Physics-Informed Neural Networks for Futures Pricing and Signal-Generation Under Stochastic Volatility

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Capstone Project

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Agenda

1. Introduction
2. Mathematical Foundations
3. PINN Architecture
4. Asset Selection and Training
5. Signal-Generation
6. Future Improvements

Introduction

1. Importance of having our own pricing model
    1. Understanding contract value
    2. Identifying investment opportunities from mispricing
2. Problems with mathematical pricing models
    1. Cost-of-Carry
    2. Black-Scholes
    3. Heston
3. Benefits of using the Physics-Informed Neural Network
    1. Captures hidden relationships
    2. Fast calculation

Mathematical Foundations

ALGO

Heston SDE

$$dS_t = rS_t dt + S_t\sqrt{v_t}\Sigma_S^{\mathrm{T}} dW_t$$
$$dv_t = \kappa(\gamma - v_t)dt + \sigma\sqrt{v_t}\Sigma_v^{\mathrm{T}} dW_t$$

Feynman-Kac PDE

$$0 = \partial_t V - rV + \mu_y^{\mathrm{T}}\nabla_y V + \frac{1}{2} trace\left(\Sigma_y \Sigma_y^{\mathrm{T}} H_y(V)\right)$$

$$0 = \frac{\partial V}{\partial t} - rV + (\mu_S \frac{\partial V}{\partial S} + \mu_v \frac{\partial V}{\partial v}) + \frac{v}{2}(S^2 \frac{\partial^2 V}{\partial S^2} + 2\rho\sigma S \frac{\partial^2 V}{\partial v \partial S} + \sigma^2 \frac{\partial^2 V}{\partial v^2})$$

Terminal Condition

$$V(T, y_t) = S_t$$

V – value
t – time
T – maturity
r – risk-free rate
S_t – spot price (S)
v_t – variance (v)
μ_S – drift for spot price
μ_v – drift for variance
κ – rate of mean reversion
γ – long-run variance
ρ – correlation between Brownian motions
σ – volatility
Σ_y – diffusion matrix

Mathematical Foundations

ALGO

Matrix Operations

$$0 = \partial_t V - rV + \mu_y^{\mathrm{T}} \nabla_y V + \frac{1}{2} trace\left(\Sigma_y \Sigma_y^{\mathrm{T}} H_y(V)\right) \qquad y = \begin{bmatrix} S \\ v \end{bmatrix}$$

$$0 = \frac{\partial V}{\partial t} - rV + \left(\mu_S \frac{\partial V}{\partial S} + \mu_v \frac{\partial V}{\partial v}\right) + \frac{v}{2}\left(S^2 \frac{\partial^2 V}{\partial S^2} + 2\rho\sigma S \frac{\partial^2 V}{\partial v \partial S} + \sigma^2 \frac{\partial^2 V}{\partial v^2}\right)$$

Drift

$$\mu_y = \begin{bmatrix} \mu_S \\ \mu_v \end{bmatrix} = \begin{bmatrix} rS \\ \kappa(\gamma - v) \end{bmatrix}$$

$$\nabla_y V = \begin{bmatrix} \frac{\partial V}{\partial S} \\ \frac{\partial V}{\partial v} \end{bmatrix}$$

$$\mu_y \nabla_y V = \mu_S \frac{\partial V}{\partial S} + \mu_v \frac{\partial V}{\partial v}$$

Trace

$$\Sigma_y = \sqrt{v} \begin{bmatrix} S & 0 \\ \rho\sigma & \sigma\sqrt{1 - \rho^2} \end{bmatrix}$$

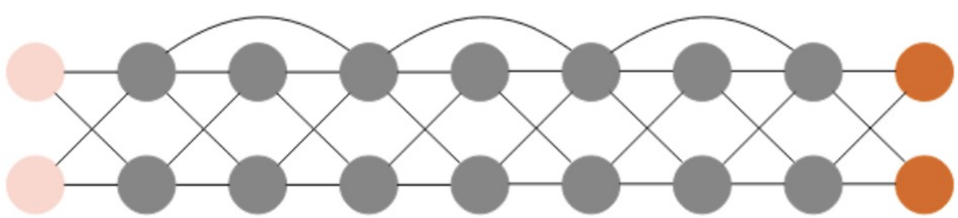
$$\mathrm{H}_y(V) = \begin{bmatrix} \frac{\partial^2 V}{\partial S^2} & \frac{\partial^2 V}{\partial S \partial v} \\ \frac{\partial^2 V}{\partial v \partial S} & \frac{\partial^2 V}{\partial v^2} \end{bmatrix}$$

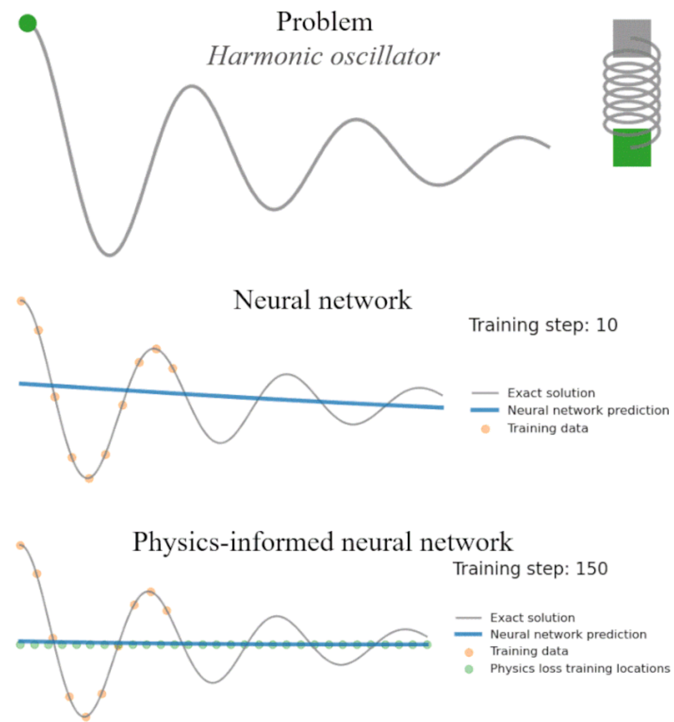
$$\Sigma_y \Sigma_y^{\mathrm{T}} = v \begin{bmatrix} S^2 & \rho\sigma S \\ \rho\sigma S & \sigma^2 \end{bmatrix}$$

$$trace\left(\Sigma_y \Sigma_y^{\mathrm{T}} H_y(V)\right) = v\left(S^2 \frac{\partial^2 V}{\partial S^2} + 2\rho\sigma S \frac{\partial^2 V}{\partial v \partial S} + \sigma^2 \frac{\partial^2 V}{\partial v^2}\right)$$

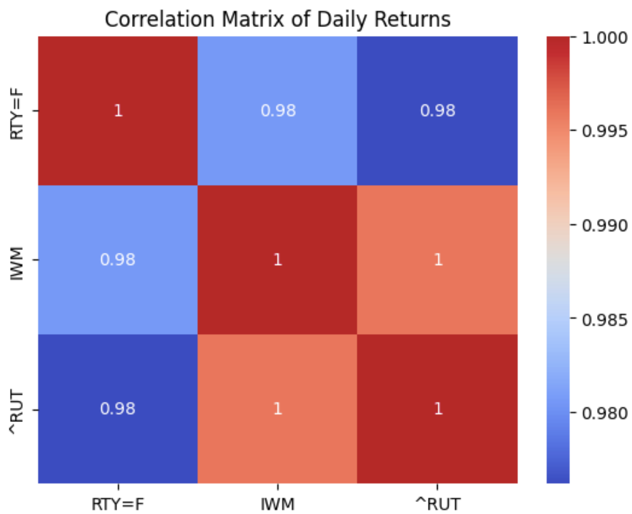
PINN Architecture

1. Input – FK variables
2. Type – Residual Neural Network
3. Activation Function – tanh
4. Loss – weighted physics loss + mean-squared error (MSE)
5. Output – futures price
6. Validation – train-test split





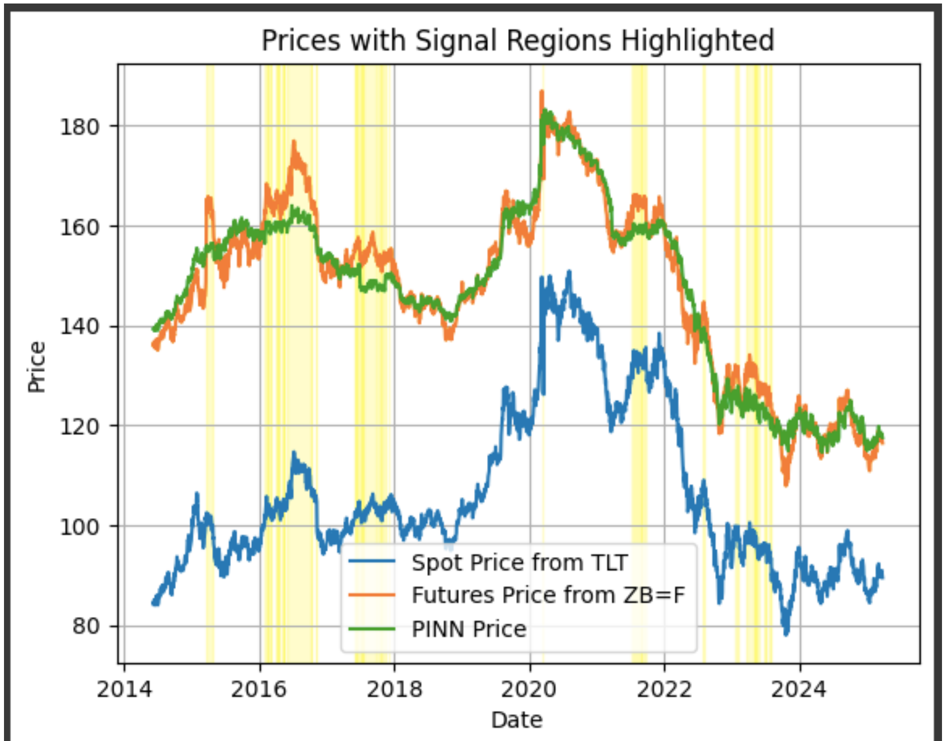
Asset Selection and Training




```
Epoch 180/250
Train Physics Loss: 0.0041, Train MSE Loss: 0.0009, Train Total Loss: 0.0021
Epoch 190/250
Train Physics Loss: 0.0034, Train MSE Loss: 0.0009, Train Total Loss: 0.0019
Epoch 200/250
Train Physics Loss: 0.0029, Train MSE Loss: 0.0008, Train Total Loss: 0.0017
Epoch 210/250
Train Physics Loss: 0.0027, Train MSE Loss: 0.0011, Train Total Loss: 0.0019
Epoch 220/250
Train Physics Loss: 0.0024, Train MSE Loss: 0.0010, Train Total Loss: 0.0017
Epoch 230/250
Train Physics Loss: 0.0022, Train MSE Loss: 0.0007, Train Total Loss: 0.0014
Epoch 240/250
Train Physics Loss: 0.0022, Train MSE Loss: 0.0007, Train Total Loss: 0.0013
Epoch 250/250
Train Physics Loss: 0.0020, Train MSE Loss: 0.0006, Train Total Loss: 0.0013
--------------------------------------------------------------------------------
Final Test Physics Loss: 0.0018
Final Test MSE Loss: 0.0006
Final Test Total Loss: 0.0011
```

Asset_Type	Asset	Futures_Ticker	Spot_Ticker	Correlation	Physics_Loss	MSE_Loss	Total_Loss
Fixed Income	10-Year T-Note Futures	ZN=F	IEF	0.92	0.0036	0.0017	0.0028
Fixed Income	30-Year T-Bond Futures	ZB=F	TLT	0.88	0.0044	0.0027	0.0040
Equities	Russell 2000	RTY=F	IWM	0.98	0.0018	0.0006	0.0011
Equities	Dow Jones	YM=F	DIA	0.97	0.0008	0.0003	0.0005
Commodities	Gold	GC=F	GLD	0.90	0.0012	0.0010	0.0013

Signal-Generation

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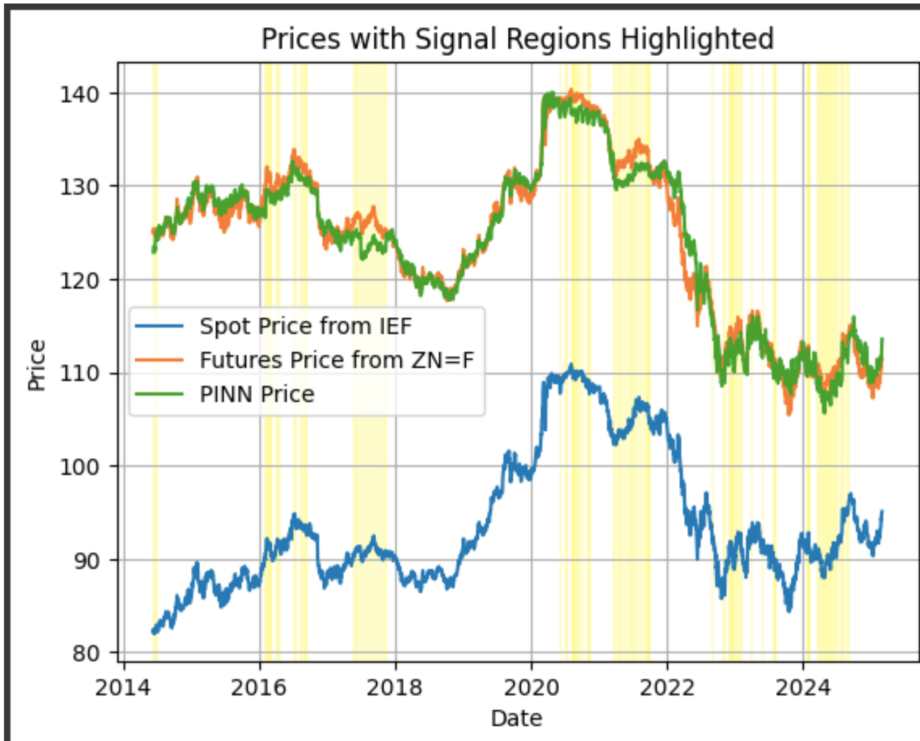
T-test (return after signal vs no signal):
  t-stat = -4.9440, p-value = 0.00000108
```

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T-test (return after signal vs no signal):
  t-stat = -2.6245, p-value = 0.00886891
```

Future Improvements

1. Finding optimal weights for physics loss
2. Conducting additional statistical tests to evaluate causal relationships
3. Backtesting strategies based on price spread signals
4. Modifying model to train up to a target loss rather than target number of epochs

Q&A