Fair Correlation Clustering

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Abstract

In this paper, we study correlation clustering under fairness constraints. Fair variants of k-median and k-center clustering have been studied recently, and approximation algorithms using a notion called fairlet decomposition have been proposed. We obtain approximation algorithms for fair correlation clustering under several important types of fairness constraints.

Our results hinge on obtaining a fairlet decomposition for correlation clustering by introducing a novel combinatorial optimization problem. We show how this problem relates to k-median fairlet decomposition and how this allows us to obtain approximation algorithms for a wide range of fairness constraints.

We complement our theoretical results with an in-depth analysis of our algorithms on real graphs where we show that fair solutions to correlation clustering can be obtained with limited increase in cost compared to the state-of-the-art (unfair) algorithms.

1 Introduction

There is a growing literature on fairness in various learning and optimization problems [33, 32, 30, 15, 45, 14, 20]. The goal of this literature is to develop criteria and algorithms to ensure that we can find solutions for optimization/learning problems that are fair with respect to a certain sensitive feature. In the case of clustering, a fundamental unsupervised learning and optimization problem, the study of fairness was initiated by Chierichetti et al. [19]. They formulated the notion of proportional fairness, and developed approximation algorithms for fair k-median and fair k-center under this notion of fairness. Follow-up work generalized their results to other clustering problems, such as k-means and facility location, and more relaxed notions of fairness [10, 1, 9, 5, 35]. Notably, the important graph clustering problem of correlation clustering has been so far not addressed by this literature.

Correlation clustering is a type of clustering that uses information about similarity as well as dissimilarity relationships among a set of objects in order to cluster them [8]. In contrast to other clustering problems such as k-median, k-means, and k-center, the number of clusters is not predetermined but rather determined based on the outcome of the optimization. This fact, as well as the fact that correlation clustering takes advantage of both similarity and dissimilarity information, makes it a desirable clustering model in many applications [37, 42, 4]. Therefore, it is natural to study this problem under fairness constraints.

The main tool introduced by Chierichetti et al. [19] for solving fair k-center and k-median problems is the notion of *fairlets*. A fairlet is a small set of elements that satisfies the fairness property. Chierichetti et al. [19] showed that fair k-median and k-center can be solved by first decomposing an instance into fairlets and then solving the clustering problem on the set of centers of these fairlets. To the best of our knowledge, this technique has been used only for *metric space* clustering problems such as k-center and k-median.

Our main result is developing a fairlet-based reduction for the *graph* clustering problem of correlation clustering. Whereas, in the case of k-center and k-median, the fairlet decomposition problem amounts to solving the same clustering problem on the same instance under the condition that each cluster is a fairlet, the situation for correlation clustering is complicated by the lack of the properties of metric spaces. To tackle this problem, we introduce a novel cost function for the correlation clustering fairlet decomposition, and prove that this cost can be approximated by a median-type clustering cost function for a carefully defined metric space. We prove that given a solution to this fairlet decomposition problem, we can reduce the fair correlation clustering instance to a regular correlation clustering instance through a graph manipulation method. Therefore, we show that any approximation algorithm for fairlet decomposition for k-median yields an approximation algorithm for fair correlation clustering. The loss in the approximation ratio depends on the size of the fairlets.

Furthermore, we prove that in many natural cases, there is a fairlet decomposition with small fairlets. This bounds the approximation ratio of our algorithm for fair correlation clustering. In addition to the theoretical bounds on the approximation ratio of our algorithm, we provide an empirical evaluation based on real data sets, showing that the algorithms often perform much better in practice than their worst-case guarantees and that they yield solutions of costs comparable to that of unfair clustering algorithms while substantially reducing the imbalance of the clusters.

Related work. Clustering is a fundamental unsupervised machine learning task with a long history (cf. [29]). Our paper spans the areas of correlation clustering, clustering in metric spaces, and fairness in clustering which are actively growing fields. For brevity, we will only focus of key works in these three areas.

Correlation clustering. Correlation clustering is a widely studied formulation of clustering with both similarity and dissimilarity information [8], with many applications in machine learning [37, 11]. Variants of the problem include complete signed graphs [8, 4] and weighted graphs [21]. We focus on the complete graph case with ± 1 weights which is APX-hard [16] but admits constant-factor algorithms [4, 17]. Distributed and streaming algorithms are also known [41, 3].

Metric space clustering. The most widely studied clustering setting is clustering in metric spaces consisting in minimizing the ℓ_p-norm of the distances between points in a cluster and their center. For $p \in \{1, 2, \infty\}$ this corresponds to

k-median, k-means, and k-center, respectively, which are NP-hard problems but admit constant-factor approximations [25, 27, 40, 2, 34].

Fairness in clustering. Fairness in machine learning is new area with a fast growing literature. Fundamental work in this area is devoted to defining notions of fairness [13, 22, 23, 32] and solving fairness-constrained problems [14, 15, 19, 30, 32, 45, 6, 33, 24, 1, 20, 26].

Chierichetti et al. [19] first introduced a notion of disparate impact for clustering and provided fair k-center algorithms for the case of two colors (or groups). We review this notion more in detail in Section 2. Following this work, the problem has been later generalized in many directions including allowing many colors [43], allowing upper bounds on the fraction of points of a given color [1] and both upper and lower bounds [9, 10]. Backurs et al. [7] designed near-linear algorithms for finding k-median fairlets, and Huang et al. [28] designed core-sets for the problem. Other variants include clustering with diversity constraints [39], proportionality constraints [18], and fair center selection [35]. Fairness has been studied in spectral clustering as well [36, 46].

To the best of our knowledge no prior work has addressed correlation clustering with fairness constraints in the cluster elements distribution. Kalhan [31] recently studied an orthogonal notion of fairness in correlation clustering which consists in bounding the maximum error for a node.

2 Problem Statement

Correlation clustering. Let $G = (V, E)$ be a complete undirected graph on $|V| = n$ vertices and $\sigma : E \mapsto \mathbb{R}$ be a function that assigns a label to each edge. The label $\sigma(e)$ for each e is either positive (indicating that the two endpoints of e are *similar*) or non-positive (indicating that they are *dissimilar*). In the *unweighted* version of the problem, $\sigma(e) \in \{-1, +1\}$ for each e. Our focus in this paper is on the unweighted problem, which is the original correlation clustering model defined by Bansal et al. [8], although we will use the weighted version in the proof. Let $E^+ = \{e \in E \mid \sigma(e) > 0\}$ be the set of positive edges and $E^- = \{e \in E \mid \sigma(e) \leq 0\}$ be the set of non-positive edges. For subsets $S, T \subseteq V$, let $E(S) = E \cap S^2$ denote the edges inside S and $E(S, T) = E \cap (S \times T)$ denote the edges between S and T.

A *clustering* is a partitioning $\mathcal{C} = \{C_1, C_2, \ldots\}$ of V into disjoint subsets. The sets of intra-cluster and inter-cluster edges in a clustering $\mathcal{C}$ are defined as $\mathrm{intra}(\mathcal{C}) = \bigcup_{C \in \mathcal{C}} E(C)$ and $\mathrm{inter}(\mathcal{C}) = E \setminus \mathrm{intra}(\mathcal{C})$. The *correlation clustering cost* of $\mathcal{C}$ is defined as:

$$\textsc{cost}(G, \mathcal{C}) = \sum_{e \in \mathrm{intra}(\mathcal{C}) \cap E^-} |\sigma(e)| + \sum_{e \in \mathrm{inter}(\mathcal{C}) \cap E^+} |\sigma(e)|.$$

In the unweighted version of the problem, this is simply $\textsc{cost}(G, \mathcal{C}) = |\mathrm{intra}(\mathcal{C}) \cap E^-| + |\mathrm{inter}(\mathcal{C}) \cap E^+|$. The goal of correlation clustering[1] is to find a clustering $\mathcal{C}$ to

[1] This is in fact the minimizing disagreements variant of correlation clustering. A maximizing

minimize $\textsc{cost}(G, \mathcal{C})$. For unweighted correlation clustering, there are constant-factor approximation algorithms for this problem (with the best known constant being 2.06) [8, 4, 17]. For the weighted case, the best known approximation factor for this problem is a $O(\log n)$-approximation [21].

Fairness constraints. In the fair version of any clustering problem, each vertex $v \in V$ has a color $c(v)$. Proportional fairness, defined by Chierichetti et al. [19], requires that in every cluster, the number of vertices of each color is proportional to the corresponding number in the whole graph. In particular, in the symmetric case where each color appears the same number of times in the graph, we require the same in each cluster. Ahmadian et al. [1] relaxed this property by requiring that each color constitutes at most an α-fraction of each cluster, for a given $\alpha \in (0, 1)$. Bera et al. [9] further generalized this notion to include lower bounds on the number of vertices of each color in each cluster.

In this paper, we give a general reduction from the fair correlation clustering to a median fairlet decomposition that works for any of these definitions of fairness, and in fact for a more general class of constraints. As long as the fairlet decomposition problem can be solved with small fairlets (which is the case for the above definitions of fairness, as we will show in Section 4), this will give us an approximation algorithm for the corresponding fair correlation clustering problem.

3 Overview of Results

In this section, we give a high-level overview of our algorithm and the proof. The main ingredient of our algorithm is a general reduction from the given constrained correlation clustering problem (as defined below) to a *fairlet decomposition* problem. We then show how the cost of a fairlet decomposition can be approximated by a median clustering cost function. This allows us to use previous results on the fair median problem to solve fairlet decomposition for the standard notions of fairness defined in the previous section. Finally, given an approximately optimal fairlet decomposition, we use our reduction to reduce the constrained correlation clustering instance to a standard correlation clustering instance, and apply known algorithms [8, 4, 17] to solve this problem.

Constrained correlation clustering. We start by defining a general class of constrained correlation clustering problems. Consider an unweighted correlation clustering instance G and let $\mathcal{F}$ be a family of subsets of V. We treat $\mathcal{F}$ as the family of feasible clusters, and assume it has the following *composability* property: for every $F_1, F_2 \in \mathcal{F}$, we have $F_1 \cup F_2 \in \mathcal{F}$. Note this property is satisfied when $\mathcal{F}$ is the collection of all fair sets under any of the definitions of fairness given in Section 2. The constrained correlation clustering problem is to define a correlation clustering $\mathcal{C}$ with minimum $\textsc{cost}(G, \mathcal{C})$ such that for all $C \in \mathcal{C}$, we have $C \in \mathcal{F}$.

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agreements version can also be defined similarly. In this paper we focus on minimizing disagreements, since the maximization version admits a trivial randomized 2-approximation that can be made fair.

Fairlet decomposition. Next, we define the notion of *fairlet decomposition* used in our reduction. A fairlet decomposition for a constrained correlation clustering problem is simply a partition $\mathcal{P} = \{P_1, P_2, \ldots\}$ of V into subsets in $\mathcal{F}$, i.e., $P_i \in \mathcal{F}$ for all i. We call each P_i a *fairlet*. The key in our reduction is a cost function $\textsc{fcost}$ that evaluates $\mathcal{P}$'s usefulness in building a correlation clustering of G. Here we define this cost function, and in Section 4.1 we show how it can be approximated by the standard k-median clustering cost function in a carefully defined metric space.

Fairlet decomposition cost. Consider a fairlet decomposition $\mathcal{P} = \{P_1, P_2, \ldots\}$. For each fairlet P_i, we let $\textsc{fcost}^{in}(P_i)$ be the number of negative edges inside P_i, i.e., $\textsc{fcost}^{in}(P_i) = |E^- \cap \mathrm{intra}(P_i)|$ and let $\textsc{fcost}^{in}(\mathcal{P}) = \sum_{P_i \in \mathcal{P}} \textsc{fcost}^{in}(P_i)$. For fairlets P_i, P_j, we let let $\textsc{fcost}^{out}(P_i, P_j)$ be the number of edges between them with the minority sign, i.e.,

$$\begin{aligned}\textsc{fcost}^{out}(P_i, P_j) \quad = \quad & \min(\left|E^- \cap E(P_i, P_j)\right|, \\ & \left|E^+ \cap E(P_i, P_j)\right|).\end{aligned}$$

Finally, we let $\textsc{fcost}^{in}(\mathcal{P}) = \sum_i \textsc{fcost}^{in}(P_i)$, $\textsc{fcost}^{out}(\mathcal{P}) = \sum_{i<j} \textsc{fcost}^{out}(P_i, P_j)$, and $\textsc{fcost}(\mathcal{P}) = \textsc{fcost}^{in}(\mathcal{P}) + \textsc{fcost}^{out}(\mathcal{P})$.

Reduced instance. Given a constrained correlation clustering instance G and a fairlet decomposition $\mathcal{P}$ for G, we define a reduced correlation clustering instance $G^{\mathcal{P}}$ as follows: $G^{\mathcal{P}}$ has $|\mathcal{P}|$ vertices, each corresponding to one fairlet in $\mathcal{P}$. We denote the vertex corresponding to the fairlet P_i by p_i. The label $\sigma(p_i, p_j)$ of the edge between p_i and p_j is the majority sign of the edges in $E(P_i, P_j)$ (with ties broken arbitrarily) multiplied by a weight that is equal to the number of edges in $E(P_i, P_j)$ with the majority sign.

Note that the instance $G^{\mathcal{P}}$ defined above is an instance of *weighted* correlation clustering, although as we will observe, the edges have weights that are within a constant factor each other, and therefore the problem can be solved using unweighted correlation clustering algorithms. Given a solution to this problem, it can be expanded into a solution of the original constrained problem. The final algorithm is sketched below.

Algorithm 1 Constrained Correlation Clustering

1: Find an approx optimal fairlet decomposition $\mathcal{P}$.
2: Compute reduced instance $G^{\mathcal{P}}$.
3: Let $\mathcal{C}$ be an approx optimal (non-constrained) correlation clustering of $G^{\mathcal{P}}$.
4: Output the clustering $\{\bigcup_{p_j \in C_i} P_j : C_i \in \mathcal{C}\}$.

To prove that the above algorithm produces an approximately optimal solution to the constrained correlation clustering problem, we need the following lemmas. The first two lemmas prove that a solution of G can be transformed to a solution of $G^{\mathcal{P}}$ and vice versa, and these transformations do not increase the cost by more than the cost of the fairlet decomposition. The third lemma bounds the cost of a fairlet decomposition in terms of the cost of the optimal solution to the constrained correlation clustering problem. The proofs of these lemmas are presented in the Supplemental Material.

Lemma 3.1. *Given a correlation clustering instance G, a fairlet decomposition $\mathcal{P}$ for G, and a clustering $\mathcal{C}$ of G, there is a clustering $\mathcal{C}'$ of $G^{\mathcal{P}}$ such that*

$$\textsc{cost}(G^{\mathcal{P}}, \mathcal{C}') \leq \textsc{cost}(G, \mathcal{C}) + \textsc{fcost}^{out}(\mathcal{P}).$$

Lemma 3.2. *Assume $\mathcal{C}$ is a clustering of $G^{\mathcal{P}}$, and let $\mathcal{C}'$ be the clustering computed in line 4 of Algorithm 1 for G. Then we have*

$$\textsc{cost}(G, \mathcal{C}') \leq \textsc{cost}(G^{\mathcal{P}}, \mathcal{C}) + \textsc{fcost}(\mathcal{P}).$$

Lemma 3.3. *For any constrained correlation clustering instance G, and any constrained clustering $\mathcal{C}$ of G, there is a fairlet decomposition $\mathcal{P}$ of G satisfying $\textsc{fcost}(\mathcal{P}) \leq \textsc{cost}(G, \mathcal{C})$.*

Putting these together, we have the following:

Theorem 3.4. *Assume we have an α-approximation algorithm A_1 for finding the minimum cost fairlet decomposition and a β-approximation algorithm A_2 for solving the unconstrained correlation clustering instance $G^{\mathcal{P}}$. Then Algorithm 1 produces a $(\beta(1+\alpha)+\alpha)$-approximation for the constrained correlation clustering instance G.*

Proof. Let OPT be an optimal solution to the constrained correlation clustering instance G. By Lemma 3.3, the fairlet decomposition problem has a solution of cost at most $\textsc{cost}(G, OPT)$, and therefore, algorithm A_1 for this problem must find a decomposition $\mathcal{P}$ with $\textsc{fcost}(\mathcal{P}) \leq \alpha \cdot \textsc{cost}(G, OPT)$. Also, by Lemma 3.1, the instance $G^{\mathcal{P}}$ has a solution of cost at most $(1+\alpha)\textsc{cost}(G, OPT)$. Therefore, algorithm A_2 can find a clustering $\mathcal{C}$ of cost at most $\beta(1+\alpha)\textsc{cost}(G, OPT)$. Thus, by Lemma 3.2, the cost of the clustering produced by Algorithm 1 is at most $(\beta(1+\alpha)+\alpha)\textsc{cost}(G, OPT)$. Finally, by the composability property of the constraints, we know that this clustering satisfies the constraints, since each of its clusters is a union of fairlets in $\mathcal{P}$. □

4 Fairlet Decomposition

In this section, we show how to solve the fairlet decomposition problem by reducing it to a fair clustering problem with the k-median cost function in an appropriately defined metric space. The reduction, presented in Section 4.1, loses a factor that is proportional to the size of the largest fairlet, but as we show in Section 4.2, in cases that we know how to solve the fairlet decomposition problem, the size of the fairlets can be guaranteed to be small.

4.1 Reduction to median cost

Consider a correlation clustering instance G and let d be a distance function that defines a metric space on the set of vertices V. For a fairlet decompostion $\mathcal{P} = \{P_1, P_2, \ldots\}$, the median cost can be defined as follows: $\textsc{mcost}(P_i) = \min_{u\in P_i} \sum_{v\in P_i} d(u,v)$ and $\textsc{mcost}(\mathcal{P}) = \sum_{P_i\in\mathcal{P}} \textsc{mcost}(P_i)$. Notice that the problem of finding the fairlet decomposition with minimum $\textsc{mcost}(\mathcal{P})$ is precisely the fairlet decomposition problem for fair k-median, as studied by [19, 9].

We now define a distance function d such that the median cost $\textsc{mcost}$ can approximate the fairlet cost $\textsc{fcost}$. We first define an embedding $\phi : V \to [0,1]^n$ as follows. For a vertex $u \in V$, let

$$\phi(u)_v = \begin{cases} 1 & \text{if } (u,v) \in E^+ \\ 0 & \text{if } (u,v) \in E^- \\ 1 & \text{if } u = v \end{cases}$$

In other words, $\phi(v)$ is the vth row of the adjacency matrix of $G(V, E^+)$ after adding a positive self-loop at every vertex. Define $d : V^2 \mapsto \mathbb{R}_{\geq 0}$ for points u, v to be the Hamming distance of their embeddings, i.e., $d(u,v) = |\phi(u) - \phi(v)|$. Intuitively, $d(u,v)$ measures the "cost" of committing to put u and v in one cluster in the correlation clustering instance.

We now prove the following two lemmas, which show that the $\textsc{fcost}$ of a fairlet decomposition is close to its $\textsc{mcost}$ with respect to the metric d. The proofs of these lemmas are presented in the Supplemental Material.

Lemma 4.1. *For any fairlet decomposition $\mathcal{P}$, we have*

$$\textsc{mcost}(\mathcal{P}) \leq 4 \cdot \textsc{fcost}(\mathcal{P}).$$

Lemma 4.2. *Let $\mathcal{P}$ be any fairlet decomposition and let $f = \max_{P\in\mathcal{P}} |P|$. Then,*

$$\textsc{fcost}(\mathcal{P}) \leq 2f \cdot \textsc{mcost}(\mathcal{P}).$$

Using the above lemmas, we have the following.

Theorem 4.3. *Assume there is a γ-approximation algorithm for fairlet decomposition with median costs. Furthermore, assume that this algorithm always produces fairlets of size at most f. Then the solution produced by this algorithm is a $(8f\gamma)$-approximation to the problem of finding a fairlet decomposition with minimum* $\textsc{fcost}$.

4.2 Algorithms for fairlet decomposition

In this section, we give algorithms for fairlet decomposition with median cost for several notions of fairness. These algorithms, together with Theorems 3.4 and 4.3, imply algorithms for the corresponding fair correlation clustering problems. We focus on three fairness constraints: an upper bound of $\alpha = \frac{1}{2}$ on the fraction of vertices of each color in each cluster; an upper bound of $\alpha = 1/C$ where C

is the number of distinct colors (this corresponds to the proportional fairness property studied in [19]); and an upper bound of $\alpha = 1/t$ for an integer t on the fraction of vertices of each color in each cluster. We give approximation algorithms in these cases (a bicriteria approximation in the third case), with an upper bound (of 3, C, and $2t-1$, respectively) on the size of fairlets.

Throughout this subsection, when we speak of the cost of a fairlet decomposition, we mean its median cost.

4.2.1 $\alpha = 1/2$

First, we show that there is an optimal fairlet decomposition with small fairlets.

Claim 4.4. *There is an optimal fairlet decomposition $\mathcal{P}^*$ where $\max_{P_i^* \in \mathcal{P}^*} |P_i^*| \leq 3$.*

Proof. Consider an optimal solution $\mathcal{P}^*$ with the largest number of fairlets. Suppose there is a part P_i^* with $|P_i^*| > 3$. Then let $v, u \in P_i^*$ be points of the most common color and the second most common color. Construct a new partition $\mathcal{P}'$ from $\mathcal{P}^*$ by removing u, v from P_i^* and adding a new partition $\{u, v\}$. By the choice of u, v, both $\{u, v\}$ and $P_i^* \setminus \{u, v\}$ still satisfy the fairness constraint. Furthermore, by the triangle inequality $d(u, v) \leq d(u, \mu_i) + d(v, \mu_i)$ where μ_i is the center of P_i^*, and hence, $\textsc{mcost}(\mathcal{P}') \leq \textsc{mcost}(\mathcal{P}^*)$. Therefore, $\mathcal{P}'$ is also an optimal fairlet decomposition and has one more fairlet than $\mathcal{P}^*$, contradicting the choice of $\mathcal{P}^*$. □

We now define a graph H on points in V as follows: two vertices u, v are connected by two edges in H if they have distinct colors. The weight of both of these edges is $d(u, v)$. Now in order to find a fairlet decomposition, we find the minimum weight 2-factor in H. Recall that a 2-factor is a subgraph where each vertex has degree 2. This problem is polynomially solvable [44, Chapter 21]. Note that the optimal cost of the 2-factor with minimum weight can be bounded as follows.

Claim 4.5. *The graph H has a 2-factor of total weight at most $2 \cdot \textsc{mcost}(\mathcal{P}^*)$, where $\mathcal{P}^*$ is the optimal fairlet decomposition.*

Proof. Using the previous claim, we know that there exists an optimal fairlet decomposition with $\max_{P_i^* \in \mathcal{P}^*} |P_i^*| \leq 3$. Each fairlet in $\mathcal{P}^*$ of size 2 can be turned into a 2-vertex 2-factor of weight equal to twice the cost of the fairlet. For each fairlet P_i of size 3, we can add the edge between the two non-center vertices to obtain a 3-vertex 2-factor. By triangle inequality, the weight of this 2-factor is at most twice the cost of P_i. Therefore, the overall cost of the 2-factor is at most $2 \cdot \textsc{mcost}(\mathcal{P}^*)$. □

It remains to show how to convert a 2-factor solution to a fairlet decomposition with small fairlets. Cycles of length 2 and 3 give fairlets of size 2 and 3; so we only need to argue how to convert cycles of length more than 3 to fairlets. If a cycle has an even length we just return one of the two sets of alternating edges, whichever has lower cost. For cycles of odd length, there must exists three consecutive vertices of pairwise distinct colors. In this case we return these three vertices as one fairlet and the rest of cycle can be partitioned into matching edges. The cost of this fairlet decomposition is at most the weight of the original 2-factor. So in this case, we get a 2-approximation for fairlet decomposition with fairlets of size at most 3.

4.2.2 $\alpha = 1/C$ with C colors

Let us first understand the structure of an optimal solution $\mathcal{P}^* = (P_1^*, P_2^*, \ldots)$. Since $\alpha = 1/C$, each part has equal number of colors. So in order to find a fair partition, we consider an ordering of colors, and solve a minimum cost matching problem between points of color c and $c+1$ in the graph G. The union of these matchings gives us a partition of V into paths of length C. Each such path defines a fairlet; let $\mathcal{P}$ denote this fairlet decomposition. We show the cost of $\mathcal{P}$ is at most twice the cost of the optimal fairlet decomposition.

Claim 4.6. *Let $\mathcal{P}^*$ be the optimal fairlet decomposition. Then,* $\textsc{mcost}(\mathcal{P}) \leq 2 \cdot \textsc{mcost}(\mathcal{P}^*)$.

Proof. Let us first bound the cost of matching edges between color c and $c+1$ using the partitions in $\mathcal{P}^*$. Since each part in $\mathcal{P}^*$ has equal number of vertices of each color, we can find points of colors c and $c+1$ within each part of $\mathcal{P}^*$. Using the triangle inequality the cost of the matching edges between vertices in P_i^*, is at most $\sum_{v:\text{color } c \text{ or } c+1} d(v, \mu(P_i^*))$, where $\mu(P_i^*)$ is the center of P_i^*. Summing up all the matching weights, we get the total matching weights are at most $2\textsc{mcost}(\mathcal{P}^*)$ since each edge of the form $(v, \mu(P_i^*))$ is charged twice, once for matching between c and $c+1$ and once for matching between $c-1$ and c. $\square$

So in this case, we get a 2-approximation with fairlets of size at most C. Note that in the motivating application of fair clustering, the color of each vertex corresponds to one possible value of a sensitive feature like race or gender, and therefore the value C tends to be small in such applications.

4.2.3 Bicriteria approximation for $\alpha = 1/t$

In this section, we focus on the case where $1/\alpha \in \mathbb{Z}_+$ and since we have already solved the case of $\alpha = 0.5$ in Section 4.2.1, we can assume $1/\alpha \geq 3$. For solving this case, we start from a solution produced by the bicriteria approximation algorithm of Bera et al. [9] and make necessary changes to make sure that maximum fairlet size is $2t-1$.

Theorem 4.7 (Theorem 1 of [9]). *Given a ρ-approximation algorithm for the k-median problem, we can return a $(\rho+2)$-approximate solution S where each part in S violates the the fairness constraint by additive factor of at most* 3.

Since median problem is a relaxation of k-median problem, using the state-of-the-art algorithm for k-median problem[12], we can get a solution S to the fairlet decomposition problem with cost at most $4.675 + \epsilon$ times the optimal solution where each fairlet 'almost' satisfies the fairness constraint. Since we need bounded fairlet sizes, we process the solution S and transform it to a solution S' where fairlet sizes are bounded using the following claim. In the following claim, we show that large fairlets can be broken into smaller fairlets. In addition, it is easy to see that if a large fairlet violates the fairness constraint by an additive factor of at most θ, then the resulting small fairlets violate the fairness condition by additive factors of at most θ.

Claim 4.8. *For any set P of size p that satisfies fairness constraint with $\alpha = 1/t$, there exists a partition of P into sets $(P_1, P_2, \dots)$ where each P_i satisfies the fairness constraint and $t \leq |P_i| < 2t$.*

Proof. Let $p = m \times t + r$ with $0 \leq r < t$, then the fairness constraints ensures that there are at most m elements of each color. Consider partitioning obtained through the following process: consider an ordering of elements where points of the same color are in consecutive places, assign points to sets $P_1, P_2, \cdots, P_m$ in a round robin fashion. So each set P_i gets at least t elements and at most $t + r < 2t$ elements assigned to it. Since there are at most m elements of each color, each set gets at most one point of any color and hence all sets satisfy the fairness constraint as $1 \leq \frac{1}{t} \cdot |P_i|$. □

Applying Claim 4.8 to the solution of Algorithm of [9], one can obtain a fairlet decomposition where fairlets have size at most $2t - 1$. Using the triangle inequality, we can show that updating the center of each fairlet to the closest vertex in that fairlet to the assigned center, will at most doubles the cost of the solution. Since the size of fairlet is at least $t \geq 3$, the fairness constraint is satisfied with a multiplicative factor of 2α. So the above arguments show that we can find a fairlet decomposition with cost at within 9.35 times the optimal cost where each fairlet is of size at least t and at most $2t - 1$ and fairness constraint is violated by at most a factor of 2α.

4.3 Solving the reduced instance

Note that the reduced correlation clustering instance $G^{\mathcal{P}}$ is a weighted correlation clustering instance. Even though in general, the best known approximation ratio for weighted correlation clustering is $O(\log n)$[21], for the fairlet decomposition algorithms presented in this section, we have the following property that allows us to get a better approximation factor: the ratio of the size of the largest fairlet to the smallest one is at most a constant. For fairlet decompositions in Section 4.2.1, 4.2.2, and 4.2.3, this constant is $3/2$, 1, and $\frac{2t-1}{t} < 2$, respectively. Now, notice that the weight of the edge between p_i and p_j in $G^{\mathcal{P}}$ is at least $|P_i| \cdot |P_j|/2$ and at most $|P_i| \cdot |P_j|$. These weights are all within a constant factor of each other (with the constants being $9/4$, 1, and 4). Therefore, by removing the weights from the instance $G^{\mathcal{P}}$ and solving the resulting unweighted instance (using the algorithm of [17], for example), we only lose an additional factor equal to this constant.

Dataset	Unfair Alg. Error		Unfair Alg. Imbalance		Fair Alg. Error		
	Local	Pivot	Local	Pivot	Match + Local	Single	Rand
amazon	0.01	0.039	0.40	0.39	0.064	0.786	0.215
reuters $\theta = 25\%$	0.095	0.156	0.65	0.62	0.229	0.754	0.255
reuters $\theta = 50\%$	0.180	0.230	0.50	0.43	0.321	0.504	0.502
reuters $\theta = 75\%$	0.188	0.286	0.15	0.20	0.199	0.252	0.746
victorian $\theta = 25\%$	0.109	0.170	0.55	0.45	0.209	0.753	0.251
victorian $\theta = 50\%$	0.182	0.254	0.31	0.25	0.324	0.502	0.499
victorian $\theta = 75\%$	0.203	0.248	0.12	0.13	0.237	0.251	0.747
mean over datasets	0.138	0.198	0.38	0.35	0.226	0.459	0.543

Table 1: Error and Imbalance in $C = 2$ color case for various datasets and different threshold θ for the quantile used for positive edges. Notice how our algorithm Match + Local has cost comparable to Pivot and not much higher than Local while reducing the imbalance from the up 65% of the unfair algorithms to 0.

5 Experiments

In this section, we present our experiments demonstrating that our algorithm solves the correlation clustering problem with fairness constraints with only a limited loss in the cost when compared to the vanilla (unfair) solution. We describe the datasets used, the algorithms evaluated, the quality measures, and our results.

Datasets. We use publicly-available datasets from the UCI Repository[2] and from the SNAP Datasets[3].

The datasets represent complete signed graphs from different domains, including both co-purchasing relationships among products, and semantic similarities among texts learned with embedding methods. The graphs are represented by complete signed matrices up to 1600x1600 in size, up to .9 million positive edges, and up to $C = 16$ colors. Here we only briefly describe the datasets, more details are in the appendix of the supplemental material. **amazon**: Nodes represents products on the Amazon website [38], the color of is the item category, and two co-reviewed items have a $+1$ weight edges (all non-co-reviewed items have -1 weight edges). We use 1000 nodes equally distributed in two 2 popular categories. **reuters** and **victorian**: These datasets are extracted from text data used in previous fair clustering work [1]. The datasets include between 50 and 100 English language texts from each of up to 16 authors.[4] Each node represent a text, the color represent the author. For each text we obtain a semantic embedding

[2] `http://archive.ics.uci.edu/ml`

[3] `http://snap.stanford.edu/data/`

[4] The datasets are available at `archive.ics.uci.edu/ml/datasets/Reuter_50_50` and `archive.ics.uci.edu/ml/datasets/Victorian+Era+Authorship+Attribution`

Algorithm	Error	Imbalance for 1/2	Imbalance for equality
Local	0.249	0.016	0.222
Pivot	0.320	0.009	0.188
Match + Local	0.252	0	0.188
Rep. Match + Local	0.306	0	0
Single	0.5	0	0
Rand	0.5	0	0

Table 2: Experimental results for the victorian $\theta = 50\%$ dataset, using $C = 8$ colors.

vector with standard methods. We use a threshold on the dot product of the embedding vectors to obtain the edges. Through this operation, we set the top $\theta \in \{25\%, 50\%, 75\%\}$ of edges via dot products as $+1$'s, and the remaining edges are assigned -1's.

Algorithms. We evaluate Algorithm 1 in two fairness scenarios: with an upper bound of $\alpha = 1/2$ of the nodes for each color, and with $\alpha = 1/C$ for equal color representation (for the two-color case, the two are equivalent). For the $\alpha = 1/2$ case, in our experiments we simplify the algorithm of Section 4.2.2 to compute a minimum-cost perfect matching (1-factor) instead of a 2-factor decomposition. This can be formally shown to be sufficient for the $C = 2$ case, or when all optimal clusters are even sized, and we observe it works well in practice in our the experiments. For $\alpha = 1/C$, we implement the repeated matching algorithm in 4.2.2 to obtain the fairlets in a similar fashion. After finding the fairlets, we use an in-house correlation clustering solver based on local search (Local). We refer to our algorithms as Match + Local for the $\alpha = 1/2$ case and as Rep. Match + Local for the $\alpha = 1/C$ case.

We also consider the following (unfair) baseline algorithms: the standard Pivot algorithm of Ailon et al. [4] for correlation clustering (for Pivot, we repeat the randomized algorithm 10 times and use the best result), and the (unfair) local search heuristic Local used as part of our algorithm. In addition, as no prior work has addressed the fair correlation clustering problem, we compare our algorithm with two simple fair baselines: the whole graph as one cluster Single, and a random fairlet decomposition Rand.

Quality measures. For each algorithm, we report the following measures. The (Error) of the correlation cost of the clustering obtained by the algorithm, presented as the ratio of the edges of the graph that are in disagreement with the clustering (i.e., inter-cluster positive edges and intra-cluster negative edges) over the number of edges (i.e., 0 corresponds to a perfect solution, whereas 1 corresponds to a completely incorrect solution). For fairness, we report the Imbalance as the total fraction of nodes that violate the α color representation constraint, i.e., nodes for each cluster and color that are above the α fraction for the size of the cluster [1]. More precisely, let P be a cluster in the solution of an α-color constraint instance. The maximum allowed number of points of a certain color in the cluster P is $\lfloor |P|\alpha \rfloor$. Let V_c be the nodes of color c on the graph and $\Delta_P = \sum_{P,c} \max(|P \cap V_c| - \lfloor |P|\alpha \rfloor, 0)$ be the nodes violating the constraint in P. We report $\sum_P \Delta_P/|V|$ as the IMBALANCE for $\alpha \in \{1/2, 1/C\}$ (i.e., 0 corresponds to no imbalance, and 1 corresponds to complete imbalance). We repeat all algorithms 10 times and report the mean results for all measures.

5.1 The $C = 2$ color case

Table 1 shows the results for the $C = 2$ color case with $\alpha = 1/2$, i.e., equal representation over the various datasets. We use the MATCH + LOCAL as a fair algorithm, which has an IMBALANCE of 0 by construction.

The table shows clearly that our algorithm MATCH + LOCAL obtains clusters that are fair and has costs comparable to the unfair PIVOT baseline (and sometimes better) and slightly worse than the LOCAL baseline. On average, over all datasets, our algorithm has an average cost of .226 vs .198 for PIVOT and .138 for LOCAL. On the other hand, our algorithm is significantly better than both the naive one-cluster and random fair clustering algorithms, which, while satisfying fairness, have exorbitantly high costs.

Notice how the LOCAL and the PIVOT baseline have very high IMBALANCE values of up to 65% of the nodes (as they are oblivious to colors), showing the importance of developing novel algorithms for the problem. This result is not surprising. If pairs of nodes of the same color are more likely to be similar, it is expected that many clusters will contain vast majorities of points with a single color.

5.2 The $C > 2$ color case

Here, we study the behavior of our algorithm in the case when more than two colors are present in the dataset. We use MATCH + LOCAL to obtain a $\alpha = 1/2$ fair solution and REP. MATCH + LOCAL to obtain a $\alpha = 1/C$ (equal representation) solution.

We report an overview of our experimental results in Table 2 for the dataset victorian with threshold $\theta = 50\%$ and $C = 8$ colors (note that this is a different graph than that produced with the previous $C = 2$ dataset). More experimental results are available in the extended material.

It is possible to see that all trends from the previous section are confirmed. Notice how the algorithm for the 1/2 case MATCH + LOCAL is only marginally worse than the best unfair solution LOCAL and much better than all other baselines. Our algorithm for the more difficult equal representation case is again better than the PIVOT baseline. Notice how all unfair algorithms are significantly far from being equally balanced. However, the presence of many colors makes it easier to get closer to the 1/2 threshold.

We confirm this observation in Figure 1. The figure shows a comparison of the ERROR for our algorithms for $\alpha = 1/2$ and $\alpha = 1/C$ vs the ERROR for the vanilla LOCAL algorithms as the number of colors goes from $C = 2$ to $C = 16$. We report the ratio of MATCH + LOCAL over LOCAL as a solid line, and the ratio for REP. MATCH + LOCAL as a dashed line. Notice how the error for our

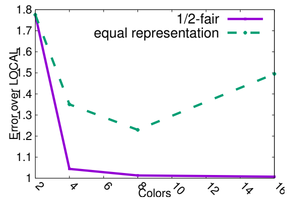

Figure 1: ERROR of our algorithms over that of the unfair LOCAL algorithms for $\alpha = 1/2$ and $\alpha = 1/C$, on a series of graphs from the victorian dataset, $\theta = 50\%$, and using $C = 2$ to $C = 16$ colors.

algorithm for the $\alpha = 1/2$ case gets closer and closer to the LOCAL output error for more colors. Again, this result is obtained because the presence of many colors makes the problem easier for the $\alpha = 1/2$ case. The performance of the algorithm for the $\alpha = 1/C$ case has a less stable pattern, but it confirms that the algorithm is quite competitive with the unfair solution (between $30 - 80\%$ higher error) even for quite a few colors. These results are significantly better than what is expected from a worst-case analysis.

Finally, we report having observed that using just MATCH or REP. MATCH fairlets without the re-clustering part of the algorithm is not sufficient for obtaining good results, as the re-clustering step is needed for obtaining clusters that do not have large errors.

6 Conclusions

In this paper we initiated the study of correlation clustering with fairness constraints. We showed a reduction to the fairlet decomposition problem with a standard k-median cost function, for a carefully chosen distance function. Using this, and old and new results on the fairlet decomposition problem with a median cost function, we obtained provable constant-factor approximation algorithms for fair correlation clustering for various notions of fairness. Our experimental evaluation shows that these algorithms perform well not only in theory but also in practice.

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