MS125: Probability
Assignment 2

To be submitted by 25th of April 2008

Question 1

The random variable x has the following discrete probability distribution:

x	1	3	5	7	9
$P(X = x)$	0.1	0.2	0.4	0.2	0.1

1. Write down cumulative probability function.
2. Calculate $E[X]$.
3. Calculate $Var[X]$.

Question 2

A machine that produces stampings for automobile engines is malfunctioning and producing 10% defectives. The defective and nondefective stampings proceed from the machine in a random manner. If the next five stampings are tested, find the probability that three of them are defective.

Question 3

Suppose the number, x, of a company's employees who are absent on Mondays has a Poisson probability distribution. Furthermore, assume that the average number of Monday absentees is 2.6.

1. Find the mean and standard deviation of x, the number of employees absent on Monday.
2. Find the probability that fewer than two employees are absent on a given Monday.
3. Find the probability that more than three employees are absent on a given Monday.
4. Find the probability that exactly four employees are absent on a given Monday.

Question 4

Suppose the research department of a steel manufacturer believes that one of the company's rolling machines is producing sheets of steel of varying thickness. The thickness is a discrete uniform random variable with values between 150 and 200 millimeters. Any sheets less than 160 millimeters must be scrapped because they are unacceptable to buyers. Calculate the fraction of steel sheets produced by this machine that have to be scrapped.

Question 5

Assume the length of time, X, between charges of a mobile phone is normally distributed with a mean of 10 hours and a standard deviation of 1.5 hours. Find the probability that the cell phone will last between 8 and 12 hours between charges.

Question 6

A fair dice is thrown three times. The result of first throw is scored as $X_1 = 1$ if the dice shows 6 and $X_1 = 0$ otherwise; X_2 and X_3 are scored likewise for the second and third throws.

Let $Y_1 = X_1 + X_2$ and $Y_2 = X_1 - X_3$.

1. Calculate the joint probabilities in the bivariate distribution of the pair (Y_1, Y_2) and display in an appropriate table.
2. Find the marginal probability distributions of Y_1 and Y_2.
3. Calculate the means and variances of Y_1 and Y_2.
4. Calculate the covariance of Y_1 and Y_2.
5. Find the conditional distribution of Y_1 given $Y_2 = 0$.
6. Find the conditional mean of Y_1 given $Y_2 = 0$.
7. State with justification if Y_1 and Y_2 are independent.

Question 7

$x_1, \ldots, x_n$ is a random sample of size n from random variable with probability density function:

$$f(x) = (1 + \theta)x^{\theta}, \quad 0 < x < 1$$

Find $\hat{\theta}$, the maximum likelihood estimator of θ.

Question 8

A manufacturer of cereal wants to test the performance of one of its filling machines. The machine is designed to discharge a mean amount of 12 ounces per box, and the manufacturer wants to detect any departure from this setting. (i) He randomly samples 100 boxes from production run and obtains that the sample mean fill of 100 boxes is $\bar{x} = 11.5$ and standard deviation is $s^2 = 6.25$. Test the null hypothesis $H_0 : \mu = 12$ against the alternative hypothesis $H_0 : \mu \neq 12$. (ii) The next day the manufacture samples 10 boxes and gets $\bar{x} = 11.6$ and $s^2 = 0.16$. Test the null hypothesis $H_0 : \mu = 12$ against the alternative hypothesis $H_0 : \mu \neq 12$.

Question 9

An economist decided to test the hypothesis that higher retail prices are being charged for Japanese automobiles in Japan than in the United States. She obtained random samples of 50 retail sales in the United States and 30 retail sales in Japan over the same time period and for the same model of automobile, converted the Japanese sales prices from yen to dollars using current conversion rates, and obtained the summary information shown in Table below.

	USA Sales	Japan Sales
Sample size	50	30
Sample mean	$ 16,545	$ 17,243
Sample standard deviation	$ 1,989	$ 1,843

Test the null hypothesis $H_0 : \mu_U = \mu_J$ against the alternative $H_0 : \mu_U > \mu_J$.

Question 10

In order to create a behavioral profile of travelers, agency interviewed 5,026 travelers in the Tampa Bay region. Two of the characteristics they investigated were the travelers' education level and their use of the Internet to seek travel information. The table below summarizes the results of the interviews.

	USE INTERNET	
	Yes	No
College Degree or More	1,072	1,287
Less than a College Degree	640	2,027

Test if travelers who use the Internet to search for travel information are likely to be people who are college educated.