Double-well TRAPS

1. $V(x) = \frac{1}{2}\omega^2 x^2 + b^2 e^{-x^2/2\sigma^2}$

$V'(x) = \omega^2 x - \frac{x b^2}{\sigma^2} e^{-x^2/2\sigma^2}$

Minimum position at $\bar{x} \neq 0$

$0 = \omega^2 - \frac{b^2}{\sigma^2} e^{-\bar{x}^2/2\sigma^2}$

$\frac{\omega^2\sigma^2}{b^2} = e^{-\bar{x}^2/2\sigma^2}$

$$\boxed{\frac{\bar{x}^2}{2\sigma^2} = -\ln\left(\frac{\omega^2\sigma^2}{b^2}\right)} \quad \frac{\omega\sigma}{b} < 1$$

The height is given by:

$V(0) - V(\bar{x}) = h$

$h = b^2 - \frac{\omega^2 \bar{x}^2}{2} - b^2 e^{\ln(\omega^2\sigma^2/b^2)}$

$$\boxed{h = (b^2 - \omega^2\sigma^2) - \frac{\omega^2 \bar{x}^2}{2}}$$

$h = (b^2 - \omega^2\sigma^2) + \omega^2\sigma^2 \ln\left(\frac{\omega^2\sigma^2}{b^2}\right)$

2. $V(x) = -\frac{a^2}{2}x^2 + \frac{b^2}{4}x^4 + \frac{a^4}{4b^2}$

$V'(x) = -a^2 x + b^2 x^3$

$0 = -a^2 + b^2 \bar{x}^2$

$$\boxed{\bar{x} = \frac{a}{b}}$$

$V(\bar{x}) = -\frac{a^4}{2b^2} + \frac{a^4}{4b^2} + \frac{a^4}{4b^2} = 0$

$V(0) = h$ is the height

$$\boxed{h = \left(\frac{a}{2}\right)^2 \left(\frac{a}{b}\right)^2 = \left(\frac{a}{2}\right)^2 \bar{x}^2}$$