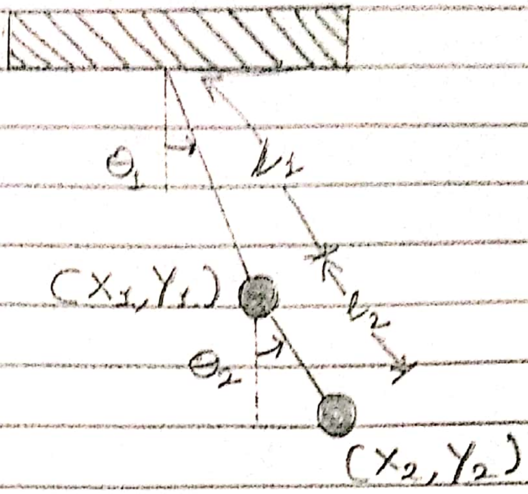


1> Co-ordinates

→ Ball 1 : $X_1 = l_1 \sin\theta_1$
$Y_1 = -l_1 \cos\theta_1$

→ Ball 2 : $X_2 = l_2 \sin\theta_2 + l_1 \sin\theta_1$
$Y_2 = -l_2 \cos\theta_2 - l_2 \cos\theta_1$

→ $\dot{X}_1 = l_1 \cos\theta_1 \cdot \dot{\theta}_1$
$\dot{Y}_1 = l_1 \sin\theta_1 \cdot \dot{\theta}_1$

→ $\dot{X}_2 = l_2 \cos\theta_2 \, \dot{\theta}_2 + l_1 \cos\theta_1 \, \dot{\theta}_1$
$\dot{Y}_2 = l_2 \sin\theta_2 \, \dot{\theta}_2 + l_1 \sin\theta_1 \, \dot{\theta}_1$

2> Kinetic Energy and Potential Energy

→ $KE_1 = \frac{1}{2} m_1 (\dot{X}_1^2 + \dot{Y}_1^2)$

$= \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2$

→ $KE_2 = \frac{1}{2} m_2 (\dot{X}_2^2 + \dot{Y}_2^2)$

$$\Rightarrow KE_2 = \frac{1}{2} m_2 \left( l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cdot [\cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2] \right)$$

$$= \frac{1}{2} m_2 ( l_1^2 \dot{\theta}_1^2 + l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2 - \theta_1))$$

→ Total Kinetic Energy

$$\boxed{KE = \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 \left( l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cdot \cos(\theta_2 - \theta_1) \right)}$$

→ Potential Energy

$$PE = m_1 g Y_1 + m_2 g Y_2$$

$$\Rightarrow \boxed{PE = -m_1 g l_1 \cos\theta_1 - m_2 g (l_2 \cos\theta_2 + l_1 \cos\theta_1)}$$

3> Lagrangian

$$L = KE - PE$$

$$= \frac{1}{2}(m_1 + m_2) l_1^2 \dot{\theta}_1^2$$

$$+ \frac{1}{2} m_2 [ l_2^2 \dot{\theta}_2^2 + 2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2 - \theta_1)]$$

$$+ g l_1 \cos\theta_1 (m_1 + m_2)$$

$$+ m_2 g l_2 \cos\theta_2$$

Or more symmetrically

$$\Rightarrow \boxed{\begin{aligned} L = & (m_1+m_2) l_1 \left[\frac{1}{2} l_1 \dot{\theta}_1^2 + g\cos\theta_1\right] \\ & + m_2 l_2 \left[\frac{1}{2} l_2 \dot{\theta}_2^2 + l_1 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_2-\theta_1) + g\cos\theta_2\right] \end{aligned}}$$

4> Equation of motion.

i> $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0$$

$\rightarrow$ $$\frac{\partial L}{\partial \dot{\theta}_1} = (m_1+m_2) l_1^2 \dot{\theta}_1 + m_2 l_1 l_2 \dot{\theta}_2 \cos(\theta_2-\theta_1)$$

$\Rightarrow$ $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) = (m_1+m_2) l_1^2 \ddot{\theta}_1 + m_2 l_1 l_2 \left[\ddot{\theta}_2 \cos(\theta_2-\theta_1) - \dot{\theta}_2 (\dot{\theta}_2 - \dot{\theta}_1) \sin(\theta_2-\theta_1)\right]$$

$\rightarrow$ $$\frac{\partial L}{\partial \theta_1} = -(m_1+m_2) l_1 g \sin\theta_1 + m_2 l_1 l_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2-\theta_1)$$

$\rightarrow$ Which simplifies to,

$$\ddot{\theta}_1 + \frac{g}{l_1}\sin\theta_1 + \left(\frac{m_2}{m_1+m_2}\right)\left(\frac{l_2}{l_1}\right)\left[\ddot{\theta}_2 \cos(\theta_2-\theta_1) - \dot{\theta}_2^2 \sin(\theta_2-\theta_1)\right] = 0$$

$\Rightarrow$ $$\boxed{\ddot{\theta}_1 + A\ddot{\theta}_2 + B = 0} \quad \text{——— (1)}$$

Where, $A = \left(\frac{m_2}{m_1+m_2}\right)\left(\frac{l_2}{l_1}\right)\cos(\theta_2-\theta_1)$

and, $B = \frac{g}{l_1} \sin\theta_1 - \left(\frac{m_2}{m_1+m_2}\right)\left(\frac{l_2}{l_1}\right) \sin(\theta_2-\theta_1)\, \dot{\theta}_2^2$

ii) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0$

$\rightarrow \frac{\partial L}{\partial \dot{\theta}_2} = m_2 l_2^2 \dot{\theta}_2 + m_2 l_1 l_2 \dot{\theta}_1 \cos(\theta_2-\theta_1)$

$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) = m_2 l_2 \left[ l_2 \ddot{\theta}_2 + l_1 \left( \ddot{\theta}_1 \cos(\theta_2-\theta_1) + \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_2-\theta_1)\right)\right]$

$\rightarrow \frac{\partial L}{\partial \theta_2} = -m_2 l_2 \left( g\sin\theta_2 + l_1 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_2-\theta_1)\right)$

→ So Equation becomes,

$$\ddot{\theta}_2 + \frac{g}{l_2}\sin\theta_2 + \left(\frac{l_1}{l_2}\right)\left(\ddot{\theta}_1 \cos(\theta_2-\theta_1) + \dot{\theta}_1^2 \sin(\theta_2-\theta_1)\right) = 0$$

→ which can be written as

$$\boxed{\ddot{\theta}_2 + C\,\ddot{\theta}_1 + D = 0} \quad — \quad (2)$$

where, $C = \left(\frac{l_1}{l_2}\right)\cos(\theta_2-\theta_1)$

$D = \left(\frac{l_1}{l_2}\right)\sin(\theta_2-\theta_1)\,\dot{\theta}_1^2 + \frac{g}{l_2}\sin\theta_2$

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→ Solving eqn ① and ②, we get

i) $\boxed{\ddot{\theta}_1 = \frac{B - AD}{AC - 1}}$ ✓

ii) $\boxed{\ddot{\theta}_2 = \frac{D - BC}{AC - 1}}$ ✓