
- \mathcal{C} is a total mapping from \mathcal{A} into $2^{\mathcal{L}} - \{\{\}\}$, where each $c \in \mathcal{C}(\alpha)$ is a *contrary* of α .

Basically, ABA frameworks can be defined for any logic specified by means of inference rules. Some of the sentences in the underlying language are assumptions, and each such sentence can be “contradicted” by any of its contraries. The following example is a simple illustration, with a set of propositions as language.

Example 5.1. Let $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ be as follows:

- $\mathcal{L} = \{a, b, c, p, q, r, s\}$ and $\mathcal{R} = \{p \leftarrow a, \quad q \leftarrow b, s, \quad r \leftarrow c, \quad s \leftarrow\}$,
- $\mathcal{A} = \{a, b, c\}$,
- $\mathcal{C}(a) = \{q\}$, $\mathcal{C}(b) = \{p\}$, $\mathcal{C}(c) = \{r, q\}$.

Given a rule $\beta_0 \leftarrow \beta_1, \dots, \beta_m$ or $\beta_0 \leftarrow$ in an ABA framework, β_0 is referred as the *head* and β_1, \dots, β_m or the empty sequence, respectively, as the *body* of the rule. ABA frameworks where no assumption occurs as the head of a rule are *flat*. As an illustration, the ABA framework in Example 5.1 is flat, as assumptions there only occur in the body of rules. In this paper, all ABA frameworks will be flat. Therefore, all remaining definitions in this section are restricted to the case of flat ABA frameworks.

In ABA, informally, arguments are deductions of claims supported by sets of assumptions, and attacks against arguments are directed at the assumptions in their supports, and are given by arguments with an assumption’s contrary as their claim. Formally, given an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$, a *deduction for $\beta \in \mathcal{L}$ supported by $S \subseteq \mathcal{L}$* is a finite tree with nodes labelled by sentences in \mathcal{L} or by τ^2 , such that

1. the root is labelled by β
2. for every node N
 - if N is a leaf then N is labelled either by a sentence in S or by τ ;
 - if N is not a leaf and β_0 is the label of N , then there is an inference rule $\beta_0 \leftarrow \beta_1, \dots, \beta_m (m \geq 0)$ and either $m = 0$ and the child of N is τ or $m > 0$ and N has m children, labelled by β_1, \dots, β_m (respectively)
3. S is the set of all sentences labelling the leaf nodes.

Then, an *argument for $\beta \in \mathcal{L}$ supported by $\Delta \subseteq \mathcal{A}$* is a deduction for β supported by Δ . We will use the shorthand $\Delta \vdash \beta$ to denote an argument for β supported by Δ . Given argument $\Delta \vdash \beta$, Δ is referred to as the *support* and β as the *claim* of the argument. Figure 2 illustrates this notion of argument for the ABA framework in Example 5.1.

Argumentation frameworks in general are useful for resolving conflicts between information, as captured by arguments. Conflicts are typically represented as *attacks* between arguments (Dung, 1995). In ABA, attacks are defined in terms of the notion of contrary of assumptions, as follows, given an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$:

2. The symbol τ is such that $\tau \notin \mathcal{L}$. τ stands for “true” and intuitively represents the empty body of rules (Dung et al., 2009).

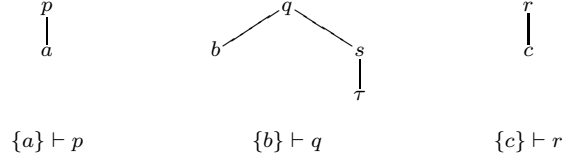


Figure 2: Arguments in Example 5.1, as trees (at the top) and using the shorthand (at the bottom).

- an argument $\Delta_1 \vdash \beta_1$ attacks an argument $\Delta_2 \vdash \beta_2$ if and only if the claim β_1 of the first argument is a contrary of one of the assumptions in the support Δ_2 of the second argument (i.e. $\exists \alpha \in \Delta_2$ such that $\beta_1 \in \mathcal{C}(\alpha)$);

This notion of attack between arguments can be lifted up to sets of arguments and sets of assumptions, as follows:

- a set of arguments **As** attacks a set of arguments **Bs** if some argument in **As** attacks some argument in **Bs**;
- a set of assumptions Δ_1 attacks a set of assumptions Δ_2 if and only if an argument supported by a subset of Δ_1 attacks an argument supported by a subset of Δ_2 .

As an illustration, for the ABA framework in Example 5.1,

- $\{a\} \vdash p$ attacks $\{b\} \vdash q$, $\{b\} \vdash q$ attacks $\{a\} \vdash p$, $\{b\} \vdash q$ attacks $\{c\} \vdash r$, and $\{c\} \vdash r$ attacks $\{c\} \vdash r$
- $\{\{a\} \vdash p\}$ attacks $\{\{b\} \vdash q, \{c\} \vdash r\}$
- $\{a\}$ attacks $\{b\}$, $\{b\}$ attacks $\{a, c\}$, $\{b\}$ attacks $\{c\}$, and $\{c\}$ attacks any set of assumptions containing c .

With argument and attack defined, all argumentation semantics given for abstract argumentation (Dung, 1995) can be applied in (flat) ABA. These semantics can be defined for assumptions, as in (Bondarenko et al., 1993, 1997), or, equivalently, for arguments (Dung et al., 2007; Toni, 2014). In this paper, given an ABA framework $AF = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$, we will use the following notions of admissibility for sets of assumptions and for individual arguments:

- a set of assumptions is *admissible* (in AF) if and only if it does not attack itself and it attacks all $\Delta \subseteq \mathcal{A}$ that attack it;
- an argument $\Delta \vdash \beta$ is *admissible* (in AF) supported by $\Delta' \subseteq \mathcal{A}$ if and only if $\Delta \subseteq \Delta'$ and Δ' is admissible (in AF).

As an illustration, for the ABA framework in Example 5.1,

- the sets of assumptions $\{a\}$, $\{b\}$, $\{\}$ are all admissible (and no other set of assumptions is admissible);
- arguments $\{a\} \vdash p$, $\{b\} \vdash q$, $\{a\} \vdash a$, $\{b\} \vdash b$ are all admissible (and no other argument is admissible).

5.1.2 DISPUTE TREES

Here, unless specified otherwise, we will assume as given a generic ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$.

We will use variants of the *abstract dispute trees* of (Dung et al., 2006) to provide explanations for recommended decisions. An *abstract dispute tree* for an argument A is a (possibly infinite) tree \mathcal{T}^a such that:³

1. every node of \mathcal{T}^a holds an argument B and is labelled by either *proponent* (P) or *opponent* (O), but not both, denoted by $L : B$, for $L \in \{P, O\}$;
2. the root of \mathcal{T}^a is a **P** node holding A ;
3. for every **P** node N holding an argument B , and for every argument C that attacks B , there exists a child of N , which is an **O** node holding C ;
4. for every **O** node N holding an argument B , there exists *at most*⁴ one child of N which is a **P** node holding an argument which attacks some assumption α in the support of B ;⁵ if N has a child attacking α , then α is said to be the *culprit* in B ;
5. there are no other nodes in \mathcal{T}^a except those given by 1-4 above.

The set of all assumptions in (the support of arguments held by) the **P** nodes in \mathcal{T}^a is called the *defence set* of \mathcal{T}^a .

Abstract dispute trees can be used as the basis for computing various argumentation semantics, including the admissibility semantics, as follows:

- Let an abstract dispute tree \mathcal{T}^a be *admissible* if and only if each **O** node has *exactly* one child and no culprit in the argument of an **O** node in \mathcal{T}^a belongs to the defence set of \mathcal{T}^a .
- The defence set of an admissible abstract dispute tree is admissible (Theorem 5.1 in (Dung et al., 2006)), and thus the root node of an admissible dispute tree is admissible.
- If an argument A is admissible then there exists an admissible abstract dispute tree for A (Theorem 5.1 in (Dung et al., 2006)).

Figure 3 gives an example of an (infinite) admissible dispute tree, with (admissible) defence set $\{a\}$, for the ABA framework in Example 5.1.

3. Here, a stands for 'abstract'. Also, 'proponent' and 'opponent' should be seen as roles/fictitious participants in a debate rather than actual agents.

4. In the original definition of abstract dispute tree (Dung et al., 2006), every **O** node is required to have *exactly* one child. We incorporate this requirement into the definition of *admissible* dispute tree given later, so that our notion of admissible abstract dispute tree and the admissible abstract dispute trees of (Dung et al., 2006) coincide.

5. An argument attacks an assumption if the argument supports a contrary of the assumption.

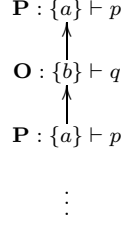


Figure 3: Admissible abstract dispute tree for the ABA framework in Example 5.1

Figure 4: Two abstract dispute trees for the argument $\{a\} \vdash a$. The tree shown on the left-hand side is maximal; the tree on the right-hand side is not.

We will use three variants of abstract dispute trees, given below.

A *maximal dispute tree* (Cyras, Satoh, & Toni, 2016) for some argument \mathbf{A} is an abstract dispute tree \mathcal{T}^a for \mathbf{A} such that for all opponent nodes $\mathbf{O} : \mathbf{B}$ in \mathcal{T}^a that are leaf nodes there is no argument \mathbf{C} such that \mathbf{C} attacks \mathbf{B} .

Note that admissible abstract dispute trees are maximal, but not all maximal dispute trees are admissible (Cyras et al., 2016). Given an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ with $\mathcal{R} = \{c \leftarrow\}$, $\mathcal{A} = \{a, b\}$, $\mathcal{C}(a) = \{b\}$, $\mathcal{C}(b) = \{c\}$, Figure 4 shows two abstract dispute trees for the argument $\{a\} \vdash a$, the tree on the left-hand side is maximal whereas the one on the right-hand side is not.

Definition 5.1. Given any abstract dispute tree \mathcal{T}^a , let $LA(\mathcal{T}^a)$ denote the set $\{\alpha | _ : \Delta \vdash _ \mid _ \text{ is a leaf node in } \mathcal{T}^a, \alpha \in \Delta\}$ and $LO(\mathcal{T}^a)$ denote the set $\{N | N = \mathbf{O} : _ \mid _ \text{ is a leaf node in } \mathcal{T}^a\}$.

- Let \mathcal{AT} be the set of all admissible abstract dispute trees for some argument \mathbf{A} . Then $\mathcal{T}^a \in \mathcal{AT}$ is *least-assumption* if and only if there is no \mathcal{T}_0^a in \mathcal{AT} such that $LA(\mathcal{T}_0^a) \subset LA(\mathcal{T}^a)$.
- Let \mathcal{MT} be the set of all maximal dispute trees for some non-admissible argument \mathbf{A} . Then $\mathcal{T}^{max} \in \mathcal{MT}$ is *best-effort* if and only if there is no \mathcal{T}_0^{max} in \mathcal{MT} such that $LO(\mathcal{T}_0^{max}) \subset LO(\mathcal{T}^{max})$.

Example 5.2. Given an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ with $\mathcal{R} = \{a \leftarrow b; a \leftarrow c; c \leftarrow\}$, $\mathcal{A} = \{b, p, q\}$, $\mathcal{C}(b) = \{z\}$, $\mathcal{C}(p) = \{q\}$, $\mathcal{C}(q) = \{a\}$, Figure 5 shows two admissible abstract dispute trees for $\{p\} \vdash p$. The tree on the left-hand side is least-assumption, as it has a single leaf node $\mathbf{P} : \{\} \vdash a$ and there is no strict subset of the empty set, so there does not exist a tree with a leaf node containing fewer assumptions than the node $\mathbf{P} : \{\} \vdash a$. On the other hand, the tree on the right-hand side is not least-assumption as it has a leaf



Figure 5: Two admissible abstract dispute trees for the argument $\{p\} \vdash p$ for Example 5.2: the tree on the left is least-assumption; the tree on the right is not.

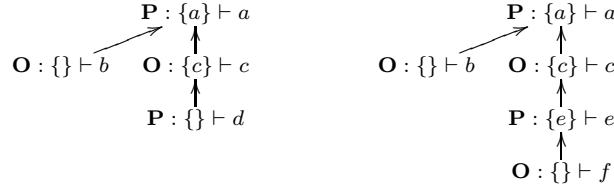


Figure 6: Two maximal abstract dispute trees for the argument $\{a\} \vdash a$ for Example 5.3: the tree on the left is best-effort; the tree on the right is not.

node $\mathbf{P} : \{b\} \vdash a$ and the tree on the left-hand side is an abstract dispute tree for the same argument $\{a\} \vdash a$ but with a smaller set of assumptions in its leaf node, i.e. $\{\} \subset \{b\}$.

Example 5.3. Given an ABA framework $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ with $\mathcal{R} = \{b \leftarrow; d \leftarrow; f \leftarrow\}$, $\mathcal{A} = \{a, c, e\}$, $\mathcal{C}(a) = \{b, c\}$, $\mathcal{C}(c) = \{d, e\}$, $\mathcal{C}(e) = \{f\}$, Figure 6 shows two non-admissible, maximal abstract dispute trees for $\{a\} \vdash a$. The tree on the left-hand side is best-effort as there is no maximal dispute tree for $a \vdash a$ which does not contain the node $\mathbf{O} : \{\} \vdash b$; thus there is no “smaller” maximal dispute tree for $a \vdash a$ than the one shown on the left-hand side. On the other hand, the tree on the right-hand side is not best-effort, as the tree on the left-hand side contains fewer opponent leaf nodes.

In the remainder we will refer to abstract dispute trees simply as dispute trees.

5.2 Argumentative Explanations for ADFs

In this section we introduce one ABA framework for each decision criterion for ADFs. These ABA frameworks share some rules, assumptions and contraries, given next. Here and in the remainder of this section we will assume as given a generic ADF $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$, unless specified otherwise.

Definition 5.2. The *core ABA framework corresponding to* $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$ is $\langle \mathcal{L}_0, \mathcal{R}_0, \mathcal{A}_0, \mathcal{C}_0 \rangle$, where⁶

6. When defining ABA frameworks, we omit to indicate the language component, as this can be easily inferred from the other components (specifically, the language is the set of all sentences occurring in rules, assumptions, and contraries).

- $\mathcal{R}_0 = \{met(d, g) \leftarrow | d \in \mathbf{D}, g \in \mathbf{G}, g \in \gamma(d)\},$
- $\mathcal{A}_0 = \{notMet(d, g) | d \in \mathbf{D}, g \in \mathbf{G}\},$
- for any $notMet(d, g) \in \mathcal{A}_0$: $\mathcal{C}_0(notMet(d, g)) = \{met(d, g)\}.$

Intuitively, a full representation, in terms of rules, of the ADF is included in \mathcal{R}_0 , and each decision can be assumed not to meet any goal (\mathcal{A}_0), unless it can be proven to meet it (\mathcal{C}_0). We illustrate the core ABA framework with the following example.

Example 5.4. (Example 3.1 continued.) The core ABA framework corresponding to $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$ is $\langle \mathcal{L}_0, \mathcal{R}_0, \mathcal{A}_0, \mathcal{C}_0 \rangle$, in which

- \mathcal{R}_0 consists of:

$$met(ic, cheap) \leftarrow \quad \quad \quad met(ic, near) \leftarrow \quad \quad \quad met(jh, near) \leftarrow$$
- \mathcal{A}_0 consists of:

$$\begin{array}{lll} notMet(jh, cheap) & notMet(ic, cheap) & notMet(ritz, cheap) \\ notMet(jh, near) & notMet(ic, near) & notMet(ritz, near) \end{array}$$
- for any $d \in \mathbf{D} = \{jh, ic, ritz\}$ and $g \in \mathbf{G} = \{cheap, near\}$:

$$\mathcal{C}_0(notMet(d, g)) = \{met(d, g)\}.$$

In this ABA framework, argument $\{notMet(jh, near)\} \vdash notMet(jh, near)$ is attacked by argument $\{\} \vdash met(jh, near)$, which cannot be attacked (as its support is empty). As another example, argument $\{notMet(jh, cheap)\} \vdash notMet(jh, cheap)$ cannot be attacked.

We then define ABA frameworks to represent the decision criteria, all incorporating the core ABA framework.

5.2.1 STRONGLY DOMINANT ABA FRAMEWORKS AND ARGUMENTATIVE EXPLANATIONS

Definition 5.3. For decisions \mathbf{D} and goals \mathbf{G} , let the *strongly dominant component* be the ABA framework with

- $\mathcal{R}_s = \{notSDom(d) \leftarrow notMet(d, g) \mid d \in \mathbf{D}, g \in \mathbf{G}\},$
- $\mathcal{A}_s = \{sDom(d) \mid d \in \mathbf{D}\},$
- for any $sDom(d) \in \mathcal{A}$: $\mathcal{C}_s(sDom(d)) = \{notSDom(d)\}.$

Then, the *strongly dominant ABA framework corresponding to* $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$ is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$, where

- $\mathcal{R} = \mathcal{R}_0 \cup \mathcal{R}_s,$
- $\mathcal{A} = \mathcal{A}_0 \cup \mathcal{A}_s,$
- for any α in \mathcal{A} : $\mathcal{C}(\alpha) = \begin{cases} \mathcal{C}_0(\alpha) & \text{if } \alpha \in \mathcal{A}_0, \\ \mathcal{C}_s(\alpha) & \text{if } \alpha \in \mathcal{A}_s. \end{cases}$

The intuition behind Definition 5.3 is as follows: given a decision d , we can assume that d is strongly dominant ($sDom(d) \in \mathcal{A}$), unless it can be proven not to be so by proving that it does not meet some goal (using the rule for d and the goal in question in \mathcal{R}_s). By definition of the core ABA framework, then, proving that a decision does not meet a goal can always be achieved, by assuming so, unless there are reasons against this assumption, by supporting its contrary. We illustrate the notion of strongly dominant ABA framework corresponding to a decision framework as follows.

Example 5.5. (Example 5.4 continued.) The strongly dominant ABA framework corresponding to $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$ is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$, in which

- \mathcal{R} consists of \mathcal{R}_0 in Example 5.4 together with:

$$\begin{array}{ll} notSDom(ic) \leftarrow notMet(ic, near) & notSDom(ic) \leftarrow notMet(ic, cheap) \\ notSDom(jh) \leftarrow notMet(jh, near) & notSDom(jh) \leftarrow notMet(jh, cheap) \\ notSDom(ritz) \leftarrow notMet(ritz, near) & notSDom(ritz) \leftarrow notMet(ritz, cheap) \end{array}$$
- \mathcal{A} consists of \mathcal{A}_0 in Example 5.4 together with:

$$sDom(jh) \quad sDom(ic) \quad sDom(ritz)$$
- for any $d \in \mathcal{D} = \{jh, ic, ritz\}$ and $g \in \mathcal{G} = \{cheap, near\}$:

$$\mathcal{C}(notMet(d, g)) = \mathcal{C}_0(notMet(d, g)) \quad \mathcal{C}(sDom(d)) = \{notSDom(d)\}$$

In this ABA framework, the argument $\{sDom(ic)\} \vdash sDom(ic)$ is attacked by

$$\{notMet(ic, cheap)\} \vdash notSDom(ic) \text{ and } \{notMet(ic, near)\} \vdash notSDom(ic)$$

which, in turn, are attacked, respectively, by arguments

$$\{\} \vdash met(ic, cheap) \text{ and } \{\} \vdash met(ic, near).$$

Further, there is no argument attacking either $\{\} \vdash met(ic, cheap)$ or $\{\} \vdash met(ic, near)$. Hence $\{sDom(ic)\} \vdash sDom(ic)$ is admissible. Instead, $\{sDom(jh)\} \vdash sDom(jh)$ is not admissible, as it is attacked by $\{notMet(jh, cheap)\} \vdash notSDom(jh)$, which cannot be attacked.

In this example, admissible arguments in the strongly dominant ABA framework and strongly dominant decisions in the ADF correspond. This correspondence holds in general:

Theorem 5.1. Let AF be the strongly dominant ABA framework corresponding to $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$. Then, for all decisions $d \in \mathcal{D}$, d is strongly dominant in $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$ if and only if $\{sDom(d)\} \vdash sDom(d)$ is admissible in AF .

Thus, using the terminology of (Cyras et al., 2021), strongly dominant ABA frameworks can be seen as *complete* (argumentation-based) representations of the underlying decision problems. Dispute trees (of various types), drawn from these ABA frameworks, can then be used to explain argumentatively why decisions are (strongly/weakly) dominant or not, as follows.

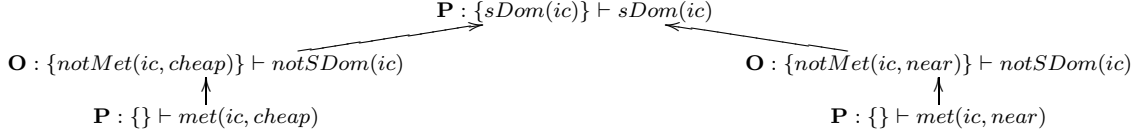


Figure 7: Least-assumption dispute tree sanctioning $\{sDom(ic)\} \vdash sDom(ic)$ as admissible (and ic as strongly dominant), for Example 5.6.

Definition 5.4. Let AF be the strongly dominant ABA framework corresponding to $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$. Then, for $d \in \mathcal{D}$:

- an *argumentative explanation* for d being strongly dominant is a least-assumption dispute tree for $\{sDom(d)\} \vdash sDom(d)$ in AF ;
- an *argumentative explanation* for d **not** being strongly dominant is a best-effort dispute tree for $\{sDom(d)\} \vdash sDom(d)$ in AF .

Example 5.6. (Example 5.5 continued.) As an illustration, Figure 7 uses a least-assumption dispute tree as a argumentative explanation for ic being strongly dominant in Example 3.1. The root of this tree puts forward the claim (supported by \mathbf{P}) that ic is a strongly dominant decision. This statement is challenged by two arguments (from \mathbf{O}): ic is not strongly dominant if it is not *cheap* and if it is not *near*. These two arguments are counter-attacked by further arguments (by \mathbf{P}): ic is cheap and near, and thus strongly dominant. This tree/explanation basically gives the reasons behind the selection of ic as strongly dominant. The explanation can be given an intuitive reading, for example, in terms of a debate between the (fictional) *proponent* (\mathbf{P}) and *opponent* (\mathbf{O}) players, as follows:

player \mathbf{P} starts by arguing that ic is strongly dominant;
player \mathbf{O} states that this is not so unless ic meets the goal *cheap*;
player \mathbf{P} replies that ic meets the goal *cheap*;
player \mathbf{O} then goes back to saying that ic is not strongly dominant if it does not meet the goal *near*;
player \mathbf{P} concludes saying that ic meets the goal *near*.

The dialogue ends successfully for \mathbf{P} , with \mathbf{O} having nothing more to contribute.

As a further illustration, Figure 8 uses a best-effort dispute tree giving an argumentative explanation for jh not being strongly dominant in Example 3.1, as an argumentative reading of the selection of jh : unlike ic , since jh does not meet the goal *cheap*, the root argument $\{sDom(jh)\} \vdash sDom(jh)$, cannot defend the attack from $\{notMet(jh, cheap)\} \vdash notSDom(jh)$. This can be read again as a dialogue between \mathbf{P} and \mathbf{O} , as follows:

\mathbf{P} states that jh is strongly dominant;
 \mathbf{O} replies that jh is not unless it meets the goal *near*;
 \mathbf{P} states that jh meets *near*;
 \mathbf{O} brings a further objection that jh is not strongly dominant if it does not meet the goal *cheap*

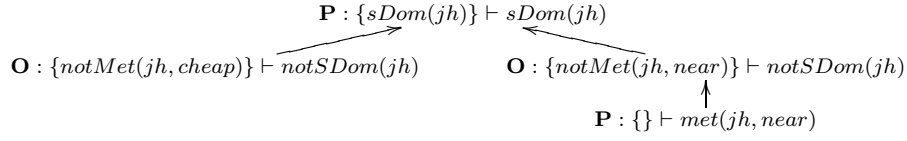


Figure 8: Best-effort dispute tree sanctioning $\{sDom(jh)\} \vdash sDom(jh)$ as not admissible (and jh as not strongly dominant) in Example 5.6.

to which \mathbf{P} cannot find a reply, thus losing to \mathbf{O} .

In addition to providing argumentative explanations per-se, (various types of) dispute trees can also be used to *extract* lean explanations for decisions (not) meeting the various decision criteria as defined in Definition 4.1.

Proposition 5.1. Let AF be the strongly dominant ABA framework corresponding to $\langle \mathbb{D}, \mathbb{G}, \gamma \rangle$. For any strongly dominant decisions $d \in \mathbb{D}$, let \mathcal{T}^a be an argumentative explanation for d being strongly dominant (in the form of least-assumption dispute tree for $\{sDom(d)\} \vdash sDom(d)$ in AF). Then

$$\{g | \mathbf{P} : \{\} \vdash met(d, g) \text{ is a leaf node in } \mathcal{T}^a\}$$

is a lean explanation for d being strongly dominant.

Example 5.7. (Example 5.6 continued.) Given the least-assumption dispute tree/ argumentative explanation in Figure 7, it is immediate to see that arguments held by the leaf nodes, $\mathbf{P} : \{\} \vdash met(ic, cheap)$ and $\mathbf{P} : \{\} \vdash met(ic, near)$, give the lean explanation $\{cheap, near\}$ for ic being strongly dominant. This can be read as: “since ic meets both goals $cheap$ and $near$, it is strongly dominant”. Whereas the lean explanation refers to the components of the decision problem only, the argumentative explanation unearths the reasoning behind ic being strongly dominant.

Proposition 5.2. Let AF be the strongly dominant ABA framework corresponding to $\langle \mathbb{D}, \mathbb{G}, \gamma \rangle$. For any $d \in \mathbb{D}$, if d is not strongly dominant, let \mathcal{T}^a be an argumentative explanation for d not being strongly dominant. Then,

$$\{g | \mathbf{O} : \{notMet(d, g)\} \vdash notSDom(d) \text{ is a leaf node in } \mathcal{T}^a\}$$

is a lean explanation for d not being strongly dominant.

Example 5.8. (Example 5.6 continued.) Given the best-effort dispute tree/argumentative explanation in Figure 8, it is immediate to see that the only leaf node labelled by \mathbf{O} is $\{notMet(jh, cheap)\} \vdash notSDom(jh)$. Thus, $\{cheap\}$ is a lean explanation for jh not being strongly dominant. This can be read as: “Since jh does not meet the goal $cheap$, it is not strongly dominant”. Again, whereas the lean explanation refers to the components of the decision problem only, the argumentative explanation unearths the reasoning behind ic not being strongly dominant.

5.2.2 DOMINANT ABA FRAMEWORKS AND ARGUMENTATIVE EXPLANATIONS

Dominant decisions can also be given an equivalent argumentative formulation, by defining a corresponding dominant ABA framework also incorporating the core ABA framework, as follows.

Definition 5.5. For decisions \mathbf{D} and goals \mathbf{G} , let the *dominant component* be the ABA framework with

- $\mathcal{R}_d = \{notDom(d) \leftarrow notMet(d, g) \mid d \in \mathbf{D}, g \in \mathbf{G}\},$
- $\mathcal{A}_d = \{dom(d) \mid d \in \mathbf{D}\} \cup \{noOthers(d, g) \mid d \in \mathbf{D}, g \in \mathbf{G}\} \cup \{notMet(d, g) \mid d \in \mathbf{D}, g \in \mathbf{G}\},$
- for any $dom(d) \in \mathcal{A}_d$: $\mathcal{C}_d(dom(d)) = \{notDom(d)\},$
 for any $noOthers(d, g) \in \mathcal{A}_d$: $\mathcal{C}_d(noOthers(d, g)) = \{met(d', g) \mid d' \in \mathbf{D}, d' \neq d\},$
 for any $notMet(d, g) \in \mathcal{A}_d$: $\mathcal{C}_d(notMet(d, g)) = \{noOthers(d, g)\}.$

Then, the *dominant ABA framework corresponding to F_a* is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$, where:

- $\mathcal{R} = \mathcal{R}_0 \cup \mathcal{R}_d,$
- $\mathcal{A} = \mathcal{A}_0 \cup \mathcal{A}_d,$
- for all α in \mathcal{A} : $\mathcal{C}(\alpha) = \begin{cases} \mathcal{C}_0(\alpha) & \text{if } \alpha \in \mathcal{A}_0 \text{ and } \alpha \notin \mathcal{A}_d, \\ \mathcal{C}_d(\alpha) & \text{if } \alpha \in \mathcal{A}_d \text{ and } \alpha \notin \mathcal{A}_0, \\ \mathcal{C}_0(\alpha) \cup \mathcal{C}_d(\alpha) & \text{if } \alpha \in \mathcal{A}_d \cap \mathcal{A}_0. \end{cases}$

Intuitively, a decision d is selected (as dominant) either if it meets all goals, or for goals that d does not meet, there is no other d' meeting them. Hence the contrary of $notMet(d, g)$ (representing that d does not meet g) is either $met(d, g)$ (representing that d meets g) or $noOthers(d, g)$ (representing that, although d does not meet g , no other decision does either).

Theorem 5.2. Let AF be the dominant ABA framework corresponding to $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$. Then for all decisions $d \in \mathbf{D}$, d is dominant if and only if $\{dom(d)\} \vdash dom(d)$ is admissible in AF .

Example 5.9. (Example 3.2 continued.) As an illustration, given the dominant ABA framework corresponding to the ADF in Example 3.2, jh is dominant since $\{dom(jh)\} \vdash dom(jh)$ is admissible. Indeed, the two counter-arguments $\{notMet(jh, g)\} \vdash notDom(jh)$, for $g = near$ and $cheap$, are attacked, respectively, by argument $\{\} \vdash met(jh, near)$ and argument $\{noOthers(ritz, cheap)\} \vdash noOthers(jh, cheap)$, neither of which can be attacked.

Definition 5.6. Let AF be the dominant ABA framework corresponding to $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$. Then, for $d \in \mathbf{D}$:

- an *argumentative explanation for d being dominant* is a least-assumption dispute tree for $\{dom(d)\} \vdash dom(d)$ in AF ;

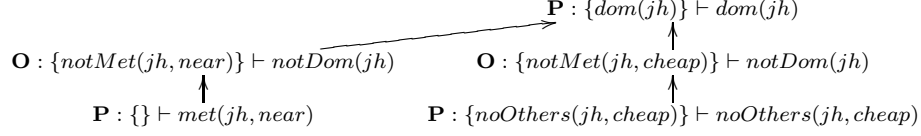


Figure 9: Least-assumption dispute tree sanctioning $\{dom(jh)\} \vdash dom(jh)$ as admissible (and jh as dominant), for Example 5.10.

- an *argumentative explanation* for d **not** being dominant is a best-effort dispute tree for $\{dom(d)\} \vdash dom(d)$ in AF .

Lean explanations for decisions being dominant can also be extracted from dispute trees, formally:

Proposition 5.3. Let AF be the dominant ABA framework corresponding to $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$. For any dominant $d \in \mathcal{D}$, let \mathcal{T}^a be an argumentative explanation for d being dominant (in the form of a least-assumption dispute tree for $\{dom(d)\} \vdash dom(d)$). Then the pair (G, F) , where

- $G = \{g | \mathbf{P} : \{\} \vdash met(d, g) \text{ is a leaf node in } \mathcal{T}^a\}$,
- $F = \{g | \mathbf{P} : \{noOthers(d, g)\} \vdash noOthers(d, g) \text{ is a leaf node in } \mathcal{T}^a\}$,

is a lean explanation for d being dominant.

Example 5.10. (Example 5.9 continued.) Figure 9 shows an argumentative explanation for jh being dominant in the decision problem from Example 3.2. We can see that since $\mathbf{P} : \{\} \vdash met(jh, near)$ and $\mathbf{P} : \{noOthers(jh, cheap)\} \vdash noOthers(jh, cheap)$ are the two proponent leaf nodes in the tree, we obtain, as G and F in Proposition 5.3, $G = \{near\}$ and $F = \{cheap\}$. Thus, a lean explanation for jh being dominant is $(\{near\}, \{cheap\})$, which can be read as: “ jh is dominant because it meets the goal $near$ and the for the goal $cheap$ which jh does not meet, no other decision meets it either”. Instead, the argumentative explanation in Figure 9 can be read, for example, as the following dialogue providing the reasons behind jh being dominant:

P: jh is dominant;
O: jh is not unless it meets the goal $near$;
P: jh meets $near$;
O: jh is not dominant unless it meets $cheap$;
P: jh does not meet $cheap$, but no other decision meets $cheap$ either.

Proposition 5.4. Let AF be the dominant ABA framework corresponding to $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$. For any $d \in \mathcal{D}$, if d is not dominant, let \mathcal{T}^a be an argumentative explanation for d not being dominant. Then

$$\{(d_i, g_j) | \mathbf{O} : \{\} \vdash met(d_i, g_j) \text{ is a leaf node in } \mathcal{T}^a\}$$

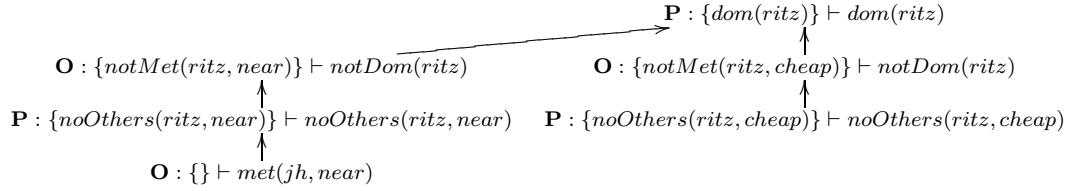


Figure 10: Best-effort dispute tree for $\{dom(ritz)\} \vdash dom(ritz)$ in Example 5.11.

is a lean explanation for d not being dominant.

Example 5.11. (Example 5.9 continued.) A best-effort dispute tree for the argument $\{dom(ritz)\} \vdash dom(ritz)$ is shown in Figure 10, providing an argumentative explanation for $ritz$ not being dominant in the decision problem from Example 3.2. From this argumentative explanation, we can see that $\{\} \vdash met(jh, near)$ is the only argument in an \mathbf{O} leaf node. Thus, $\{(jh, near)\}$ is a lean explanation for $ritz$ not being dominant. This can be read as: “ $ritz$ is not dominant because it does not meet the goal $near$ yet jh meets it”. As in the previous illustrations, the argumentative explanation can be given a dialogical reading indicating the reasoning leading to sanctioning $ritz$ as not being dominant.

5.2.3 WEAKLY DOMINANT ABA FRAMEWORKS AND ARGUMENTATIVE EXPLANATIONS

As for strongly dominant and dominant decisions, ABA can be used to explain weakly dominant decisions.

Definition 5.7. For decisions \mathbf{D} and goals \mathbf{G} , let the *weakly dominant component* be the ABA framework with

- $\mathcal{R}_w = \{notWDom(d) \leftarrow met(d', g), notMet(d, g), notMore(d, d') \mid d, d' \in \mathbf{D}, d \neq d', g \in \mathbf{G}\} \cup \{more(d, d') \leftarrow met(d, g), notMet(d', g) \mid d, d' \in \mathbf{D}, d \neq d', g \in \mathbf{G}\},$
- $\mathcal{A}_w = \{wDom(d) \mid d \in \mathbf{D}\} \cup \{notMore(d, d') \mid d, d' \in \mathbf{D}, d \neq d'\},$
- for any $wDom(d) \in \mathcal{A}_w$: $\mathcal{C}_w(wDom(d)) = \{notWDom(d)\},$
for any $notMore(d, d') \in \mathcal{A}_w$: $\mathcal{C}_w(notMore(d, d')) = \{more(d, d')\}.$

Then, the *weakly dominant ABA framework corresponding to* $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$ is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$, where

- $\mathcal{R} = \mathcal{R}_0 \cup \mathcal{R}_w,$
- $\mathcal{A} = \mathcal{A}_0 \cup \mathcal{A}_w,$
- for all α in \mathcal{A} , $\mathcal{C}(\alpha) = \begin{cases} \mathcal{C}_0(\alpha) & \text{if } \alpha \in \mathcal{A}_0, \\ \mathcal{C}_w(\alpha) & \text{if } \alpha \in \mathcal{A}_w. \end{cases}$

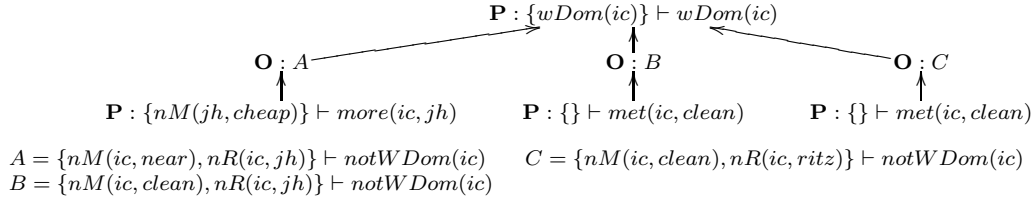


Figure 11: Least-assumption dispute tree sanctioning $\{wDom(ic)\} \vdash wDom(ic)$ as admissible (and ic as weakly dominant) in Example 5.13. Here, nM and nR are short-hands for $notMet$ and $notMore$, respectively.

Intuitively, a decision d is selected (as weakly dominant) if and only if there is no other decision d' such that the set of goals d' meets is a superset of the set of goals d meets. This can be tested for all $d' \neq d$ by definition of contrary of $wDom(d)$, as $notWDom(d)$, and by the rule with head such a contrary. Indeed, if we assume d to be weakly dominant, to attack it, one needs to find another decision d' and a goal g such that $met(d', g)$, $notMet(d, g)$, and $notMore(d, d')$ all hold. Also, to attack assumption $notMore(d, d')$, one needs to show that there exists some goal g' , such that $met(d', g')$ and $notMet(d, g')$.

Theorem 5.3. Let AF be the weakly dominant ABA framework corresponds to $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$. Then for all decisions $d \in \mathbf{D}$, d is dominant if and only if $\{wDom(d)\} \vdash wDom(d)$ is admissible in AF .

Example 5.12. (Example 3.3 continued.) As an illustration, given the weakly dominant ABA framework corresponding to the ADF in Example 3.3, ic is weakly dominant since $\{wDom(ic)\} \vdash wDom(ic)$ is admissible. Indeed, the (two) arguments A and B and the argument C , all attacking $\{wDom(ic)\} \vdash wDom(ic)$, are all counter-attacked (the first by argument $\{notMet(jh, cheap)\} \vdash more(ic, jh)$, and the latter two by $\{\} \vdash met(ic, clean)$, none of which can be attacked.)

Definition 5.8. Let AF be the weakly dominant ABA framework corresponding to $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$. Then, for $d \in \mathbf{D}$:

- an *argumentative explanation* for d being weakly dominant is a least-assumption dispute tree for $\{wDom(d)\} \vdash wDom(d)$ in AF ;
- an *argumentative explanation* for d **not** being weakly dominant is a best-effort dispute tree for $\{wDom(d)\} \vdash wDom(d)$ in AF .

Leaf nodes in dispute trees also give lean explanations for weakly dominant decisions, formally:

Proposition 5.5. Let AF be the weakly dominant ABA framework corresponding to $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$. For any weakly dominant $d \in \mathbf{D}$, let \mathcal{T}^a be an argumentative explanation for d being weakly dominant (in the form of a least-assumption dispute tree for $\{wDom(d)\} \vdash wDom(d)$). Then the pair (G, M) , where

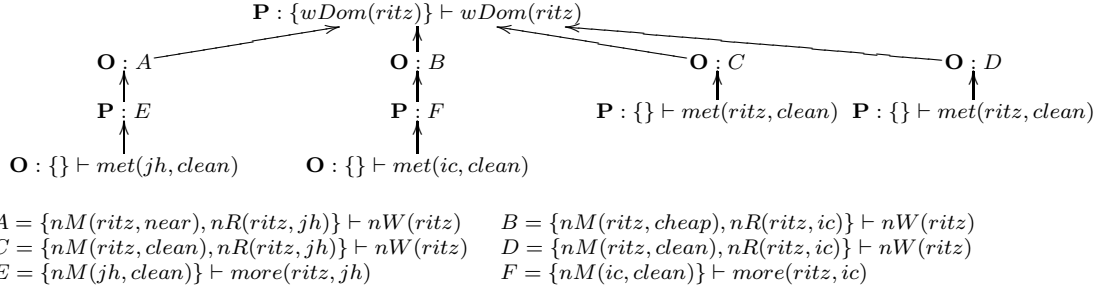


Figure 12: A best-effort dispute tree for $\{wDom(ritz)\} \vdash wDom(ritz)$ for Example 5.14. Here, nM , nR and nW are short-hands for $notMet$, $notMore$ and $notWDom$, respectively.

- $G = \{g | \mathbf{P} : \{ \} \vdash \{met(d, g)\} \text{ or } \mathbf{P} : \{notMet(-, g)\} \vdash more(d, -) \text{ is a node in } \mathcal{T}^a\}$ and
- $M = \{(d', g) | \mathbf{P} : \{notMet(d', g)\} \vdash more(d, d') \text{ is a leaf node in } \mathcal{T}^a\}$

is a lean explanation for d being weakly dominant.

Example 5.13. (Example 5.12 continued.) Figure 11 shows an argumentative explanation for ic being weakly dominant in the decision problem from Example 3.3. Here, we can see that the leaf nodes are $\mathbf{P} : \{notMet(jh, cheap)\} \vdash more(ic, jh)$, $\mathbf{P} : \{ \} \vdash met(ic, clean)$ and $\mathbf{P} : \{ \} \vdash met(ic, clean)$. Thus, $(\{clean, cheap\}, \{(jh, cheap)\})$ is a lean explanation for ic being weakly dominant, which can be read as “A reason for ic being weakly dominant is that ic meets goals $clean$ and $cheap$, in which $cheap$ is not met by jh ”.

Proposition 5.6. Let AF be the weakly dominant ABA framework corresponding to $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$. For any $d \in \mathcal{D}$, if d is not weakly dominant, let \mathcal{T}^a be an argumentative explanation for d not being dominant (in the form of a best-effort dispute tree for $\{wDom(d)\} \vdash wDom(d)$). Then the set

- $\{d' | \mathbf{O} : \{ \} \vdash met(d', g) \text{ a leaf node in } \mathcal{T}^a \text{ or } \mathbf{O} : \{notMet(d, g), notMore(d, d')\} \vdash notWDom(d) \text{ a leaf node in } \mathcal{T}^a\}$

is a lean explanation for d not being weakly dominant.

Example 5.14. (Example 5.12 continued.) An argumentative explanation for $ritz$ not being weakly dominant in the decision problem of Example 3.3 is given in Figure 12. Here, both $\{ \} \vdash met(jh, clean)$ and $\{ \} \vdash met(ic, clean)$ are held by leaf nodes labelled by \mathbf{O} . Thus $\{ic, jh\}$ is a lean explanation for $ritz$ not being weakly dominant, which can be read as: “ $ritz$ is not weakly dominant as jh and ic meet all goals met by $ritz$ and more”.

5.3 Argumentative Explanations for PDFs

The problem of determining g-preferred decisions in a PDF can also be equivalently understood as the problem of determining admissible arguments in an ABA framework, defined as follows:

Definition 5.9. Let \mathbf{S} be the comparable goal set in $F_e = \langle \mathbf{D}, \mathbf{G}, \mathbf{T}_{\mathbf{DG}}, \leq_{\mathbf{g}} \rangle$, the *g-preferred component* is an ABA framework $\langle \mathcal{L}_p, \mathcal{R}_p, \mathcal{A}_p, \mathcal{C}_p \rangle$, where

- $\mathcal{R}_p = \{pfr(s_t, s_r) \leftarrow |s_t, s_r \in \mathbf{S}, s_r <_{\mathbf{g}} s_t\} \cup \{notMetS(d, s) \leftarrow notMet(d, g) | d \in \mathbf{D}, s \in \mathbf{S}, g \in s\} \cup \{better(d, d', s) \leftarrow metS(d, s'), notMetS(d', s'), pfr(s', s) | d, d' \in \mathbf{D}, d \neq d', s, s' \in \mathbf{S}, s \neq s'\} \cup \{notGP(d) \leftarrow metS(d', s), notMetS(d, s), notBetter(d, d', s) | d, d' \in \mathbf{D}, d \neq d', s \in \mathbf{S}\};$
- $\mathcal{A}_p = \{gP(d), metS(d, s), notBetter(d, d', s) | d, d' \in \mathbf{D}, d \neq d', s \in \mathbf{S}\};$
- for any $gP(d) \in \mathcal{A}$: $\mathcal{C}_p(gP(d)) = \{notGP(d)\};$
for any $metS(d, s) \in \mathcal{A}$: $\mathcal{C}_p(metS(d, s)) = \{notMetS(d, s)\};$
for any $notBetter(d, d', s) \in \mathcal{A}$: $\mathcal{C}_p(notBetter(d, d', s)) = \{better(d, d', s)\}.$

Rules in a g-preferred component can be read as follows.

- $pfr(s_t, s_r) \leftarrow$ represents that a comparable goal set s_t is preferred to another comparable goal set s_r .
- $notMetS(d, s) \leftarrow notMet(d, g)$ for $g \in s$ states that a decision d does not meet a comparable goal set s if d does not meet some goal g in s .
- $better(d, d', s) \leftarrow metS(d, s'), notMetS(d', s'), pfr(s', s)$ states that decision d is better than d' with respect to a comparable goal set s if d meets some goal set s' that is not met by d' and s' is preferred to s .
- $notGP(d) \leftarrow metS(d', s), notMetS(d, s), notBetter(d, d', s)$ states that decision d is not g-preferred if (1) there exists another decision d' meeting all goals in a comparable goal set s , (2) d does not meet all goals in s and (3) comparing with d' , d does not meet any more preferred goal set than s .

All decisions are assumed to be g-preferred as $gP(d)$ is an assumption in the ABA framework for all decision d . Then all such assumptions are examined by arguments for $notGP(d)$, constructed with relevant rules and assumptions.

With the g-preferred component defined, we are ready to obtain the ABA framework for g-preferred decisions by taking the “union” of the g-preferred component and the core ABA framework.

Definition 5.10. Let $F_e = \langle \mathbf{D}, \mathbf{G}, \mathbf{T}_{\mathbf{DG}}, \leq_{\mathbf{g}} \rangle$, $\langle \mathcal{L}_p, \mathcal{R}_p, \mathcal{A}_p, \mathcal{C}_p \rangle$ the g-preferred component and let $\langle \mathcal{L}_0, \mathcal{R}_0, \mathcal{A}_0, \mathcal{C}_0 \rangle$ be the core ABA framework corresponding to $\langle \mathbf{D}, \mathbf{G}, \gamma \rangle$. Then, the *g-preferred ABA framework corresponding to F_e* is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ where:

- $\mathcal{R} = \mathcal{R}_0 \cup \mathcal{R}_p;$
- $\mathcal{A} = \mathcal{A}_0 \cup \mathcal{A}_p;$
- for all α in \mathcal{A} : $\mathcal{C}(\alpha) = \begin{cases} \mathcal{C}_0(\alpha) & \text{if } \alpha \in \mathcal{A}_0, \\ \mathcal{C}_p(\alpha) & \text{if } \alpha \in \mathcal{A}_p. \end{cases}$

	<i>jh</i>	<i>ic</i>
cheap	0	1
near	1	1
quiet	1	0

Table 2: T_{DG} for Example 5.15.

The definition of g-preferred ABA framework (corresponding to F_e) is given in the same spirit as for the definitions of corresponding ABA frameworks given earlier for (strongly/weakly) dominant decisions, to guarantee an argumentative reading of g-preferred decisions, as follows:

Theorem 5.4. Let AF be the g-preferred ABA framework corresponding to $\langle D, G, T_{DG}, \leq_g \rangle$. Then, for all $d \in D$, d is g-preferred if and only if $\{gP(d)\} \vdash gP(d)$ is admissible in AF .

The following example illustrates the argumentative re-interpretation of g-preferred decisions.

Example 5.15. Consider $\langle D, G, T_{DG}, \leq_g \rangle$ with T_{DG} in Table 2 and \leq_g such that

$$\{quiet\} <_g \{cheap\} <_g \{near\} <_g \{quiet, near\}$$

The comparable goal set is $S = \{\{quiet, near\}, \{near\}, \{cheap\}, \{quiet\}\}$. Let qnS, nS, cS and qS stand for the elements of S (respectively).

Also, we use b , nB , nM and nS as short-hands for *better*, *notBetter*, *notMet* and *notMetS*, respectively. Then \mathcal{R}_0 consists of

$$met(ic, cheap) \leftarrow \quad met(ic, near) \leftarrow \quad met(jh, near) \leftarrow \quad met(jh, quiet) \leftarrow$$

\mathcal{R}_p consists of

$$\begin{aligned} &\{notGP(ic) \leftarrow metS(jh, s), nS(ic, s), nB(ic, jh, s) | s \in \{qnS, nS, cS, qS\}\}; \\ &\{notGP(jh) \leftarrow metS(ic, s), nS(jh, s), nB(jh, ic, s) | s \in \{qnS, nS, cS, qS\}\}; \\ &\{nS(ic, s) \leftarrow nM(ic, g) | g \in \{near, quiet, cheap\}, s \in \{qnS, nS, cS, qS\}, g \in s\}; \\ &\{nS(jh, s) \leftarrow nM(jh, g) | g \in \{near, quiet, cheap\}, s \in \{qnS, nS, cS, qS\}, g \in s\}; \\ &\{b(d, d', qS) \leftarrow metS(d, s'), nS(d', s') | d \in \{ic, jh\}, d' \in \{ic, jh\} \setminus \{d\}, s' \in \{cS, nS, qnS\}\}; \\ &\{b(d, d', cS) \leftarrow metS(d, s'), nS(d', s') | d \in \{ic, jh\}, d' \in \{ic, jh\} \setminus \{d\}, s' \in \{nS, qnS\}\}; \\ &\{b(d, d', nS) \leftarrow metS(d, qnS), nS(d', qnS) | d \in \{ic, jh\}, d' \in \{ic, jh\} \setminus \{d\}\}; \end{aligned}$$

\mathcal{A} consists of the following elements, with \mathcal{C} as in Definition 5.10:

$gP(ic)$	$nM(ic, near)$	$nM(ic, cheap)$	$nM(ic, quiet)$
$gP(jh)$	$nM(jh, near)$	$nM(jh, cheap)$	$nM(jh, quiet)$
$metS(ic, qnS)$	$metS(ic, cS)$	$metS(ic, qS)$	$metS(ic, nS)$
$metS(jh, qnS)$	$metS(jh, cS)$	$metS(jh, qS)$	$metS(jh, nS)$
$nB(ic, jh, qnS)$	$nB(ic, jh, cS)$	$nB(ic, jh, nS)$	$nB(ic, jh, qS)$
$nB(jh, ic, qnS)$	$nB(jh, ic, cS)$	$nB(jh, ic, nS)$	$nB(jh, ic, qS)$

In this example, it is easy to see that $\{gP(jh)\} \vdash gP(jh)$ is admissible, as well as the root of the admissible (least-assumption) dispute tree in Figure 13. The root argument claims that jh is a g-preferred decision, and the tree provides an argumentative explanation, in the same spirit as for other decision criteria earlier in this section. This explanation can be provided, for example, the following intuitive reading. The root argument is attacked by

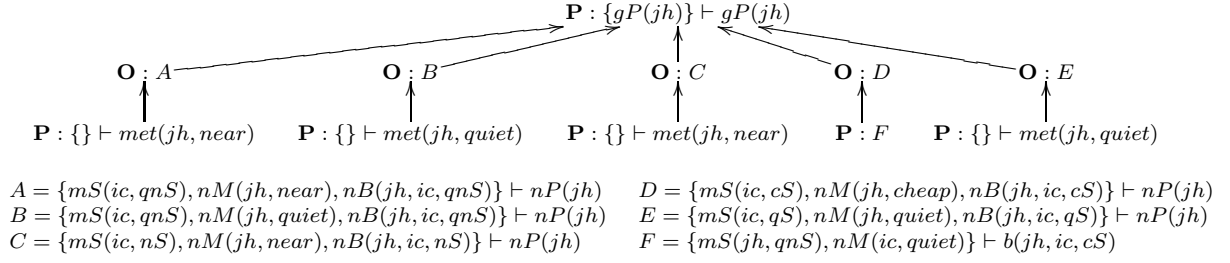


Figure 13: Least-assumption dispute tree for Example 5.15, showing that $\{gP(jh)\} \vdash jh$ is admissible. Here, mS , nM , nB and nP are short-hands for $metS$, $notMet$, $notBetter$, and $notGP$ respectively.

five arguments, $A - E$ in the tree, stating various reasons against jh being g-preferred. For instance, argument A can be read as the following objection

jh is not a g-preferred decision if the alternative decision ic meets both goals $near$ and $quiet$, jh does not meet $near$, and jh does not meet more preferred goals than $near$ and $quiet$.

Each of the attacking arguments $A - E$ is counterattacked, by some argument which is a leaf of the tree. For instance, the leaf argument $\{\} \vdash met(jh, near)$, counter-attacking both A and C , can be read as

but it is a fact that jh meets goal $near$.

The argument $F = \{metS(jh, qnS), notMet(ic, quiet)\} \vdash better(jh, ic, cS)$ can be read as

jh is better than ic as it meets more preferred goals than $cheap$, since jh meets both $near$ and $quiet$ (which are together more preferred than $cheap$ alone, as indicated in the specification of the decision problem) yet ic does not meet $quiet$.

For this same example, Figure 14 gives a maximal (best-effort) dispute tree with root $\{gP(ic)\} \vdash gP(ic)$, showing that this argument is non-admissible. This tree in provides an argumentative explanation, which can be intuitively read dialogically as follows, for example:

the proponent \mathbf{P} claims that ic is g-preferred;
 this is challenged by \mathbf{O} putting forward arguments that ic does not meet the goal $near$, which are in the set $\{quiet, near\}$ met by jh , and ic does not meet more preferred goals than the set $\{quiet, near\}$;
 \mathbf{P} challenges that jh does not meet $quiet$, so it does not meet the set $\{quiet, near\}$.
 \mathbf{O} confirms that jh indeed meets $quiet$. (Left-most branch of the tree.) Then the debate moves to whether ic meets $quiet$, etc.

This example illustrates how least-assumption dispute trees (as in Figure 13) and best-effort dispute tree (as in Figure 14) can be used as argumentative explanations for g-preferred decisions. Formally:

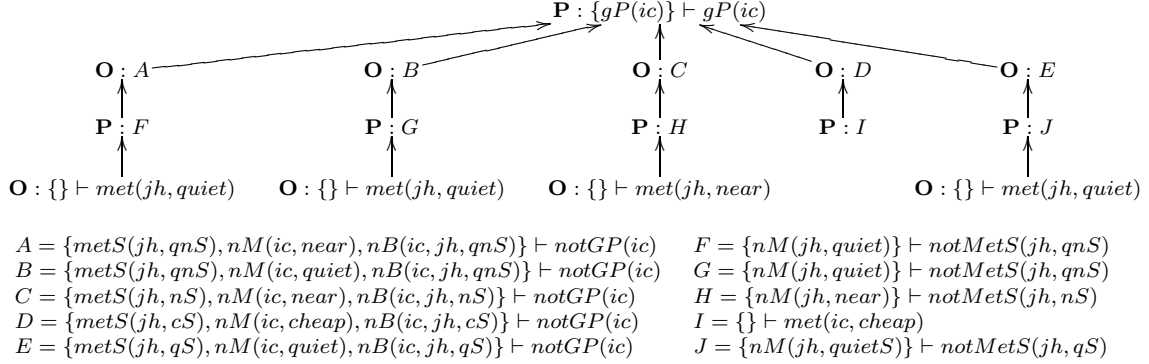


Figure 14: Best-effort dispute tree for Example 5.15, showing that $\{gP(ic)\} \vdash ic$ is not admissible. Here, nM and nB are short-hands for $notMet$ and $notBetter$, respectively.

Definition 5.11. Let AF be the g-preferred ABA framework corresponding to $\langle D, G, T_{DG}, \leq_g \rangle$. Then, for $d \in D$:

- an *argumentative explanation* for d being g-preferred is a least-assumption dispute tree for $\{gP(d)\} \vdash gP(d)$ in AF ;
- an *argumentative explanation* for d **not** being g-preferred is a best-effort dispute tree for $\{gP(d)\} \vdash gP(d)$ in AF .

As in ADFs, in addition to using dispute tree to explain decisions, we can explicitly extract, from these trees, lean explanations for g-preferred decisions.

Proposition 5.7. Let AF be the g-preferred ABA framework corresponding to $\langle D, G, T_{DG}, \leq_g \rangle$. For any g-preferred $d \in D$, let \mathcal{T}^a be an argumentative explanation for d being g-preferred. Then (G, M) , where

- $G = \{g | \mathbf{P} : \{\} \vdash \{met(d, g)\}\}$ is a leaf node in \mathcal{T}^a
- $M = \{(S, d') | \mathbf{P} : \{metS(d, S), notMet(d', _)\} \vdash better(d, d', _)\}$ is a leaf node in \mathcal{T}^a

is a lean explanation for d being g-preferred.

Example 5.16. (Example 5.15 continued.) For the argumentative explanation for jh being g-preferred in Figure 13, the leaf nodes are $\mathbf{P} : \{metS(jh, qnS), notMet(ic, quiet)\} \vdash better(jh, ic, cS)$, $\mathbf{P} : \{\} \vdash met(jh, near)$ and $\mathbf{P} : \{\} \vdash met(jh, quiet)$. Thus,

$$(\{near, quiet\}, \{(ic, qnS)\})$$

is a lean explanation for jh being g-preferred, which can be read as “ jh is g-preferred as it meets *near* and *quiet*, and the goal set $\{near, quiet\}$ is more preferred than any set of goals met by ic ”.

Proposition 5.8. Let AF be the g-preferred ABA framework corresponding to $\langle D, G, T_{DG}, \leq_g \rangle$. For any $d \in D$, if d is not g-preferred, let \mathcal{T}^a be an argumentative explanation for d not being g-preferred. Then

$$\{d' | \mathbf{O} : \{\} \vdash met(d', g) \text{ a leaf node in } \mathcal{T}^a\}$$

is a lean explanation for d not being g-preferred.

Example 5.17. (Example 5.15 continued.) For the argumentative explanation for ic not being g-preferred in Figure 14, we can see that $\{\} \vdash met(jh, near)$ and $\{\} \vdash met(jh, quiet)$ label \mathbf{O} leaf nodes, thus, $\{jh\}$ is a lean explanation for ic not being g-preferred, which can be read as: “A reason for ic not being g-preferred is that jh meets goals that are more preferred than the ones met by ic ”.

6. Explainable Decision Making with Decision Graphs (DGs)

ADFs and PDFs are very high-level formulations of decision problems, assuming the existence of “tabular” information linking decisions and goals directly. Richer representations, such as graphs, have been long used in decision making, e.g. in the form of Bayesian (or belief) networks (Ben-Gal, 2007) and influence diagrams (Howard & Matheson, 2005). In this section we will explore instantiations of ADFs and PDFs with *Decision Graphs (DGs)*, giving a logical structure to the decision domain and revealing the decision maker’s uncertainties and constraints (Matt & Toni, 2008). We first define (various forms of) decision making with DGs (Section 6.1), followed by ABA mappings to capture this (varied) decision making (Section 6.2), to conclude with explanations (Section 6.3).

6.1 Instantiating ADFs, PDFs and Lean Explanations with DGs

DGs are generalised *and-or trees* (Luger, 2008). In a DG, nodes are of three types: *decisions*, *goals*, and *intermediates*, representing, respectively, candidate decisions, goals, and attributes in the modelled decision problem. Edges represent relations amongst nodes, e.g. an edge from a decision to an intermediate represents that the decision “has” the intermediate attribute; an edge from an intermediate to a goal represents that the intermediate “satisfies” the goal; an edge from an intermediate to another intermediate represents that the former “leads to” the latter. Edges are of two types: *standard* and *defeasible*. A standard edge from a node to another represents that the former *definitely* leads to the latter, whereas a defeasible edge represents a *tentative* association, to be dropped if some *defeasible condition* (expressed in a suitable formal language) for the edge are satisfied. Finally, edges (of whichever type) are *tagged* with a natural number: edges (from node a to node b) provide (i) stand-alone support (from a to b) if they are tagged with 1, and (ii) conjoined support with other edges with the same tag, if greater than 1. Formally:

Definition 6.1. A *Decision Graph (DG)* is a tuple $\langle N, E, B \rangle$, in which

- $\langle N, E \rangle$ is a directed acyclic graph with
 - $N = D \cup N_{int} \cup G$ a set of *nodes* such that D , N_{int} and G are pairwise disjoint and $D \neq \emptyset$ is a set of *decisions*; N_{int} is a set of *intermediates*; $G \neq \emptyset$ is a set of *goals*;

- $E = E_s \cup E_d$ a set of *edges* such that E_s and E_d are pairwise disjoint and E_s is a set of *standard* edges, E_d is a set of *defeasible* edges; edges are of the form $[n_i \triangleright n_j]$ for $n_i, n_j \in \mathbb{N}$ such that $[n_i \triangleright n_j]$ is in E if and only if either $n_i \in D$ and $n_j \in N_{int} \cup G$, or $n_i \in N_{int}$ and $n_j \in N_{int} \cup G$; each $e \in E$ is *tagged* with a number i such that $i \in \mathbb{N}$, denoted by $t(e) = i$ (if the tag of an edge is 1, then it is often omitted);
- B (referred to as *defeasible condition*) is a (finite) set of implications of the form:

$$t_n \wedge \dots \wedge t_1 \rightarrow t_0$$

for $n \geq 0$ and $t_i \in \mathcal{L}_B$ (for $i = 0, \dots, n$) with \mathcal{L}_B a formal language such that for each defeasible edge $e = [n \triangleright n'] \in E_d$ with $t(e) = i$ there is a sentence $\neg dEdge(n, n', i) \in \mathcal{L}_B$.

We modify Example 3.1 to illustrate DGs.

Example 6.1. Figure 15 shows an example of a DG. The agent wants this accommodation to be convenient and cheap. The two candidates are Imperial College London Student Accommodation (*ic*) and Ritz Hotel (*ritz*). *ic* is £50 a night and in South Kensington (*inSK*). Ritz is £200 a night and in Piccadilly (*inPic*). Ritz is also having a promotion discount. £50 per night is *cheap* so *ic* is cheap. £200 a night with the current promotion discount makes *ritz* *cheap* as well. Hence both places are *cheap*. However, South Kensington is *near* so it is *convenient* whereas Piccadilly is not. Intuitively, *ic* is the better choice. Here:

- the decisions are: $D = \{ic, ritz\}$;
- the goals are: $G = \{convenient, cheap\}$;
- the intermediates are: $N_{int} = \{inSK, 50, inPic, 200, discount, near\}$.

The edges from *ic* to 50 and *ic* to *inSK* are defeasible (shown with dashed arrows in the figure), meaning that it is defeasibly known that *ic* is £50 a night and in South Kensington. The remaining edges are strict (shown with solid arrows in the figure), meaning that they are standard. Also, we let the defeasible condition B be:

$$B = \{\rightarrow termTime; termTime \rightarrow \neg dEdge(ic, 50, 1)\}$$

with language $\mathcal{L}_B = \{termTime, \neg dEdge(ic, 50, 1), \neg dEdge(ic, inSK, 1)\}$. The two implications in B state that: it is currently term time and if it is term time, then *ic* is not £50 per night. Finally, all edges are tagged with 1 (mostly omitted, as per convention), except for two edges (tagged with 2): they indicate that *cheap* is supported by the conjunction of 200 and *discount*; instead, for example, 50 alone supports *cheap*.

To instantiate the notion of *decisions-meeting-goals* with DGs, we define a notion of *reachability* from a set of nodes to a node, taking into account that defeasible edges may be *blocked*, depending on whether this can be inferred from the defeasible condition. We will assume that \mathcal{L}_B is equipped with an inference mechanism \vdash_{MP} amounting to the repeated application of modus ponens with implications: formally, for $s \in \mathcal{L}_B$, $B \vdash_{MP} s$ if and only if there is a sequence $s_1, \dots, s_m = s$ of sentences in \mathcal{L}_B and m implications in B such that, for each $i \in \{1, \dots, m\}$, either $\rightarrow s_i \in B$ or $t_1, \dots, t_n \rightarrow s_i \in B$ and $\{t_1, \dots, t_n\} \subseteq \{s_1, \dots, s_{i-1}\}$.

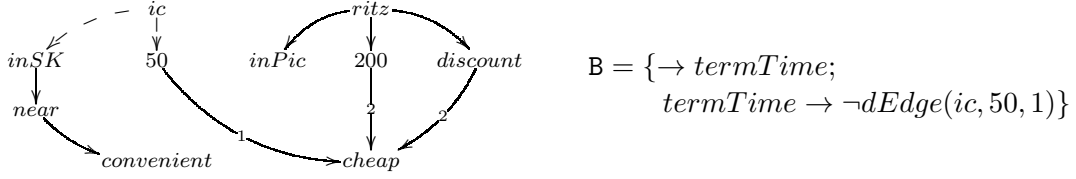


Figure 15: DG for Example 6.1. Solid arrows (\longrightarrow) represent strict edges whereas dashed arrows (\dashrightarrow) represent defeasible edges.

Definition 6.2. Given a DG $G = \langle N, E, B \rangle$ with $E = E_s \cup E_d$, let the *blocked edges* of G be $\mu(G) \subseteq E_d$ such that: $\mu(G) = \{e \in E_d \mid e = [n_i \triangleright n_j], t(e) = k, B \vdash_{MP} \neg dEdge(n_i, n_j, k)\}$. Then, for $n \in N, N \subseteq N$, we say that n is *reachable* from N if and only if one of the following two conditions holds:

- C1. there exists a tag k such that $N = \{n_i \in N \mid e_i = [n_i \triangleright n] \in E \setminus \mu(G) \text{ and } t(e_i) = k\}$; or
- C2. there exists some $N' \subseteq N$ such that n is reachable from N' and for each $n' \in N'$, n' is reachable from N .

Definition 6.2 is given recursively with C1 being the base case. C1 specifies that if there is a set of nodes N all having edges with the same tag pointing to some node n and none being blocked, then n is reachable from N . C2 specifies that if there is some set of nodes N' satisfying C1 (n being reachable from N') and every single node n' in N' is reachable from N , then n is reachable from N .

Example 6.2. (Example 6.1 continued.) There are two defeasible edges in this example: $[ic \triangleright inSK]$ and $[ic \triangleright 50]$. It is easy to see that $[ic \triangleright 50]$ is blocked whereas $[ic \triangleright inSK]$ is not. Thus,

- $inSK$ is reachable from $\{ic\}$, whereas 50 is not, since the edge $[ic \triangleright 50]$ is blocked,
- $inPic, 200, discount$ are reachable from $\{ritz\}$,
- $near$ is reachable from $\{inSK\}$ and $\{ic\}$,
- $convenient$ is reachable from $\{near\}, \{inSK\}$ and $\{ic\}$,
- $cheap$ is reachable from $\{50\}, \{200, discount\}$, and $\{ritz\}$, but not $\{ic\}$ since the edge $[ic \triangleright 50]$ is blocked.

With reachability defined, decisions-meeting-goals in DGs is defined as follows.

Definition 6.3. Given a DG $\langle N, E, B \rangle$, $N = D \cup N_{int} \cup G$, with D the decisions and G the goals, a decision $d \in D$ *meets* a goal $g \in G$, if and only if g is reachable from $\{d\}$. We again use $\gamma(d) \subseteq G$ to denote the set of goals met by d .

Example 6.3. (Example 6.2 continued.) The decision ic meets the goal $convenient$ but not the goal $cheap$, whereas the decision $ritz$ meets the goal $cheap$.

Proposition 6.1. For any DG $G = \langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$, there is an ADF $F = \langle \mathbf{D}, \mathbf{G}, \gamma \rangle$ such that a decision $d \in \mathbf{D}$ meets a goal $g \in \mathbf{G}$ in G if and only if d meets g in F .

The proof of this and all other results in this section are also in Appendix B.

Note that Proposition 6.1 sanctions that DGs are instances of ADFs. It is easy to see that ADFs are also, trivially, instances of DGs, namely:

Proposition 6.2. For any ADF $F = \langle \mathbf{D}, \mathbf{G}, \gamma \rangle$, there is a DG G such that a decision $d \in \mathbf{D}$ meets a goal $g \in \mathbf{G}$ in F if and only if d meets g in G .

By virtue of Proposition 6.1, decision criteria, i.e. (strong / weak) dominance, directly apply in DGs.

Example 6.4. (Example 6.3 continued.) Given that *ic* meets the goal *convenient* and *ritz* meets the goal *cheap*, both *ic* and *ritz* are weakly dominant.

Also, preferences over goals can be added to DGs, using partial orders over sets of goals as in the case of ADFs:

Definition 6.4. A *Preferential Decision Graph (PDG)* is a tuple $\langle \mathbf{N}, \mathbf{E}, \mathbf{B}, \leq_{\mathbf{g}} \rangle$, in which $\langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$ is a DG with $\mathbf{N} = \mathbf{D} \cup \mathbf{N}_{int} \cup \mathbf{G}$ and $\leq_{\mathbf{g}}$ is a partial order over $2^{\mathbf{G}}$.

Proposition 6.3. For any PDG $G = \langle \mathbf{N}, \mathbf{E}, \mathbf{B}, \leq_{\mathbf{g}} \rangle$, there is a PDF $F = \langle \mathbf{D}, \mathbf{G}, \gamma, \leq_{\mathbf{g}} \rangle$ such that a decision $d \in \mathbf{D}$ meets a goal $g \in \mathbf{G}$ in G if and only if d meets g in F .

Then, by virtue of Proposition 6.3, Definition 3.5 can be directly applied to define *g-preferred decisions in PDGs* as illustrated next.

Example 6.5. (Example 6.4 continued.) We add preference to the DG shown in Figure 15: between the two goal nodes, *convenient* and *cheap*, we let $\{\text{convenient}\} \leq_{\mathbf{g}} \{\text{cheap}\}$. Since both *ic* and *ritz* are weakly dominant but $\{\text{convenient}\} \leq_{\mathbf{g}} \{\text{cheap}\}$, *ritz* is g-preferred.

Finally, note that all notions of lean explanations are directly applicable to DGs and PDGs, given that they give rise to ADFs and PDFs, respectively. For illustration, in Example 6.4, $(\{\text{convenient}\}, \{(cheap, ritz)\})$ is a lean explanation for *ic* being weakly dominant, which can be read as “decision *ic* is weakly dominant because it meets the goal *convenient*, although the decision *ritz* meets another goal *cheap* not met by *ic*”; and, in Example 6.5, $(\{\text{cheap}\}, (\{cheap\}, ic))$ is a lean explanation for *ritz* being g-preferred, which can be read as “*ritz* is g-preferred as it meets the goal *cheap*, and the alternative decision *ic* does not meet anything more preferred than *cheap*”.

Note that other definitions of lean explanations are possible when using DGs, e.g. of the kind illustrated in the Introduction with the illustrative toy example, whereby the reasons for goals being met are accommodated within the explanations. We leave the exploration of other forms of lean explanation with DGs as future work. Instead, in the remainder of this section we will focus on argumentative explanations, based on ABA mappings that also incorporate the notion of reachability, as defined next.

6.2 (Strongly, Weakly) Dominant and G-Preferred ABA Frameworks with DGs

To build an ABA counterpart for explanation of decisions with DGs and PDGs, we need a change in the decisions-meeting-goals relation to reflect the “reasoning” with DGs. We first define the *core ABA framework corresponding to* DGs without defeasible edges and thus with empty defeasible condition B , as follows.

Definition 6.5. The *core ABA framework corresponding to* $\langle N, E, \emptyset \rangle$, $N = D \cup N_{int} \cup G$ and $E = E_s$ with E_s the strict edges, is $\langle \mathcal{L}_g, \mathcal{R}_g, \mathcal{A}_g, \mathcal{C}_g \rangle$, where

- $\mathcal{R}_g = \{ \text{edge}(n_1, n_2, t(e)) \leftarrow [n_1, n_2 \in N, e = [n_1 \triangleright n_2] \in E] \} \cup$
 $\{ \text{reach}(n_1, n_2) \leftarrow \text{edge}(n_1, n_2, t(e)) | n_1, n_2 \in N, e = [n_1 \triangleright n_2] \in E \} \cup$
 $\{ \text{reach}(n_1, n_2) \leftarrow \text{reach}(n_1, n_3), \text{edge}(n_3, n_2, t(e)), \neg \text{unreachableSib}(n_3, n_2, t(e), n_1) | n_1,$
 $n_2, n_3 \in N, n_3 \notin D, n_1 \neq n_2, n_1 \neq n_3, e = [n_3 \triangleright n_2] \in E, [n_1 \triangleright n_2] \notin E \} \cup$
 $\{ \text{unreachableSib}(n_3, n_2, t(e), n_1) \leftarrow \text{edge}(n_4, n_2, t(e)), \neg \text{reach}(n_1, n_4) | n_1, n_2, n_3, n_4 \in$
 $N, n_1 \neq n_2, n_1 \neq n_3, n_4 \neq n_3, e = [n_3 \triangleright n_2] \in E, [n_4 \triangleright n_2] \in E \} \cup$
 $\{ \text{met}(n_1, n_2) \leftarrow \text{reach}(n_1, n_2) | n_1 \in D, n_2 \in G \};$
- $\mathcal{A}_g = \{ \neg \text{unreachableSib}(n_2, n_3, t(e), n_1) | \neg \text{unreachableSib}(n_2, n_3, t(e), n_1) \text{ is in the}$
 $\text{body of a rule in } \mathcal{R}_g \} \cup$
 $\{ \neg \text{reach}(n_1, n_2) | \neg \text{reach}(n_1, n_2) \text{ is in the body of a rule in } \mathcal{R}_g \} \cup$
 $\{ \text{notMet}(d, g) | d \in D, g \in G \};$
- for any $\neg \text{unreachableSib}(n_3, n_2, t, n_1)$ in \mathcal{A}_g , $\mathcal{C}_g(\neg \text{unreachableSib}(n_3, n_2, t, n_1)) =$
 $\{ \text{unreachableSib}(n_3, n_2, t, n_1) \};$
 for any $\neg \text{reach}(n_1, n_2)$ in \mathcal{A}_g , $\mathcal{C}_g(\neg \text{reach}(n_1, n_2)) = \{ \text{reach}(n_1, n_2) \};$
 for any $\text{notMet}(d, g) \in \mathcal{A}_g$: $\mathcal{C}_g(\text{notMet}(d, g)) = \{ \text{met}(d, g) \}.$

Although the construction of the core ABA framework underpinning this definition is somewhat tedious, its underlying intuition is straightforward, describing decisions meeting goals in DGs. Intuitively, Definition 6.5 can be understood as follows. A decision d meets a goal g if the node g is reachable from the singleton set $\{d\}$. We know that a node n_2 is reachable from a singleton set $\{n_1\}$ under one of the two conditions:

1. if there is an edge from n to n_2 (regardless of the tag of the edge), represented with the rule: $\text{reach}(n_1, n_2) \leftarrow \text{edge}(n_1, n_2, t(e))$, or
2. if there is a node n_3 such that both of the following two conditions hold:
 - (a) n_3 is reachable from $\{n_1\}$, there is an edge from n_3 to n_2 with some tag t , and
 - (b) it is not the case that there exists some node $n_4 \neq n_3$, such that there is an edge from n_4 to n_2 with tag t , and n_4 is not reachable from $\{n_1\}$, represented by rules: $\text{reach}(n_1, n_2) \leftarrow \text{reach}(n_1, n_3), \text{edge}(n_3, n_2, t), \neg \text{unreachableSib}(n_3, n_2, t, n_1)$; and $\text{unreachableSib}(n_4, n_2, t, n_1) \leftarrow \text{edge}(n_4, n_2, t), n_4 \neq n_3, \neg \text{reach}(n_1, n_3).$

We give an illustration of core ABA framework for a DG without defeasible information in Example C.1 in Appendix C.

If now we consider also defeasible information (i.e. defeasible edges and defeasible condition) we obtain the following augmented core ABA framework:

Definition 6.6. Given a DG $\langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$, $\mathbf{E} = \mathbf{E}_d \cup \mathbf{E}_s$ with \mathbf{E}_d the defeasible edges, \mathbf{E}_s the strict edges, let $\langle \mathcal{L}_g, \mathcal{R}_g, \mathcal{A}_g, \mathcal{C}_g \rangle$ be the core ABA framework corresponding to $\langle \mathbf{N}, \mathbf{E}_s, \emptyset \rangle$. Then, the core ABA framework corresponding to $\langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$ is $\langle \mathcal{L}_e, \mathcal{R}_e, \mathcal{A}_e, \mathcal{C}_e \rangle$, where

- $\mathcal{R}_e = \mathcal{R}_g \cup$
 $\{edge(n_1, n_2, t(e)) \leftarrow dEdge(n_1, n_2, t(e)) \mid e = [n_1 \triangleright n_2] \in \mathbf{E}_d\} \cup$
 $\{t_0 \leftarrow t_1, \dots, t_n \mid t_n \wedge \dots \wedge t_1 \rightarrow t_0 \in \mathbf{B}\};$
- $\mathcal{A}_e = \mathcal{A}_g \cup \{dEdge(n_1, n_2, t(e)) \mid e = [n_1 \triangleright n_2] \in \mathbf{E}_d\};$
- for all $dEdge(n, n', t(e)) \in \mathcal{A}_e$, $\mathcal{C}_e(dEdge(n, n', t(e))) = \{\neg dEdge(n, n', t(e))\};$
for all $\neg unreachableSib(n_2, n_3, t(e), n_1), \neg reach(n_1, n_2), notMet(d, g) \in \mathcal{A}_e$,
 $\mathcal{C}_e(\neg unreachableSib(n_2, n_3, t(e), n_1)) = \mathcal{C}_g(\neg unreachableSib(n_2, n_3, t(e), n_1));$
 $\mathcal{C}_e(\neg reach(n_1, n_2)) = \mathcal{C}_g(\neg reach(n_1, n_2));$
 $\mathcal{C}_e(notMet(d, g)) = \mathcal{C}_g(notMet(d, g)).$

Definition 6.6 is based on Definition 6.5 with added structure to handle defeasible edges and defeasible condition. Namely, all defeasible edges are represented as assumptions $dEdge(n_1, n_2, t)$ with contraries $\neg dEdge(n_1, n_2, t)$ for some n_1, n_2 and t . Moreover, all implications in defeasible condition \mathbf{B} are represented as rules. Hence, for a defeasible edge $e = [n_1 \triangleright n_2]$ with $t(e) = i$, if e is blocked, then $\neg dEdge(n_1, n_2, i)$ holds in the core ABA framework corresponding to the graph. It is immediate to see that the core ABA framework corresponding to $\langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$, as given by Definition 6.6, is the same as the one given by Definition 6.5 when no defeasible information is present in $\langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$.

We give an illustration of core ABA framework for a DG without defeasible information in Example C.2 in Appendix C.

As before, we can take the “union” of core ABA frameworks and strongly dominant, dominant and weakly dominant components to obtain ABA frameworks drawn from DGs. As such components, we can use exactly those we defined for ADFs. Formally,

Definition 6.7. Given a DG $G = \langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$, let $\langle \mathcal{L}_g, \mathcal{R}_g, \mathcal{A}_g, \mathcal{C}_g \rangle$ be the core ABA framework corresponding to G and let $\langle \mathcal{L}_s, \mathcal{R}_s, \mathcal{A}_s, \mathcal{C}_s \rangle$, $\langle \mathcal{L}_d, \mathcal{R}_d, \mathcal{A}_d, \mathcal{C}_d \rangle$, $\langle \mathcal{L}_w, \mathcal{R}_w, \mathcal{A}_w, \mathcal{C}_w \rangle$ be the strongly dominant, dominant, and weakly dominant components, as given in Definitions 5.3, 5.5 and 5.7 respectively. Then the *Strongly Dominant*, *Dominant*, *Weakly Dominant ABA Frameworks drawn from G* are $\langle \mathcal{L}_s^g, \mathcal{R}_s^g, \mathcal{A}_s^g, \mathcal{C}_s^g \rangle$, $\langle \mathcal{L}_d^g, \mathcal{R}_d^g, \mathcal{A}_d^g, \mathcal{C}_d^g \rangle$ and $\langle \mathcal{L}_w^g, \mathcal{R}_w^g, \mathcal{A}_w^g, \mathcal{C}_w^g \rangle$, respectively, in which:

- $\mathcal{R}_s^g = \mathcal{R}_g \cup \mathcal{R}_s$; $\mathcal{R}_d^g = \mathcal{R}_g \cup \mathcal{R}_d$; $\mathcal{R}_w^g = \mathcal{R}_g \cup \mathcal{R}_w$;
- $\mathcal{A}_s^g = \mathcal{A}_g \cup \mathcal{A}_s$; $\mathcal{A}_d^g = \mathcal{A}_g \cup \mathcal{A}_d$; $\mathcal{A}_w^g = \mathcal{A}_g \cup \mathcal{A}_w$;

- for all α in \mathcal{A}_s^g : $\mathcal{C}_s^g(\alpha) = \begin{cases} \mathcal{C}_g(\alpha) & \text{if } \alpha \in \mathcal{A}_g, \\ \mathcal{C}_s(\alpha) & \text{if } \alpha \in \mathcal{A}_s. \end{cases}$
- for all α in \mathcal{A}_d^g : $\mathcal{C}_d^g(\alpha) = \begin{cases} \mathcal{C}_g(\alpha) & \text{if } \alpha \in \mathcal{A}_g \text{ and } \alpha \notin \mathcal{A}_d, \\ \mathcal{C}_d(\alpha) & \text{if } \alpha \in \mathcal{A}_d \text{ and } \alpha \notin \mathcal{A}_g, \\ \mathcal{C}_0(\alpha) \cup \mathcal{C}_d(\alpha) & \text{if } \alpha \in \mathcal{A}_d \cap \mathcal{A}_g. \end{cases}$
- for all α in \mathcal{A}_w^g : $\mathcal{C}_w^g(\alpha) = \begin{cases} \mathcal{C}_g(\alpha) & \text{if } \alpha \in \mathcal{A}_g, \\ \mathcal{C}_w(\alpha) & \text{if } \alpha \in \mathcal{A}_w. \end{cases}$

As before, strongly dominant, dominant and weakly dominant decisions correspond to admissible arguments in their corresponding ABA frameworks.

Proposition 6.4. Given a DG $G = \langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$, $\mathbf{N} = \mathbf{D} \cup \mathbf{N}_{int} \cup \mathbf{G}$ with \mathbf{D} the decisions, \mathbf{G} the goals, let AF_s , AF_d and AF_w be the strongly dominant, dominant and weakly dominant ABA frameworks, respectively, drawn from G . Then for all decisions $d \in \mathbf{D}$, d is strongly dominant, dominant, weakly dominant in G if and only if $\{sDom(d)\} \vdash sDom(d)$, $\{dom(d)\} \vdash dom(d)$ and $\{wDom(d)\} \vdash wDom(d)$ is admissible in AF_s , AF_d and AF_w , respectively.

Also as before, ABA can be used to obtain g-preferred decisions in PDGs. We define the *g-preferred ABA framework corresponding to a PDG* as the “union” of the core ABA framework and the g-preferred component, as follows:

Definition 6.8. Given a PDG $G_p = \langle \mathbf{N}, \mathbf{E}, \mathbf{B}, \leq_g \rangle$, let $\langle \mathcal{L}_e, \mathcal{R}_e, \mathcal{A}_e, \mathcal{C}_e \rangle$ be the core ABA framework corresponding to $\langle \mathbf{N}, \mathbf{E}, \mathbf{B} \rangle$, $\langle \mathcal{L}_p, \mathcal{R}_p, \mathcal{A}_p, \mathcal{C}_p \rangle$ the g-preferred component from Definition 5.9. Then, the *g-preferred ABA framework corresponding to G_p* is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ where:

- $\mathcal{R} = \mathcal{R}_e \cup \mathcal{R}_p$;
- $\mathcal{A} = \mathcal{A}_e \cup \mathcal{A}_p$;
- for all α in \mathcal{A} : $\mathcal{C}(\alpha) = \begin{cases} \mathcal{C}_e(\alpha) & \text{if } \alpha \in \mathcal{A}_e, \\ \mathcal{C}_p(\alpha) & \text{if } \alpha \in \mathcal{A}_p. \end{cases}$

Proposition 6.5. Given $G_p = \langle \mathbf{N}, \mathbf{E}, \mathbf{B}, \leq_g \rangle$, with $\mathbf{N} = \mathbf{D} \cup \mathbf{N}_{int} \cup \mathbf{G}$, let AF be the g-preferred ABA framework corresponding to G_p . Then for all decisions $d \in \mathbf{D}$, d is g-preferred if and only if $\{gP(d)\} \vdash gP(d)$ is admissible in AF .

In the next section we will show how the ABA reformulations for our various forms of decision making with DGs can serve as the basis for obtaining argumentative explanations for the “goodness” (or “badness”) of decisions. Before we do so, though, we note that these ABA formulations can be seen as decision making abstractions in their own right, and thus provide forms of argumentation-based decision making. Indeed, they support the direct definition of reasons for goals being met, and one can easily think of further variants of DGs, e.g. where the defeasible condition can be defined in more general formal languages (of the kinds afforded by ABA) whereby, for example, one could allow arguments against the applicability of defeasible conditions. We leave these more general forms of argumentation-based decision making as future work.

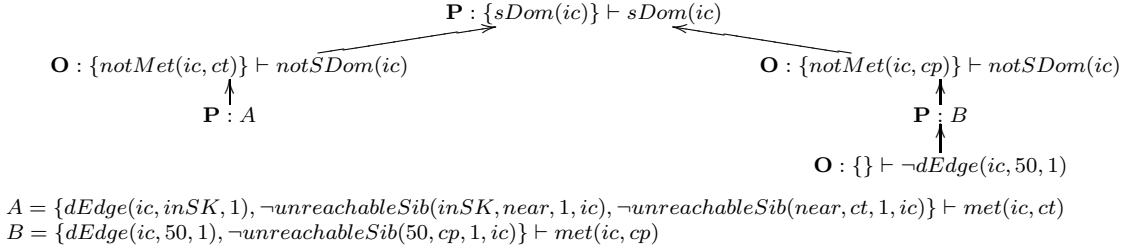


Figure 16: Best-effort dispute tree for $\{sDom(ic)\} \vdash sDom(ic)$ in Example 6.6. Here, cp and ct are short-hands for *cheap* and *convenient*, respectively.

6.3 Argumentative Explanations with DGs

As in the abstract setting of ADFs and PDFs, argumentative explanations for DGs can be defined in terms of (least-assumption and best-effort) dispute trees with respect to ABA frameworks drawn from DGs.

Example 6.6. (Example 6.2 continued.) For the weakly dominant ABA framework drawn from the DG in Figure 15, $\{sDom(ic)\} \vdash sDom(ic)$ is not admissible as shown by the best-effort dispute tree in Figure 16. Indeed, as the argument B , stating ic meets the goal *cheap* because ic is £50 per night and £50 per night leads to *cheap*, is attacked by $\{\} \vdash \neg dEdge(ic, 50, 1)$, stating that ic is not £50 per night, as it is known that during term time, ic is not £50 per night and it is currently term time. Note that, in this illustration, the information presented in B affects the decision, by providing an opponent argument defeating the proponent's argument B .

As in Section 4 argumentative explanations for decisions (not) meeting the various decision criteria integrate information about coresponding lean explanations, as follows.

Proposition 6.6. Let AF be the strongly dominant ABA framework corresponding to a DG G .

For any strongly dominant decision d in G , let \mathcal{T}^a be a least-assumption dispute tree for $\{sDom(d)\} \vdash sDom(d)$. Then,

$\{g|N = \mathbf{P} : _ \vdash met(d, g) \text{ is a leaf node in } \mathcal{T}^a \text{ or is not the ancestor of any } \mathbf{O} \text{ leaf nodes}\}$

a lean explanation for d being strongly dominant.

For any non-strongly dominant decision d , let \mathcal{T}^a be a best-effort dispute tree for $\{sDom(d)\} \vdash sDom(d)$. Then,

$\{g|N = \mathbf{O} : \{notMet(d, g)\} \vdash notSDom(d) \text{ is a leaf node in } \mathcal{T}^a \text{ or is not the ancestor of any } \mathbf{O} \text{ leaf nodes}\}$

is a lean explanation for d' not being strongly dominant.

Example 6.7. We illustrate Proposition 6.6 with the DG shown in Figure 17 (note that all edges are strict in this DG, and the accompanying B is empty). Here, we have two decisions

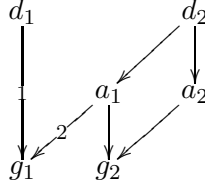
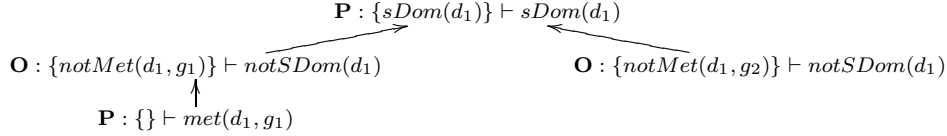


Figure 17: DG for Example 6.7.


 Figure 18: Best-effort dispute tree for $\{sDom(d_1)\} \vdash sDom(d_1)$ for Example 6.7.

d_1 and d_2 along with two goals g_1 and g_2 . d_1 meets g_1 , d_2 meets both g_1 (via a_1) and g_2 (via both a_1 and a_2). Thus, d_1 is not strongly dominant but d_2 is.

The ABA framework corresponding to this setting is given in Appendix C. With this ABA framework, dispute trees for both $\{sDom(d_1)\} \vdash sDom(d_1)$ and $\{sDom(d_2)\} \vdash sDom(d_2)$ are shown in Figures 18 and 19 respectively. From these trees, we can easily see that a lean explanation for d_1 not being strongly dominant is $\{g_2\}$ (as g_2 is not met by d_1) and a lean explanation for d_2 being strongly dominant is $\{g_1, g_2\}$ (as both g_1 and g_2 are met by d_1).

Proposition 6.7. Let AF be the dominant ABA framework corresponding to G .

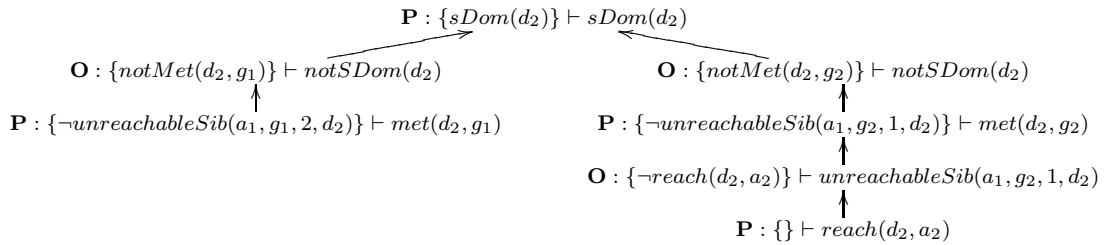
For any dominant decision d in G , let \mathcal{T}^a be a least-assumption dispute tree for $\{dom(d)\} \vdash dom(d)$. Then the pair (G, F) , where

- $G = \{g | \mathbf{P} : \{\} \vdash met(d, g) \text{ is not ancestor of any } \mathbf{O} \text{ leaf nodes in } \mathcal{T}^a\}$,
- $F = \{g | \mathbf{P} : \{noOthers(d, g)\} \vdash noOthers(d, g) \text{ is not ancestor of any } \mathbf{O} \text{ leaf nodes in } \mathcal{T}^a\}$,

is a lean explanation for d being dominant.

For any non-dominant decision d , let \mathcal{T}^a be the best-effort dispute tree for $\{dom(d)\} \vdash dom(d)$. Then

- $\{g | \mathbf{O} : \{notMet(d, g)\} \vdash notDom(d) \text{ is ancestor of an } \mathbf{O} \text{ leaf node}\}$


 Figure 19: Least-assumption dispute tree for $\{sDom(d_2)\} \vdash sDom(d_2)$ for Example 6.7.

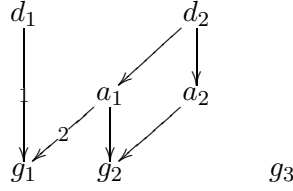


Figure 20: DG for Example 6.8. (This is the same as the one in Figure 17 except for the presence of goal g_3 .)

is a lean explanation for d not being dominant.

Example 6.8. We illustrate Proposition 6.7 with the DG shown in Figure 20. This example is adapted from Example 6.7 with a newly added goal g_3 that is not met by either d_1 or d_2 . Thus, neither of the two is strongly dominant any more. However, since d_2 still meets all goals met by d_1 and more, d_2 is dominant and d_1 is not. The ABA framework corresponding to this setting is given in Appendix C. For this ABA framework, Figures 21 and 22 show argumentative explanations in the form of dispute trees for $\{dom(d_1)\} \vdash dom(d_1)$ and $\{dom(d_2)\} \vdash dom(d_2)$, respectively. $\{g_2\}$ is a lean explanation for d_1 not being dominant as $\mathbf{O} : nM(d_1, g_2) \vdash nD(d_1)$ is ancestor of the leaf node $\mathbf{O} : \{\} \vdash reach(d_2, a_2)$. We read this as

d_1 is not dominant because it does not meet the goal g_2 , which is met by some other decision.

$\{g_1, g_2\}, \{g_3\}$ is a lean explanation for d_2 being dominant as $\mathbf{P} : \{\neg uS(a_1, g_1, 2, d_2)\} \vdash met(d_2, g_1)$ is a leaf node and $\mathbf{P} : \{\neg uS(a_1, g_2, 1, d_2)\} \vdash met(d_2, g_2)$ is not ancestor of any \mathbf{O} leaf node. Moreover, $\mathbf{P} : \{nO(d_2, g_3)\} \vdash nO(d_2, g_3)$ is also a leaf node in the tree.

d_2 is dominant as it meets both goals g_1 and g_2 and for the goal g_3 it does not meet, g_3 is not met by the other decision d_1 .

Proposition 6.8. Let AF be the weakly dominant ABA framework corresponding to G .

For any d in G , if d is weakly dominant, let \mathcal{T}^a be a least-assumption dispute tree for the argument $\{wDom(d)\} \vdash wDom(d)$. Then the pair (G, M) , where

- $G = \{g | \mathbf{P} : _ \vdash \{met(d, g)\} \text{ or } \mathbf{P} : \{notMet(d, g)\} \vdash more(d, _) \text{ is not ancestor of an } \mathbf{O} \text{ leaf node in } \mathcal{T}^a\}$,
- $M = \{(d', g) | \mathbf{P} : \{notMet(d', g)\} \vdash more(d, d') \text{ is not ancestor of an } \mathbf{O} \text{ leaf node in } \mathcal{T}^a\}$

is a lean explanation for d being weakly dominant.

For any non-weakly dominant decision d , let \mathcal{T}^a be a best-effort dispute tree for the argument $\{wDom(d)\} \vdash wDom(d)$. Then

- $\{d' | \mathbf{O} : \{_, notMore(d', d)\} \vdash notWDom(d) \text{ is ancestor of an } \mathbf{O} \text{ leaf node in } \mathcal{T}^a\}$.

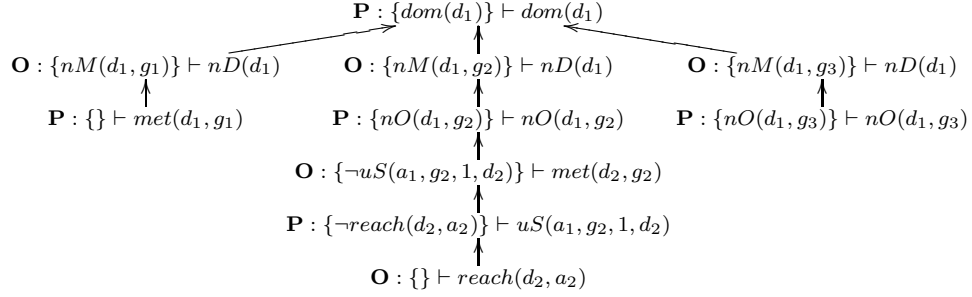


Figure 21: Best-effort dispute tree for $\{dom(d_1)\} \vdash dom(d_1)$ for Example 6.8. Here, nM , nD , nO and uS are short-hands for *notMet*, *notDom*, *noOthers* and *unreachableSib*, respectively.

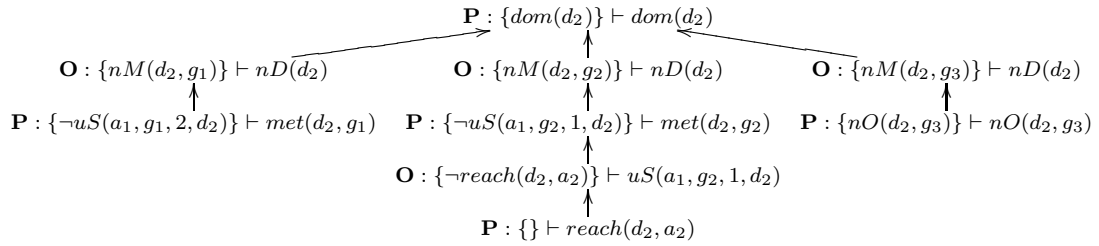


Figure 22: Least-assumption dispute tree for $\{dom(d_2)\} \vdash dom(d_2)$ for Example 6.8. Here, nM , nD , nO and uS are short-hands for *notMet*, *notDom*, *noOthers* and *unreachableSib*, respectively.

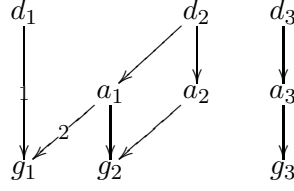


Figure 23: DG for Example 6.9. (This is the same as the one in Figure 20 except for the presence of decision d_3 reaching g_3 via a_3 .)

is a lean explanation for d not being weakly dominant.

Example 6.9. We illustrate Proposition 6.8 with the DG shown in Figure 23. This example is adapted from Example 6.7 and 6.8 with a newly added decision d_3 meeting the goal g_3 (via a_3). Thus, both d_2 and d_3 are weakly dominant whereas d_1 is not. The ABA framework corresponding to this setting is given in Appendix C. For this ABA framework, Figures 24, 25 and 26 show three argumentative explanations in the form of dispute trees for $\{wDom(d_1)\} \vdash wDom(d_1)$, $\{wDom(d_2)\} \vdash wDom(d_2)$ and $\{wDom(d_3)\} \vdash wDom(d_3)$, respectively. From these trees, we can extract lean explanations for decisions as follows.

From Figure 24, $\{d_2\}$ is a lean explanation for d_1 not being weakly dominant (because $\mathbf{O} : \{\neg uS(a_1, g_2, 1, d_2), nM(d_1, g_2), nE(d_1, d_2)\} \vdash nW(d_1)$ is ancestor of the leaf node $\mathbf{O} : \{\} \vdash reach(d_2, a_2)$). We can read this as:

Since d_2 meets more goals than d_1 , d_1 is not weakly dominant.

From Figure 25, since both argument $\{\neg uS(a_1, g_1, 2, d_2)\} \vdash met(d_2, g_1)$ and argument $\{\neg uS(a_1, g_1, 2, d_2), notMet(d_3, g_1)\} \vdash more(d_1, d_3)$ are not ancestors of any \mathbf{O} leaf node, $G = \{g_1\}$ and $M = \{(d_3, g)\}$. Thus, $(\{g_1\}, \{(d_3, g)\})$ is a lean explanation for d_2 being weakly dominant. We can read this as

d_2 is weakly dominant because it meets the goal g_1 , which is not met by g_3 .

From Figure 26, since both argument $\{\neg uS(a_3, g_3, 1, d_3), notMet(d_2, g_3)\} \vdash more(d_3, d_2)$ and argument $\{\neg uS(a_3, g_3, 1, d_3), notMet(d_1, g_3)\} \vdash more(d_3, d_1)$ are not ancestors of any \mathbf{O} leaf node, $\exp \{g_3\} \{(d_1, g_3), (d_1, g_3)\}$ is a lean explanation for d_3 being weakly dominant. We can read this as

The decision d_3 is weakly dominant because it meets the goal g_3 , which is not met by d_1 or d_2 .

Proposition 6.9. Let AF be the g-preferred ABA framework corresponding to a decision graph G .

For any g-preferred decision d in G , let \mathcal{T}^a be a least-assumption dispute tree for $gP(d) \vdash gP(d)$. Then, the pair (G, M) , where

- $G = \{g | \mathbf{P} : _ \vdash \{met(d, g)\} \text{ is not ancestor of any } \mathbf{O} \text{ leaf node in } \mathcal{T}^a\}$ and

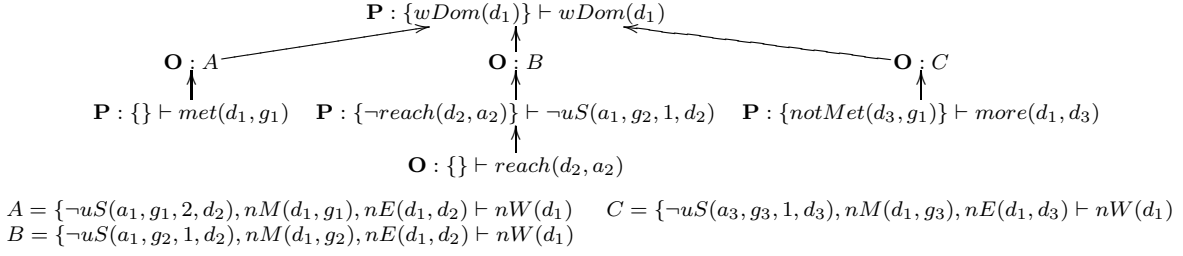


Figure 24: Best-effort dispute tree for $\{wDom(d_1)\} \vdash wDom(d_1)$ for Example 6.9. Here, nM , nE , nW and uS are short-hands for *notMet*, *notMore*, *notW Dom* and *unreachableSib*, respectively.

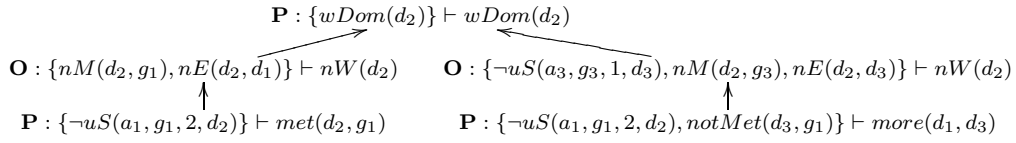


Figure 25: Least-assumption dispute tree for $\{wDom(d_2)\} \vdash wDom(d_2)$ for Example 6.9. Here, nM , nE , nW and uS are short-hands for *notMet*, *notMore*, *notW Dom* and *unreachableSib*, respectively.

- $M = \{(S, d') \mid \mathbf{P} : \{metS(d, S), notMet(d', -)\} \vdash better(d, d', -) \text{ is not ancestor of any } \mathbf{O} \text{ leaf node in } \mathcal{T}^a\}$

is a lean explanation for d being g-preferred.

For any non-g-preferred decision d in G , let \mathcal{T}^a be a best-effort dispute tree for $gP(d) \vdash gP(d)$. Then,

- $\{d' \mid \mathbf{O} : -, notBetter(d, d', -) \vdash notGP(d) \text{ is ancestor of an } \mathbf{O} \text{ leaf node in } \mathcal{T}^a\}$

is a lean explanation for d being not g-preferred.

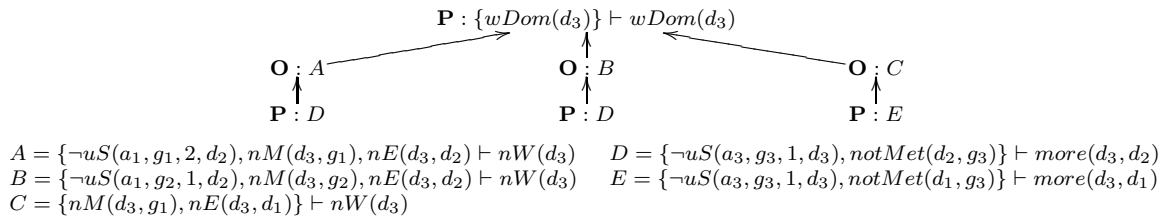


Figure 26: Least-assumption dispute tree for $\{wDom(d_3)\} \vdash wDom(d_3)$ for Example 6.9. Here, nM , nE , nW and uS are short-hands for *notMet*, *notMore*, *notW Dom* and *unreachableSib*, respectively.

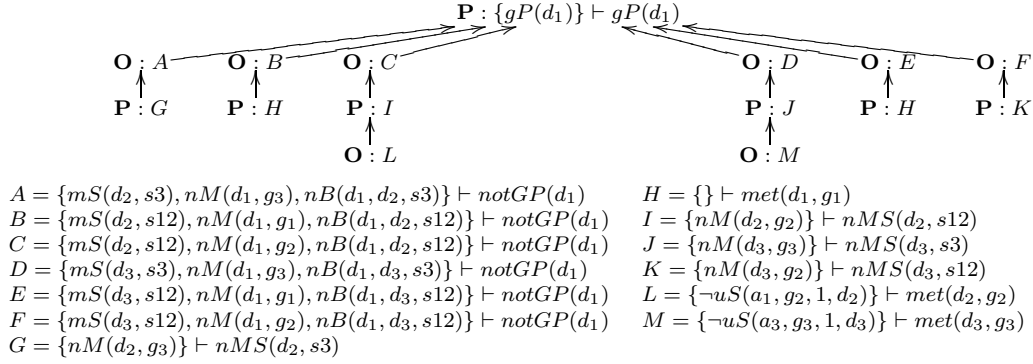


Figure 27: Best-effort dispute tree for $\{gP(d_1)\} \vdash gP(d_1)$ for Example 6.10. Here, mS , nM , nB , nMS and uS are short-hands for $metS$, $notMet$, $notBetter$, $notMetS$ and $unreachableSib$, respectively.

Example 6.10. We illustrate Proposition 6.9 again with the DG shown in Figure 20. However, we impose the preference relation $\{g_3\} \leq_g \{g_1, g_2\}$. As shown in Example 6.9, both d_2 and d_3 are weakly dominant and d_2 meets $\{g_1, g_2\}$ and d_3 meets $\{g_3\}$. Thus, d_3 is preferred to d_2 so d_3 is a g-preferred decision whereas d_2, d_1 are not. (d_1 is not even weakly dominant thus it cannot be g-preferred.)

The ABA framework corresponding to this setting is given in Appendix C. With this ABA framework, Figures 27, 28 and 29 show three dispute trees for the arguments $\{gP(d_1)\} \vdash gP(d_1)$, $\{gP(d_2)\} \vdash gP(d_2)$ and $\{gP(d_3)\} \vdash gP(d_3)$, respectively, providing argumentative explanations for d_1 and d_2 not being g-preferred and for d_3 being g-preferred, respectively, and from which lean explanations can be drawn as follows:

- d_1 : $\{d_2, d_3\}$ is the lean explanation for d_1 not being g-preferred; this is obtained by extracting d_2 from the argument $C = \{mS(d_1, s12), nM(d_3, g2), nB(d_3, d_1, s12)\} \vdash notGP(d_3)$ in the node $\mathbf{O} : C$, an ancestor of the node $\mathbf{O} : L$, and d_3 from the argument $D = \{mS(d_3, s3), nM(d_2, g3), nB(d_2, d_3, s3)\} \vdash notGP(d_2)$ in the node $\mathbf{O} : D$, an ancestor of the node $\mathbf{O} : M$;
- d_2 : $\{d_3\}$ is the lean explanation for d_3 not being g-preferred; this is obtained by extracting d_3 from the argument $D = \{mS(d_3, s3), nM(d_2, g3), nB(d_2, d_3, s3)\} \vdash notGP(d_2)$ in the node $\mathbf{O} : D$, an ancestor of the node $\mathbf{O} : M$;
- d_3 : $(\{g_3\}, \{(g_3, d_1), (g_3, d_2)\})$ is the lean explanation for d_3 being g-preferred, extracted from arguments $G = \{\neg uS(a_3, g3, 1, d_3)\} \vdash met(d_3, g3)$ in the node $\mathbf{P} : G$, $H = \{mS(d_3, s3), nM(d_1, g3)\} \vdash b(d_3, g1, s12)$ in the node $\mathbf{P} : H$ and $I = \{mS(d_3, s3), nM(d_2, g3)\} \vdash b(d_3, g2, s12)$ in the node $\mathbf{P} : I$.

7. Conclusion

In this paper, we focused on two representations for decision making, of different abstraction levels, as summarised in Table 3, and on a number of normative decision criteria for

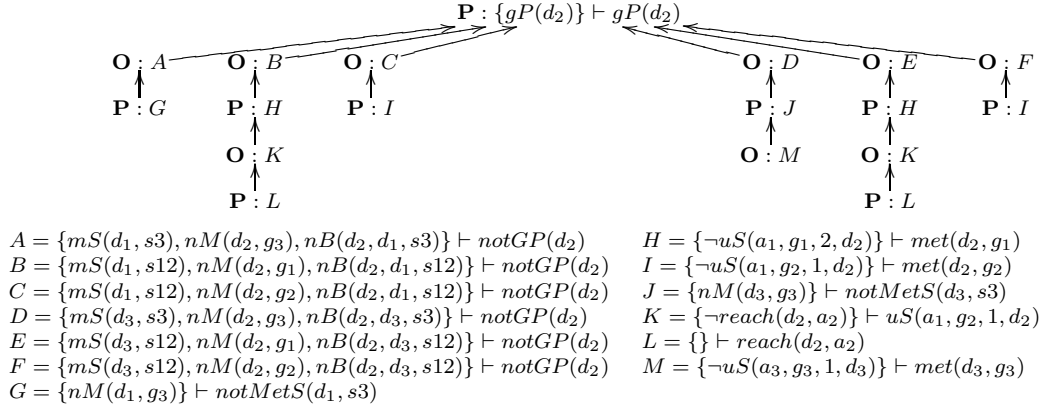


Figure 28: Best-effort dispute tree for $\{gP(d_2)\} \vdash gP(d_2)$ for Example 6.10. Here, mS , nM , nB , nMS and uS are short-hands for metS , notMet , notBetter , notMetS and unreachableSib , respectively.

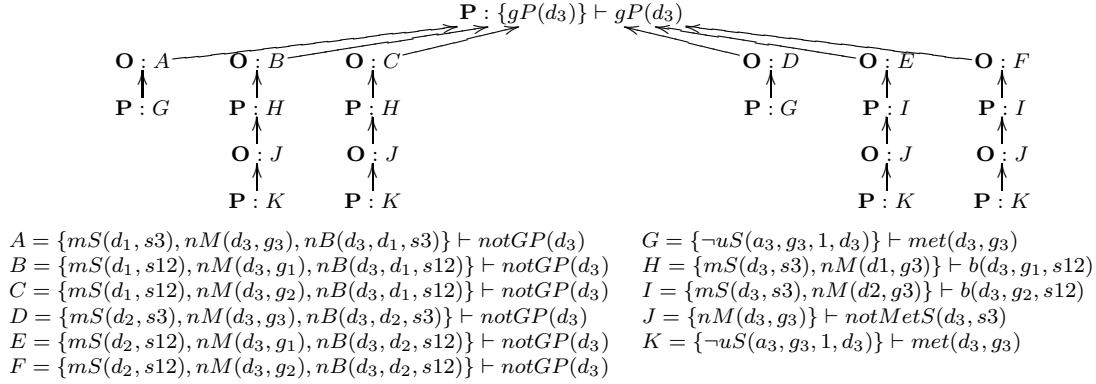


Figure 29: Least-assumption dispute tree for $\{gP(d_2)\} \vdash gP(d_2)$ for Example 6.10. Here, mS , nM , nB , nMS , b and uS are short-hands for metS , notMet , notBetter , notMetS , better and unreachableSib , respectively.

identifying “good” decisions therein, as summarised in Table 4. For each decision making setting of interest (i.e. representation and criterion) we provided two forms of explanation: *lean* explanations, based solely on justifying “good” decisions in terms of the ingredients of the underlying representation, and *argumentative* explanations, based on spelling out the reasoning underpinning the considered decision criterion, while also incorporating the same information present in lean explanations. To define argumentative explanations, we used Assumption-based argumentation (ABA) as an equivalent representation (to the representation for decision making and decision criterion). Specifically, in the ABA reformulations, “good” decisions are claims of *admissible arguments* and vice versa. The argumentative explanations are special (and novel) forms of dispute trees in ABA: *least-assumption dispute trees* (a restricted form of admissible dispute trees in ABA) for explaining why decisions are

Abstract Decision Frameworks (ADFs): $\langle \mathcal{D}, \mathcal{G}, \gamma \rangle$ <ul style="list-style-type: none"> amounting to decisions \mathcal{D}, goals \mathcal{G} and a decision-meets-goal relation γ.
Preferential Decision Frameworks (PDFs): $\langle \mathcal{D}, \mathcal{G}, \gamma, \leq_g \rangle$ <ul style="list-style-type: none"> amounting to ADFs with preferences over goals.
Decision Graphs (DGs): $\langle \mathcal{N}, \mathcal{E}, \mathcal{B} \rangle$ <ul style="list-style-type: none"> amounting to relations (\mathcal{E}) between decisions, goals, and intermediates (all in \mathcal{N}), with defeasible condition \mathcal{B}.
Preferential Decision Graphs (PDGs): $\langle \mathcal{N}, \mathcal{E}, \mathcal{B}, \leq_g \rangle$ <ul style="list-style-type: none"> amounting to DGs with preferences over goals.

Table 3: Overview of decision making representations studied in this paper.

Decision Criterion	Brief Description
Strongly Dominant	Select decisions meeting all goals.
Dominant	Select decisions meeting all goals ever met.
Weakly Dominant	Select decisions meeting goals not met by others.
G-Preferred	Select decisions meeting the most preferred goal sets.

Table 4: Overview of decision criteria studied in this paper.

deemed “good”, and *best-effort dispute trees* (a restricted form of maximal, non-admissible dispute trees in ABA) for explaining why decisions are not deemed “good”.

Our paper opens several avenues for future work, including those briefly outlined next.

Our decision settings (i.e. representations and criteria) correspond to some existing decision making frameworks (as shown in Appendix A); thus, our novel forms of explanation can be used to explain (“good” or “bad”) decisions in those frameworks as well. It would be interesting to see whether our approach could be adapted to provide explanations in other settings, e.g. when preferences over decisions or, in the case of decision graphs, intermediates are present or in decision settings not restricted to a single fixed world state, such as decision making problems studied with Markov decision processes (Bellman, 1957) or Sequential Decision Making (Barto, Sutton, & Watkins, 1989), with constantly evolving world states.

Our argumentative explanations make use of novel, restricted types of dispute trees. It would be interesting to study their stand-alone properties in ABA and their usefulness in other settings where ABA or its instances are used for explanation via forms of dispute trees (e.g. as in (Cyras, Birch, Guo, Toni, Dulay, Turvey, Greenberg, & Hapuarachchi, 2019)).

In the case of decision graphs, we have focused on lean explanations matching exactly (by instantiating) those for ADFs and PDFs. However, as already mentioned at the end of Section 6.1, “deeper” lean explanations could be defined whereby the reasons for goals being met, in terms of reachability of goals from decisions in the graphs, are accommodated within the explanations themselves.

We have defined argumentative explanations as dispute trees, providing a range of intuitive readings thereof as disputes between fictional proponent and opponent players, in the

spirit of dispute derivations in ABA (e.g. as defined in (Toni, 2013)). Alternative readings may also be possible, for example as summaries of the salient points in the reasoning leading to sanctioning selected decisions (as “good” or “bad”). Also, we leave as future work the automated extraction of these readings from templates, e.g. in the spirit of (Cyras et al., 2019; Rago, Cocarascu, Bechlivanidis, & Toni, 2020).

We have defined argumentative explanations as static dispute trees. However, fragments of these trees could be extracted providing partial information on demand within interactive exchanges with the users, in the spirit of (Rago et al., 2020). This could be also useful to unearth, when decision graphs are used, the reasoning leading to establishing that a decision meets a goal (whereas in our current argumentative explanations this reasoning is “hidden” within arguments which are nodes of dispute trees).

Finally, as mentioned at the end of Section 6.2, our ABA formulations can be seen as decision making abstractions in their own right, and thus provide forms of argumentation-based decision making. We leave these more general forms of argumentation-based decision making as future work.

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Appendix A. Relations with Decision Making Methods

In this section, we compare decision making with ADFs, DGs and several other decision making methods. We show correspondences between several of the proposed decision making criteria and existing decision making methods in the literature.

In this appendix, in line with existing works, we use a tabular representation T_{DG} of γ in ADFs and PDFs, whereby T_{DG} is a table of size $n \times l$ and for all $d \in D$, $g \in G$, $T_{DG}[d, g] = 1$ if and only if $g \in \gamma(d)$, and $T_{DG}[d, g] = 0$ otherwise.

A.1 Strongly Dominant Decisions and Multi-Attribute Decision Making

Multi-Attribute Decision Making (MADM) refers to decision making over available alternatives characterised by multiple, usually conflicting, attributes (Hwang & Yoon, 1981). Here we show that our approach models certain MADM problems well. These are MADM problems where information is presented directly on attributes, equipped with *non-compensatory methods*, where, quoting from (Yoon & Hwang, 1995):

A compensatory or non-compensatory distinction is made on the basis of whether advantages of one attribute can be traded for disadvantages of another or not. A choice strategy is compensatory if trade-offs among attribute values are permitted, otherwise it is non-compensatory.

In particular, MADM under the (non-compensatory) *conjunctive method* (Yoon & Hwang, 1995) can be formulated as follows. There is a set of *alternatives* L such that each $l \in L$ can be represented as a set of *attributes* I_l . There is a set of requirements R , such that each attribute t can be tested against one requirement $r_t \in R$ (r_t is the *matching requirement* for t). There is a boolean *requirement function* \mathcal{B} which maps alternatives and requirements to 1s and 0s. For an alternative l with attributes I_l , l is selected by the conjunctive method if and only if for all attributes $t \in I_l$ and $r_t \in R$ (r_t is the matching requirement for t), $\mathcal{B}(t, r_t) = 1$, i.e. the conjunctive method selects the alternatives whose attributes meet all

Students	TOEFL	GRE	GPA
A_1	582	1420	2.8
A_2	563	1250	3.5
A_3	620	1080	3.2
A_4	558	1280	3.0
A_5	600	1210	3.6
Minimum	550	1200	3.0

Table 5: Graduate School Admission for International Students (Yoon & Hwang, 1995).

	$TOEFL \geq 550$	$GRE \geq 1200$	$GPA \geq 3.0$
A_1	1	1	0
A_2	1	1	1
A_3	1	0	1
A_4	1	1	1
A_5	1	1	1

Table 6: T_{DG} in the ADF for Example A.1. Here, $T_{DG}[d_i, g_j] = 1$ if and only if d_i meets the minimum requirement for g_j , e.g. $T_{DG}[A_1, TOEFL] = 1$ as the TOEFL score of student A_1 is 582 and the minimum requirement is 550.

requirements. For lack of a better word, we use the term *conjunctive framework* to denote the tuple $\langle L, R, \mathcal{B} \rangle$.

The following example illustrates conjunctive frameworks and the conjunctive method by adapting an example from (Yoon & Hwang, 1995).

Example A.1. A graduate school screens international applicants for admission. To pass the screening, an applicant whose native language is not English must meet the minimum scores on three tests: TOEFL, GRE and GPA. Table 5 indicates the tests' results for five students. This decision problem can be represented as a conjunctive framework with $L = \{A_1, \dots, A_5\}$, for any $i = 1, \dots, 5$, I_{A_i} is the set of assignments to TOEFL, GRE, GPA given in Table 5 (e.g. $I_{A_1} = \{TOEFL = 582, GRE = 1420, GPA = 2.8\}$), and $R = \{TOEFL \geq 550, GRE \geq 1200, GPA \geq 3.0\}$. Finally, the matching requirement for $T = v$ is $T \geq \min_T$, for any T amongst $TOEFL, GRE, GPA$, with \min_T as in Table 5, and $\mathcal{B}(T = v, T \geq \min_T) = 1$ iff $v \geq \min_T$. Given this conjunctive framework $\langle L, R, \mathcal{B} \rangle$, the conjunctive method selects students A_2, A_4, A_5 , whereas, quoting from (Yoon & Hwang, 1995):

Students A_1 and A_3 are rejected due to their low GPA and GRE scores, respectively. Note that A_1 was rejected even though the candidate has a very high GRE score.

We can model this problem also using a decision framework $\langle D, G, T_{DG} \rangle$, with $D = \{A_1, \dots, A_5\}$, $G = \{TOEFL \geq 550, GRE \geq 1200, GPA \geq 3.0\}$ and T_{DG} given in Table 6. Then, A_2, A_4 and A_5 are strongly dominant decisions as they meet all three goals, and no other decision is strongly dominant.

In general, there is a mapping from any conjunctive framework to an “equivalent” ADF such that the alternatives selected by the conjunctive method in the former are strongly dominant decisions in the latter. Formally:

Proposition A.1. Given a conjunctive framework $\langle L, R, \mathcal{B} \rangle$, let $\langle D, G, T_{DG} \rangle$ be such that $D = L$, $G = R$, $T_{DG}[l, r] = 1$ if $\mathcal{B}(t, r) = 1$, for r the matching requirement of t and t attribute of l , $T_{DG}[l, r] = 0$, otherwise. Then $\langle D, G, T_{DG} \rangle$ is an ADF and the set of all strongly dominant decisions in $\langle D, G, T_{DG} \rangle$ is the set of alternatives in $\langle L, R, \mathcal{B} \rangle$ selected by the conjunctive method.

The other direction of Proposition A.1 holds as well, as follows.

Proposition A.2. Given an ADF $\langle D, G, T_{DG} \rangle$, let $\langle L, R, \mathcal{B} \rangle$ be such that $L = D$, $R = G$, $\mathcal{B}(t, r) = 1$, for r the matching requirement of t and t an attribute of l , if $T_{DG}[l, r] = 1$; $\mathcal{B}(t, r) = 0$, otherwise. Then $\langle L, R, \mathcal{B} \rangle$ is a conjunctive framework and the set of alternatives in $\langle L, R, \mathcal{B} \rangle$ selected by the conjunctive method is the set of strongly dominant decisions in $\langle D, G, T_{DG} \rangle$.

A.2 Weakly Dominant Decisions and Pareto Optimisation

Efficiency in Pareto optimisation is one of the criteria used in classical decision making (Emmerich & Deutz, 2006). In this section, we show that *weakly dominant* decisions correspond to *efficient* solutions in Pareto optimisation.

We first summarise the notion of efficient solutions in Pareto optimisation (Emmerich & Deutz, 2006). Two spaces are considered: the *decision space* \mathbb{S} and the *objective space* \mathbb{Y} , with \prec an order over \mathbb{Y} . A vector-valued objective function $\mathbf{f} : \mathbb{S} \mapsto \mathbb{Y}$ provides a mapping from the decision space to the objective space. The set of *feasible solutions* \mathcal{X} is a subset of the decision space, and \mathcal{Y} is the image of \mathcal{X} under \mathbf{f} . The *Pareto front* \mathcal{Y}_N is defined as the set of *non-dominated solutions* in $\mathcal{Y} = \mathbf{f}(\mathcal{X})$, i.e. $\mathcal{Y}_N = \{\mathbf{y} \in \mathcal{Y} \mid \nexists \mathbf{y}' \in \mathcal{Y} : \mathbf{y}' \prec \mathbf{y}\}$. Finally, the *efficient set* is defined as the pre-image of the Pareto front, $\mathcal{X}_E = \mathbf{f}^{-1}(\mathcal{Y}_N)$, and a decision is an efficient solution if it belongs to the efficient set.

We use the term *Pareto framework* to denote the tuple $\langle \mathbb{S}, \mathbb{Y}, \mathbf{f}, \prec \rangle$ consisting of a decision space \mathbb{S} , an objective space \mathbb{Y} , an objective function \mathbf{f} , and an order \prec over the objective space. The following example illustrates Pareto frameworks and efficient solutions by recasting the decision problem of Example 3.3.

Example A.2. Let the decision space \mathbb{S} be $\{jh, ic, ritz\}$ and the objective space \mathbb{Y} be $2^{\{cheap, near\}}$, with \prec defined as \supset . Also, let the objective function \mathbf{f} be such that:

$$\mathbf{f}(jh) = \{near\}, \quad \mathbf{f}(ic) = \{cheap\}, \quad \mathbf{f}(ritz) = \{\}.$$

Assume that the set feasible solutions \mathcal{X} is the same as \mathbb{S} . Since $\mathcal{Y} = \mathbf{f}(\mathcal{X})$, $\mathcal{Y} = \{\{near\}, \{cheap\}, \{\}\}$. By letting \prec be \supset , we have $\mathbf{f}(jh) \prec \mathbf{f}(ritz)$ and $\mathbf{f}(ic) \prec \mathbf{f}(ritz)$. Thus, the Pareto front \mathcal{Y}_N is $\{\{near\}, \{cheap\}\}$, and the efficient set is $\{jh, ic\}$. As a result, both jh and ic are efficient solutions.

In the example, weakly dominant decisions correspond to efficient solutions. This is the case in general, in the sense that there is a mapping from any ADF to an “equivalent” Pareto framework such that weakly dominant decisions in the former are efficient solutions in the latter (see Table 7 for a full overview of the mapping). Formally:

Our notions	Pareto Optimisation
ADFs	Pareto Frameworks
decisions (\mathcal{D})	decision space (\mathcal{S})
sets of goals ($2^{\mathcal{G}}$)	objective space (\mathcal{Y})
goals met by decisions (γ)	objective function (\mathbf{f})
set of all weakly dominant decisions (\mathcal{S}_w)	efficient set (\mathcal{X}_E)
goals met by weakly dominant decisions	Pareto front (\mathcal{Y}_N)

Table 7: From weak dominance in ADFs to Pareto Optimisation.

Proposition A.3. Given an ADF $\langle \mathcal{D}, \mathcal{G}, \mathbf{T}_{\mathcal{DG}} \rangle$ with γ as in Definition 3.1, let $\langle \mathcal{S}, \mathcal{Y}, \mathbf{f}, \prec \rangle$ be such that $\mathcal{S} = \mathcal{D}, \mathcal{Y} = \mathcal{G}, \mathbf{f} = \gamma, \prec = \supset$. Then, $\langle \mathcal{S}, \mathcal{Y}, \mathbf{f}, \prec \rangle$ is a Pareto framework and $\{d \in \mathcal{D} | d \text{ is weakly dominant in } \langle \mathcal{D}, \mathcal{G}, \mathbf{T}_{\mathcal{DG}} \rangle\}$ is the efficient set of $\langle \mathcal{S}, \mathcal{Y}, \mathbf{f}, \prec \rangle$.

The other direction of Proposition A.3 holds as well, as follows.

Proposition A.4. Given a Pareto Framework $\langle \mathcal{S}, \mathcal{Y}, \mathbf{f}, \prec \rangle$ with $\prec = \supset$, let $\langle \mathcal{D}, \mathcal{G}, \mathbf{T}_{\mathcal{DG}} \rangle$ be an ADF with $\mathcal{D} = \mathcal{S}, \mathcal{G} = \mathcal{Y}, \mathbf{T}_{\mathcal{DG}}$ be such that $\mathbf{T}_{\mathcal{DG}}[d, g] = 1$ if and only if $g \in \mathbf{f}(d)$. Then, the efficient set of $\langle \mathcal{S}, \mathcal{Y}, \mathbf{f}, \prec \rangle$ is the set of all weakly dominant decisions in $\langle \mathcal{D}, \mathcal{G}, \mathbf{T}_{\mathcal{DG}} \rangle$.

A.3 G-Preferred Decisions and the Lexicographic Method

One MADM method for selecting decisions with preference is the *Lexicographic method*, where, quoting from (Yoon & Hwang, 1995):

[The lexicographic method] uses one lexicographic attribute at a time to examine alternatives for elimination. After all the alternatives are examined, if more than one still remains, the process is repeated using another lexicographic attribute. ... The Lexicographic method examines alternatives in the order of attribute importance.

MADM under the Lexicographic method (Yoon & Hwang, 1995) can be formulated as follows. There is a set of *alternatives* (\mathbf{A}) and each alternative has some *lexicographic attributes* (\mathbf{X}) from the set $\mathbf{X} = \{x_1, \dots, x_m\}$. Let $x_1 \in \mathbf{X}$ be the most important lexicographic attribute, $x_2 \in \mathbf{X}$ be the second most important one, and so on. Then a set of alternatives \mathbf{A}^1 is selected such that

$$\mathbf{A}^1 = \{a_i | \max_i a_i \text{ has } x_1\}, \text{ for } a_i \in \mathbf{A}.$$

If \mathbf{A}^1 is a singleton, then the alternative in \mathbf{A}^1 is the most preferred alternative. If there remain multiple alternatives, consider

$$\mathbf{A}^2 = \{a_i | \max_i a_i \text{ has } x_2\}, \text{ for } a_i \in \mathbf{A}^1.$$

If \mathbf{A}^2 is a singleton, then stop and select the alternative in \mathbf{A}^2 . Otherwise continue this process until either some singleton \mathbf{A}^k is found or all lexicographic attributes have been

considered. If the final set contains more than one alternative, then they are considered to be equivalent. In this process, if for some lexicographic attribute x there is no alternative has x , then no alternative is eliminated in this iteration and the next important lexicographic attribute is examined.

Example A.3. We again consider the decision making problem shown in [Example 3.5](#). We let the set of alternatives be $\{d_1, d_2\}$ and the set of lexicographic attributes be $\{g_1, g_2, g_3, g_4, g_5\}$. We let the importance amongst lexicographic attributes be $g_1 > g_2 > g_3 > g_4 > g_5$. The lexicographic method works as follows.

Since neither d_1 nor d_2 has g_1 , no alternative is eliminated in the first iteration.

$$A^1 = \{d_1, d_2\}.$$

Then, we move to g_2 , the second most important lexicographic attribute, since both of the two alternatives d_1 and d_2 have g_2 , we have:

$$A^2 = \{d_i | \max_i d_i \text{ has } g_2\}, \text{ for } d_i \in \{d_1, d_2\} = \{d_i | d_i \text{ has } g_2\} = \{d_1, d_2\}.$$

Then we move to the third important lexicographic attribute, g_3 . Since only d_2 has g_3 , we have

$$A^3 = \{d_i | \max_i d_i \text{ has } g_3\}, \text{ for } d_i \in \{d_1, d_2\} = \{d_i | d_i \text{ has } g_3\} = \{d_2\}.$$

Since A^3 is a singleton set containing only d_2 , the lexicographic method terminates with d_2 being the selected alternative.

We can see that our modelling of preference ranking over goals generalises the Lexicographic method, formally:

Proposition A.5. Given a set of alternatives A and a set of lexicographic attributes X , let $\langle D, G, T_{DG}, \leq_g \rangle$ be an PDF with $D = A$, $G = X$, $T_{DG}[d, g] = 1$ if and only if the alternative corresponds to d has the attribute corresponds to g , and \leq_g be such that $\{g_1\} \leq_g \{g_2\}$ for $g_1, g_2 \in G$ if and only if the attribute corresponds to g_1 is more important than the attribute corresponds to g_2 . Then alternatives selected by the Lexicographic method is the set of g -preferred decisions.

The other direction of Proposition A.5 holds as well.

Proposition A.6. Given an PDF $\langle D, G, T_{DG}, \leq_g \rangle$ such that \leq_g is a total order over $\{\{g\} | g \in G\}$, if we let $A = D$, $X = G$ and the importance of each $a \in X$ as described by \leq_g , then g -preferred decisions in the PDF are alternatives selected by the Lexicographic method with alternatives A and attributes X .

g -preferred decisions selected in DGs also correspond to decisions selected with the Lexicographic method.

Proposition A.7. Given a set of alternatives A and a set of lexicographic attributes X , let $\langle N, E, B, \leq_g \rangle$ be an PDG with $N = D \cup N_{int} \cup G$ such that

- $D = A$,
- $N_{int} = \{\}$,
- $G = X$,
- E be such that for $a \in A$, $x \in X$, $[a \triangleright x] \in E$ if and only if a has x ,
- $B = \{\}$,
- \leq_g be such that $\{g_1\} \leq_g \{g_2\}$ for $g_1, g_2 \in G$ if and only if the lexicographic attribute corresponds to g_1 is more important than the lexicographic attribute corresponds to g_2 .

Then alternatives selected by the Lexicographic method is the set of g-preferred decisions in PDG.

Proposition A.8. Given an PDG $\langle N, E, B, \leq_g \rangle$ with $N = D \cup N_{int} \cup G$, such that \leq_g is a total order over $\{\{g\} | g \in G\}$, if we let

- $A = D$;
- $X = G$;
- an alternative $a \in A$ has an lexicographic attribute $x \in X$ if and only if x can be reached from $\{a\}$ in $\langle N, E, B, \leq_g \rangle$; and
- the importance of each $x \in X$ is such that for $x_1, x_2 \in X$, x_1 is less important than x_2 if and only if $\{x_1\} \leq_g \{x_2\}$;

then g-preferred decisions in the PDF are alternatives selected by the Lexicographic method with alternatives A and attributes X .

A.4 Proofs for all results in Appendix A

Proof of Proposition A.1 $\langle D, G, T_{DG} \rangle$ is an ADF by construction and Definition 3.1. It is easy to see for each alternative l , l is selected in C if and only if l satisfies all requirements R . This implies that each $d \in D$ meets all goals in G . Thus, by definition of strong dominance, each selected alternative is a strongly dominant decision. \square

Proof of Proposition A.2 To show strongly dominant decisions are the set of alternatives selected by the conjunctive method, we only need to show that all dominant decisions are alternatives meeting all requirements. This is easy to see as a strongly dominant decision meets all goals in its corresponding ADF (Definition 3.2). Thus strongly dominant decisions are alternatives selected by the conjunctive method. \square

Proof of Proposition A.3 Follows trivially from the definitions. \square

Proof of Proposition A.4 Let D be the efficient set. We need to show that there is no decision $d \in D$ meeting strictly fewer goals than some other decision $d' \in D$. This is the case as by definition of efficient set: since efficient decisions meet goals in in the Pareto front, there is no decision meeting strictly more goals than an efficient decision. Therefore efficient decisions are weakly dominant.

Then the other direction. Let D be the set of all weakly dominant decisions, by Definition 3.2, there is no $d \in D$ meeting strictly fewer goals than some $d' \in D$. Thus, the set $S = \{\gamma(d) | d \in D\}$ is the Pareto front and for each element E in S , $\gamma^{-1}(E) = d$ is an efficient solution.

Proof of Proposition A.5 To show that the set of alternatives selected by the lexicographic method are g-preferred decisions is to show that each selected decision d is (1) weakly dominant, and (2) goals met by other decisions are no more preferred.

(1) holds as if d is not weakly dominant, then there exists a decision d' such that d' meets more goals (with respect to \subseteq) than d . This cannot be the case as if such d' exists, then the Lexicographic method would select d' instead upon examining some goal g met by d' but not d .

(2) holds as if there exists d' meeting goal g not met by d and d meets no other goal g' preferred to g , then d will not be selected by the Lexicographic method as upon examining decisions meeting g , d would be eliminated. \square

Proof of Proposition A.6 It is easy to show that the Lexicographic method selects g-preferred decisions if \leq_g is a total order over $\{\{g\} | g \in G\}$. Since the Lexicographic method examines lexicographic attributes in the order of their importance, for any g-preferred decision d , d exists in all A^i . \square

Proof of Proposition A.7 As shown in the proof of Proposition A.5, g-preferred decisions in ADFs can be used to model alternatives selected by the lexicographic method. The construction of PDG in Proposition A.7 is given in the same spirit as the ADF construction given in Proposition A.5. Since DG generalises ADF (Proposition 6.2), it is easy to see that the correspondance between lexicographic method and g-preferred decisions in PDG holds. \square

Proof of Proposition A.8 As shown in the proof of Proposition A.6, g-preferred decisions in ADF can be mapped to selected alternatives by the lexicographic method, as DG generalises ADF (Proposition 6.2), it is easy to see that g-preferred decisions in PDG can also be mapped to selected alternatives by the lexicographic method. \square

Appendix B. Proofs for the Results in Sections 3–6

Proof of Proposition 3.1 Follows from Definition 3.2.

Proof of Proposition 3.2 Follows from Definition 3.2.

Proof of Proposition 3.3 First we prove that if $S_s \neq \{\}$, then $S_s = S_d$. By Proposition 3.2, $S_s \subseteq S_d$. We show that there is no d such that $d \in S_d$, $d \notin S_s$. Assuming otherwise, (1) since $d \notin S_s$, $\gamma(d) \neq G$, hence there is some $g \in G$ and $g \notin \gamma(d)$; (2) since $S_s \neq \{\}$, there is $d' \in S_s$ such that $\gamma(d') = G$, therefore $g \in \gamma(d')$. By (1) and (2), $d \notin S_d$. Contradiction.

Then we prove that $S_s = S_w$. Assume by contradiction that $S_s \subset S_w$. Then, there exists $d \in S_w$, $d \notin S_s$. Since $S_s \neq \{\}$, there exists $d' \in S_s$ and $\gamma(d') = \mathbf{G}$. Since $d \notin S_s$, $\gamma(d) \subset \mathbf{G}$. Hence $\gamma(d) \subset \gamma(d')$. Then, by Definition 3.2, $d \notin S_w$. Contradiction. \square

Proof of Proposition 3.4 By Proposition 3.2, we know that $S_d \subseteq S_w$. We need to show $S_w \subseteq S_d$. Assume otherwise, i.e. there exists $d \in S_w$ and $d \notin S_d$. Since $S_d \neq \{\}$, there exists $d' \in S_d$, such that $\gamma(d') \supseteq \gamma(d'')$, for all $d'' \in \mathbf{D}$. Hence $\gamma(d) \subseteq \gamma(d')$. Since $d \notin S_d$, $\gamma(d) \neq \gamma(d')$. Therefore, $\gamma(d) \subset \gamma(d')$. Then, by Definition 3.2, $d \notin S_w$. Contradiction. \square

Proof of Theorem 3.1 We will use the following lemma:

Lemma B.1. If, for all $d, d' \in \mathbf{D}$, $\gamma(d) = \gamma(d')$, then the set of all dominant decisions is \mathbf{D} .

Proof: Since for all $d, d' \in \mathbf{D}$, $\gamma(d) = \gamma(d')$, then there is no $g \in \mathbf{G}$ such that $g \in \gamma(d)$ and $g \notin \gamma(d')$. Hence, by Definition 3.2, all $d \in \mathbf{D}$ are dominant. \square

We now prove the theorem. Since $S_d = \{\}$, by Lemma B.1, $|\mathbf{D}| > 1$ and $|S_w| > 1$. Assume that for all $d, d' \in S_w$, with $d \neq d'$, $\gamma(d) = \gamma(d')$. Then there are two cases, both leading to contradiction:

1. there is no $d'' \in \mathbf{D} \setminus S_w$. Then $S_w = \mathbf{D}$, and, by the absurd assumption, $\gamma(d) = \gamma(d')$ for all $d, d' \in \mathbf{D}$. Then, by Lemma B.1, for all $d \in S_w$, $d \in S_d$. But we also have $S_d = \{\}$, hence contradiction.
2. there exists some $d'' \in \mathbf{D} \setminus S_w$. Then there are five possibilities, and they all give contradictions, as follows:
 - (a) $\gamma(d) \supset \gamma(d'')$. Not possible, as if so there would exist $d^* \in \mathbf{D}$ such that d^* is dominant (with d a possible such d^*).
 - (b) $\gamma(d) \subset \gamma(d'')$. Not possible, as if so there would exist $d^* \in \mathbf{D} \setminus S_w$ such that d^* is dominant (d'' would be a possible such d^*).
 - (c) $\gamma(d) = \gamma(d'')$. Not possible, as if so d'' would be in S_w .
 - (d) None of (a)(b)(c) but $\gamma(d) \cap \gamma(d'') \neq \{\}$. Not possible, as if so there would exist $g \in \gamma(d'')$, $g \notin \gamma(d)$, hence there would exist $d^* \in \mathbf{D} \setminus S_w$ with d^* weakly dominant (d'' would be a possible such d^*), but S_w is the set of weakly dominant decisions and $d^* \notin S_w$.
 - (e) None of (a)(b)(c), but $\gamma(d) \cap \gamma(d'') = \{\}$. Same as case 2(d).

Both cases 1 and 2 give contradictions, and thus the theorem holds. \square

Proof of Proposition 4.1 Given an ADF, for each decision d , since d either meets a decision criterion or not, it is always possible to have a lean explanation for d being (strongly/weakly) dominant or not.

If d is not weakly dominant, since for any decision d , the set of goals met by d , $\gamma(d)$, is unique, then the lean explanation as given in Definition 4.1 is unique.

Proof of Proposition 4.2 Similar to the proof of Proposition 4.1, since d is either g-preferred or not, d always has a lean explanation. Since $\gamma(d)$ is unique, if d is not g-preferred, then it has a unique lean explanation.

Proof of Theorem 5.1 Let $AF = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$.

(Part I.) We first show that if d is strongly dominant, then $\{sDom(d)\} \vdash sDom(d)$ is admissible. This is to show:

1. $\{sDom(d)\} \vdash sDom(d)$ is an argument;
2. there exists $\Delta \subseteq \mathcal{A}$ such that $\{sDom(d)\} \subseteq \Delta$ and Δ withstands all attacks;
3. Δ does not attack itself.

Since $sDom(d) \in \mathcal{A}$, $\{sDom(d)\} \vdash sDom(d)$ is an argument, and 1. holds trivially.

The contrary of $sDom(d)$ is $notSDom(d)$, and the only rules with head $notSDom(d)$ are of the form $notSDom(d) \leftarrow notMet(d, g), isD(d), isG(g)$ for some $g \in \mathbf{G}$. Hence attackers of $\{sDom(d)\} \vdash sDom(d)$ are of the form $\{notMet(d, g)\} \vdash notSDom(d)$. Since d is strongly dominant, $\gamma(d) = \mathbf{G}$. Hence, for every $g \in \mathbf{G}$, d meets g and $met(d, g) \leftarrow \in \mathcal{R}$. So, for any $g \in \mathbf{G}$, we can construct an argument $\{\} \vdash met(d, g)$ and such argument can not be attacked (as it is supported by the empty set). Hence, for all g , $\{notMet(d, g)\} \vdash notSDom(d)$ is counter-attacked and 2. also holds, with $\Delta = \{sDom(d)\}$.

Since no argument for $notSDom(d)$ can be obtained with support a subset of Δ , Δ does not attack itself, and 3. also holds.

(Part II.) We then show that if $\{sDom(d)\} \vdash sDom(d)$ is admissible then d is strongly dominant, namely it meets all goals.

As shown in Part I, attackers against $\{sDom(d)\} \vdash sDom(d)$ are of the form $\{notMet(d, g)\} \vdash notSDom(d)$, for all g . Since $\{sDom(d)\} \vdash sDom(d)$ is admissible, it withstands all attacks. Hence, since $\mathcal{C}(notMet(d, g)) = \{met(d, g)\}$, by definition of \mathcal{R} , for all g , $\{\} \vdash met(d, g)$ is an argument. Furthermore, because $met(d, g) \leftarrow$ is a fact, d meets g , for any $g \in \mathbf{G}$, and thus d is strongly dominant. \square

Proof of Proposition 5.1 This is easy to see as the lean explanation for d needs to be $\{\gamma(d)\} = \mathbf{G}$ (Definition 4.1). For each \mathbf{P} node $\mathbf{P} : \{\} \vdash met(d, g)$ in \mathcal{T}^a , since it does not have any \mathbf{O} child, by Definition 5.3, we know that d meets g . Since \mathcal{T}^a is admissible, meaning that for all $g \in \mathbf{G}$, there exists a \mathbf{P} node $\mathbf{P} : \{\} \vdash met(d, g)$ in \mathcal{T}^a , we conclude that the set $\{g | \mathbf{P} : \{\} \vdash met(d, g) \text{ is a leaf node in } \mathcal{T}^a\}$ must be \mathbf{G} . Note that in this case, the fact that \mathcal{T}^a is least-assumption does not play a role; indeed, every admissible dispute tree with the same root as \mathcal{T}^a is necessarily least-assumption. \square

Proof of Proposition 5.2 To show that $S = \{g | \mathbf{O} : \{notMet(d, g)\} \vdash notSDom(d) \text{ is a leaf node in } \mathcal{T}^a\}$ is the lean explanation for d not being strongly dominant, we need to show that $S = \mathbf{G} \setminus \gamma(d)$. Since \mathcal{T}^a is best-effort, meaning that although \mathcal{T}^a is not admissible, it contains as few \mathbf{O} leaf nodes as possible (Definition 5.1). Thus, by Definition 5.3, we know that for each $g \in \mathbf{G}$, if d does not meet g , $\mathbf{O} : \{notMet(d, g)\} \vdash notSDom(d)$ must be a leaf node in \mathcal{T}^a . Therefore, $S = \mathbf{G} \setminus \gamma(d)$. \square

Proof of Theorem 5.2 Let $AF = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$.

(Part I.) To prove that dominance implies admissibility is to prove:

1. $\{dom(d)\} \vdash dom(d)$ is an argument;
2. there exists $\Delta \subseteq \mathcal{A}$ such that $\{dom(d)\} \subseteq \Delta$ and Δ withstands all attacks;

3. Δ does not attack itself.

Since $dom(d) \in \mathcal{A}$, $\{dom(d)\} \vdash dom(d)$ is an argument, and 1. holds trivially.

Let $\Delta = \{dom(d), noOthers(d, g) | g \in \mathbf{G}, g \notin \gamma(d)\}$. Attacks against $\{dom(d)\}$ are of the form $\{notMet(d, g)\} \vdash notDom(d)$, for all $g \in \mathbf{G}$. Since d is dominant, d meets all goals met by any decision in \mathbf{D} . Hence, for each goal $g \in \mathbf{G}$, either

1. d meets g , hence $\{\} \vdash met(d, g)$ exists and is not attacked; or
2. d does not meet g , but there is no argument $\{\} \vdash met(d', g)$ for any $d' \in \mathbf{D}$ (indicating that g is not met by any d'); therefore $\{noOthers(d, g)\} \vdash noOthers(d, g)$ can not be attacked.

Whichever the case, $\{notMet(d, g)\} \vdash notDom(d)$ is counter-attacked by Δ so $\{dom(d)\} \vdash dom(d)$ withstands all attacks. Also, since arguments of the form $\{noOthers(d, g)\} \vdash noOthers(d, g)$, such that g is not met by d , cannot be attacked, 2. also holds.

It is easy to see Δ does not attack itself, and 3. also holds.

(Part II.) We then show that admissibility implies dominance. Since $dom(d)$ belongs to an admissible set, all arguments $\{notMet(d, g)\} \vdash notDom(d)$ are attacked for all $g \in \mathbf{G}$. Since the contrary of $notMet(d, g)$ is $met(d, g)$ or $noOthers(d, g)$, $\{dom(d)\} \vdash dom(d)$ withstanding all of its attacks means, for each g , either $met(d, g)$ or $noOthers(d, g)$ can be “proved”. Hence, either g is met by d or there is no $d' \in \mathbf{D}$ meeting g and, by Definition 3.2, d is dominant. \square

Proof of Proposition 5.3 Since \mathcal{T}^a is admissible and least-assumption, for each node $\mathbf{O} : \{notMet(d, g)\} \vdash notDom(d)$ in \mathcal{T}^a , it must have a \mathbf{P} node child $\mathbf{P} : \{\} \vdash met(d, g)$. Jointly, they give the collection of goals met by d . Therefore, the set $\{g | \mathbf{P} : \{\} \vdash met(d, g)\}$ is a leaf node in $\mathcal{T}^a = \gamma(d)$.

Moreover, for those nodes $\mathbf{O} : \{notMet(d, g)\} \vdash notDom(d)$ without a $\mathbf{P} : \{\} \vdash met(d, g)$ as its child, (since \mathcal{T}^a is admissible), each of them must have a child of the form $\mathbf{P} : \{noOthers(d, g)\} \vdash noOthers(d, g)$, stating that although d does not meet g , there is no other decision d' meets g . Thus, gs in those \mathbf{P} nodes give nodes not met by d .

Proof of Proposition 5.4 Since \mathcal{T}^a is best effort, for each goal g not met by d , there must be a node $\mathbf{P} : \{noOthers(d, g)\} \vdash noOthers(d, g)$ claiming that there is no other decision d' meets g . Since \mathcal{T}^a is not admissible, there must be some g in those nodes met by some other decision d' . Nodes containing those gs thus have children of the form $\mathbf{O} : \{\} \vdash met(d', g)$. Those gs are met by d' s but not d . \square

Proof of Theorem 5.3 Let $AF = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$.

(Part I.) To prove that weak dominance implies admissibility is to prove:

1. $\{wDom(d)\} \vdash wDom(d)$ is an argument;
2. there exists $\Delta \subseteq \mathcal{A}$ such that $\{wDom(d)\} \subseteq \Delta$ and Δ withstands all attacks;
3. Δ does not attack itself.

Since $wDom(d) \in \mathcal{A}$, $\{wDom(d)\} \vdash wDom(d)$ is an argument, and 1. trivially holds.

Let $\Delta = \{wDom(d), notMet(d', g) | d' \in \mathbb{D}, d' \neq d, g \in \mathbb{G}, g \in \gamma(d), g \notin \gamma(d')\}$. Attacks against $\{wDom(d)\}$ are of the form $\{notMet(d, g), notMore(d, d')\} \vdash notWDom(d)$. Since d is weakly dominant, then there is no d' such that $\gamma(d) \subset \gamma(d')$. Hence, given any $d' \neq d$, for each $g \in \gamma(d')$, either

1. $g \in \gamma(d)$ or
2. $g \notin \gamma(d)$, but there exists some $g' \in \mathbb{G}$ such that $g' \in \gamma(d)$ and $g' \notin \gamma(d')$.

In the first case, $notMet(d, g)$ is attacked by $\{\} \vdash met(d, g)$; in the second case, $notMore(d, d')$ there is some g' met by d but not d' (so $more(d, d')$ can be “proved” from Δ). Hence, whichever the case, $\{wDom(d)\} \vdash wDom(d)$ withstands attacks towards it. Also, no $notMet(d', g) \in \Delta$ can be attacked, and 2. also holds.

It is easy to see that Δ does not attack itself, and 3. also holds.

(**Part II.**) We then show that if admissibility implies weak dominance.

Since $\{wDom(d)\} \vdash wDom(d)$ is admissible, all of its attackers are counterattacked. Hence, arguments for $notWDom(d)$ are counter-attacked. By definition of \mathcal{R} in the *AF*, if $notWDom(d)$ is attacked, then, for any g and $d' \neq d$:

1. it is not the case that $met(d', g)$ and $notMet(d, g)$ both hold or
2. there is some g' such that $met(d, g')$ and $notMet(d', g')$ (so $more(d, d')$ holds and $notMore(d, d')$ is attacked).

Whichever the case, $\gamma(d)$ is not a subset of $\gamma(d')$. Therefore d is weakly dominant. \square

Proof of Proposition 5.5 Since \mathcal{T}^a is least-assumption, for each goal g met by d , there must be a **P** node $\mathbf{P} : \{\} \vdash met(d, g)$ in a leaf of \mathcal{T}^a . Moreover, for each node $\mathbf{P} : \{notMet(_, g)\} \vdash more(d, _)$ in \mathcal{T}^a , from Definition 5.7, we can see that d meets g . Jointly, the two collections of goals g in these nodes are met by d .

For each leaf node $\mathbf{P} : \{notMet(d', g)\} \vdash more(d, d')$ in \mathcal{T}^a , as it has no **O** child, we can see that d' does not meet g but d meets. Thus, the collection of these (d', g) pairs gives the set of goals met by d but not d' s.

Overall, we can see that for each d' , let $g' \in \gamma(d')$, if d meets g' , then $\mathbf{P} : \{\} \vdash met(d, g')$ would be a leaf node in \mathcal{T}^a ; otherwise, $\mathbf{P} : \{notMet(d', g)\} \vdash more(d, d')$ would be a leaf node in \mathcal{T}^a for some $g, g \in \gamma(d)$ and $g \notin \gamma(d')$.

Proof of Proposition 5.6 An explanation for some decision d not being weakly dominant is the set of decisions meeting strictly more goals than d (with respect to \subset). As \mathcal{T}^a is best effort, all nodes N of the form $\mathbf{O} : \{notMet(d, g), notMore(d, d')\} \vdash notWDom(d)$ have children $\mathbf{P} : \{\} \vdash met(d, g)$, if d meets g . Only for the goals g' , which d does not meet, arguments of the form $\{notMet(d', g')\} \vdash more(d, d')$ are used to label **P** nodes as children of N . If these arguments cannot be attacked, then indeed d meets more goals than d' . However, since d is not weakly dominant, some of these arguments must be attacked by arguments of the form $\{\} \vdash met(d', g')$. In these cases, d' meets strictly more goals than d (g' is one such example). It is easy to see that nodes of the form $\mathbf{O} : \{\} \vdash met(d', g')$ are leaf nodes of \mathcal{T}^a so the collection of d' s in these nodes give the explanation for d .

When d does not meet any goal, no node of the form $\mathbf{P} : \{notMet(d', g')\} \vdash more(d, d')$ can be added to \mathcal{T}^a , (to construct these arguments, d has to meet g'). In this case, leaf nodes of \mathcal{T}^a would be in the form of $\mathbf{O} : \{notMet(d, g), notMore(d, d')\} \vdash notWDom(d)$, which shows that d' meets g and d does not. Thus, the collection of d' in these leaf nodes also constitutes the explanation for d not being weakly dominant.

Proof of Theorem 5.4 The proof of this theorem is similar to the proofs of Theorems 5.1, 5.2 and 5.3. Let $AF = \langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$.

(**Part I.**) We first prove if d is g-preferred, then $\{gP(d)\} \vdash gP(d)$ is in an admissible extension. To show $\{gP(d)\} \vdash gP(d)$ is admissible, we need to show:

1. $\{gP(d)\} \vdash gP(d)$ is an argument.
2. With help of the set of arguments Δ , $\{gP(d)\} \vdash gP(d)$ withstands its attacks.
3. $\{\{gP(d)\} \vdash gP(d)\} \cup \Delta$ is conflict-free.

Since $gP(d)$ is an assumption, $\{gP(d)\} \vdash gP(d)$ is an argument.

To show $\{gP(d)\} \vdash gP(d)$ withstands its attacks, we show its attackers are defeated. Since $\mathcal{C}(gP(d)) = \{notGP(d)\}$, attackers of $\{gP(d)\} \vdash gP(d)$ are arguments with claim $notGP(d)$. Since rules with head $notGP(d)$ are of the form

$$notGP(X) \leftarrow metS(X_1, S), notMetS(X, S), notBetter(X, X_1, S);$$

attackers of $\{gP(d)\} \vdash gP(d)$ are arguments of the form

$$\{metS(d', s), notMet(d, g), notBetter(d, d', s)\} \vdash notGP(d) \text{ (for some } g \in s).$$

Hence we need to show for all $s \in \mathbf{S}, d' \in \mathbf{D}$, $\{gP(d)\} \vdash gP(d)$ withstands (with help) attacks from $\{metS(d', s), notSet(d, g), notBetter(d, d', s)\} \vdash notGP(d)$.

Because $metS(d', s)$, $notMet(d, g)$ and $notBetter(d, d', s)$ are assumptions, if there are arguments \mathbf{Arg} for contraries of them and \mathbf{Arg} is either not attacked or withstands its attacks, then we conclude that the argument $\{metS(d', s), notMet(d, g), notBetter(d, d', s)\} \vdash notGP(d)$ is defeated and $\{gP(d)\} \vdash gP(d)$ holds. We show such \mathbf{Arg} exists when d is a g-preferred.

For all $d' \in \mathbf{D}, d' \neq d$, for any $s \in \mathbf{S}$ there are two possibilities:

1. it is the case that $s \not\subseteq \gamma(d)$ and $s \subseteq \gamma(d')$; and
2. it is not the case that $s \not\subseteq \gamma(d)$ and $s \subseteq \gamma(d')$, i.e. one of the following holds:
 - (a) $s \not\subseteq \gamma(d)$ and $s \not\subseteq \gamma(d')$,
 - (b) $s \subseteq \gamma(d)$ and $s \subseteq \gamma(d')$,
 - (c) $s \subseteq \gamma(d)$ and $s \not\subseteq \gamma(d')$.

In case 1, since d is g-preferred, by Definition 3.5, there exists $s' \in \mathbf{S}$, such that

$$(1) s' \geq s \in \leq_g, (2) s' \subseteq \gamma(d), \text{ and } (3) s' \not\subseteq \gamma(d').$$

From the three conditions above, we see that:

- Since $s' \geq s$ in \mathbf{P} , $pfr(s', s) \leftarrow$ is in \mathcal{R} .
- Since $s' \subseteq \gamma(d)$, $\{\} \vdash met(d, g)$ is an argument for all $g \in s'$.
- Since $s' \not\subseteq \gamma(d')$, there is no argument for $met(d', g')$ for some $g' \in s'$, hence the assumption $notMet(d', g')$ is not attacked (the contrary of $notMet(d', g')$ is $met(d', g')$).

Jointly, we know that there is an argument $\{notMet(d', g')\} \vdash better(d, d', s)$, and this argument is not attacked. Therefore, since the contrary of $notBetter(d, d', s)$ is $better(d, d', s)$, the argument $\{notMet(d, g), notBetter(d, d', s)\} \vdash notPG(d)$, cannot withstand its attacks.

In case 2(a), $s \not\subseteq \gamma(d')$, there is some $g \in s$ such that $g \notin \gamma(d')$. There is an argument $\{notMet(d', g)\} \vdash notMetS(d', s)$ defeating the argument

$$\{metS(d', s), notMet(d, g'), notBetter(d, d', s)\} \vdash notPG(d).$$

In case 2(b) and 2(c), $s \subseteq \gamma(d)$, hence for each $g' \in s$, arguments of the form $\{\} \vdash met(d, g')$ can be constructed and not attacked. Such arguments attack

$$\{metS(d', s), notMet(d, g'), notBetter(d, d', s)\} \vdash notPG(d).$$

In case 1 or 2, $\{pG(d)\} \vdash pG(d)$ withstands its attacks.

It is easy to see that $\{\{pG(d)\} \vdash pG(d)\} \cup \Delta$ is conflict-free by checking through all assumption and contrary mappings.

Since $\{gP(d)\} \vdash gP(d)$ is an argument and, with help from a conflict-free set of arguments, withstands all of its attacks, $\{gP(d)\} \vdash gP(d)$ is admissible.

(Part II.) We show: if $\{gP(d)\} \vdash gP(d)$ belongs to an admissible set of arguments, then d is a g-preferred. To show d is g-preferred, we need to show for all $d' \in \mathbf{D} \setminus \{d\}$, the following holds:

- ★ for all $s \in \mathbf{S}$, if $s \not\subseteq \gamma(d)$ and $s \subseteq \gamma(d')$, then there exists $s' \in \mathbf{S}$ such that: (1) $s' \geq s$ in \mathbf{P} , (2) $s' \subseteq \gamma(d)$, and (3) $s' \not\subseteq \gamma(d')$.

Since there is Δ such that $\{pG(d)\} \vdash pG(d) \cup \Delta$ is an admissible set,

1. $\{gP(d)\} \vdash gP(d)$ is an argument;
2. with help of Δ , $\{gP(d)\} \vdash gP(d)$ withstands all attacks towards it.

With the reasoning shown in Part I in reverse, we can see that $\{gP(d)\} \vdash gP(d)$ being admissible implies d being g-preferred. \square

Proof of Proposition 5.7 To show (G, M) is an explanation for d being g-preferred, we first need to show that $G \subset \gamma(d)$. This is easy to see as G is the set of goals gs in leaf \mathbf{P} nodes labelled by $\{\} \vdash \{met(d, g)\}$. By Definition 5.9, d meets these gs . Secondly, for each pair (S, d') such that $N = \mathbf{P} : \{metS(d, S), notMet(d', -)\} \vdash better(d, d', -)$ is a leaf node in \mathcal{T}^a , we know that d meets S and d' does not meet anything more preferred than S (as if it does, N would not be a leaf node). \square

Proof of Proposition 5.8 The proof of this proposition is similar to the one for Proposition 5.6. Since \mathcal{T}^a is best effort, all arguments labelling leaf \mathbf{O} nodes in \mathcal{T}^a capture decisions meeting more preferred goals than d with g being a goal in such a goal set. \square

Proof of Proposition 6.1 Given a DG $G = \langle N, E, B \rangle$ with $N = D \cup N_{int} \cup G$, $F = \langle D, G, \gamma \rangle$ with, let, for $n_1 \in D$ and $n_2 \in G$, $n_2 \in \gamma(n_1)$ if and only if n_2 is reachable from $\{n_1\}$. Then, it is immediate to see that F is an equivalent ADF to G , such that decision $d \in D$ meets a goal $g \in G$ in F if and only if d meets g in G . \square

Proof of Proposition 6.2 Given an ADF $F = \langle D, G, \gamma \rangle$, we can create an equivalent DG $G = \langle N, E \rangle$ where $N = \{n | n \in D \cup G\}$ and $E = \{[n_1 \triangleright n_2] | n_2 \in \gamma(n_1)\}$. It is easy to see that a decision $d \in D$ meets a goal $g \in G$ in G if and only if d meets g in AF . \square

Proof of Proposition 6.3 The proof of this proposition is similar to the one for Proposition 6.1. For any PDG $G = \langle N, E, B, \leq_g \rangle$ with $N = D \cup N_{int} \cup G$, we can have $F = \langle D, G, \gamma, \leq_g \rangle$ such that for $n_1 \in D$ and $n_2 \in G$, $n_2 \in \gamma(n_1)$ if and only if n_2 is reachable from $\{n_1\}$. This ensures that a decision d meets a goal g in G if and only if d meets g in F .

Proof of Proposition 6.4 The proof of this proposition is similar to the proof of Theorems 5.1, 5.2 and 5.3. The difference is that upon examining $met(d, g)$ for some $d \in D$ and $g \in G$, instead of checking γ directly, it uses the rule

$$met(D, S) \leftarrow reach(D, S)$$

which further invokes rules

$$reach(X, Y) \leftarrow edge(X, Y, T)$$

$$reach(X, Y) \leftarrow reach(X, Z), edge(Z, Y, T), \neg unreachableSib(Z, Y, T, X)$$

$$unreachableSib(Z, Y, T, X) \leftarrow edge(W, Y, T), \neg reach(X, W), W \neq Z$$

$$edge(n_1, n_2, t(e)) \leftarrow dEdge(n_1, n_2, t(e))$$

$$edge(n_1, n_2, t(e)) \leftarrow$$

and assumptions $\neg unreachableSib(Z, Y, T, X)$, $reach(X, W)$ and their contraries. The defeasibility is captured by assumptions of the form $dEdge(n_1, n_2, t(e))$, which have contraries $\neg dEdge(n_1, n_2, t(e))$ that can be derived from rules generated from defeasible conditions. Thus, the condition of a decision d meeting a goal g (both d and g are nodes in the graph) is now subject to the test on defeasibility on the path from d to g . The structure of the rest of the proof remains unchanged and the conclusions hold. \square

Proof of Proposition 6.5 As in the proof of Proposition 6.4, the proof of this proposition is similar to the one for Theorem 5.4. Since DG generalises ADF and allows richer description on the decision-meeting-goal relation, the rule

$$met(d, g) \leftarrow$$

in ABA frameworks corresponding to ADFs are replaced by

$$met(d, g) \leftarrow reach(d, g)$$

and additional rules for proving $reach(d, g)$, e.g.,

$$reach(n_1, n_2) \leftarrow edge(n_1, n_2, t(e))$$

and

$$reach(n_1, n_2) \leftarrow reach(n_1, n_3), edge(n_3, n_2, t(e)), \neg unreachableSib(n_3, n_2, t(e), n_1).$$

However, these rules do not affect the preference relations between sets of goals, i.e., the rules, assumptions and contraries described in the g-preferred component. Thus, with reasoning similiar to Theorem 5.4, this proposition holds. \square

Proof of Proposition 6.6 To prove $S = \{g | \mathbf{P} : \{-\} \vdash met(d, g)\}$ is a leaf node in \mathcal{T}^a is an explanation for d being strongly dominant we need to show that $S = \gamma(d)$. This is easy to see as for each goal g in G , by Definition 6.7, there is an \mathbf{O} node of the form $\mathbf{O} : \{notMet(d, g)\} \vdash notSDom(d)$ attacking the root node $\mathbf{P} : \{sDom(d)\} \vdash sDom(d)$. These \mathbf{O} nodes can only be attacked by \mathbf{P} nodes of the form $\mathbf{P} : \{-\} \vdash met(d, g)$. Thus, if these \mathbf{P} nodes are leaf nodes, it means that d meets g is undisputed. Since \mathcal{T}^a is admissible, all gs are met by d . Thus, S is an explanation for d being strongly dominant.

With the same reasoning, we see that for decisions that are not strongly dominant, $\{g | N = \mathbf{O} : \{notMet(d, g)\} \vdash notSDom(d)\}$ is a leaf node in \mathcal{T}^a or N is ancestor of an \mathbf{O} leaf node is an explanation as either N is a leaf node or N is ancestor of an \mathbf{O} leaf node, it means that g is not met by d . The collection of all such goals gs is an explanation for d not being strongly dominant. \square

Proof of Proposition 6.7 The proof of this proposition is similiar to the ones for Proposition 5.3 and 5.4. The differences are:

- Since the rule $met(d, g) \leftarrow$ in the ABA framework corresponding to ADF is now replaced by $met(d, g) \leftarrow reach(d, g)$ and rules with head $reach(d, g)$, arguments $\{-\} \vdash met(d, g)$ labelling leaf nodes are replaced by $\{-\} \vdash met(d, g)$ for admissible dispute trees.
- Moreover, since $met(d, g)$ in the ABA framework corresponding to DG can be claims for arguments supported by assumptions, when d does not meet g , it might be the case that $\{notMet(d, g)\} \vdash notDom(d)$ is no longer a leaf node in \mathcal{T}^a . Thus, the condition for d not meeting g for explaining d not being dominant is changed to there is a path from an \mathbf{O} leaf node to $\mathbf{O} : \{notMet(d, g)\} \vdash notDom(d)$. \square

Proof of Proposition 6.8 The proof of this proposition is similiar to the ones for Proposition 5.5 and 5.6. As in the proof for Proposition 6.7, $\{-\} \vdash met(d, g)$ are replaced by $\{-\} \vdash met(d, g)$. Since $\{-\} \vdash met(d, g)$ are now subject to possible attacks, in order to establish its validity, we impose that nodes labelled by these arguments must not be ancestors of \mathbf{O} leaf nodes. \square

Proof of Proposition 6.9 The proof of this proposition is similiar to the ones for Proposition 5.5 and 5.6 with the same modification shown in proofs for Proposition 6.7 and 6.8. \square

Appendix C. ABA Frameworks for some of the Examples in Section 6

Example C.1. Consider the DG without defeasible information in Figure 30. Let uS , dt and ct be short-hands for *unreachableSib*, *discount* and *convenient*, respectively, the core ABA framework corresponding to it is $\langle \mathcal{L}_g, \mathcal{R}_g, \mathcal{A}_g, \mathcal{C}_g \rangle$, where

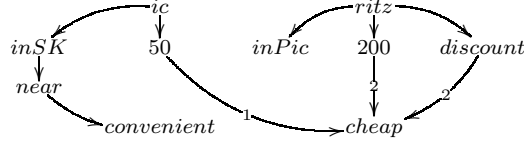


Figure 30: DG for Example C.1.

- \mathcal{R}_g consists of:

$$\begin{aligned}
& \text{edge}(ic, inSK, 1) \leftarrow \text{edge}(ic, 50, 1) \leftarrow \text{edge}(ritz, inPic, 1) \leftarrow \\
& \text{edge}(ritz, dt, 1) \leftarrow \text{edge}(ritz, 200, 1) \leftarrow \text{edge}(inSK, near, 1) \leftarrow \\
& \text{edge}(near, ct, 1) \leftarrow \text{edge}(50, cheap, 1) \leftarrow \text{edge}(200, cheap, 2) \leftarrow \\
& \text{edge}(dt, cheap, 2) \leftarrow \\
& \text{reach}(ic, inSK) \leftarrow \text{edge}(ic, inSK, 1) \quad \text{reach}(ic, 50) \leftarrow \text{edge}(ic, 50, 1) \\
& \text{reach}(ritz, inPic) \leftarrow \text{edge}(ritz, inPic, 1) \quad \text{reach}(ritz, 200) \leftarrow \text{edge}(ritz, 200, 1) \\
& \text{reach}(inSK, near) \leftarrow \text{edge}(inSK, near, 1) \quad \text{reach}(50, cheap) \leftarrow \text{edge}(50, cheap, 1) \\
& \text{reach}(200, cheap) \leftarrow \text{edge}(200, cheap, 2) \\
& \text{reach}(near, ct) \leftarrow \text{edge}(near, ct, 1) \\
& \text{reach}(ritz, dt) \leftarrow \text{edge}(ritz, dt, 1) \\
& \text{reach}(dt, cheap) \leftarrow \text{edge}(dt, cheap, 2) \\
& \text{reach}(ic, near) \leftarrow \text{reach}(ic, inSK), \text{edge}(inSK, near, 1), \neg uS(inSK, near, 1, ic) \\
& \text{reach}(ic, ct) \leftarrow \text{reach}(ic, near), \text{edge}(near, ct, 1), \neg uS(near, ct, 1, ic) \\
& \text{reach}(ic, cheap) \leftarrow \text{reach}(ic, 50), \text{edge}(50, cheap, 1), \neg uS(50, cheap, 1, ic) \\
& \text{reach}(ritz, cheap) \leftarrow \text{reach}(ritz, 200), \text{edge}(200, cheap, 2), \neg uS(200, cheap, 2, ritz) \\
& \text{reach}(ritz, cheap) \leftarrow \text{reach}(ritz, dt), \text{edge}(dt, cheap, 2), \neg uS(dt, cheap, 2, ritz) \\
& uS(dt, cheap, 2, ritz) \leftarrow \text{edge}(200, cheap, 2), \neg \text{reach}(ritz, 200) \\
& uS(200, cheap, 2, ritz) \leftarrow \text{edge}(dt, cheap, 2), \neg \text{reach}(ritz, dt) \\
& \{met(n_1, n_2) \leftarrow \text{reach}(n_1, n_2) | n_1 \in \{ic, ritz\}, n_2 \in \{ct, cheap\}\}.
\end{aligned}$$

- \mathcal{A}_g consists of:

$$\begin{aligned}
& \neg uS(inSK, near, 1, ic) \quad \neg uS(near, ct, 1, ic) \quad \neg uS(50, cheap, 1, ic) \\
& \neg uS(200, cheap, 2, ritz) \quad \neg uS(dt, cheap, 2, ritz) \quad \neg \text{reach}(ritz, 200) \\
& \neg \text{reach}(ritz, dt) \quad \{\text{notMet}(d, g) | d \in \{ic, ritz\}, g \in \{ct, cheap\}\}
\end{aligned}$$

- Let $AN = \{ic, ritz, ct, cheap, inSK, near, 50, inPic, 200, dt\}$.

For $n_1, n_2, n_3 \in AN$, $\mathcal{C}_g(\neg uS(n_3, n_2, t, n_1)) = \{uS(n_3, n_2, t, n_1)\}$;

For $n_1, n_2 \in AN$, $\mathcal{C}_g(\neg \text{reach}(n_1, n_2)) = \{\text{reach}(n_1, n_2)\}$;

For $d \in \{ic, ritz\}$, $g \in \{ct, cheap\}$, $\mathcal{C}_g(\text{notMet}(d, g)) = \{\text{met}(d, g)\}$.

Example C.2. Consider the DG (with defeasible information) in Figure 23. The core ABA framework corresponding to it is as shown in Example C.1 with the following modifications.

$$\text{edge}(ic, inSK, 1) \leftarrow \text{edge}(ic, 50, 1) \leftarrow$$

replaced by

$$termTime \leftarrow \neg dEdge(ic, 50, 1) \leftarrow termTime.$$

$edge(ic, inSK, 1)$ and $edge(ic, 50, 1)$ added to \mathcal{A}_g , such that $\mathcal{C}_g(edge(ic, inSK, 1)) = \neg edge(ic, inSK, 1)$ and $\mathcal{C}_g(edge(ic, 50, 1)) = \neg edge(ic, 50, 1)$, respectively.

ABA framework for Example 6.7. The ABA framework for this example is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ where

- \mathcal{R} consists of:

$$\begin{aligned} & edge(d_1, g_1, 1) \leftarrow edge(d_2, a_1, 1) \leftarrow edge(d_2, a_2, 1) \leftarrow \\ & edge(a_1, g_1, 2) \leftarrow edge(a_1, g_2, 1) \leftarrow edge(a_2, g_2, 1) \leftarrow \\ & reach(d_1, g_1) \leftarrow edge(d_1, g_1, 1) \quad reach(d_2, a_1) \leftarrow edge(d_2, a_1, 1) \\ & reach(d_2, a_2) \leftarrow edge(d_2, a_2, 1) \quad reach(a_1, g_1) \leftarrow edge(a_1, g_1, 1) \\ & reach(a_1, g_2) \leftarrow edge(a_1, g_2, 1) \quad reach(a_2, g_2) \leftarrow edge(a_2, g_2, 1) \\ & reach(d_1, g_2) \leftarrow reach(d_1, a_1), edge(a_1, g_2, 1), \neg unreachableSib(a_1, g_2, 1, d_1) \\ & reach(d_1, g_2) \leftarrow reach(d_1, a_2), edge(a_2, g_2, 1), \neg unreachableSib(a_2, g_2, 1, d_1) \\ & reach(d_2, g_1) \leftarrow reach(d_2, a_1), edge(a_1, g_1, 2), \neg unreachableSib(a_1, g_1, 2, d_2) \\ & reach(d_2, g_2) \leftarrow reach(d_2, a_1), edge(a_1, g_2, 1), \neg unreachableSib(a_1, g_2, 1, d_2) \\ & reach(d_2, g_2) \leftarrow reach(d_2, a_2), edge(a_2, g_2, 1), \neg unreachableSib(a_2, g_2, 1, d_2) \\ & unreachableSib(a_1, g_2, 1, d_1) \leftarrow edge(a_2, g_2, 1), \neg reach(d_1, a_2) \\ & unreachableSib(a_2, g_2, 1, d_1) \leftarrow edge(a_1, g_2, 1), \neg reach(d_1, a_1) \\ & unreachableSib(a_1, g_2, 1, d_2) \leftarrow edge(a_2, g_2, 1), \neg reach(d_2, a_2) \\ & unreachableSib(a_2, g_2, 1, d_2) \leftarrow edge(a_1, g_2, 1), \neg reach(d_2, a_1) \\ & met(d_1, g_1) \leftarrow reach(d_1, g_1) \quad met(d_1, g_2) \leftarrow reach(d_1, g_2) \\ & met(d_2, g_1) \leftarrow reach(d_2, g_1) \quad met(d_2, g_2) \leftarrow reach(d_2, g_2) \\ & notSDom(d_1) \leftarrow notMet(d_1, g_1) \quad notSDom(d_1) \leftarrow notMet(d_1, g_2) \\ & notSDom(d_2) \leftarrow notMet(d_2, g_1) \quad notSDom(d_2) \leftarrow notMet(d_2, g_2) \end{aligned}$$

- \mathcal{A} consists of:

$$\begin{aligned} & \neg unreachableSib(a_1, g_2, 1, d_1) \quad \neg unreachableSib(a_2, g_2, 1, d_1) \\ & \neg unreachableSib(a_1, g_1, 2, d_2) \quad \neg unreachableSib(a_1, g_2, 1, d_2) \\ & \neg unreachableSib(a_2, g_2, 1, d_2) \\ & \neg reach(d_1, a_2) \quad \neg reach(d_1, a_1) \quad \neg reach(d_2, a_2) \quad \neg reach(d_2, a_1) \quad sDom(d_1) \\ & notMet(d_1, g_1) \quad notMet(d_1, g_2) \quad notMet(d_2, g_1) \quad notMet(d_2, g_2) \quad sDom(d_2) \end{aligned}$$

- Let $AN = \{d_1, d_2, a_1, a_2, g_1, g_2\}$.

For $n_1, n_2, n_3 \in AN$, $\mathcal{C}(\neg unreachableSib(n_3, n_2, t, n_1)) = \{unreachableSib(n_3, n_2, t, n_1)\}$.

For $n_1, n_2 \in AN$, $\mathcal{C}(\neg reach(n_1, n_2)) = \{reach(n_1, n_2)\}$.

For $d \in \{d_1, d_2\}$, $g \in \{g_1, g_2\}$, $\mathcal{C}(notMet(d, g)) = \{met(d, g)\}$.

For $d \in \{d_1, d_2\}$, $\mathcal{C}(sDom(d)) = \{notSDom(d)\}$.

ABA framework for Example 6.8. The ABA framework for this example is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ where

- \mathcal{R} is all rules in the ABA framework for Example 6.7 with

$$\begin{array}{ll} notSDom(d_1) \leftarrow notMet(d_1, g_1) & notSDom(d_1) \leftarrow notMet(d_1, g_2) \\ notSDom(d_2) \leftarrow notMet(d_2, g_1) & notSDom(d_2) \leftarrow notMet(d_2, g_2) \end{array}$$

replaced by

$$\begin{array}{ll} notDom(d_1) \leftarrow notMet(d_1, g_1) & notDom(d_1) \leftarrow notMet(d_1, g_2) \\ notDom(d_2) \leftarrow notMet(d_2, g_1) & notDom(d_2) \leftarrow notMet(d_2, g_2) \\ met(d_1, g_3) \leftarrow reach(d_1, g_3) & met(d_2, g_3) \leftarrow reach(d_2, g_3) \end{array}$$

- \mathcal{A} is all assumptions in the ABA framework for Example 6.7 with

$$sDom(d_1) \quad sDom(d_2)$$

replaced by

$$\begin{array}{llll} dom(d_1) & dom(d_2) & notMet(d_1, g_3) & notMet(d_2, g_3) \\ noOthers(d_1, g_1) & noOthers(d_1, g_2) & noOthers(d_1, g_3) & \\ noOthers(d_2, g_1) & noOthers(d_2, g_2) & noOthers(d_2, g_3) & \end{array}$$

- For any α in \mathcal{A} , $\mathcal{C}(\alpha)$ is as defined in the ABA framework for Example 6.7 along with

$$\begin{array}{ll} \mathcal{C}(dom(d_1)) = \{notDom(d_1)\} & \mathcal{C}(dom(d_2)) = \{notDom(d_2)\} \\ \mathcal{C}(noOthers(d_1, g_1)) = \{met(d_2, g_1)\} & \mathcal{C}(noOthers(d_1, g_2)) = \{met(d_2, g_2)\} \\ \mathcal{C}(noOthers(d_1, g_3)) = \{met(d_2, g_3)\} & \mathcal{C}(noOthers(d_2, g_1)) = \{met(d_1, g_1)\} \\ \mathcal{C}(noOthers(d_2, g_2)) = \{met(d_1, g_2)\} & \mathcal{C}(noOthers(d_2, g_3)) = \{met(d_1, g_3)\} \end{array}$$

$$\text{For } d \in \{d_1, d_2\}, g \in \{g_1, g_2, g_3\}, \mathcal{C}(notMet(d, g)) = \{met(d, g), noOthers(d, g)\}.$$

ABA framework for Example 6.9. The ABA framework for this example is $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$, where

- \mathcal{R} is all rules in the ABA framework for Example 6.7 with

$$\begin{array}{ll} notSDom(d_1) \leftarrow notMet(d_1, g_1) & notSDom(d_1) \leftarrow notMet(d_1, g_2) \\ notSDom(d_2) \leftarrow notMet(d_2, g_1) & notSDom(d_2) \leftarrow notMet(d_2, g_2) \end{array}$$

replaced by

$$\begin{array}{ll} edge(d_3, a_3, 1) \leftarrow & edge(a_3, g_3, 1) \leftarrow \\ reach(d_3, a_3) \leftarrow edge(d_3, a_3, 1) & reach(a_3, g_3) \leftarrow edge(a_3, g_3, 1) \\ met(d_1, g_3) \leftarrow reach(d_1, g_3) & met(d_2, g_3) \leftarrow reach(d_2, g_3) \\ met(d_3, g_1) \leftarrow reach(d_3, g_1) & met(d_3, g_2) \leftarrow reach(d_3, g_2) \\ met(d_1, g_3) \leftarrow reach(d_1, g_3) & \\ reach(d_1, g_3) \leftarrow reach(d_1, a_3), edge(a_3, g_3, 1), \neg unreachableSib(a_3, g_3, 1, d_1) & \\ reach(d_2, g_3) \leftarrow reach(d_2, a_3), edge(a_3, g_3, 1), \neg unreachableSib(a_3, g_3, 1, d_2) & \\ reach(d_3, g_3) \leftarrow reach(d_3, a_3), edge(a_3, g_3, 1), \neg unreachableSib(a_3, g_3, 1, d_3) & \end{array}$$

$$\text{For } d, d' \in \{d_1, d_2, d_3\}, d \neq d', g, g' \in \{g_1, g_2, g_3\},$$

$$notWDom(d) \leftarrow met(d', g), notMet(d, g), notMore(d, d'),$$

$$more(d, d') \leftarrow met(d, g), notMet(d', g).$$

- \mathcal{A} is all assumptions in the ABA framework for Example 6.7 with

$$sDom(d_1) \quad sDom(d_2)$$

replaced by

$$\begin{array}{llll} wDom(d_1) & wDom(d_2) & wDom(d_3) & notMet(d_1, g_3) \\ notMet(d_2, g_3) & notMet(d_3, g_1) & notMet(d_3, g_2) & notMet(d_3, g_3) \\ notMore(d_1, d_2) & notMore(d_1, d_3) & notMore(d_2, d_1) & notMore(d_2, d_3) \\ notMore(d_3, d_1) & notMore(d_3, d_2) & \neg unreachableSib(a_3, g_3, 1, d_1) & \\ \neg unreachableSib(a_3, g_3, 1, d_2) & & \neg unreachableSib(a_3, g_3, 1, d_3) & \end{array}$$

- For any α in \mathcal{A} , $\mathcal{C}(\alpha)$ is as defined in the ABA framework for Example 6.7 along with

$$\begin{array}{l} \mathcal{C}(wDom(d_1)) = \{notWDom(d_1)\} \quad \mathcal{C}(wDom(d_2)) = \{notWDom(d_2)\} \\ \mathcal{C}(wDom(d_3)) = \{notWDom(d_3)\} \end{array}$$

$$\text{For } d \in \{d_1, d_2, d_3\}, g \in \{g_1, g_2, g_3\}, \mathcal{C}(notMet(d, g)) = \{met(d, g)\}$$

$$\text{For } d, d' \in \{d_1, d_2, d_3\}, d \neq d', \mathcal{C}(notMore(d, d')) = \{more(d, d')\}$$

$$\text{For } d \in \{d_1, d_2, d_3\}, \mathcal{C}(\neg unreachableSib(a_3, g_3, 1, d)) = \{unreachableSib(a_3, g_3, 1, d)\}$$

ABA framework for Example 6.10. The ABA framework for this example $\langle \mathcal{L}, \mathcal{R}, \mathcal{A}, \mathcal{C} \rangle$ where

- \mathcal{R} is all rules in the ABA framework for Example 6.9 with

for all $d, d' \in \{d_1, d_2, d_3\}, d \neq d', g \in \{g_1, g_2, g_3\}$

$$notWDom(d) \leftarrow met(d', g), notMet(d, g), notMore(d, d')$$

$$more(d, d') \leftarrow met(d, g), notMet(d', g)$$

replaced by

$$- \quad pfr(s3, s12) \leftarrow$$

$$- \quad \text{for all } d \in \{d_1, d_2, d_3\},$$

$$notMetS(d, s3) \leftarrow notMet(d, g_3) \quad notMetS(d, s12) \leftarrow notMet(d, g_1)$$

$$notMetS(d, s12) \leftarrow notMet(d, g_2)$$

$$- \quad \text{for all } d, d' \in \{d_1, d_2, d_3\}, d \neq d', s, s' \in \{s3, s12\}, s \neq s',$$

$$better(d, d', s) \leftarrow metS(d, s'), notMetS(d', s'), pfr(s', s)$$

$$notGP(d) \leftarrow metS(d', s), notMetS(d, s), notBetter(d, d', s)$$

- \mathcal{A} is all assumptions in the ABA framework for Example 6.9 with for $d, d' \in \{d_1, d_2, d_3\}, d \neq d'$

$$wDom(d) \quad notMore(d, d')$$

replaced by

$$notBetter(d, d', s3) \quad notBetter(d, d', s12) \quad metS(d, s3) \quad metS(d, s12) \quad gP(d)$$

- For all $\alpha \in \mathcal{A}$, $\mathcal{C}(\alpha)$ is as given in the ABA framework for Example 6.9, along with

$$- \quad \text{for } d \in \{d_1, d_2, d_3\}, s \in \{qs3, q1q2S\},$$

$$\mathcal{C}(gP(d)) = \{notGP(d)\} \quad \mathcal{C}(metS(d, s)) = \{notMetS(d, s)\}$$

- for $d, d' \in \{d_1, d_2, d_3\}, d \neq d'$
 - $\mathcal{C}(\text{notBetter}(d, d', s3)) = \{\text{better}(d, d', s3)\}$
 - $\mathcal{C}(\text{notBetter}(d, d', s12)) = \{\text{better}(d, d', s12)\}$