The Incentives that Shape Behaviour

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Abstract

Which variables does an agent have an incentive to *control* with its decision, and which variables does it have an incentive to *respond* to? We formalize these incentives, and demonstrate unique graphical criteria for detecting them in any single-decision causal influence diagram. To this end, we introduce structural causal influence models, a hybrid of the influence diagram and structural causal model frameworks. Finally, we illustrate how these incentives predict agent incentives in both fairness and AI safety applications.

1 Introduction

Incentives are a useful language for analysing the behaviour of humans and artificial agents. For example:

- A chess player (or chess-playing AI) often has an incentive to capture its opponent's queen.
- A pilot (or autopilot) has an incentive to move to a safe altitude.
- A walking human (or humanoid robot) has an incentive to pay attention to obstacles on the ground.

When modelling these human or artificial agents, incentive analysis allows us to (imperfectly) predict an agent's behaviour without knowing its implementation details, but just assuming that it is "smart enough" to do what will help it to attain its goals. What causes goal-attainment in one environment will not necessarily do so in another: a safe altitude is not the same for an aeroplane as for a helicopter. So to talk about incentives precisely, it is useful to describe them with reference to an environment with a specific causal structure.¹

In order to discuss incentives consistently, a critical step is to formally describe them. In the setting of a causal influence diagram, Everitt et al. (2019b) define two types of incentives: intervention incentives and observation incentives. An intervention incentive is present for a variable X if the agent can obtain more utility given the ability to arbitrarily manipulate the variable

— i.e. the value of control of X is greater than zero (Matheson, 1990). An observation incentive is present for a variable X if the agent can obtain more utility if it is allowed to observe X before making a decision than if it cannot — i.e. the value of information of X is greater than zero (Howard, 1966). Usefully, Everitt et al. also provide sound and complete criteria for the presence of intervention and observation incentives in single-decision influence diagrams, which have been used in fairness and AI safety analyses (Cohen et al., 2020; Everitt et al., 2019a,b; Everitt and Hutter, 2019).

However, the analysis of Everitt et al. (2019b) has one shortcoming: it defines incentives so as to include events that an agent cannot feasibly achieve. For example, a chess-playing AI would always have an intervention incentive on its opponent's immediate resignation, even if it had no way to bring about that outcome. A pilot would have an observation incentive for its altitude even if the altitude sensors are irreparably broken. As such, the incentives described by Everitt et al. (2019b) overstate the incentives shaping an agent's policy.

This paper aims to characterize the incentives that shape the agent's decisions. To this end, we present a new framework that combines the best elements from (causal) influence diagrams and structural causal models (Section 2). Using this framework, we define control incentives and response incentives, and demonstrate unique (Sections 3 and 4). We also illustrate how control and response incentives can illuminate fairness and manipulation issues (Section 5). Finally, we review related work (Section 6), and conclude (Section 7).

2 Setup

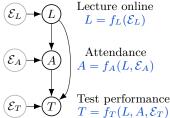
2.1 Structural Causal Models

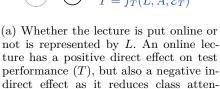
In this section, we recap structural causal models (SCMs), a main building block of our theory. An SCM is a type of causal model where all randomness is consigned to *exogenous* variables. The variables of interest (called *endogenous* variables) are all deterministic functions of each other and the exogenous variables.

Definition 1 (Structural causal model; Pearl, 2009, Chapter 7). A structural causal model (with independent errors) is a tuple $M = \langle \mathcal{E}, V, \mathcal{F}, P \rangle$ where:

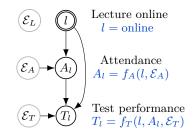
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¹In fact, Miller et al. (2019) argue that it is impossible to infer incentives without causal assumptions.

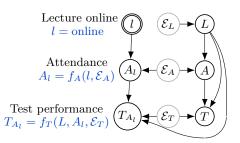




dance (A).



(b) The effect of putting the lecture online do(L=l) is reflected in the *potential responses* A_l and T_l for attendance and test performance.



(c) The nested potential response T_{A_l} is the test performance that would result if lecture online took its true value L (not online, say) but attendance was A_l , i.e. as if the lecture was online.

Figure 1: An example of a structural causal model (a) with (nested) potential responses (b,c), which can be used to describe a wide range of hypothetical events.

- V is a set of structural variables with finite domains.
- $\mathcal{E} = \{\mathcal{E}_V\}_{V \in \mathbf{V}}$ is a set of finite-domain exogenous variables, one for each structural variable.
- $F = \{f_V\}_{V \in V}$, $f_V : \operatorname{dom}(\mathbf{Pa}_V \cup \{\mathcal{E}_V\}) \to \operatorname{dom}(V)$ is a set of *structural functions* that specify the value of each variable in terms of the corresponding exogenous variable \mathcal{E}_V and structural parents, $\mathbf{Pa}_V \subset V$, where these functional dependencies are acyclic. Here, $\operatorname{dom}(V)$ denotes the domain of a variable V and $\operatorname{dom}(W) = X_{W \in W} \operatorname{dom}(W)$ denotes the domain of a set of variables W.
- P is a probability distribution for \mathcal{E} such that the individual exogenous variables \mathcal{E}_V are mutually independent.

An SCM defines a probability distribution over its variables. For any values $\mathcal{E} = \varepsilon$ of the exogenous variables, the value of any set of variables $W \subseteq V$ is given by recursive application of the structural functions F and is denoted by $W(\varepsilon)$. Combined with the distribution $P(\varepsilon)$ on the exogenous variables, this induces a joint distribution $\Pr(W = w) = \sum_{\{\varepsilon \mid W(\varepsilon) = w\}} P(\varepsilon)$.

As an illustration, we model the effects of putting a lecture online in Figure 1a. We will use this example throughout the paper to illustrate concepts as we introduce them.

An SCM supports *causal interventions* that set variables to particular values while breaking the functional dependencies of those variables on their parents. These are defined via submodels:

Definition 2 (Submodel; Pearl, 2009, Chapter 7). Let $M = \langle \mathcal{E}, V, F, P \rangle$ be an SCM, X a set of variables in V, and x a particular realization of X. The submodel M_x represents the effects of an intervention do(X = x), and is formally defined as the SCM $\langle \mathcal{E}, V, F_x, P \rangle$ where $F_x = \{f_V | V \notin X\} \cup \{X = x\}$. That is to say, the original functional relationships of $X \in X$ are replaced with the constant functions X = x.

If W is a random variable in an SCM M, then W_x refers to the same random variable in the submodel M_x . W_x is called a *potential response variable*, as it represents the hypothetical value that W would take under an intervention do(X = x). Potential response variables are illustrated in Figure 1b.

More elaborate hypotheticals can be described with nested potential responses, in which the intervention is itself a potential response variable. For example, in our running example, we may want to consider the value of test performance T if the effect of setting the lecture online, do(L = l), is propagated *only* through attendance A. This hypothetical quantity is represented with the nested potential response T_{A_l} , as illustrated in Figure 1c. Formally, for arbitrary variables X, Y, Z, X_{Y_z} is the random variable $X_{Y_z(\varepsilon)}(\varepsilon)$ for $\varepsilon \sim \mathcal{E}$.

2.2 Structural Causal Influence Models

To describe incentives, we will need other types of variables: "utility" and "decision" nodes that are not present in structural causal models. A causal influence diagram is a graphical representation of a decision-making problem that contains a decision-maker whose goal is to maximize summed expected utility (see Figure 2).

Definition 3 (Causal Influence Diagram (CID)). A causal influence diagram is a directed acyclic graph \mathcal{G} where the vertex set V is partitioned into structure nodes X, decision nodes D, and utility nodes U. The utility nodes have no children.

We use $\mathbf{Pa}_V^{\mathcal{G}}$, $\mathbf{Ch}_V^{\mathcal{G}}$, $\mathbf{Anc}_V^{\mathcal{G}}$, $\mathbf{Desc}_V^{\mathcal{G}}$ respectively to denote the parents, children, ancestors and descendants of a node V in V. For a decision D, we call $\mathbf{Pa}_D^{\mathcal{G}}$ the decision context of D. We use $\mathbf{Fa}_V^{\mathcal{G}} \coloneqq \{V\} \cup \mathbf{Pa}_V^{\mathcal{G}}$ to denote the family of V. When the graph is obvious from context the superscript is omitted. An edge from node X to node Y is denoted $X \to Y$, a directed path (of length at least zero) is denoted $X \dashrightarrow Y$, and an undirected

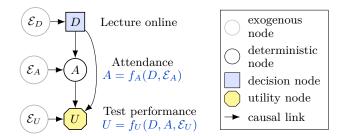


Figure 2: A structural causal influence model displaying a lecturer's decision regarding how to choose whether to place a lecture online (D) given its direct and indirect effects on students' test performance (U). A policy $D = \pi(\mathcal{E}_D)$ specifies the missing structural function for the decision node.

path by X - Y. For sets of variables, $X \longrightarrow Y$ means that $X \longrightarrow Y$ holds for some $X \in X$, $Y \in Y$.

In order to model specific decision scenarios, we need to augment the influence diagram with specific functions and probability distributions (sometimes called a "realization" of the diagram). To do this we introduce the *structural causal influence model* (SCIM, pronounced 'skim'), a generalization of SCMs to causal influence diagrams. A SCIM specifies the value of each CID variable as a deterministic function of its parents and an additional set of exogenous variables and supports potential response inference via the structural causal model.

SCIMs were anticipated by functional influence diagrams (Dawid, 2002) and Howard canonical form (Heckerman and Shachter, 1995), as discussed further in Section 6.

Definition 4 (Structural causal influence model (SCIM)). A structural causal influence model is a tuple $\mathcal{M} = \langle \mathcal{G}, \mathcal{E}, F, P \rangle$ where:

- \mathcal{G} is a causal influence diagram on a set of variables V (partitioned into X, D, and U) with finite domains where utility variable domains are a subset of \mathbb{R} . We say that that \mathcal{M} is compatible with \mathcal{G} .
- $\mathcal{E} = \{\mathcal{E}_V\}_{V \in \mathbf{V}}$ is a set of finite-domain exogenous variables, one for each structural variable.
- $F = \{f_V\}_{V \in V}$, $f_V : \text{dom}(\mathbf{Pa}_V \cup \{\mathcal{E}_V\}) \to \text{dom}(V)$ is a set of *structural functions* that specify the value of each structure and utility variable in terms of the values of its parents.
- P is a probability distribution for \mathcal{E} such that the individual exogenous variables \mathcal{E}_V are mutually independent.

A single-decision SCIM has $|\mathbf{D}| = 1$. For the present work, incentive definitions will be restricted to this case and we take D to be the unique element of \mathbf{D} .

For example, we can view the lecturing problem from Figure 1a as a decision-making problem for the lecturer, using a SCIM as in Figure 2.

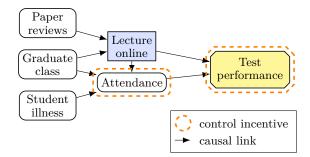


Figure 3: An extended version of the lecture example labeled with control incentives. Attendance and test-performance are both feasible and desirable to control. Although student illness and the number of graduate students also affect attendance, they are not downstream of any decisions and therefore lack a control incentive. The lecturer knows the reviews of their recent paper but controlling the paper reviews is neither possible nor useful for improving test performance. It therefore lacks a control incentive as well.

Definition 5 (Policy). A *policy* for a decision D in a SCIM is a function $\pi_D : \text{dom}(\mathbf{Pa}_D \cup \{\mathcal{E}_D\}) \to \text{dom}(D)$.

When we specify the policy of a decision variable, it becomes a structural variable and a single-decision SCIM becomes an SCM. For a policy π , \mathcal{M}_{π} is defined as the SCM $\langle \mathcal{E}, V, F \cup \{\pi\}, P \rangle$. We use \Pr_{π} and \mathbb{E}_{π} to denote probabilities and expectations with respect to \mathcal{M}_{π} . When X is a set of variables that do not descend from D, then $\Pr_{\pi}(x)$ is independent of π and we simply write $\Pr(x)$. With this, we can define an *optimal policy* for a SCIM as any policy π that maximizes $\mathbb{E}_{\pi}[\mathcal{U}]$, where $\mathcal{U} := \sum_{U \in \mathcal{U}} \mathcal{U}$. As with variables in the model, a potential response \mathcal{U}_x of \mathcal{U} is defined with respect to its value in the submodel \mathcal{M}_{π} .

3 Control incentives

This section and the next define two types of incentives in terms of expected utility and potential responses in a structural causal model. The sections also establish graphical criteria for detecting possible control incentives directly from a causal influence diagram. These incentive definitions are compared against those of Everitt et al. (2019b) in Table 1.

3.1 Definition

This section asks which variables a decision-maker would use its decision to control. We approach the question of how to define these *control incentives* in two steps, using the context of our running example to aid intuition (Figure 3).

First, suppose the lecturer was magically given the ability to control a variable X. Would they want to use it?² The answer is that it is advantageous for an agent to

²This is the question of intervention incentives, explored

| Incentive | Expected Utility Definition | Graphical Criterion |
|-------------------------------------|---|--|
| Intervention (old) | $\exists g^X : \max_{\pi} \mathbb{E}_{\pi}[\mathcal{U}] < \max_{\pi, \boldsymbol{x}} \mathbb{E}_{\pi}[\mathcal{U}_{g^X}]$ | exists path $X \dashrightarrow U$ in the reduced graph |
| Control (new) | $\forall \pi^* \exists \mathbf{pa}_D, d : \mathbb{E}_{\pi^*}[\mathcal{U} \mathbf{pa}_D] \neq \mathbb{E}_{\pi^*}[\mathcal{U}_{X_d} \mathbf{pa}_D]$ | exists path $D \dashrightarrow X \dashrightarrow U$ |
| Observation (old) Response (new) | $\max_{\pi \in \Pi^{-}} \mathbb{E}_{\pi}[\mathcal{U}] < \max_{\pi \in \Pi^{+}} \mathbb{E}_{\pi}[\mathcal{U}]$ $\forall \pi^{*} \ \exists x, \varepsilon : \ D(\varepsilon) \neq D_{x}(\varepsilon) \text{ in } \mathcal{M}_{\pi^{*}}$ | $X \not\perp \boldsymbol{U} \mid \mathbf{Fa}_D \setminus \{X\}, \ U \in \mathrm{Desc}(D)$ exists path $X \dashrightarrow W \to D \dashrightarrow \boldsymbol{U}$ and $W \not\perp \boldsymbol{U} \mid \mathbf{Fa}_D \setminus \{W\}$ |

Table 1: Comparison of control and response incentives with intervention and observation incentives from Everitt et al. (2019b). A diagram is "compatible with" the incentive in the left column on a variable X if and only if the node for X matches the graphical criterion in the right column. In the expected utility definitions, π^* denotes an optimal policy, g^X : dom(\mathbf{Pa}_X) \to dom(X) is a soft intervention replaces the structural function f^X in \mathcal{M} , Π^{-i} is the set of policies that do not depend on X, and Π^+ is the set of policies that can depend on X. A reduced graph, per Everitt et al., is a graph that is modified by removing edges from decision parents that lack an observation incentive.

control X if X influences its utility, or, more formally, if there is an intervention x on X such that $\mathcal{U}_x > \mathcal{U}$. Here \mathcal{U}_x represents the utility attained under intervention x, and \mathcal{U} the baseline utility attained without an intervention. In fact, the lecturer could benefit from magically intervening on any of the nodes in Figure 3. In contrast, an agent can never gain utility from intervening on a node X if there exists no directed path $X \dashrightarrow \mathcal{U}$.

The lecturer, however, does not have magical abilities, and can only affect variables X through their decision D. It is necessary, therefore, to consider the ways D can affect X, formally represented by the potential response X_d . When there is no directed path $D \dashrightarrow X$, then $X_d = X$, which means that the decision cannot influence X. For example, the lecturer in Figure 3 is unable to control whether the students are sick or the number of graduate students in their class, but they can control the attendance by (not) putting the lecture online.

Putting these components together, the nested potential response \mathcal{U}_{X_d} represents how \mathcal{U} is influenced by D via the variable X. To a first approximation, if $\mathcal{U}_{X_d} \neq \mathcal{U}$ for some decision d, then the agent can benefit from controlling X (or lose utility from losing the ability to control X).

Definition 6 (Control Incentive). Let \mathcal{M} be a single-decision SCIM. There is a *control incentive* on a variable $X \in \mathbf{V}$ in \mathcal{M} if, for every optimal policy π^* , there exists a decision context \mathbf{pa}_D with $\Pr(\mathbf{pa}_D) > 0$ and an alternative decision $d \in \text{dom}(D)$ such that $\mathbb{E}_{\pi^*}[\mathcal{U}_{X_d} \mid \mathbf{pa}_D] \neq \mathbb{E}_{\pi^*}[\mathcal{U} \mid \mathbf{pa}_D]$.

We say that a CID \mathcal{G} is compatible with a control incentive on X if \mathcal{G} is compatible with a SCIM \mathcal{M} that has a control incentive on X.

3.2 Graphical criterion

The following theorem establishes a graphical criterion for detecting whether a causal influence diagram is compatible with a control incentive on a given variable X. Figure 3 illustrates the application of the theorem, showing how it predicts control incentives for test result and class attendance, but not for student illness.

Theorem 7 (Control Incentive Graphical Crierion). A single-decision causal influence diagram \mathcal{G} is compatible with a control incentive on $X \in \mathbf{V}$ if and only if \mathcal{G} has a directed path from the decision D to a utility node $U \in \mathbf{U}$ that passes through X, i.e. has a path of the form $D \dashrightarrow X \dashrightarrow \mathbf{U}$.

Proof. Follows from Lemmas 19 and 20 in Appendix A.

4 Response incentive

Assume a decision is made to optimize expected utility. Which changes in the world does the decision have both ability and interest in responding to? This is the motivating question behind response incentives. First, we will give an expected utility definition of response incentives, then we will establish a criterion for detecting response incentives directly from a graphical representation.

4.1 Definition

Variables with a response incentive are ones that an optimal policy must be causally responsive to. The decision value selected by a policy in response to an intervention do(X = x) is the potential response D_x . Formally, we ask whether, for every optimal policy, there is some D_x that differs from the natural decision distribution D.

Definition 8 (Response Incentive). Let \mathcal{M} be a singledecision SCIM. There is a response incentive on a variable $X \in \mathbf{X}$ if for every optimal policy π^* , there exists some intervention do(X = x) and some setting $\mathbf{\mathcal{E}} = \varepsilon$, such that $D_x(\varepsilon) \neq D(\varepsilon)$ in \mathcal{M}_{π^*} .

in detail by Everitt et al. (2019b). See Table 1 for a comparison of control and intervention incentives.

³For those familiar with causal mediation, this is equivalent to the statement that under all optimal policies there exists an alternative decision in some decision context that has a nonzero average natural indirect effect on utility through X (Robins and Greenland, 1992; Pearl, 2001).

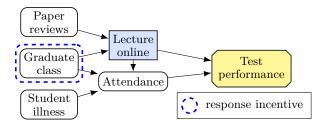


Figure 4: The lecture example from Figure 3 labelled with response incentives. There is both a means, and a reason to let the number of graduate students influence the decision whether to put the lecture online, so this variable has a response incentive. In contrast, the lecturer has no incentive to take into account the recent paper reviews for the decision. Whether students get ill would be of interest, but cannot be known in advance. Therefore both student illness and paper reviews lack a response incentive.

A CID \mathcal{G} is *compatible with* a response incentive on X if there exists a compatible SCIM that has a response incentive on X.

4.2 Graphical criterion

Just as for control incentives, there is a graphical criterion for detecting whether a given causal influence diagram is compatible with a response incentive to some node X. The graphical criterion makes use of the standard notion of d-separation (Pearl, 2009).

Definition 9 (d-separation). A path p is said to be d-separated by a set of nodes Z if and only if

- 1. p contains a chain $X \to W \to Y$ or a fork $X \leftarrow W \to Y$ such that W is in \mathbb{Z} , or
- 2. p contains a collider $X \to W \leftarrow Y$ such that the middle node W is not in Z and such that no descendant of W is in Z.

A set Z is said to d-separate X from Y, written $(X \perp Y \mid Z)$ if and only if Z d-separates every path from a node in X to a node in Y. Sets that are not d-separated are called d-connected.

Theorem 10 (Response Incentive Graphical Criterion). A single-decision causal influence diagram \mathcal{G} is compatible with a response incentive on $X \in \mathbf{X}$ if and only if

- there is a directed path from X to an observation W ∈ Pa_D.
- 2. there is a directed path from D to a utility variable U, and
- W is d-connected to U given the remaining Fa_D,
 i.e. W ⊥ U | Fa_D \ {W}.

The conditions of the theorem ensure that changes to X must be able to influence an observation W (condition 1.) that all optimal policies must depend on (conditions 2. and 3.). The proof follows from Lemmas 21 and 22 in Appendix A.

5 Applications

This section shows how the results developed in Sections 3 and 4 relate to fairness and AI safety.

5.1 Incentivized unfairness

Fairness is a growing concern of machine learning systems making life-impacting decisions (O'Neill, 2016). One formal interpretation of fairness is *counterfactual fairness*, which requires decisions not to be causally responsive to protected attributes such as *race* or *gender*.

Definition 11 (Counterfactual fairness; Kusner et al., 2017). A policy π is counterfactually fair with respect to a protected attribute A if⁴

$$\Pr_{\pi} \left(D_{a'} = d \mid \mathbf{pa}_D, a \right) = \Pr_{\pi} \left(D = d \mid \mathbf{pa}_D, a \right) \tag{1}$$

for every decision $d \in \text{dom}(D)$, every context $\mathbf{pa}_D \in \text{dom}(\mathbf{Pa}_D)$, and every pair of attributes $a, a' \in \text{dom}(A)$ with $\Pr(\mathbf{pa}_D, a) > 0$.

Kusner et al. (2017, Lemma 1) establish that a decision can only be unfair if the protected attribute is an ancestor of the decision. For example, consider a toy example in which a classifier predicts the likelihood of a car accident (Figure 5). In this scenario, the predictor can be unfair with respect to marital status or race, which are ancestors of the decision, but not age, which is not.

Kusner et al.'s Lemma 1 only indicates whether counterfactual fairness is *possible*, not whether it is *incentivized*. The latter question is determined by the response incentive to the protected attribute, as established by the following theorem (proven in Appendix A.3).

Theorem 12 (Counterfactual fairness and response incentives). There exists an optimal policy π^* that is counterfactually fair with respect to a protected attribute A if and only if there is no response incentive on A.

Using Theorem 12, we can use the graphical criterion for response incentives (Theorem 10) to rule out incentives for counterfactual unfairness.

First, age is ruled out, because age is not an ancestor of accident prediction.

⁴To compute counterfactual probabilities on the left-hand-side, we first "abduct" (update the \mathcal{E} -distribution based on \mathbf{pa}_D and a), and then "act" (intervene with do(A=a')) (Pearl, 2009, Ch. 7). The question at hand is whether the "act" step changes the distribution over the decision D. The protected attribute A may or may not be part of \mathbf{Pa}_D .

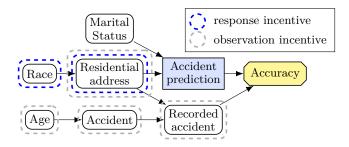


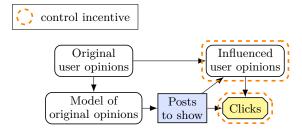
Figure 5: A toy example for how a car accident risk-estimator may come to make unfair decisions based on race. In this hypothetical, we consider that one of the protected attributes, age, affects accident rates. We assume that race affects residential address, which affects the rates of recorded accidents, but not the rates of accidents themselves. We assume that marital status does not affect any of these. Lastly, we assume that only marital status and residential address are available to the system. Since race influences residential area in this model, this may inadvertently create a (potentially unfair) response incentive for race. (For ease of comparison, we also illustrate observation incentives in gray.)

• More interestingly, perhaps, there can be no incentive to be counterfactually unfair to marital status, because the family of accident prediction d-separates marital status from accuracy. So although counterfactual unfairness regarding marital status is possible in Figure 5, an optimal predictor can be blind to marital status.

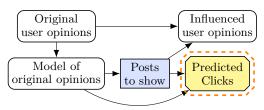
Thus, the only protected attribute that there can be an incentive to respond to is race.

Beyond counterfactual fairness. Counterfactual fairness has been criticized as being too liberal in deeming decisions unfair, when the effect of the protected attribute on the decision is mediated by a resolving variable (Kilbertus et al., 2017). For example, one might argue that the influence of race on accident prediction in Figure 5 is unproblematic, since it is mediated by residential address, which is a reasonable feature to use in predicting accident risk. Path-specific counterfactual fairness refines counterfactual fairness, by considering a decision unfair only if the protected attribute affects the decision along an unfair pathway (Chiappa, 2019; Chiappa and Isaac, 2019). A path-specific version of a response incentive is left to future work.

Everitt et al. (2019b) propose an alternative to manually specifying which paths are fair and not. They propose a graphical criterion called the *observation incentive* criterion to detect whether a classifier has an incentive to infer a protected attribute, and argue that if a classifier lacks an incentive to infer a protected attribute, then it lacks an incentive for disparate treatment of that attribute. (Indicated with grey in Figure 5.) The observation incentive criterion may be combined with the response incentive criterion. The combined criterion says



(a) Control incentive on user opinion.



(b) No control incentive on user opinion.

Figure 6: A social media content-selecting algorithm optimizes clicks, which leads to a control incentive for the user's opinion (a). This can be avoided with a change to the objective (b).

that a decision lacks unfairness incentives unless it has both an observation incentive and a response incentive for a protected attribute. This would imply that the decision problem in Figure 5 lacks unfair incentives, because race and marital status have no observation incentive, while race and age have no response incentive. Note that the combined criterion is more specific than either one alone, because only response incentives exclude unfair incentives towards age, and only observation incentives rule out unfair incentives towards race.

Taking a step back, the virtue of an incentive-based approach to fairness is that rather than evaluating the fairness of individual trained models, we can analyze whether a whole training regime (or class of training regimes) is likely to give rise to unfair models.

5.2 Controlling a user's preferences

A worry in AI safety (Russell, 2019) is that systems will find a way to manipulate their objective, in order to get more reward. This can be viewed as a type of control incentive (Everitt and Hutter, 2019). In the context of "filter bubbles" in social media, Stuart Russell says "Reinforcement learning changes the state of the world to maximize the reward. The state of the world in this case is your brain . . . [so] it changes you in a way that makes you more predictable so that it can then send you stuff that it knows you're going to click on." (Brand et al., 2019).

This dynamic is modelled in Figure 6a. Here the target of the content-selecting algorithm is to maximize user clicks over some time window, and these clicks depend on the influenced user opinions. From the diagram and the graphical control incentive crite-

rion (Theorem 7), we see that the content-selecting algorithm may have a control incentive for user opinion. For example, it may be easier to predict what content a more emotional user will click on and thereby achieve a higher click rate.

How could one design a content-selecting algorithm without such an incentive? The control incentive on user opinion relies on two things: (1) that the selected content influences user opinion, and (2) that the updated user opinion influences the objective of the content-selecting algorithm (i.e. the clicks). While there is not much we can do about (1), we can change the objective of the content-selecting algorithm. As shown in Figure 6b, we could redesign the system so that instead of being reward for the true click rate, it is rewarded for the predicted clicks on posts based on a model of the original user opinions. An agent trained in this way would view any modification of user opinions as irrelevant for improving its performance.

To work in practice, the click prediction must not itself include the effect of user opinion modification. We might accomplish this by using a prediction model that assumes independence between posts, or one that is learned by only showing one post to each user. This speaks to an important consideration when reasoning about incentives: the lack of an incentive on a variable is only practically meaningful if none of the variables act as "proxies" for one another. Otherwise, a control incentive on some variable X might systematically induce the same kinds of decisions that a control incentive on another variable Y would induce, even if there is no control incentive on Y.

Similarly to the fairness setting, we see here how an incentive analysis can be used to find training regimes that are likely to generate models with desirable properties. Everitt and Hutter (2019) provide a more detailed discussion of how to avoid undesirable control incentives.

6 Related Work

In this section, we will review related work on (1) AI safety, (2) incentive analysis in influence diagrams, as well as (3) the relationship between influence diagrams and (structural) causal models.

6.1 AI safety

The incentives of intelligent agents have been a core concern in AI safety since at least Omohundro (2008) and Bostrom (2012). A wide range of works, including works on cooperative inverse RL (Hadfield-Menell et al., 2016), reward corruption (Everitt et al., 2017), interruptibility (Soares et al., 2015; Orseau and Armstrong, 2016), oracles (Armstrong et al., 2012), and boxing (Cohen et al., 2020) have all focused on engineering desirable incentives of various types of AI systems. Our work aims to put this line of analysis on firmer footing in the causality and influence diagram literatures.

6.2 Value of information and control

Influence diagrams have long been used to compute the value of information or control — the degree to which utility can be gained by setting or observing a variable (Matheson, 1990). Subsequently, graphical criteria were developed for detecting requisite information links (Nielsen and Jensen, 1999; Lauritzen and Nilsson, 2001; Fagiuoli and Zaffalon, 1998), and "relevant" actions (Nielsen and Jensen, 1999). More recently, Everitt et al. (2019b) reframed this discussion in terms of agent incentives in AI and machine learning and established a graphical criteria for nodes that would be valuable to control. Our work offers an important complement, as it formalizes the incentives shaping the policy of the decision nodes, rather than giving the agent hypothetical additional observations or decision nodes.

6.3 Influence diagrams and structural causal models

Variants of structural causal influence models have been considered previously. A SCIM can be understood as a functional influence model (Dawid, 2002) augmented with utility nodes. It can also be viewed as a successor to the Howard canonical form influence diagram (Howard, 1990; Heckerman and Shachter, 1995). A SCIM requires all intrinsic nodes, rather than just descendants of the decision, to be deterministic so deterministic potential responses may be computed on all intrinsic nodes (not just descendants of the decision). This feature is instrumental in defining response incentives (Definition 8).

7 Conclusion

With the theory of incentives, it is possible to take a decision-making task, and analyze what behaviours agents are likely to exhibit. This analysis is independent of the agent's architecture and instead relies on the agent being sufficiently capable of attaining utility.

Formalizing agent environments and objectives with structural causal influence models, this paper defined two types of incentives that shape agent behaviour. Control incentives inform us of an agent's instrumental goals, an important consideration for AI safety. Meanwhile, response incentives indicate which variables an optimal decision is sensitive to, with implications for algorithmic fairness. Ultimately, we hope that researchers will find formal incentive analysis useful for designing fair and safe AI systems.

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A Proofs

A.1 Preliminaries

The proofs rely on some fundamental results from Pearl (2009).

Definition 13 (Path Interception). Let $(X \rightarrow Y | Z)_{\mathcal{G}}$ stand for the statement "Every directed path from X to Y in graph \mathcal{G} contains at least one element in Z." \mathcal{G} is omitted if the graph is clear.

Definition 14 (Causal Irrelevance). X is causally irrelevant to Y, given Z, written $(X \not\to Y|Z)$ if, for every set W disjoint of $X \cup Y \cup Z$, we have

$$\forall \varepsilon, \boldsymbol{z}, \boldsymbol{x}, \boldsymbol{x}', \boldsymbol{w} \qquad \boldsymbol{Y_{xzw}}(\varepsilon) = \boldsymbol{Y_{x'zw}}(\varepsilon)$$

Lemma 15. For every model \mathcal{M} compatible with a DAG \mathcal{G} ,

$$(X \not\rightarrow Y|Z)_{\mathcal{G}} \Rightarrow (X \not\rightarrow Y|Z)$$

Proof. By induction over variables, as in Galles and Pearl (1997, Lemma 12).

Lemma 16. For any disjoint subsets of variables W, X, Y, Z in the DAG \mathcal{G} , $\mathbb{E}(Y_x|z, w) = \mathbb{E}(Y_x|w)$ if $Y \perp Z|(X, W)$ in the graph \mathcal{G}' formed by deleting all incoming edges to X.

Proof. Follows from Pearl (2009), Thm. 3.4.1, Rule 1.

Lemma 17. For any three disjoint subsets of nodes (X, Y, Z) in a DAG \mathcal{G} , $(X \perp_{\mathcal{G}} Y|Z)$ if and only if $(X \perp \!\!\!\perp Y|Z)_P$ for every probability function P compatible with \mathcal{G} .

Proof. See Pearl (2009), Theorem 1.2.4

Lemma 18. d-separation obeys the intersection property. That is, for all disjoint sets of variables W, X, Y, and Z,

$$(W \perp X|Y,Z) \wedge (W \perp Y|X,Z)$$

$$\Rightarrow (W \perp (X \cup Y)|Z)$$

Proof. Suppose that that the RHS is false, so there is a path from W to $X \cup Y$ conditional on Z. This path must have a sub-path that passes from W to $X \in X$ without passing through Y or to $Y \in Y$ without passing through X (it must traverse one set first). But this implies that W is d-connected to X given Y, Z or to Y given X, Z, meaning the LHS is false. So if the LHS is true, then the RHS must be true.

A.2 Graphical criteria

Control incentives

Lemma 19 (CI Graphical Criterion Soundness). If a single-decision CID \mathcal{G} does not contain a path of the form $D \dashrightarrow X \dashrightarrow U$ then there is no control incentive on X in any SCIM \mathcal{M} compatible with \mathcal{G} .

Proof. Let \mathcal{M} be any SCIM compatible with \mathcal{G} and π any policy for \mathcal{M} . We consider variables in the SCM \mathcal{M}_{π} .

If there is no directed path $D \dashrightarrow X \dashrightarrow U$ in \mathcal{G} , then either $D \dashrightarrow X$ or $X \dashrightarrow U$. If $D \dashrightarrow X$, then $(D \dashrightarrow X \mid \mathbf{Pa}_D)$ as well. By Lemma 15, $X_d \mid \mathbf{pa}_D = X \mid \mathbf{pa}_D$ for any context \mathbf{pa}_D and decision d. Therefore, $\mathcal{U} \mid \mathbf{pa}_D = \mathcal{U}_{X_d} \mid \mathbf{pa}_D$. Similarly, if $X \dashrightarrow U$ then $U \mid \mathbf{pa}_D = U_x \mid \mathbf{pa}_D$ for every $x \in \text{dom}(X)$ and $U \in U$ so $\mathcal{U} \mid \mathbf{pa}_D = \mathcal{U}_{X_d} \mid \mathbf{pa}_D$. In either case, $\mathbb{E}_{\pi}[\mathcal{U} \mid \mathbf{pa}_D] = \mathbb{E}_{\pi}[\mathcal{U}_{X_d} \mid \mathbf{pa}_D]$ and there is no control incentive on X.

Lemma 20 (CI Graphical Criterion Completeness). If a single-decision CID \mathcal{G} contains a path of the form $D \longrightarrow X \longrightarrow U$ then there is a control incentive on X in at least one SCIM \mathcal{M} compatible with \mathcal{G} .

Proof. Assume that \mathcal{G} contains a directed path $D = Z_0 \to Z_1 \to \cdots \to Z_n = U$ where $U \in U$ and $Z_i = X$ for some $i \in \{0, \dots, n\}$. We prove Lemma 20 by explicitly constructing a compatible SCIM for which there is a control incentive on X. Consider the model $\mathcal{M} = \langle \mathcal{G}, \mathcal{E}, \mathbf{F}, P \rangle$ where P is arbitrary, all variables have domain $\{0, 1\}$, and $\mathbf{F} = \{f_V \mid V \in \mathbf{X} \cup \mathbf{U}\}$ where

$$f_V(\mathbf{pa}_V, \varepsilon_V) = \begin{cases} Z_{i-1} & \text{if } V = Z_i, 1 \le i \le n \\ 0 & \text{otherwise} \end{cases}$$

That is, all variables along the path $Z_0 \to \ldots \to Z_n$ are set equal to their predecessor (except $Z_0 = D$, which has no structure function) and all other variables are set to 0. In this model, $U = D \in \{0,1\}$ and all other utility variables are always 0 so the only optimal policy is $\pi^*(\mathbf{pa}_D) = 1$, which gives $\mathbb{E}_{\pi^*}[\mathcal{U} \mid \mathbf{Pa}_D = \mathbf{0}] = 1$. Meanwhile, $U_{X_d} = d$ so for d = 0 we have $\mathbb{E}_{\pi^*}[\mathcal{U}_{X_d} \mid \mathbf{Pa}_D = \mathbf{0}] = 0$.

Response incentive

Lemma 21 (RI Graphical Criterion Soundness). If a single-decision CID \mathcal{G} does not satisfy the response incentive graphical criterion $(D \dashrightarrow U) \land (X \dashrightarrow W) \land (W \perp U | \mathbf{Fa}_D \setminus \{W\})$ for any $U \in \mathbf{U}$, $W \in \mathbf{Pa}_D$ then there is no response incentive on X in any SCIM \mathcal{M} compatible with \mathcal{G} .

Proof. Assume that \mathcal{G} does not satisfy the response incentive graphical criterion on X. Let \mathcal{M} be any SCIM compatible with \mathcal{G} and let π^* be an optimal policy for \mathcal{M} . We show that there is no response incentive on X in \mathcal{M} by constructing an optimal policy $\tilde{\pi}$ such that $D(\varepsilon) = D_x(\varepsilon)$ for every $\varepsilon \in \text{dom}(\mathcal{E})$ and $x \in \text{dom}(X)$. Partition \mathbf{Pa}_D into

$$\mathbf{Pa}_D^X = \mathbf{Pa}_D \cap \mathbf{Desc}_X$$

 $\mathbf{Pa}_D^{ar{X}} = \mathbf{Pa}_D \setminus \mathbf{Pa}_D^X$

and select any value $\tilde{\mathbf{pa}}_D^X \in \text{dom}(\mathbf{Pa}_D^X)$ for which $\Pr_{\pi^*}(\mathbf{Pa}_D^X = \tilde{\mathbf{pa}}_D^X) > 0$. Define

$$\tilde{\pi}(\mathbf{p}\mathbf{a}_D^X,\mathbf{p}\mathbf{a}_D^{\bar{X}},\varepsilon_D) \coloneqq \pi^*(\tilde{\mathbf{p}}\mathbf{a}_D^X,\mathbf{p}\mathbf{a}_D^{\bar{X}},\varepsilon_D).$$

By design, X is causally irrelevant to D in the SCM $\mathcal{M}_{\tilde{\pi}}$ so for any x and ε , $D_x(\varepsilon) = D(\varepsilon)$. All that remains is to show that $\tilde{\pi}$ is an optimal policy.

Partition U into $U^D = U \cap \mathbf{Desc}_D$ and $U^{\bar{D}} = U \setminus \mathbf{Desc}_D$. D is causally irrelevant for every $U \in U^{\bar{D}}$ so every policy π (in particular, $\tilde{\pi}$) is optimal with respect to $\mathcal{U}^{\bar{D}} := \sum_{U \in U^{\bar{D}}} U$.

We now consider $\mathcal{U}^D \coloneqq \sum_{U \in \mathcal{U}^D} U$. Since the response incentive graphical criterion is not satisfied, it must be the case that $W \perp \mathcal{U}^D \mid \{D\} \cup \mathbf{Pa}_D \setminus \{W\}$ for every $W \in \mathbf{Pa}_D^X$. By inductively applying the intersection property of d-separation over elements of \mathbf{Pa}_D^X we get that $\mathbf{Pa}_D^X \perp \mathcal{U}^D \mid \{D\} \cup \mathbf{Pa}_D^{\overline{X}}$ and therefore

$$\mathbf{Pa}_D^X \perp \!\!\! \perp \mathcal{U}^D \mid \{D\} \cup \mathbf{Pa}_D^{\bar{X}}. \tag{2}$$

Since $\Pr_{\pi}(\mathbf{pa}_D)$ is independent of the policy π , we show that $\mathbb{E}_{\tilde{\pi}}[\mathcal{U}^D] = \mathbb{E}_{\pi^*}[\mathcal{U}^D]$ by showing that $\mathbb{E}_{\tilde{\pi}}[\mathcal{U}^D \mid \mathbf{pa}_D] = \mathbb{E}_{\pi^*}[\mathcal{U}^D \mid \mathbf{pa}_D]$ for every $\mathbf{pa}_D \in \text{dom}(\mathbf{Pa}_D)$ with $\Pr(\mathbf{pa}_D) > 0$. First, the expected utility of $\tilde{\pi}$ on any \mathbf{pa}_D^X is equal to the expected utility of π^* on input $\tilde{\mathbf{pa}}_D^X$:

$$\mathbb{E}_{\tilde{\pi}}[\mathcal{U}^D \mid \mathbf{P}\mathbf{a}_D^X = \mathbf{p}\mathbf{a}_D^X, \mathbf{P}\mathbf{a}_D^{\bar{X}} = \mathbf{p}\mathbf{a}_D^{\bar{X}}]$$

$$= \sum_{u,d} u \Pr(\mathcal{U}^D = u \mid d, \mathbf{p}\mathbf{a}_D^X, \mathbf{p}\mathbf{a}_D^{\bar{X}})$$

$$\Pr_{\tilde{\pi}}(D = d \mid \mathbf{p}\mathbf{a}_D^X, \mathbf{p}\mathbf{a}_D^{\bar{X}})$$

$$= \sum_{u,d} u \Pr(\mathcal{U}^D = u \mid d, \tilde{\mathbf{p}}\tilde{\mathbf{a}}_D^X, \mathbf{p}\mathbf{a}_D^{\bar{X}})$$

$$\Pr_{\pi^*}(D = d \mid \tilde{\mathbf{p}}\tilde{\mathbf{a}}_D^X, \mathbf{p}\mathbf{a}_D^{\bar{X}})$$

where the last equality follows from (2) and the definition of $\tilde{\pi}$. Second, the expected utility of π^* on input $\tilde{\mathbf{pa}}_D^X$ is the same as its expected utility on any input \mathbf{pa}_D^X :

$$\begin{split} &= \mathbb{E}_{\pi^*}[\mathcal{U}^D \mid \mathbf{P}\mathbf{a}_D^X = \tilde{\mathbf{p}}\tilde{\mathbf{a}}_D^X, \mathbf{P}\mathbf{a}_D^{\bar{X}} = \mathbf{p}\mathbf{a}_D^{\bar{X}}] \\ &= \max_d \mathbb{E}[\mathcal{U}_d^D \mid \mathbf{P}\mathbf{a}_D^X = \tilde{\mathbf{p}}\tilde{\mathbf{a}}_D^X, \mathbf{P}\mathbf{a}_D^{\bar{X}} = \mathbf{p}\mathbf{a}_D^{\bar{X}}] \\ &= \max_d \mathbb{E}[\mathcal{U}_d^D \mid \mathbf{P}\mathbf{a}_D^X = \mathbf{p}\mathbf{a}_D^X, \mathbf{P}\mathbf{a}_D^{\bar{X}} = \mathbf{p}\mathbf{a}_D^{\bar{X}}] \\ &= \mathbb{E}_{\pi^*}[\mathcal{U}^D \mid \mathbf{P}\mathbf{a}_D^X = \mathbf{p}\mathbf{a}_D^X, \mathbf{P}\mathbf{a}_D^{\bar{X}} = \mathbf{p}\mathbf{a}_D^{\bar{X}}] \end{split}$$

where the second last equality follows from Lemma 16. Therefore, $\tilde{\pi}$ is optimal for $\mathcal{U}^{\bar{D}}$ with $\mathbb{E}_{\tilde{\pi}}[\mathcal{U}^{\bar{D}}] = \mathbb{E}_{\pi^*}[\mathcal{U}^{\bar{D}}]$. Since $\tilde{\pi}$ is also optimal for $\mathcal{U}^{\bar{D}}$, $\tilde{\pi}$ is an optimal policy and so there is no response incentive on X in \mathcal{M} . \square

Lemma 22 (RI Graphical Criterion Completeness). If a single-decision CID \mathcal{G} satisfies the response incentive graphical criterion $(D \dashrightarrow U) \land (X \dashrightarrow W) \land (W \perp U | \mathbf{Fa}_D \setminus \{W\})$ for some $U \in \mathbf{U}$, $W \in \mathbf{Pa}_D$ then is a response incentive on X in at least one SCIM \mathcal{M} compatible with \mathcal{G} .

Proof. Assume that a CID \mathcal{G} satisfies the response incentive graphical criterion for X with respect to $U \in U$ and $W \in \mathbf{Pa}_D$. The variables W and U are d-connected given $\mathbf{Fa}_D \setminus \{W\}$ so there are models in

which they are conditionally dependent. Consider a (hypothetical) model \mathcal{M} , outlined in Figure 7, in which variables have domain $\{-1,0,1\}$, X=1, $U=D\cdot Z$ for $Z\sim \mathrm{Unif}(\{-1,1\})$ and, when conditioning on $\mathbf{Pa}_D\setminus\{W\}$, $W=X\cdot Z\cdot c$ where $c\in\{-1,1\}$ is a function of $\mathbf{pa}_D^{\setminus W}\in\mathrm{dom}(\mathbf{Pa}_D\setminus\{W\})$. If X and W are the same node we instead set $W=Z\cdot c$. In this model, $D=W\cdot c$ is an optimal policy and yields a utility of $U=X\cdot Z^2\cdot c^2=1$.

Now consider the intervention that sets X=0 and consequently $W_{X=0}=0$. Without the information about Z contained in W, Z is independent of $(\mathbf{Pa}_D)_{X=0}$ and hence independent of $D_{X=0}$ regardless of the selected policy. Therefore, $\mathbb{E}_{\pi}[U_{D_{X=0}}] = \mathbb{E}_{\pi}[Z \cdot D_{X=0}] = 0$ for every policy π . In particular, for any optimal policy π^* , $\mathbb{E}_{\pi^*}[U_{D_{X=0}}] \neq \mathbb{E}_{\pi^*}[U_D] = \mathbb{E}_{\pi^*}[U] = 1$ so there must be some ε such that $D_{X=0}(\varepsilon) \neq D(\varepsilon)$. Therefore, if this \mathcal{M} exists then there is a response incentive on X.

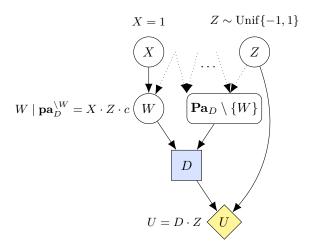


Figure 7: Outline of the variables involved in the response incentive model construction. Arrows represent directed paths and the dotted path d-connects W and Z given $\mathbf{Pa}_D \setminus \{W\}$. Every graph that satisfies the response incentive graphical criterion contains this structure when allowing X and Z to represent the same variable as W. $\pi(\mathbf{pa}_D, \varepsilon_D) = W \cdot c$ is an optimal policy for the model described by the node labels.

The remainder of this proof consists of explicitly constructing the model \mathcal{M} and is similar to the proof of the observation incentive criterion by Everitt et al. (2019b, Theorem 18). In \mathcal{G} there exists an undirected path $\overline{WU} = Z_0 Z_1 \cdots Z_n$ that is unblocked given $\mathbf{Fa}_D \setminus W$ where $Z_0 = W$ and $Z_n = U$. For each Z_i , if $Z_{i-1} \leftarrow Z_i \rightarrow Z_{i+1}$ then label Z_i a "source" node S_l and if $Z_{i-1} \rightarrow Z_i \leftarrow Z_{i+1}$ then label it a "collider" node O_l . By the definition of conditional d-separation, $O_l \dashrightarrow D$. With this labelling, we can write \overline{WU} as



for $m \geq 0$ where the first S_0 is optional; we give W the label S_0 if S_0 does not otherwise exist. The path cannot end with $O_m \leftarrow U$ because there is a directed path $O_m \rightarrow D \rightarrow U$ in \mathcal{G} so the existence of a path $U \rightarrow O_m$ would create a cycle. Therefore, S_m always exists although it may be the case that $S_m = W$ when m = 0.

We construct $\mathcal{M} = \langle \mathcal{G}, \mathcal{E}, F, P \rangle$ as follows: The exogenous variables \mathcal{E} have domain $\{-1,1\}$ and P gives each an independent uniform distribution. The decision and utility variables have domain $\{-1,0,1\}$ and all other variables have domain $\{-1,0,1\}^{m+4}$. Conceptually, we would like each variable to have domain $\{-1,0,1\}$ but we need to define the functional structure along many possibly intersecting paths so we define each path structure in its own dimension. For a variable $V \in \mathbf{V}$ and a structure function f_V , we write

$$V = (V^*, V^{\overrightarrow{XW}}, V^{\overrightarrow{DU}}, V^{\overrightarrow{WU}}, V^{\overrightarrow{O_1D}}, \dots, V^{\overrightarrow{O_mD}})$$

$$f_V = (f_V^*, f_V^{\overrightarrow{XW}}, f_V^{\overrightarrow{DU}}, f_V^{\overrightarrow{WU}}, f_V^{\overrightarrow{O_1D}}, \dots, f_V^{\overrightarrow{O_mD}})$$

where V^* is the canonical dimension that encodes the values described in the previous outline of \mathcal{M} (and also refers to the sole dimension of D and U), $V^{\overrightarrow{AB}}$ is used for the directed path $A \dashrightarrow B$, and $V^{\overline{AB}}$ is used for the undirected path $A \dashrightarrow B$. For each dimension of the form \overrightarrow{AB} , let $A = Y_0 \to Y_1 \to \cdots \to Y_n = B$ be a path in \mathcal{G} and define

$$f_{V}^{\overrightarrow{AB}}(\mathbf{pa}_{V}, \varepsilon_{V}) = \begin{cases} Y_{0}^{*} & \text{if } V = Y_{1} \\ Y_{i-1}^{\overrightarrow{AB}} & \text{if } V = Y_{i}, 2 \leq i \leq n \\ 0 & \text{otherwise} \end{cases}$$

for $V \in \boldsymbol{X} \cup \boldsymbol{U}$. Since utilities have scalar domain, we don't include $f_{U}^{\overrightarrow{AB}}$ in the model, only f_{U}^{*} , but it is helpful to define $f_{U}^{\overrightarrow{AB}}$ in order to refer to it later. With these structure functions, $B^{\overrightarrow{AB}} = f_{B}^{\overrightarrow{AB}}(\mathbf{pa}_{B}, \varepsilon_{B}) = A^{*}$ whenever $n \geq 1$.

Along the \overline{WU} dimension, define

$$\begin{split} f_V^{\overline{WU}}(\mathbf{pa}_V, \varepsilon_V) \\ = \begin{cases} Z_{i-1}^{\overline{WU}} & \text{if } V = Z_i, \ Z_{i-1} \to Z_i \to Z_{i+1} \\ Z_{i+1}^{\overline{WU}} & \text{if } V = Z_i, \ Z_{i-1} \leftarrow Z_i \leftarrow Z_{i+1} \\ Z_{i-1}^{\overline{WU}} \cdot Z_{i+1}^{\overline{WU}} & \text{if } V = Z_i, \ Z_{i-1} \to Z_i \leftarrow Z_{i+1} \\ \varepsilon_V & \text{if } V = Z_i, \ Z_{i-1} \leftarrow Z_i \to Z_{i+1} \\ \varepsilon_V & \text{if } V = Z_0, \ Z_0 \to Z_1 \\ 0 & \text{otherwise} \end{cases} \end{split}$$

for $V \in X \cup U$. This gives us

$$S_l^{\overline{WU}} \sim \text{Unif}(\{-1,1\})$$

 $O_l^{\overline{WU}} = S_{l-1}^{\overline{WU}} \cdot S_l^{\overline{WU}}.$

Finally, we define the structure functions for the canonical dimension:

$$f_U^*(\mathbf{p}\mathbf{a}_U,\varepsilon_U) = f_U^{\overrightarrow{DU}}(\mathbf{p}\mathbf{a}_U,\varepsilon_U) \cdot f_U^{\overline{WU}}(\mathbf{p}\mathbf{a}_U,\varepsilon_U)$$

which implies $U = D \cdot S_m^{\overline{WU}}$, and

$$f_W^*(\mathbf{p}\mathbf{a}_W, \varepsilon_W) = \begin{cases} f_W^{\overline{XW}}(\mathbf{p}\mathbf{a}_W, \varepsilon_W) & \text{if } X \neq W \\ \cdot f_W^{\overline{WU}}(\mathbf{p}\mathbf{a}_W, \varepsilon_W) & \text{otherwise} \end{cases}$$

$$f_X^*(\mathbf{p}\mathbf{a}_X, \varepsilon_X) = 1 \quad \text{if } X \neq W$$

$$f_V^*(\mathbf{p}\mathbf{a}_V, \varepsilon_V) = 0 \quad \text{for all other } V \in \mathbf{V} \setminus \{U, W, X\}$$

meaning $W^* = X^* \cdot S_0^{\overline{WU}}$ if $X \neq W$ and $W^* = S_0^{\overline{WU}}$ if X = W.

This construction matches the desired structure of \mathcal{M} where $Z = S_m^{\overline{WU}}$ and $c = \prod_l O_l^{\overline{WU}} = S_0^{\overline{WU}} S_m^{\overline{WU}}$. Note that the value of $O_l^{\overline{WU}}$ is represented in $\mathbf{Pa}_D^{\overline{O_lD}}$ via the path $O_l \dashrightarrow D$ so c is indeed a function of \mathbf{Pa}_D . Therefore, there is a response incentive on X in \mathcal{M} . \square

A.3 Counterfactual fairness

Proof of Theorem 12. Assume that there exists an optimal policy π that is counterfactually fair. Let $\operatorname{supp}_{\pi}(D \mid \mathbf{pa}_D) = \{d \mid \operatorname{Pr}_{\pi}(D = d \mid \mathbf{pa}_D) > 0\}$ and $\operatorname{supp}_{\pi}(D_a \mid \mathbf{pa}_D) = \{d \mid \operatorname{Pr}_{\pi}(D_a = d \mid \mathbf{pa}_D) > 0\}$. As a first step, we will show that for any $\varepsilon \in \operatorname{dom}(\mathcal{E})$ and any intervention a on A,

$$\operatorname{supp}_{\pi}(D \mid \mathbf{Pa}_{D}(\boldsymbol{\varepsilon})) = \operatorname{supp}_{\pi}(D_{a} \mid \mathbf{Pa}_{D}(\boldsymbol{\varepsilon})). \tag{3}$$

By way of contradiction, suppose there exist a decision

$$d \in \operatorname{supp}_{\pi}(D \mid \mathbf{Pa}_D(\varepsilon)) \setminus \operatorname{supp}_{\pi}(D_a \mid \mathbf{Pa}_D(\varepsilon)).$$
 (4)

Since $d \in \operatorname{supp}_{\pi}(D \mid \mathbf{Pa}_D(\varepsilon))$, we have

$$\Pr_{\pi} (D = d \mid \mathbf{Pa}_D(\varepsilon), A(\varepsilon)) > 0.$$
 (5)

And since $d \notin \operatorname{supp}_{\pi}(D_a \mid \mathbf{Pa}_D(\varepsilon))$, there exists no ε' with positive probability such that $\mathbf{Pa}_D(\varepsilon') = \mathbf{Pa}_D(\varepsilon)$, $A(\varepsilon') = A(\varepsilon)$, and $D_a(\varepsilon') = d$, which gives

$$\Pr_{\pi} \left(D_a = d \mid \mathbf{Pa}_D(\varepsilon), A(\varepsilon) \right) = 0. \tag{6}$$

Equations (5) and (6) violate the counterfactual fairness property (1), which shows that (4) is impossible. An analogous argument shows that $d \in \operatorname{supp}_{\pi}(D_a \mid \operatorname{\mathbf{Pa}}_D(\varepsilon)) \setminus \operatorname{supp}_{\pi}(D \mid \operatorname{\mathbf{Pa}}_D(\varepsilon))$ also violates the counterfactual fairness property (1). We have thereby established (3).

Select now an arbitrary ordering of the elements of dom(D) and define a new policy π^* such that $\pi^*(\mathbf{pa}_D)$ is the minimal element of $\operatorname{supp}_{\pi}(D \mid \mathbf{pa}_D)$. Then π^* is optimal because π is optimal. Further, π^* will make the same decision in decision contexts $\mathbf{Pa}_D(\varepsilon)$ and $(\mathbf{Pa}_D)_a(\varepsilon)$ because of (3). In other words, $D_a(\varepsilon) = D(\varepsilon)$ in \mathcal{M}_{π^*} for the optimal policy π^* , which means that there is no response incentive on A.

Now we prove the reverse direction — that if there is no response incentive then some optimal π^* is counterfactually fair. Choose any optimal policy π^* where

 $D_a(\varepsilon) = D(\varepsilon)$ for all ε . Since an intervention a cannot change D in any setting, $P(D_a = d \mid \cdot) = P(D = d \mid \cdot)$ for any condition and any decision d, hence π^* is counterfactually fair.