Imbalanced classification: an objective-oriented review

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Abstract

A common issue for classification in scientific research and industry is the existence of imbalanced classes. When sample sizes of different classes are imbalanced in training data, naively implementing a classification method often leads to unsatisfactory prediction results on test data. Multiple resampling techniques have been proposed to address the class imbalance issues. Yet, there is no general guidance on when to use each technique. In this article, we provide an objective-oriented review of the common resampling techniques for binary classification under imbalanced class sizes. The learning objectives we consider include the classical paradigm that minimizes the overall classification error, the cost-sensitive learning paradigm that minimizes a cost-adjusted weighted type I and type II errors, and the Neyman-Pearson paradigm that minimizes the type II error subject to a type I error constraint. Under each paradigm, we investigate the combination of the resampling techniques and a few state-of-the-art classification methods. For each pair of resampling techniques and classification methods, we use simulation studies to study the performance under different evaluation metrics. From these extensive simulation experiments, we demonstrate under each classification paradigm, the complex dynamics among resampling techniques, base classification methods, evaluation metrics, and imbalance ratios. For practitioners, the take-away message is that with imbalanced data, one usually should consider all the combinations of resampling techniques and the base classification methods.

Keywords: Binary classification, Imbalanced data, Resampling methods, Imbalance ratio, Classical Classification (CC) paradigm, Neyman-Pearson (NP) paradigm, Cost-Sensitive (CS) learning paradigm.

1 Introduction

Classification is a widely studied type of supervised learning problems with extensive applications. A myriad of classification methods, which we refer to as the base classification methods in this paper, have been developed to deal with all different kinds of distributions of data [Kotsiantis et al., 2007]. However, in the case where the classes are of different sizes (i.e., the imbalanced classification scenario), naively applying the existing methods could lead to undesirable results. Some prominent applications include medical diagnosis, fraud detection, spam email filtering, text

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categorization, oil spills detection in satellite radar images. To address the class size imbalance scenario, there has been extensive research on developing different methods [Sun et al., 2009, López et al., 2013, Guo et al., 2017]. Some popular tools include resampling techniques [López et al., 2013], direct methods [Ling et al., 2004, Sun et al., 2007, Zhou and Liu, 2005], post-processing methods [Castro and Braga, 2013], as well as different combinations of these tools. The most common and understandable class of approaches is resampling techniques. However, there is a lack of consensus about when and how to use them.

In this work, we aim to provide some guidelines on using resampling techniques for imbalanced binary classification. We first disentangle the general claims of undesirability in classification results under imbalanced classes, via listing a few common learning objectives and evaluation metrics. To decide which resampling technique to use, we need to be clear on the learning objectives as well as the preferred evaluation metrics. Sometimes, the chosen learning objective as the evaluation metric is not compatible, which makes the problem unsolvable by any technique. When they are, we will show that the optimal resampling technique depends on both the learning objective and the base classification method (e.g., logistic regression, support vector machines, random forest, neural networks).

There are different degrees of data imbalance. We characterize this degree by the *imbalance* ratio (IR) [García et al., 2012b], which is the ratio of the sample size of the majority class and that of the minority class. In real applications, IR can range from 1 to more than 1,000. For instance, a rare disease occurs only in 0.1% of the human population [Beaulieu et al., 2014]. We will show that different IRs might demand different combinations of resampling techniques and base classification methods.

This review conducts extensive simulation experiments to concretely illustrate the dynamics among data distributions, IR, base classification methods, learning objectives, and resampling techniques. This is the *first* time that such dynamics are explicitly examined. To the best of our knowledge, this is also the first time that a review paper uses running simulation examples to demonstrate the advantages and disadvantages of the reviewed methods. Through simulation, we give practitioners a look into the complicated nature of the imbalanced problem in classification, even if we narrow our search to the resampling techniques only. For important applications where data distributions can be approximately simulated, practitioners are encouraged to mimic our simulation studies and properly evaluate the combinations of resampling techniques and base classification methods. However, for most practitioners and most analytic jobs, the take-away message is that with imbalanced data, one should consider all the combinations of resampling techniques and the base classification methods.

The rest of the review is organized as follows. In Section 2, we describe three classification paradigms with different objectives. Then, we introduce a matrix of classification algorithms as pairs of resampling techniques and the base classification methods in Section 3. Section 4 provide a list of commonly used evaluation metrics for imbalanced classification. In Section 5, we conduct a systematic simulation study to evaluate the performance of different combinations of resampling techniques and base classification methods, under different objectives, data distributions, and IRs, in terms of various evaluation metrics. We conclude the review with a short discussion in Section

2 Three Classification Paradigms with Different Learning Objectives

In this section, we review three classification paradigms with different learning objectives. Concretely, we consider the Classification (CC) paradigm that minimizes the overall classification error (Section 2.1), the Cost-Sensitive (CS) learning paradigm that minimizes the cost-adjusted weighted type I and type II errors (Section 2.2), and the Neyman-Pearson (NP) paradigm that minimizes the type II error subject to a type I error constraint (Section 2.3).

Assume $X \in \mathcal{X} \subset \mathbb{R}^d$ is a random vector of d features, and $Y \in \{0,1\}$ is the class label. Let $\mathbb{P}(Y=0) = \pi_0$ and $\mathbb{P}(Y=1) = \pi_1 = 1 - \pi_0$. Throughout the article, we label the minority class as 0 and the majority class as 1 (i.e., $\pi_0 \leq \pi_1$). Also, for language consistency, we call class 0 as the negative class and class 1 as the positive class. Please note that the minority class might be referred to as "positive" in medical applications.

2.1 Classical Classification paradigm

A classifier is defined as $\phi: \mathcal{X} \to \{0,1\}$, which is a mapping from the feature space to the label space. The overall classification error (risk) is naturally defined as $R(\phi) = \mathbb{E}[\mathbb{I}(\phi(X) \neq Y)] = \mathbb{P}(\phi(X) \neq Y)$, where $\mathbb{I}(\cdot)$ is the indicator function. In binary classification, most existing classification methods focus on the minimization of the overall classification error (risk) [Hastie et al., 2009, James et al., 2013]. In this article, this objective is referred to as Classification (CC) Paradigm. Under this paradigm, the CC oracle ϕ^* is a classifier that minimizes the population risk; that is,

$$\phi^* = \operatorname*{arg\,min}_{\phi} R(\phi) \,.$$

It is well known that $\phi^* = \mathbb{I}(\eta(x) > 1/2)$, where $\eta(x) = \mathbb{E}(Y|X=x)$ is the regression function [Koltchinskii, 2011]. In practice, we construct a classifier $\hat{\phi}$ based on finite sample $\{(X_i, Y_i), i = 1, \dots, n\}$ using some classification method.

Popular the CC paradigm is, it may not be the ideal choice when the class sizes are imbalanced. By the *law of total probability*, we decompose the overall classification error as a weighted sum of type I and II errors, that is,

$$R(\phi) = \pi_0 R_0(\phi) + \pi_1 R_1(\phi) \,,$$

where $R_0(\phi) = \mathbb{P}(\phi(X) \neq Y|Y=0)$ denotes the (population) type I error (the conditional probability of misclassifying a class 0 observation as class 1); and $R_1(\phi) = \mathbb{P}(\phi(X) \neq Y|Y=1)$ denotes the (population) type II error (the conditional probability of misclassifying a class 1 observation as class 0). However, in many practical applications, we may want to treat type I and II errors differently under two common scenarios. One is the asymmetric error importance scenario. In this scenario, making one type of error (e.g., type I error) is more serious than making the other type of error (e.g., type II error). For instance, in severe disease diagnosis, misclassifying a diseased patient as healthy could lead to missing the optimal treatment window while misclassifying

a healthy patient as diseased can lead to patient anxiety and incur additional medical costs. The other is the *imbalanced class proportion* scenario. Under this scenario, π_0 is much smaller than π_1 , and minimizing the overall classification error could sometimes result in a larger type I error. For applications that fit these two scenarios, the overall classification error, which implies equal weights on the two types of classification errors, may not be the optimal choice to serve the users' purpose, either as an optimization criterion or as an evaluation metric. Next, we will introduce two other paradigms that have been used the address the asymmetric error importance and imbalanced class proportion issues.

2.2 Cost-Sensitive learning paradigm

In the asymmetric error importance and imbalanced class proportion scenarios introduced at the end of Section 2.1, the cost of type I error is usually higher than that of type II error. For example, in spam email filtering, the cost of misclassifying a regular email as spam is much higher than the cost of misclassifying spam as a regular email. A popular approach to incorporate different costs for these two types of errors is the Cost-Sensitive (CS) learning paradigm [Elkan, 2001, Zadrozny et al., 2003]. Let $C(\phi(X), Y)$ being the cost function for classifier ϕ at observation pair (X, Y). Let $C_0 = C(1, 0)$ and $C_1 = C(0, 1)$ being the costs of type I and II errors, respectively. For the correct classification result, we have C(0, 0) = C(1, 1) = 0. Then, CS learning minimizes the expected misclassification cost [Kuhn and Johnson, 2013]:

$$\begin{split} R_c(\phi) &= \mathbb{E}C(\phi(X), Y) \\ &= C_0 \mathbb{P}(\phi(X) = 1, Y = 0) + C_1 \mathbb{P}(\phi(X) = 0, Y = 1) \\ &= C_0 \mathbb{P}(\phi(X) = 1 | Y = 0) \mathbb{P}(Y = 0) + C_1 \mathbb{P}(\phi(X) = 0 | Y = 1) \mathbb{P}(Y = 1) \\ &= C_0 \pi_0 R_0(\phi) + C_1 \pi_1 R_1(\phi) \,. \end{split}$$

There are primarily two types of approaches in the literature on CS learning. The first type is called *direct methods*, which builds a cost-sensitive learning classifier by incorporating the different misclassification costs into the training process of the base classification method. For instance, there has been much work on CS decision tree [Ling et al., 2004, Bradford et al., 1998, Turney, 1994], CS boosting [Sun et al., 2007, Wang and Japkowicz, 2010, López et al., 2015], and CS neural network [Zhou and Liu, 2005]. The second type is usually referred to as *postprocessing methods*, in such a way that we adjust the decision threshold with the base classification algorithm unmodified. An example of this can be found in Domingos [1999]. Some additional references on cost-sensitive learning include López et al. [2012, 2013], Guo et al. [2017], Voigt et al. [2014], Zhang et al. [2016], Zou et al. [2016].

In this review, we focus on the *postprocessing methods* as it combines well with any existing base classification algorithm without the need to change its internal mechanism, which is also better understood among practitioners. In addition, it serves the purpose of making an informative comparison among different learning paradigms across different classification methods. On the

population level, with the knowledge of C_0 and C_1 , the CS oracle is

$$\phi_c^* = \operatorname*{arg\,min}_{\phi} R_c(\phi) = \mathbb{I}\left(\eta(x) > \frac{C_0}{C_0 + C_1}\right),$$

which reduces to the CC oracle ϕ^* when $C_0 = C_1$.

Although CS learning has its merits on the control of asymmetric errors, its drawback is also apparent because it is sometimes difficult or immoral to assign the value of costs C_0 and C_1 . In most applications, including the severe disease classification, these costs are unknown and cannot be easily provided by experts. One way to extricate from this dilemma is to set the majority class misclassification cost $C_1 = 1$ and the minority class misclassification cost $C_0 = \pi_1/\pi_0$ [Castro and Braga, 2013].

2.3 Neyman-Pearson paradigm

Besides requiring the knowledge of costs for different misclassification errors, the CS learning paradigm does not provide an explicit probabilistic control on type I error under a pre-specified level. Even if the practitioner tunes the empirical type I error equal to the pre-specified level, the population-level type I error still has a non-trivial chance of exceeding this level [Tong, Feng, and Zhao, 2016, Tong, Feng, and Li, 2018]. To deal with this issue, another emerging statistical framework to control asymmetric error is called Neyman-Pearson (NP) paradigm [Cannon et al., 2002, Rigollet and Tong, 2011, Tong, 2013, Tong et al., 2016, 2018], which aims to minimize type II error $R_1(\phi)$ while controlling type I error $R_0(\phi)$ under a desirable level. The corresponding NP oracle is

$$\phi_{\alpha}^* = \underset{\phi: R_0(\phi) \leq \alpha}{\operatorname{arg \, min}} R_1(\phi),$$

where α is a targeted upper bound for type I error. It can be shown that $\phi_{\alpha}^*(\cdot) = \mathbb{I}(\eta(\cdot) > D_{\alpha}^*)$ for some properly chosen D_{α}^* . Unlike 1/2 or $C_0/(C_0 + C_1)$, D_{α}^* is not known unless one has access to the distribution information. Tong et al. [2018] proposed an umbrella algorithm for NP classification, which adapts existing scoring-type classification methods (e.g., logistic regression, support vector machines, random forest) by choosing an order-statistics based thresholding level so that the resulting classifier has type I error bounded from above by α with high probability. This thresholding mechanism, along with the thresholds 1/2 and $C_0/(C_0 + C_1)$ for CC and CS paradigms respectively, will be systematically studied in combination with several state-of-the-art base classification methods in numerical studies.

2.4 A summary of three classification paradigms

For readers' convenience, we summarize the three classification paradigms with their corresponding objectives and oracle classifiers in Table 1.

3 A Matrix of Algorithms for Imbalanced Classification

In this section, we introduce a matrix of algorithms for imbalanced classification, which consists of combinations of resampling techniques and base classification methods.

Table 1: Three types of classification paradigms in binary classification.

Paradigm	Objective	Oracle Classfier
Classical	Minimize the overall classification error	$\phi^* = \arg\min_{\phi} R(\phi)$
Cost-Sensitive	Minimize the expected misclassification cost	$\phi_c^* = \arg\min_{\phi} R_c(\phi)$
Neyman-Pearson	Minimize type II error while controlling	$\phi_{\alpha}^* = \arg\min R_1(\phi)$
	type I error under α	$\phi: R_0(\phi) \leq \alpha$

To fix idea, assume among the n observation pairs $\{(X_i, Y_i), i = 1, \dots, n\}$, there are n_0 observations with $Y_i = 0$ (the minority class) and n_1 observations with $Y_i = 1$ (the majority class). Then, the *imbalance ratio* IR = n_1/n_0 .

3.1 Resampling techniques

To address the imbalanced classification problem under one of the three classification paradigms described in Section 2, resampling techniques are often used to create a new training dataset by balancing the number of data points in the minority and majority classes in order to alleviate the effect of class size imbalance in the process of classification. López et al. [2013] pointed out that about one-third of their reviewed papers have used resampling techniques. They are usually divided into three categories: undersampling, oversampling, and hybrid methods.

The undersampling methods directly discard a subset of observations of the majority class. It includes two main versions: the cluster-based undersampling and random undersampling [Yen and Lee, 2009, Kumar et al., 2014, Sun et al., 2015, Guo et al., 2017]. In the cluster-based undersampling, a clustering algorithm is applied to cluster the majority class such that the number of clusters is equal to that of the data points in the minority class (i.e., n_0 clusters). Nevertheless, the clustering process could be quite slow when n_1 is large. Random undersampling is a simpler and more efficient approach, which randomly eliminates the data points from the majority class to make it of size n_0 . By undersampling, the processed training data set is a combination of n_0 randomly chosen data points from the majority class and all (n_0) data points from the minority class. However, undersampling may lead to loss of information as a large portion of the data from the majority class is discarded.

The oversampling method, on the other hand, increases the number of data points in the minority class from n_0 to n_1 while keeping the observations from the majority class intact. The leading two approaches are random oversampling and SMOTE [Han et al., 2005, He et al., 2008, García et al., 2012a, Beaulieu et al., 2014, Nekooeimehr and Lai-Yuen, 2016]. Random oversampling, as a counterpart of random undersampling, is perhaps the most straightforward approach to duplicate the data points of minority class randomly. One version of the approach samples $n_1 - n_0$ observations with replacement from the minority class and add them to the new training set. The approach SMOTE is the acronym for the "Synthetic Minority Over-sampling Technique" proposed by Chawla et al. [2002]. It generates $n_1 - n_0$ new synthetic data points for the minority class by interpolating pairs of k nearest neighbors. We review the details of SMOTE in Algorithm 1. Compared with undersampling, oversampling methods usually require longer training time and could

cause over-fitting. A popular extension of SMOTE is the Borderline-SMOTE [Han et al., 2005], which only oversamples the minority observations near the borderline.

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Algorithm 1: SMOTE [Chawla et al., 2002]
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For any data point of minority class X_i = (X_{i1}, X_{i2}, \dots, X_{id})^{\top}, the multiple N = IR - 1, number of nearest neighbors K

Step 1: Find the K nearest neighbor points of X_i in the minority class: X_{i_1}, \dots, X_{i_k};

Step 2: for j = 1 : N do

| randomly choose one of the K nearest neighbor points: X_{i_j} = (X_{i_j1}, X_{i_j2}, \dots, X_{i_jd})^{\top};
| generate a random number r_s \sim Unif[0, 1];
| generate the synthetic data point for the minority class as X_j^* = (X_{j1}^*, X_{j2}^*, \dots, X_{jd}^*)^{\top},
| where X_{js}^* = X_{is} + r_s * (X_{i_js} - X_{is}), s = 1, \dots, d.

end

Return X_1^*, \dots, X_N^* as new synthetic data points.
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The hybrid method is just a combination of undersampling and oversampling methods [Cao et al., 2014, Cateni et al., 2014, Díez-Pastor et al., 2015, Sáez et al., 2015]. It simultaneously decreases the number of data points from the majority class and increases the number of data points from the minority class to n_h , where the above described undersampling and oversampling methods can be used. The hybrid method could serve as an option that balances the goodness of fit, computational cost as well as the robustness of the classifier.

3.2 Classification methods

Using any of the resampling methods, we will arrive at a new training dataset that has balanced classes. Naturally, we can apply any existing base classification method on this new dataset coupled with one of the learning objectives described in Section 2.

Many classification methods have been extensively studied. The well-known ones include decision trees (DT) [Safavian and Landgrebe, 1991], k-nearest neighbors (KNN) [Altman, 1992], Linear discriminant analysis (LDA) [McLachlan, 2004], logistic regression (LR) [Nelder and Wedderburn, 1972], naïve bayes (NB) [Rish et al., 2001], neural network (NN) [Rumelhart et al., 1985], random forest (RF) [Breiman, 2001], support vector machine (SVM) [Cortes and Vapnik, 1995], and XGBoost (XGB) [Chen and Guestrin, 2016], among others.

To learn more about these methods, we refer the readers to a review of classifications methods [Kotsiantis et al., 2007] and a book on statistical learning [Hastie et al., 2009].

3.3 A summary of the matrix of algorithms

In numerical studies, we consider a matrix of classification algorithms shown in Figure 1, as combinations of resampling techniques described in Section 3.1 and five (out of many) state-of-the-art classification methods described in Section 3.2. where "Original" refers to no resampling, "Under" refers to random undersampling and "Hybrid" refers to a hybrid of random undersampling and SMOTE. Note here we chose random undersampling and SMOTE as representatives of undersampling and oversampling methods due to their popularity among practitioners. The readers can easily study other types of resampling technique and classification method combinations by

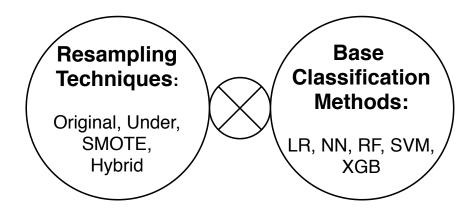


Figure 1: A summary of the matrix of algorithms.

adapting the companion code from this review.

In the numerical studies, we will conduct a comparative study on those 20 combinations described in Figure 1 under each of the three objectives introduced in Section 2 with the IR varying from 1 to 128, in terms of different evaluation metrics which will be introduced in the next section. A flowchart demonstrating our imbalanced classification system can be found in Figure 2.

4 Evaluation Metrics

In this section, we will review several popular evaluation metrics to compare the performance of different classification algorithms.

For a given classifier, suppose that it classifies the *i*-th observation X_i to \hat{Y}_i where Y_i is the true label. Then, the classification results can be summarized into the four terms: True Positives $\text{TP} = \sum_{i=1}^{n} \mathbb{I}(Y_i = 1, \hat{Y}_i = 1)$, False Positives $\text{FP} = \sum_{i=1}^{n} \mathbb{I}(Y_i = 0, \hat{Y}_i = 1)$, False Negatives $\text{FN} = \sum_{i=1}^{n} \mathbb{I}(Y_i = 1, \hat{Y}_i = 0)$, and True Negatives $\text{TN} = \sum_{i=1}^{n} \mathbb{I}(Y_i = 0, \hat{Y}_i = 0)$. These four terms are usually summarized in the so-called confusion matrix (Table 2). Note that in Table 2, the class

Table 2: Confusion matrix for a two-class problem.

	Predicted Class 0	Predicted Class 1
True Class 0	TN	FP
True Class 1	FN	TP

0 is being regarded as the "negative class". In practice, sometime we may need to set class 0 as the "positive class".

Then, the *empirical risk* can be denoted as

$$\hat{R} = \hat{\pi}_0 \hat{R}_0 + \hat{\pi}_1 \hat{R}_1 = \frac{FP + FN}{TP + FP + TN + FN},$$

where $\hat{\pi}_0 = (TN + FP)/(TP + FP + TN + FN)$, $\hat{\pi}_1 = (FN + TP)/(TP + FP + TN + FN)$ are the empirical proportions of Class 0 and 1; \hat{R}_0 and \hat{R}_1 are the empirical type I and II errors, respectively, that is,

$$\hat{R}_0 = \frac{FP}{TN + FP}, \quad \hat{R}_1 = \frac{FN}{FN + TP}.$$

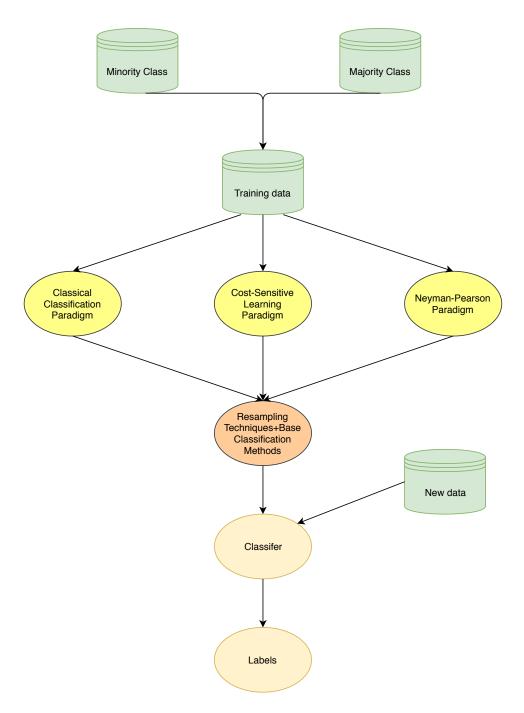


Figure 2: A flow chart for imbalanced classification with an objective-oriented view.

Similarly, for given costs C_0 and C_1 , the empirical misclassification cost is expressed as

$$\hat{R}_c = C_0 \hat{\pi}_0 \hat{R}_0 + C_1 \hat{\pi}_1 \hat{R}_1.$$

Another popular synthetic metric in the imbalanced classification literature is the F-score (also F_1 -score or F-measure, [Bradley, 1997]) for class 0, which is harmonic mean of Precision and Recall:

$$F\text{-score (class 0)} = \frac{2}{Precision_0^{-1} + Recall_0^{-1}},$$

where $Precision_0 = TN/(TN + FN)$ and $Recall_0 = TN/(TN + FP)$. Similarly, we can also define F-score for class 1 as

$$F\text{-score (class 1)} = \frac{2}{Precision_1^{-1} + Recall_1^{-1}},$$

where $Precision_1 = TP/(TP+FP)$ and $Recall_1 = TP/(TP+FN)$. Here, we set F-score (class 0) or F-score (class 1) to 0 if the corresponding precision or recall is undefined or equal to 0.

When the parameter in a classification method (e.g., the threshold of scoring functions) is varied, we usually get different trade-offs between type I and type II errors. A popular tool to visualize these trade-offs is the Receiver Operating Characteristic (ROC) curve [Bradley, 1997, Huang and Ling, 2005]. The area under the ROC curve (ROC-AUC) provides an aggregated measure for the method's performance. ROC-AUC has been used extensively to compare the performance of different classification methods. However, when the data is highly imbalanced, the ROC curves can present an overly optimistic view of classifiers' performance [Davis and Goadrich, 2006]. Precision-Recall (PR) curves and their AUCs (PR-AUC) have been advocated as an alternative metric when dealing with imbalanced data [Goadrich et al., 2004, Singla and Domingos, 2005]. Note that we also have two versions of PR-AUC, depending on which class we call "positive": PR-AUC (class 0) and PR-AUC (class 1).

Now, we summarize all of the metrics discussed in Table 3.

Table 3: Various evaluation metrics. Formula Metric (FP+FN)/(TP+FP+TN+FN)Risk Type I error(\hat{R}_0) FP/(TN + FP)FN/(FN+TP)Type II $error(R_1)$ Cost $C_0\hat{\pi}_0\hat{R}_0 + C_1\hat{\pi}_1\hat{R}_1$ $2/(Precision_0^{-1} + Recall_0^{-1})$ $2/(Precision_1^{-1} + Recall_1^{-1})$ F-score (class 0) F-score (class 1) ROC-AUC The area under the ROC curve PR-AUC (class 0) The area under the PR curve when class 0 is negative PR-AUC (class 1) The area under the PR curve when class 0 is positive

5 Simulation

In this section, we conduct extensive simulation studies to compare the matrix of 20 classification methods introduced in Section 3 under each of the three classification paradigm described in Section

2 when the IR varies, using evaluation metrics reviewed in Section 4.

5.1 Data generation process

We consider the following two examples with different data generation mechanisms.

Example 1. The conditional distributions for each class are multivariate Gaussian distributions with a common covariance matrix but different mean vectors.

Class
$$\theta: X | (Y = 0) \sim \mathcal{N}(\mu^0, \Sigma)$$
, Class $1: X | (Y = 1) \sim \mathcal{N}(\mu^1, \Sigma)$,

where $\mu^0 = (0,0,0,0,0)^{\top}$, $\mu^1 = (2,2,2,0,0)^{\top}$ and

$$\Sigma = \begin{pmatrix} 1 & 0.5 & 0.25 & 0 & 0 \\ 0.5 & 1 & 0.5 & 0 & 0 \\ 0.25 & 0.5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

To have a precise control on the imbalance ratio (IR), we explicitly generate $n_0 = 300$ observations from the minority class (class 0) and n_1 observations from the majority class, where $IR = n_1/n_0$ is a pre-specified value varying in $\{2^i, i = 0, 1, \dots, 7\}$. This leads to a training sample $\{(X_i, Y_i), i = 1, \dots, n\}$ where $n = n_0 + n_1$. Following the same mechanism, we also generate a test sample with size m consisting of $m_0 = 2000$ and $m_1 = m_0 \times IR$ observations from class 0 and 1, respectively. This generation mechanism guarantees the same IR for both training and test samples.

Since the Bayes classifier in Example 1 is linear, we expect the linear classifiers (e.g., logistic regression) to work better than the nonlinear ones. Next, we move away from the linear decision boundary.

Example 2. The conditional distributions for each class are multivariate Gaussian vs. a mixture of multivariate Gaussian. In particular,

Class 0:
$$X|(Y=1) \sim \mathcal{N}\left(\frac{1}{2}(\mu^0 + \mu^1), \Sigma\right),$$
 (1)

Class 1:
$$X|(Y=0) \sim \frac{1}{2} \mathcal{N}\left(\mu^0, \Sigma\right) + \frac{1}{2} \mathcal{N}\left(\mu^1, \Sigma\right),$$
 (2)

where μ^0 , μ^1 and Σ are the same as Example 1. Here, we consider $IR \in \{1, 128\}$. The remaining data generation mechanism is the same as in Example 1.

5.2 Implementation details

Regarding the resampling methods, we consider the following four options.

- No resampling (Original): we use the training dataset as it is without any modificcation.
- Random undersampling (Under): we keep all the n_0 observations in the minority class and randomly sample n_0 observations without replacement from the majority class. Then, we have a balanced data set in which each class is of size n_0 .

- Oversampling (SMOTE): we keep all the n_1 observations in the majority class. We use SMOTE (R Package **smotefamliy**, v1.3.1, Siriseriwan 2019) to generate new synthetic data for the minority class until the new training set is balanced. Then, we have a balanced data set in which each class is of size n_1 . Following the default choice in **smotefamily**, we set the number of nearest neighbors K = 5 in the oversampling process.
- Hybrid methods (Hybrid): we conduct a combination of random undersampling and SMOTE with the final training set consists of n_h minority and majority observations with $n_h = \lfloor \sqrt{n_0 * n_1} / n_0 \rfloor * n_0$ where $\lfloor \cdot \rfloor$ is the floor function.

Regarding the base classification methods, we apply the following R packages or functions with their default parameters.

- Logistic regression (glm function in base R).
- Neural network (R Package **deepNN**, v0.3, Taylor 2019). Here, we use a neural network with one hidden layer consisting of five neurons. We use the ReLU activation function [Nair and Hinton, 2010] in the input layer and cross-entropy loss for the output.
- Random forest (R Package randomForest, v4.6.14, Liaw and Wiener 2002).
- Support vector machine (R Package e1071, v1.7.2, Meyer et al. 2019).
- XGBoost (R Package **xgboost**, v0.90.0.2, Chen et al. 2019).

Regarding the classification paradigms, some specifics are listed as below.

- CS learning paradigm: we specify the cost $C_0 = IR$ and $C_1 = 1$.
- NP paradigm: we use the NP umbrella algorithm as implemented in R package **nproc** v2.1.4, and set $\alpha = 0.05$ and the tolerance level $\delta = 0.5$.

Denote by |S| the cardinality of a set S. Let $O = \{CC, CS, NP\}$, $T = \{Original, Under, SMOTE, Hybrid\}$, $C = \{LR, NN, RF, SVM, XGB\}$ and $B = \{2^i, i = 0, 1, 2, ..., 7\}$. Hence, there are $|O| \times |T| \times |C| \times |B|$ (480) classification systems studied in this paper for a given imbalanced classification problem, as illustrated in Figure 3.

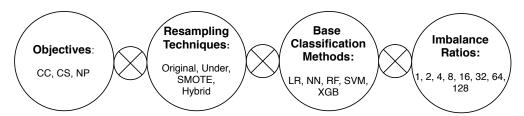


Figure 3: All of the classification systems studied in this paper.

For each ensemble system, we evaluate the performance of different classifiers in terms the following metrics reviewed in Section 4: overall classification error (Risk), type I error, type II

error, expected misclassification cost (Cost), F-score (class 0) and F-score (class 1). When the threshold varies for each classification method, we also report the area under ROC curve (ROC-AUC) and the area under PR curve (PR-AUC (class 0) and PR-AUC (class 1)).

We repeat the experiment 100 times and report the average performance in terms of mean and standard errors for each metric and classification paradigm combination. The results are summarized in Figures 4 to 24 as well as in Tables 4, 5 and 6.

5.3 Results and interpretations

For each figure, we present the results of a matrix of classification methods under each IR in the first five panels with the last panel showing the optimal combination of resampling technique and base classification method under each IR.

Next, we provide some interpretations and insights from the figures and tables under each classification paradigm.

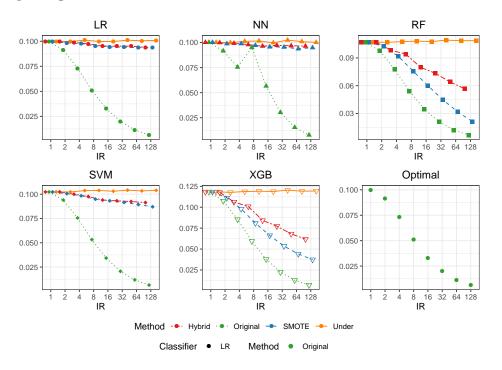


Figure 4: Risk of different methods under the CC paradigm in Example 1. The minimum and maximum of standard error: LR(0, 0.0009), NN(0, 0.0026), RF(0, 0.0012), SVM(0, 0.0011), XGB(0, 0.0012).

5.3.1 Classical classification paradigm.

We first focus on analyzing the results for Example 1. Figure 4 exhibits the risk of different methods. We observe that the empirical risk of all classifiers without resampling is smaller than that with any resampling technique, and decreases as IR increases. This is in line with our intuition that if the risk is the primary measure of interest, we would be better off not applying any resampling techniques. In addition, we observe that only undersampling leads to a stable risk when the IR

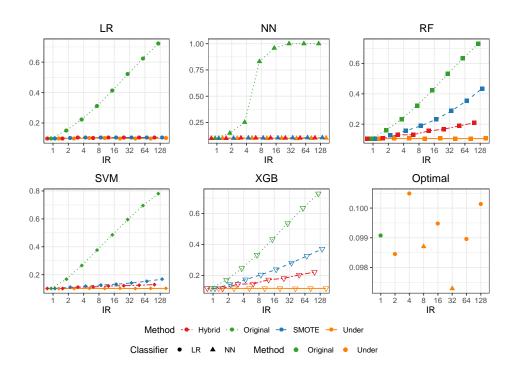


Figure 5: Type I error of different methods under the CC paradigm in Example 1. The minimum and maximum of standard error: $LR(0.0009,\,0.0021)$, $NN(0,\,0.0275)$, $RF(0.0012,\,0.0023)$, $SVM(0.0011,\,0.0025)$, $XGB(0.0013,\,0.0022)$.

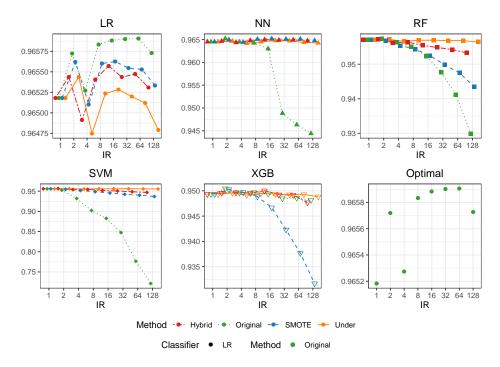


Figure 6: ROC-AUC of different methods under the CC paradigm in Example 1. The minimum and maximum of standard error: $LR(0.0001,\ 0.0002),\ NN(0.0002,\ 0.0106),\ RF(0.0002,\ 0.0005),\ SVM(0.0003,\ 0.0033),\ XGB(0.0003,\ 0.0007).$

increases for all five base classification methods considered. We report the range of the standard errors for each base classification method in the caption of Figure 4, and they are all reasonably small. Finally, the last panel shows Original (no resampling) combined with LR leads to the optimal performance across all IRs.

As mentioned in Section 2, minimizing the risk with imbalanced data could lead to large type I errors, demonstrated clearly in Figure 5. By using the resampling techniques, however, we can have much better control over type I error as IR increases. In particular, SMOTE and Hybrid work well for LR and NN, while undersampling works well for all five classifiers. Lastly, we note that the optimal choice when IR > 1 involves resampling techniques.

Now, we look at the ROC-AUC in Figure 6 as an overall measure of classification methods without the need to specify the classification paradigm. First of all, LR is surprisingly stable for all resampling techniques across all IRs. Then, from the panels corresponding to NN, RF, SVM, and XGB, we suggest that it is essential to apply specific resampling techniques to keep the ROC-AUC at a high value when IR increases.

Now, we present the results for Example 2 in Figure 7¹. This figure shows a similar message as in Example 1 that we do not need any resampling if the goal is to minimize the risk. On the other hand, applying certain resampling techniques is critical to bring down the type I error and increase the ROC-AUC value. Overall, we see that in this example, undersampling combined with SVM leads to the smallest type I error and the largest ROC-AUC value.

For readers' convenience, we summarize in Table 4 the optimal combination of resampling techniques and classification methods in terms of each evaluation metric when IR equals 1 and 128 in Examples 1 and 2. Clearly, the optimal choices differ for different evaluation metrics, IRs, and data generation mechanisms.

Table 4: Optimal combination of resampling technique and base classification method under CC paradigm.

	Example 1		Example 2	
Metric	IR=1	IR=128	IR=1	IR=128
Risk	Original+LR	Original+LR	Original+SVM	Original+LR
Type I error	Original+LR	Under+LR	Original+SVM	Under+SVM
Type II error	Original+LR	Original+NN	Original+SVM	Original+LR
F-score (class 0)	Original+LR	Original+LR	Original+SVM	Under+SVM
F-score (class 1)	Original+LR	Original+LR	Original+SVM	Original+LR
ROC-AUC	Original+LR	Original+LR	Original+SVM	Under+SVM
PR-AUC (class 0)	Original+LR	Original+LR	Original+SVM	Under+SVM
PR-AUC (class 1)	Original+LR	Original+LR	${\bf Original + SVM}$	Under+SVM

 $^{^{1}}$ The minimum and maximum of standard error of risk: LR(0, 0.0015), RF(0, 0.0021), SVM(0, 0.0018), XGB(0, 0.0017). The minimum and maximum of standard error of type I error: LR(0, 0.0022), RF(0, 0.0025), SVM(0, 0.0025), XGB(0, 0.0023). The minimum and maximum of standard error of ROC-AUC: LR(0.0005, 0.0008), RF(0.0008, 0.0010), SVM(0.0007, 0.0016), XGB(0.0008, 0.0013).

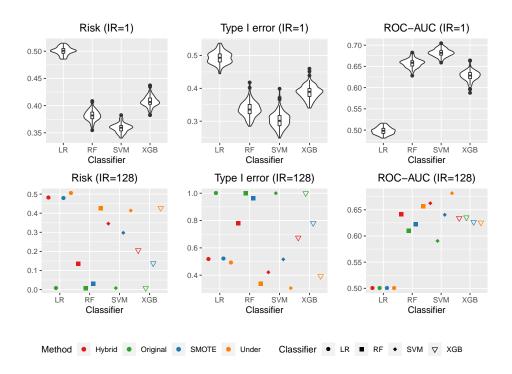


Figure 7: The risk, type I error and ROC-AUC under CC paradigm in Example 2.

5.3.2 Cost-Sensitive learning paradigm.

When we are in the CS learning paradigm, the objective is to minimize the expected total misclassification cost. We again first look at the results from Example 1. Naturally, we would like to see the impact of the resampling techniques on different classification methods in terms of empirical cost, which is summarized in Figure 8. From the figure, we observe that no resampling leads to the smallest cost for LR and RF. When IR is large, undersampling leads to the smallest cost for NN, SVM, and XGB. It is worth noting that LR, coupled with no resampling, works the best again across all IRs.

Now, we look at the results for type I error in Figure 9, where we discover that all classification methods benefit significantly from resampling techniques with undersampling being the best choice for most scenarios. One interesting observation is that without resampling, only NN has a low type I error when IR is very large.

We report the results for type II error (Figure 14), F-score (class 0) (Figure 15), F-score (class 1) (Figure 16) in the Appendix.

For Example 2, the results are summarized in Figure 10². The figure shows that without any resampling leads to a reasonably small cost. However, applying SMOTE can further reduce the cost for RF and XGB. As in the CC paradigm, applying resampling techniques can reduce the type I error.

²The minimum and maximum of standard error of cost: LR (0, 0.0011), RF(0, 0.0019), SVM(0, 0.0024), XGB(0.0010, 0.0019). The minimum and maximum of standard error of type I error: LR (0, 0.0046), RF(0, 0.0045), SVM(0, 0.0081), XGB(0.005, 0.0034).

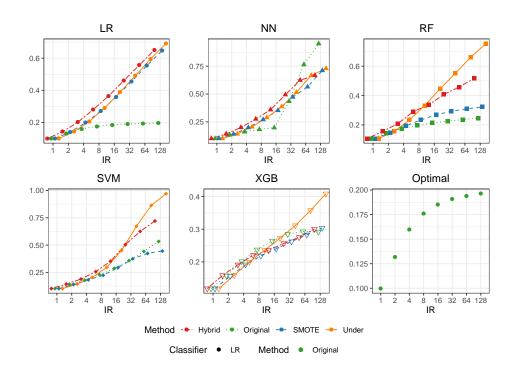


Figure 8: Cost of different methods under CS learning Paradigm in Example 1. The minimum and maximum of standard error: $LR(0.0004,\,0.0053)$, $NN(0.0005,\,0.0245)$, $RF(0.0005,\,0.0059)$, $SVM(0.0005,\,0.0158)$, $XGB(0.0007,\,0.0039)$.

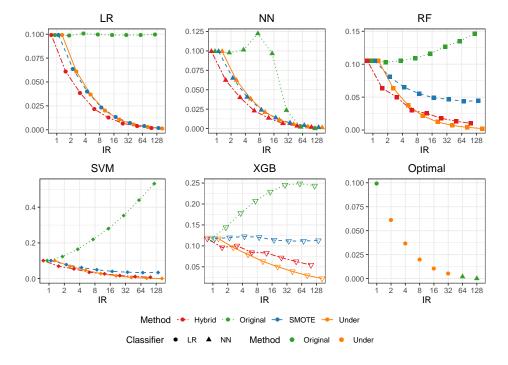


Figure 9: Type I error of different methods under CS learning Paradigm in Example 1. The minimum and maximum of standard error: LR(0.0001, 0.0010), NN(0, 0.0045), RF(0.0002, 0.0012), SVM(0.0001, 0.0022), XGB(0.0006, 0.0019).

Again, we summarize in Table 5 for the CS learning paradigm the optimal combination of resampling techniques and classification methods in terms of each evaluation metric when IR equals 1 and 128.

Table 5: Optimal combinations of resampling technique and base classification method under CS learning paradigm.

	Example 1		Example 2	
Metric	IR=1	IR=128	IR=1	IR=128
Cost	Original+LR	Original+LR	Original+SVM	SMOTE+XGB
Type I error	Original+LR	Original+NN	Original+SVM	Under+SVM
Type II error	Original+LR	Original+LR	Original+SVM	Original+XGB
F-score (class 0)	Original+LR	Original+LR	Original+SVM	SMOTE+XGB
F-score (class 1)	Original+LR	Original+LR	Original+SVM	Original + XGB

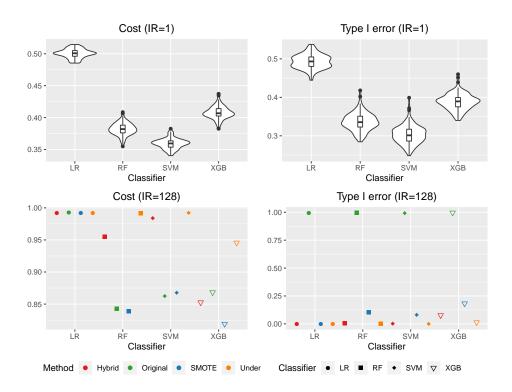


Figure 10: The cost and type I error under CS learning paradigm in Example 2.

5.3.3 Neyman-Pearson paradigm.

The NP paradigm aims to minimize type II error while controlling type I error under a target level α . In the current implementation, we set $\alpha = 0.05$. From Figures 11 and 13, we observe that the type I errors are well-controlled under α throughout all IRs for all base classification methods in both Examples 1 and 2.

When we look at Figure 12, the benefits that resampling techniques can bring are apparent. Except for LR, all four methods have their type II error decreased significantly by specific re-

sampling techniques. The most significant improvement can be observed in RF and SVM, where no resampling results in a type II error close to 1 at IR = 128, while undersampling or hybrid resampling leads to a type II error well under control.

For Example 1, we report the results for risk (Figure 22), F-score (class 0) (Figure 23), F-score (class 1) (Figure 24) in the Appendix.

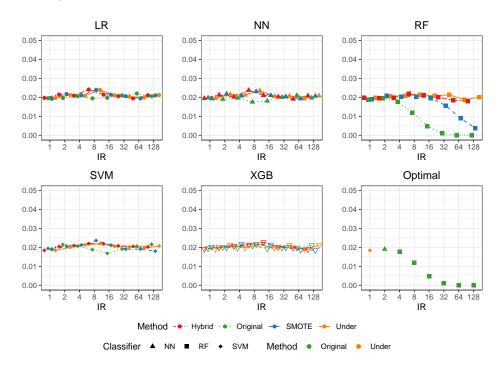


Figure 11: Type I error of different methods under NP paradigm in Example 1. The minimum and maximum of standard error: LR(0.0009, 0.0016), NN(0.0009, 0.0015), RF(0, 0.0015), SVM(0.0008, 0.0014), XGB(0.0009, 0.0013).

For Example 2, we report the results of type I and II errors in Figure 13³. Like in Example 1, resampling techniques help to reduce type II error with the type I error well-controlled under α .

Lastly, we summarize in Table 6 the optimal combination of resampling techniques and classification methods in terms of each evaluation metric when IR equals 1 and 128. It shows that for most evaluation metrics, LR with no resampling is the choice for Example 1, and RF with SMOTE (IR=1) or undersampling (IR=128) works the best for Example 2.

6 Discussion

In this paper, we review the imbalanced classification with an objective-oriented view. The main message from the review is that there is no single best approach to imbalanced classification. The optimal choice for resampling techniques and base classification methods highly depend on the

³The minimum and maximum of standard error of type I error: LR (0.0011, 0.0013), RF(0, 0.0013), SVM(0.0009, 0.0015), XGB(0.0011, 0.0014). The minimum and maximum of standard error of type II error: LR (0.0013, 0.0025), RF(0, 0.0044), SVM(0.0025, 0.0046), XGB(0.0028, 0.0037).

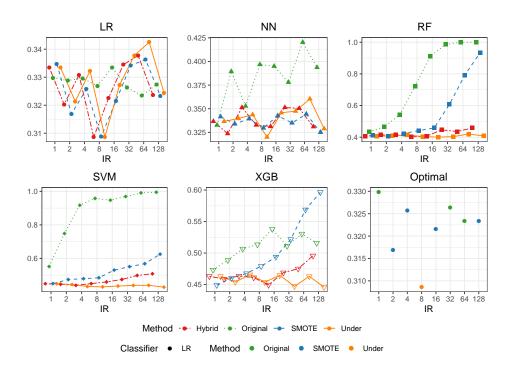


Figure 12: Type II error of different methods under NP paradigm in Example 1. The minimum and maximum of standard error: $LR(0.0079,\,0.0106)$, $NN(0.0079,\,0.0246)$, $RF(0,\,0.0295)$, $SVM(0.0012,\,0.0201)$, $XGB(0.0112,\,0.0154)$.

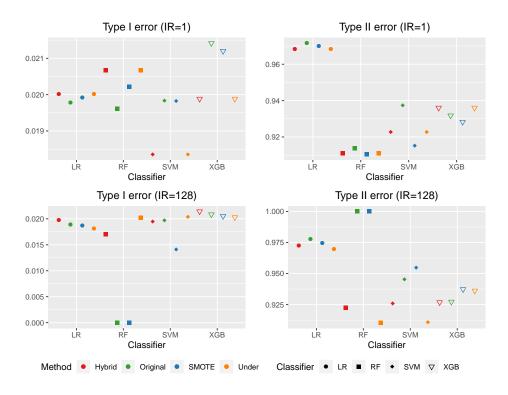


Figure 13: The type I and II errors under NP paradigm in Example 2.

Table 6: Optimal combination of resampling technique and base classification method under Neyman-Pearson paradigm in Example 1.

	Example 1		Example 2	
Metric	IR=1	IR=128	IR=1	IR=128
Type I error	Under+SVM	Original+RF	Under+SVM	Original+RF
Type II error	Original+LR	Original+NN	SMOTE+RF	Under+RF
Risk	Original+LR	Original+LR	SMOTE+RF	Under+RF
F-score (class 0)	Original+LR	Original+LR	SMOTE+RF	Under+RF
F-score (class 1)	Original+LR	SMOTE+LR	SMOTE+RF	Under+RF

classification objectives, evaluation metric, as well as the severity of imbalanceness (imbalance ratio).

Admitted, we only considered a selective list of resampling techniques and base classification methods. There are many other combinations that are worth further consideration. In addition, we presented results from two data generation processes, which could be unrepresentative for specific applications. We suggest practitioners to adapt our analysis process for evaluating different choices for imbalanced classification to align with their data generation mechanism.

Lastly, we focused on binary classification throughout the review. We expect similar interpretations and conclusions from multi-class imbalanced classification.

References

Naomi S Altman. An introduction to kernel and nearest-neighbor nonparametric regression. *The American Statistician*, 46(3):175–185, 1992.

Chandree L Beaulieu, Jacek Majewski, Jeremy Schwartzentruber, Mark E Samuels, Bridget A Fernandez, Francois P Bernier, Michael Brudno, Bartha Knoppers, Janet Marcadier, David Dyment, et al. Forge canada consortium: outcomes of a 2-year national rare-disease genediscovery project. The American Journal of Human Genetics, 94(6):809–817, 2014.

Jeffrey P Bradford, Clayton Kunz, Ron Kohavi, Cliff Brunk, and Carla E Brodley. Pruning decision trees with misclassification costs. In *European Conference on Machine Learning*, pages 131–136. Springer, 1998.

Andrew P Bradley. The use of the area under the roc curve in the evaluation of machine learning algorithms. *Pattern recognition*, 30(7):1145–1159, 1997.

Leo Breiman. Random forests. Machine learning, 45(1):5–32, 2001.

Adam Cannon, James Howse, Don Hush, and Clint Scovel. Learning with the neyman-pearson and min-max criteria. Los Alamos National Laboratory, Tech. Rep. LA-UR, pages 02–2951, 2002.

Peng Cao, Jinzhu Yang, Wei Li, Dazhe Zhao, and Osmar Zaiane. Ensemble-based hybrid probabilistic sampling for imbalanced data learning in lung nodule cad. *Computerized Medical Imaging and Graphics*, 38(3):137–150, 2014.

- Cristiano L Castro and Antônio P Braga. Novel cost-sensitive approach to improve the multilayer perceptron performance on imbalanced data. *IEEE transactions on neural networks and learning systems*, 24(6):888–899, 2013.
- Silvia Cateni, Valentina Colla, and Marco Vannucci. A method for resampling imbalanced datasets in binary classification tasks for real-world problems. *Neurocomputing*, 135:32–41, 2014.
- Nitesh V Chawla, Kevin W Bowyer, Lawrence O Hall, and W Philip Kegelmeyer. Smote: synthetic minority over-sampling technique. *Journal of artificial intelligence research*, 16:321–357, 2002.
- Tianqi Chen and Carlos Guestrin. Xgboost: A scalable tree boosting system. In *Proceedings of the 22nd acm sigkdd international conference on knowledge discovery and data mining*, pages 785–794. ACM, 2016.
- Tianqi Chen, Tong He, Michael Benesty, Vadim Khotilovich, Yuan Tang, Hyunsu Cho, Kailong Chen, Rory Mitchell, Ignacio Cano, Tianyi Zhou, Mu Li, Junyuan Xie, Min Lin, Yifeng Geng, and Yutian Li. *xgboost: Extreme Gradient Boosting*, 2019. URL https://CRAN.R-project.org/package=xgboost. R package version 0.90.0.2.
- Corinna Cortes and Vladimir Vapnik. Support-vector networks. *Machine learning*, 20(3):273–297, 1995.
- Jesse Davis and Mark Goadrich. The relationship between precision-recall and roc curves. In *Proceedings of the 23rd international conference on Machine learning*, pages 233–240. ACM, 2006.
- José F Díez-Pastor, Juan J Rodríguez, César García-Osorio, and Ludmila I Kuncheva. Random balance: ensembles of variable priors classifiers for imbalanced data. *Knowledge-Based Systems*, 85:96–111, 2015.
- Pedro Domingos. Metacost: A general method for making classifiers cost-sensitive. In *KDD*, volume 99, pages 155–164, 1999.
- Charles Elkan. The foundations of cost-sensitive learning. In *International joint conference on artificial intelligence*, volume 17, pages 973–978. Lawrence Erlbaum Associates Ltd, 2001.
- Vicente García, José Salvador Sánchez, Raúl Martín-Félez, and Ramón Alberto Mollineda. Surrounding neighborhood-based smote for learning from imbalanced data sets. *Progress in Artificial Intelligence*, 1(4):347–362, 2012a.
- Vicente García, José Salvador Sánchez, and Ramón Alberto Mollineda. On the effectiveness of preprocessing methods when dealing with different levels of class imbalance. *Knowledge-Based Systems*, 25(1):13–21, 2012b.
- Mark Goadrich, Louis Oliphant, and Jude Shavlik. Learning ensembles of first-order clauses for recall-precision curves: A case study in biomedical information extraction. In *International Conference on Inductive Logic Programming*, pages 98–115. Springer, 2004.

- Haixiang Guo, Yijing Li, Jennifer Shang, Mingyun Gu, Yuanyue Huang, and Bing Gong. Learning from class-imbalanced data: Review of methods and applications. *Expert Systems with Applications*, 73:220–239, 2017.
- Hui Han, Wen-Yuan Wang, and Bing-Huan Mao. Borderline-smote: a new over-sampling method in imbalanced data sets learning. In *International conference on intelligent computing*, pages 878–887. Springer, 2005.
- Trevor Hastie, Robert Tibshirani, and Jerome Friedman. The elements of statistical learning: data mining, inference, and prediction. Springer Science & Business Media, 2009.
- Haibo He, Yang Bai, Edwardo A Garcia, and Shutao Li. Adasyn: Adaptive synthetic sampling approach for imbalanced learning. In 2008 IEEE International Joint Conference on Neural Networks (IEEE World Congress on Computational Intelligence), pages 1322–1328. IEEE, 2008.
- Jin Huang and Charles X Ling. Using auc and accuracy in evaluating learning algorithms. *IEEE Transactions on knowledge and Data Engineering*, 17(3):299–310, 2005.
- Gareth James, Daniela Witten, Trevor Hastie, and Robert Tibshirani. *An introduction to statistical learning*, volume 112. Springer, 2013.
- Vladimir Koltchinskii. Oracle Inequalities in Empirical Risk Minimization and Sparse Recovery Problems: Ecole dEté de Probabilités de Saint-Flour XXXVIII-2008, volume 2033. Springer Science & Business Media, 2011.
- Sotiris B Kotsiantis, I Zaharakis, and P Pintelas. Supervised machine learning: A review of classification techniques. *Emerging artificial intelligence applications in computer engineering*, 160:3–24, 2007.
- Max Kuhn and Kjell Johnson. Applied predictive modeling, volume 26. Springer, 2013.
- N Santhosh Kumar, K Nageswara Rao, A Govardhan, K Sudheer Reddy, and Ali Mirza Mahmood. Undersampled k-means approach for handling imbalanced distributed data. *Progress in Artificial Intelligence*, 3(1):29–38, 2014.
- Andy Liaw and Matthew Wiener. Classification and regression by randomforest. R News, 2(3): 18–22, 2002. URL https://CRAN.R-project.org/doc/Rnews/.
- Charles X Ling, Qiang Yang, Jianning Wang, and Shichao Zhang. Decision trees with minimal costs. In *Proceedings of the twenty-first international conference on Machine learning*, page 69. ACM, 2004.
- Victoria López, Alberto Fernández, Jose G Moreno-Torres, and Francisco Herrera. Analysis of preprocessing vs. cost-sensitive learning for imbalanced classification. open problems on intrinsic data characteristics. *Expert Systems with Applications*, 39(7):6585–6608, 2012.

- Victoria López, Alberto Fernández, Salvador García, Vasile Palade, and Francisco Herrera. An insight into classification with imbalanced data: Empirical results and current trends on using data intrinsic characteristics. *Information sciences*, 250:113–141, 2013.
- Victoria López, Sara Del Río, José Manuel Benítez, and Francisco Herrera. Cost-sensitive linguistic fuzzy rule based classification systems under the mapreduce framework for imbalanced big data. Fuzzy Sets and Systems, 258:5–38, 2015.
- Geoffrey J McLachlan. Discriminant analysis and statistical pattern recognition, volume 544. John Wiley & Sons, 2004.
- David Meyer, Evgenia Dimitriadou, Kurt Hornik, Andreas Weingessel, and Friedrich Leisch. e1071: Misc Functions of the Department of Statistics, Probability Theory Group (Formerly: E1071), TU Wien, 2019. URL https://CRAN.R-project.org/package=e1071. R package version 1.7-2.
- Vinod Nair and Geoffrey E Hinton. Rectified linear units improve restricted boltzmann machines. In *Proceedings of the 27th international conference on machine learning (ICML-10)*, pages 807–814, 2010.
- Iman Nekooeimehr and Susana K Lai-Yuen. Adaptive semi-unsupervised weighted oversampling (a-suwo) for imbalanced datasets. *Expert Systems with Applications*, 46:405–416, 2016.
- John Ashworth Nelder and Robert WM Wedderburn. Generalized linear models. *Journal of the Royal Statistical Society: Series A (General)*, 135(3):370–384, 1972.
- Philippe Rigollet and Xin Tong. Neyman-pearson classification, convexity and stochastic constraints. *Journal of Machine Learning Research*, 12(Oct):2831–2855, 2011.
- Irina Rish et al. An empirical study of the naive bayes classifier. In *IJCAI 2001 workshop on empirical methods in artificial intelligence*, volume 3, pages 41–46, 2001.
- David E Rumelhart, Geoffrey E Hinton, and Ronald J Williams. Learning internal representations by error propagation. Technical report, California Univ San Diego La Jolla Inst for Cognitive Science, 1985.
- José A Sáez, Julián Luengo, Jerzy Stefanowski, and Francisco Herrera. Smote—ipf: Addressing the noisy and borderline examples problem in imbalanced classification by a re-sampling method with filtering. *Information Sciences*, 291:184–203, 2015.
- S Rasoul Safavian and David Landgrebe. A survey of decision tree classifier methodology. *IEEE transactions on systems, man, and cybernetics*, 21(3):660–674, 1991.
- Parag Singla and Pedro Domingos. Discriminative training of markov logic networks. In AAAI, volume 5, pages 868–873, 2005.
- Wacharasak Siriseriwan. smotefamily: A Collection of Oversampling Techniques for Class Imbalance Problem Based on SMOTE, 2019. URL https://CRAN.R-project.org/package=smotefamily. R package version 1.3.1.

- Yanmin Sun, Mohamed S Kamel, Andrew KC Wong, and Yang Wang. Cost-sensitive boosting for classification of imbalanced data. *Pattern Recognition*, 40(12):3358–3378, 2007.
- Yanmin Sun, Andrew KC Wong, and Mohamed S Kamel. Classification of imbalanced data: A review. *International Journal of Pattern Recognition and Artificial Intelligence*, 23(04):687–719, 2009.
- Zhongbin Sun, Qinbao Song, Xiaoyan Zhu, Heli Sun, Baowen Xu, and Yuming Zhou. A novel ensemble method for classifying imbalanced data. *Pattern Recognition*, 48(5):1623–1637, 2015.
- Benjamin Taylor. deepNN: Deep Learning, 2019. URL https://CRAN.R-project.org/package=deepNN. R package version 0.3.
- Xin Tong. A plug-in approach to neyman-pearson classification. The Journal of Machine Learning Research, 14(1):3011–3040, 2013.
- Xin Tong, Yang Feng, and Anqi Zhao. A survey on neyman-pearson classification and suggestions for future research. Wiley Interdisciplinary Reviews: Computational Statistics, 8(2):64–81, 2016.
- Xin Tong, Yang Feng, and Jingyi Jessica Li. Neyman-pearson classification algorithms and np receiver operating characteristics. *Science advances*, 4(2):eaao1659, 2018.
- Peter D Turney. Cost-sensitive classification: Empirical evaluation of a hybrid genetic decision tree induction algorithm. *Journal of artificial intelligence research*, 2:369–409, 1994.
- Tobias Voigt, Roland Fried, Michael Backes, and Wolfgang Rhode. Threshold optimization for classification in imbalanced data in a problem of gamma-ray astronomy. *Advances in Data Analysis and Classification*, 8(2):195–216, 2014.
- Benjamin X Wang and Nathalie Japkowicz. Boosting support vector machines for imbalanced data sets. *Knowledge and information systems*, 25(1):1–20, 2010.
- Show-Jane Yen and Yue-Shi Lee. Cluster-based under-sampling approaches for imbalanced data distributions. Expert Systems with Applications, 36(3):5718–5727, 2009.
- Bianca Zadrozny, John Langford, and Naoki Abe. Cost-sensitive learning by cost-proportionate example weighting. In *ICDM*, volume 3, page 435, 2003.
- Chenggang Zhang, Wei Gao, Jiazhi Song, and Jinqing Jiang. An imbalanced data classification algorithm of improved autoencoder neural network. In 2016 Eighth International Conference on Advanced Computational Intelligence (ICACI), pages 95–99. IEEE, 2016.
- Zhi-Hua Zhou and Xu-Ying Liu. Training cost-sensitive neural networks with methods addressing the class imbalance problem. *IEEE Transactions on knowledge and data engineering*, 18(1): 63–77, 2005.
- Quan Zou, Sifa Xie, Ziyu Lin, Meihong Wu, and Ying Ju. Finding the best classification threshold in imbalanced classification. *Big Data Research*, 5:2–8, 2016.

Appendix

The appendix contains additional figures from the simulation results.

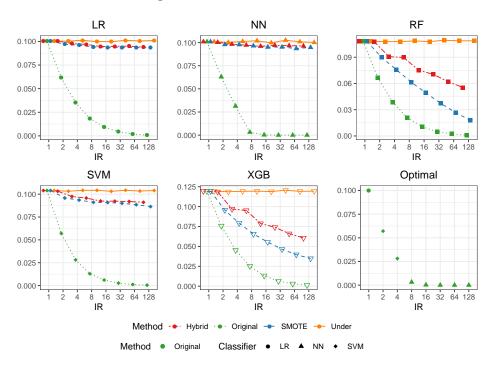


Figure 14: Type II error of different methods under CC paradigm in Example 1. The minimum and maximum of standard error: LR(0, 0.0011), NN(0, 0.0013), RF(0, 0.0014), SVM(0, 0.0013), XGB(0, 0.0015).

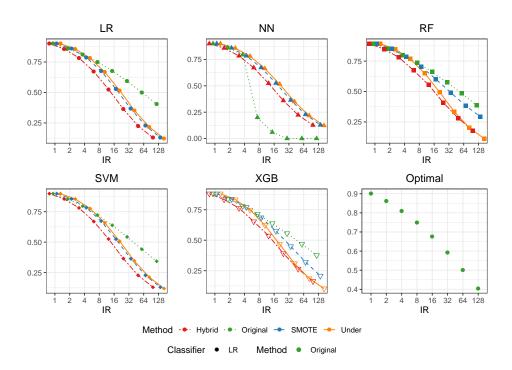


Figure 15: F-score (class 0) of different methods under CC paradigm. The minimum and maximum of standard error: LR(0.0005, 0.0017), NN(0, 0.0322), RF(0.0005, 0.0020), SVM(0.0005, 0.0025), XGB(0.0007, 0.0020).

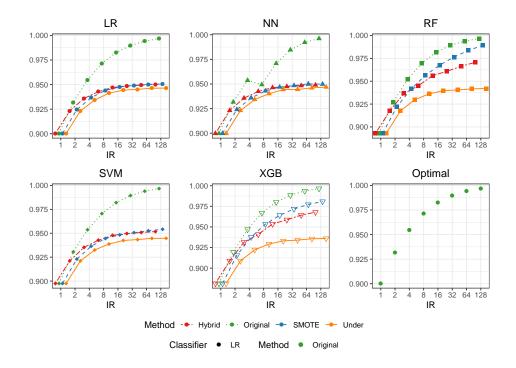


Figure 16: F-score (class 1) of different methods under CC paradigm. The minimum and maximum of standard error: LR(0, 0.0005), NN(0, 0.0013), RF(0, 0.0007), SVM(0, 0.0006), XGB(0, 0.0007).

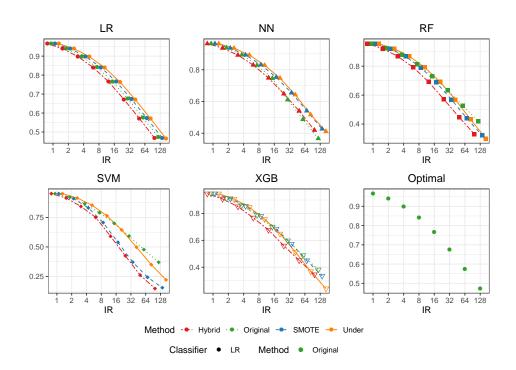


Figure 17: PR-AUC (Class 0) of different methods under CC paradigm in Example 1. The minimum and maximum of standard error: LR(0.0002, 0.0014), NN(0.0003, 0.0083), RF(0.0004, 0.0030), SVM(0.0005, 0.0041), XGB(0.0006, 0.0048).

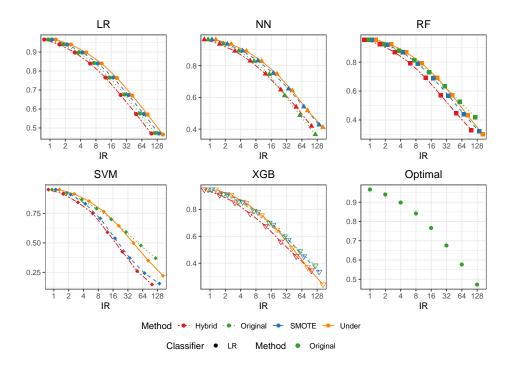


Figure 18: PR-AUC (Class 1) of different methods under CC paradigm in Example 1. The minimum and maximum of standard error: LR(0, 0.0003), NN(0, 0.0006), RF(0, 0.0004), SVM(0, 0.0005), XGB(0, 0.0006).

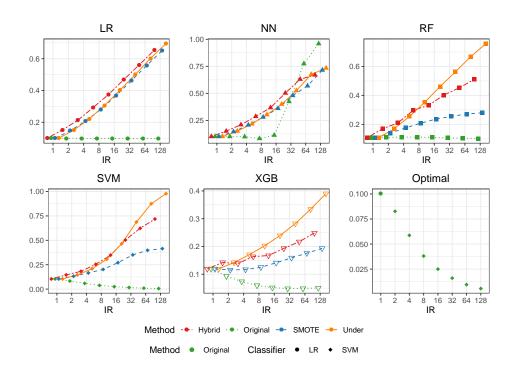


Figure 19: Type II error of different methods under CS learning paradigm in Example 1. The minimum and maximum of standard error: LR(0.0006, 0.0054), NN(0.0010, 0.0271), RF(0.0006, 0.0062), SVM(0.0001, 0.0165), XGB(0.0004, 0.0042).

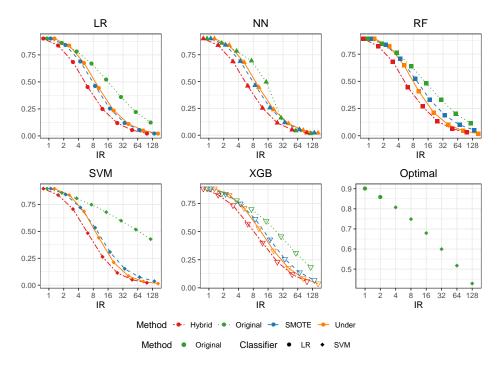


Figure 20: F-score (class 0) of different methods under CS learning paradigm in Example 1. The minimum and maximum of standard error: LR(0.0002, 0.0021), NN(0.0001, 0.0074), RF(0.0002, 0.0022), SVM(0.0001, 0.0034), XGB(0.0004, 0.0021).

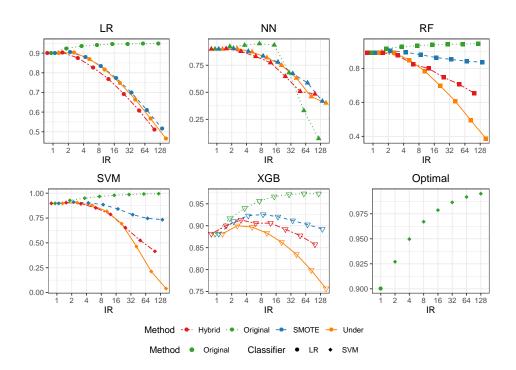


Figure 21: F-score (class 1) of different methods under CS learning paradigm in Example 1. The minimum and maximum of standard error: LR(0.0003 , 0.0063), NN(0.0004, 0.0295), RF(0.0003, 0.0076), SVM(0, 0.0198), XGB(0.0002, 0.0033).

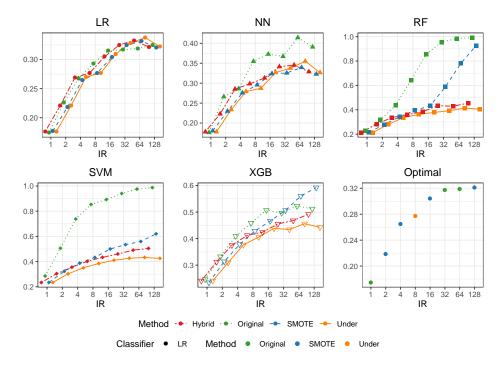


Figure 22: Risk of different methods under NP paradigm in Example 1. The minimum and maximum of standard error: LR(0.0036, 0.0104), NN(0.0035, 0.0242), RF(0, 0.0283), SVM(0.001, 0.0199), XGB(0.0054, 0.0152).

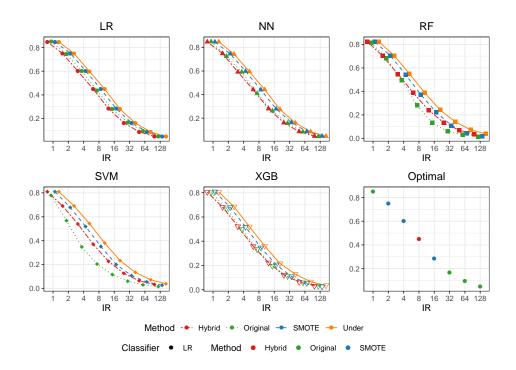


Figure 23: F-score (class 0) of different methods under NP paradigm in Example 1. The minimum and maximum of standard error: LR(0.0011, 0.0065), NN(0.0011, 0.0087), RF(0, 0.0092), SVM(0, 0.0086), XGB(0.0005, 0.0072).

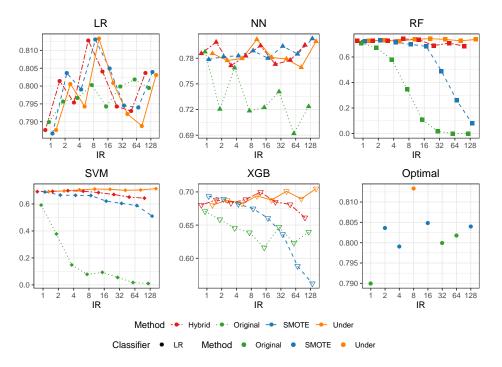


Figure 24: F-score (class 1) of different methods under NP paradigm in Example 1. The minimum and maximum of standard error: LR(0.0056, 0.0083), NN(0.0056, 0.0273), RF(0, 0.0368), SVM(0.0022, 0.0245), XGB(0.0099, 0.0151).