HW3

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Exercise A - Data Collection and Coding

For my data, I decided to collect iPhone text messages with my roommate that pertained to moving our cars around in our driveway. iMessage only had texts by the minute so I was unable to check for exact seconds in response times, but either way, I found that all of our responses were equal to or longer than 1 minute. This was collected and inputted into a csv named Andria_Chen_data_HW3.csv.

Exercise B - Modeling with an Exponential Distribution

I used Mathematica's Manipulate feature to explore the different possible distributions for the prior on λ. I consulted with PyMC documentation to determine which continuous distributions were wide and low informative considering that the mean of exponential distributions is $1/\lambda$. This means that if λ is small, the mean response time is large and vice versa.

I looked at both Gamma and Uniform distributions but ultimately ended up choosing Gamma because Uniform would require me to set an arbitrary upper bound. Gamma is relatively flat and weakly informative which matches realistic expectations that text message response times may range from a few minutes to multiple hours depending on day-to-day situations.

After playing around with the sliders in mathematica by varying α and β, I determined that Gamma($\alpha = 2,\ \beta = 0.05$) provided an appropriate prior. Using Mathematica documentation of scale parameterization, this gives a mean of $\alpha * \beta = 0.1$ for λ which corresponds to a mean response time of around 10 minutes. In PyMC documentation of rate parameterization, we have Gamma($\alpha = 2, \beta = 20$) where the mean is $\alpha/\beta = 2/20 = 0.1$. This allowed for a wide range of possible values and it still drops off quickly as it approaches 0.

The posterior predictive distribution shows that the exponential model learned from the data, concentrating probability mass at shorter response times. However, the fit is not perfect as the model underestimates the frequency of the short response times and doesn't fully capture the heavy right tail of occasional long delays in response time. This may be because the Exponential distribution assumes a constant probability of responding regardless of wait time which isn't reflective of actual messaging behavior. The Exponential distribution does not fully correspond with actual response time behavior which motivates exploring other distributions that may provide a better fit.

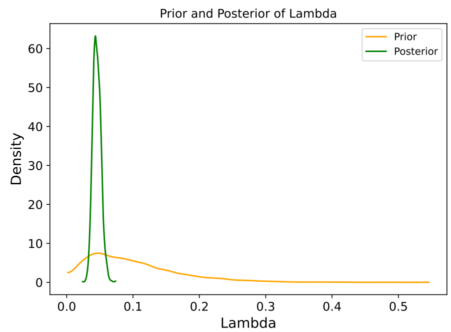

Figure 1: Prior and Posterior of Lambda

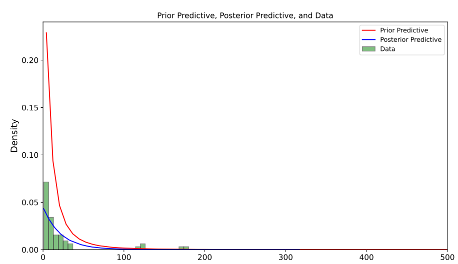

Figure 2: Prior Predictive, Posterior Predictive, and Data

Exercise C - Modeling with a Gamma Distribution

I approached exercise C in a similar manner. I chose Gamma distributions as priors for both α and β because both parameters must only be positive values which matches the domain constraints of Gamma distributions. It is also flexible which accounts for variability. For α, the shape parameter, I used Gamma(2,1) according to Mathematica documentation which gives us a mean of 2 (2 * 1) and allows α to be weakly informative while staying in a reasonable range for the response time data. For β, the rate parameter, I used Gamma(3, 0.02) in Mathematica which has a mean of 0.06 (3 * 0.02). According to PyMC documentation, a prior with $\alpha = 2$ and $\beta = 0.06$ implies a mean response time of $2/0.06 = 33.34$ minutes while remaining wide enough to allow the data to update my beliefs. I explored other posiitve valued distributions in Mathematica like Exponential and LogNormal but I found Gamma to be the most flexible and interpretable.

The Gamma model showed a substantial improvement in terms of fit over the Exponential model. As seen in the figures below, the posterior predictive distribution captures the shape of the data more accurately. Specifically, the concentration of short response times and the gradual tail formed. The posterior for α converged around 0.5 suggesting that the distribution is more exponential in nature than highly peaked. The posterior mean response time of approximately 26 minutes seems to correspond to the observed data mean (I estimated about 25-30 minutes after I was done collecting my data).

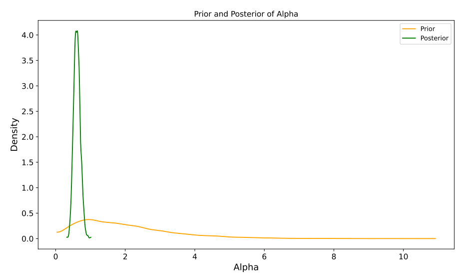

Figure 3: Prior and Posterior for Alpha

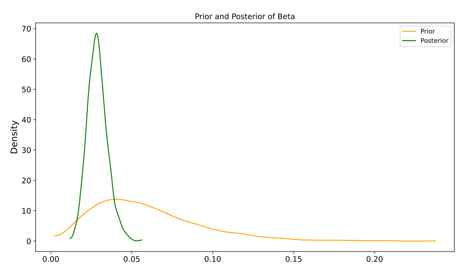

Figure 4: Prior and Posterior for Beta

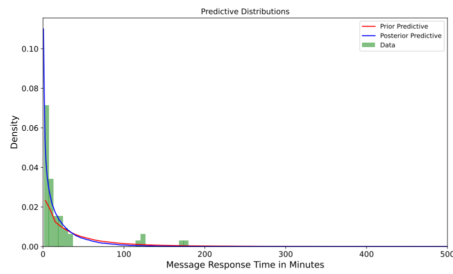

Figure 5: Prior Predictive, Posterior Predictive, and Data

Exercise D - Modeling with a Weibull Distribution

Never having worked with a Weibull Distribution before, I began by consulting both Mathematica and PyMC documentation to get a better understanding of the parameters. Considering that the domain constraint must only have positive numbers and my priors must be weakly informative, I also decided to model the priors for this exercise with a Gamma distribution especially because I already modeled the other two exercises' priors with Gamma as well. Especially given that it naturally constrains values to be positive while allowing for flexibility in shape.

Again, after playing around with the Manipulate sliders in Mathematica, I determined appropriate values for the Weibull distribution parameters α (shape) and β (scale). For α, I chose Gamma(2,0.5) in Mathematica's scale parameterization because this gives us a mean of 1 (2 * 0.5). This allows for a large range of values which reasonably covers message response time behaviors and expresses weak belief that response behavior is exponential. For β, I chose Gamma(3,10) in Mathematica which has a mean of 30 (3 * 10). This allows for a flexible range as well while still remaining weakly informative and accommodating for quick and slower response patterns. With PyMC's rate parameterization, these priors become Gamma(alpha=2, beta=2) for α and Gamma(alpha=3, beta=0.1) for β.

Upon analyzing these graphs, the Weibull model seems to provide a better fit to the true nature of message response behavior. The posterior distribution of the Weibull shape shows that as time passes without a response, the probability of receiving a response in the next moment decreases. This pattern makes intuitive sense for text messaging behavior because if someone hasn't responded quickly, they are likely busy or otherwise engaged. The longer the delay, the more "settled" they are in a non-responding state, thus making an immediate response less likely.

The failure of the Exponential model confirms that message response behavior is time-dependent. People's likelihood of responding depends on how long they've already been waiting, not just a fixed rate parameter. The Weibull model's flexibility in capturing this time-varying behavior makes it a better choice for modeling message response times.

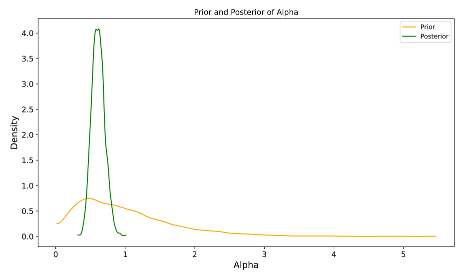

Figure 6: Prior and Posterior for Alpha

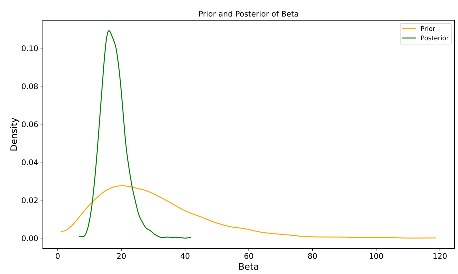

Figure 7: Prior and Posterior for Beta

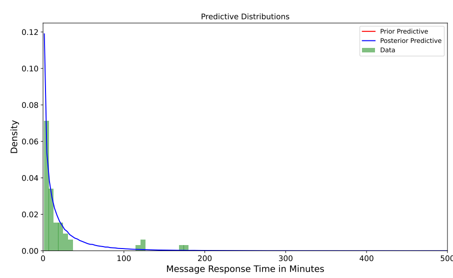

Figure 8: Prior Predictive, Posterior Predictive, and Data

Exercise E - Comparison and Discussion

Code Output:

The data is more likely under the 'Gamma' than the 'Exponential'

BF_Gamma_Exponential=88.59329214856527

The data is more likely under the ’Weibull’ than the ’Exponential’
BF_Weibull_Exponential=3978.9766415386453

The data is more likely under the ’Weibull’ than the ’Gamma’
BF_Weibull_Gamma=44.91284323045635

Qualitative Assessment: Upon visual inspection of the posterior predictive plots, the Exponential model (Exercise B) showed the poorest fit to the data, with the posterior predictive failing to capture the concentration of short response times and underestimating the tail. The Gamma model (Exercise C) showed improvement to fit, better capturing the overall shape of the distribution. The Weibull model (Exercise D) provided the best qualitative fit, closely matching both the high density of short responses and the gradual decay toward longer response times.

Quantitative Assessment: The Bayes Factor analysis strongly confirms this qualitative assessment. The Gamma model is approximately 89 times more likely than the Exponential model (BF = 88.6), indicating that there is strong evidence against the constant rate assumption of the Exponential distribution. The Weibull model dramatically outperforms the Exponential model by a factor of approximately 4,000 (BF = 3,979), demonstrating overwhelming evidence for time-varying rates. Finally, the Weibull model is approximately 45 times more likely than the Gamma model (BF = 44.9), confirming that the Weibull provides the overall best fit to the data. The quantitative evaluation matches the qualitative assessment perfectly with Weibull > Gamma > Exponential.

Finally, some conclusions we can draw regarding the nature of message response times:

1. Message responses are not memory-less processes. The overall poor fit of the Exponential model (BF 4,000 in favor of Weibull) demonstrates that response behavior fundamentally depends on elapsed time. Unlike random events with constant probability, the likelihood of receiving a message response changes as time passes. This suggests that human communication behavior is influenced by temporal factors and ongoing life activities.

2. Response probability decreases as waiting time increases. The Weibull model’s posterior parameter suggests a general pattern in messaging behavior: people tend to respond either quickly (within minutes) or then in a longer time frame. Once someone has failed to respond immediately, they are likely engaged in an activity that prevents them from checking their messages, and this engagement persists over time. The longer someone waits to respond, the less likely they are to respond soon, perhaps because they become mentally ”settled” in their current task or forget about responding to the message entirely.

3. Message response behavior exhibits multi stages. The fact that the Gamma model (which can represent waiting for multiple successive events) outperformed the Exponential model with a Bayes factor of 89 suggests that re-

sponding to a message may involve multiple behavioral stages. For example: 1 - seeing the notification, 2 - reading the message, 3 - deciding to respond, and 4 - composing the response. Each stage may have its own respective completion time and the overall response time reflects the sum of these stages.

4. There is significant heterogeneity in response patterns. The data shows many very quick responses (1-5 minutes) alongside occasional long delays (hours). This suggests that message response behavior depends heavily on context: availability, current activity, relationship to sender, and message importance. The Weibull distribution's flexibility in modeling this makes it the best at capturing this real world complexity.