



Step-1

→ Co-ordinates of Ball 1

$$\begin{aligned} x_1 &= l \sin \theta_1 \\ y_1 &= -l \cos \theta_1 \end{aligned} \Rightarrow \begin{aligned} \dot{x}_1 &= l \cos \theta_1 \dot{\theta}_1 \\ \dot{y}_1 &= l \sin \theta_1 \dot{\theta}_1 \end{aligned}$$

→ Co-ordinates of Ball 2

$$\begin{aligned} x_2 &= l \sin \theta_2 + d \\ y_2 &= -l \cos \theta_2 \end{aligned} \Rightarrow \begin{aligned} \dot{x}_2 &= l \cos \theta_2 \dot{\theta}_2 \\ \dot{y}_2 &= l \sin \theta_2 \dot{\theta}_2 \end{aligned}$$

Step-2

$$\begin{aligned} \rightarrow KE &= \frac{1}{2} m (\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2} m (\dot{x}_2^2 + \dot{y}_2^2) \\ &= \frac{1}{2} m l^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2) \end{aligned}$$

$$\begin{aligned} \rightarrow PE &= mg y_1 + mg y_2 + \frac{1}{2} K (\Delta d)^2 \\ &= -mg l \cos \theta_1 - mg l \cos \theta_2 + \frac{1}{2} K \left(\frac{1}{2} \sin(\theta_2 - \theta_1) \right)^2 \end{aligned}$$

∴ Lagrangian

$$L = KE - PE$$

$$= \frac{1}{2} ml^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2) + mgl (\cos \theta_1 + \cos \theta_2) - \frac{1}{8} Kl^2 \sin^2(\theta_2 - \theta_1)$$

Step-3

→ Eqⁿ of Motion for Ball 1

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = 0$$

$$\Rightarrow ml^2 \ddot{\theta}_1 = -mgl \sin \theta_1 - \frac{1}{8} Kl^2 \sin(2(\theta_2 - \theta_1)) \quad (-1)$$

$$\Rightarrow \boxed{\ddot{\theta}_1 = -\frac{g}{l} \sin \theta_1 + \frac{K}{8m} \sin(2(\theta_2 - \theta_1))}$$

→ Eqⁿ of Motion for Ball 2

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0$$

$$\Rightarrow ml^2 \ddot{\theta}_2 = -mgl \sin \theta_2 - \frac{1}{8} \frac{K}{m} \sin(2(\theta_2 - \theta_1))$$

$$\Rightarrow \boxed{\ddot{\theta}_2 = -\frac{g}{l} \sin \theta_2 - \frac{K}{8m} \sin(2(\theta_2 - \theta_1))}$$