A general $s-z$ fit routine: ZSRFTV

JADE Computer Note 95

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ABSTRACT. The new routine ZSRFTV can be used for refitting individual tracks as well as for performing common z fits for subsets of tracks going through specified common points in $r-\varphi$. The fits are helix fits in the sense that linear $s-z$ rather than $r-z$ relation is used where s is the track length along the circle in $r-\varphi$. For FADC multihadron data, the resolution in the fit parameters has been improved by modifying ZSFIT and the hit classification. A further improvment is provided by a new single track fitting routine ZSRFT1 used by ZSRFTV. FADC and DL8 data are compared.

The subroutine ZSRFTV (MODE, IOPT)

may be called to perform the following tasks:

MODE =

0 Overwrite old 'PATR' bank with new results.

1 Create new 'PATR' bank with new results.

IOPT =

1 $s-z$ fits separately for all tracks.

2 $s-z$ fits separately for all tracks and subsequently a common z fit for those ones which extrapolate to within 15 mm to the run vertex in $r-\varphi$ and have $|z(r \approx 0)| <$ 800 mm.

4 Common z fits for user specified (up to 5) sets of tracks going through user specified common (x, y) points or $s-z$ fit for a single track. The single track fits are performed for tracks occuring in the sets only. If a requested track is not within 15 mm to the specified common (x, y) point or $|z(x, y)| > 1600$ mm, it will be excluded from the common fit.

In case of using IOPT=4, you must specify your request in the COMMON /CZSSTE/ NSETZS, NTSTZS(5), KTSTZS(100,2,5), XYSTZS(2,5), SQCHZS(3,5). The following variables must be properly set prior to the call: Number of track sets NSETZS for which a common fit is to be done; number of tracks in each set NTSTZS(·); sequence number (in 'PATR') of tracks requested for the I-th set KTSTZS(·,1,I); x and y of common point for the I-th set XYSTZS(·,I).

Refitting of a single track may be requested by setting only

$$\text{NSETZS} = 1; \quad \text{NTSTZS}(1) = 1; \quad \text{KTSTZS}(1,1,1) = \text{track seq. nr.}$$

nothing else, besides IOPT=4, is needed.

In addition to the results in the 'PATR' bank, the routine provides you with the following information:

KTSTZS(·,2,·) are flags with the following meaning:

- 2 Track was used in a common z fit.
- 1 Single track fit was successful but the track was not used in a common fit.
- 0 Track was not considered for refit.
- <0 Single track fit was attempted but failed.

(Please, for the exact conditions, look at the code.)

SQCHZS(2,·) is set to the total number of hits (R*4 (!)) used in the common fit; SQCHZS(1,·) will hold the sum of weighted residual squares (average weight close to 1) divided by the number of degrees of freedom $n_{hits} - (n_{tracks} + 1)$. The normalization of the latter to get the χ^2/dof is left to the user. It should be done differently for the DL8 and FADC data. SQCHZS(3,·) will contain the common z at the common (x, y) point. All three values are set to 0.0 if the common fit was actually not performed.

In case IOPT=1 or 2, you need not care about the COMMON unless you want to get the information just described. For these values of IOPT, the track flags are given in KTSTZS(·,2,1) (KTSTZS(ITR,1,1)=ITR is implicitly assumed without being set) and, in addition for IOPT=2, SQCHZS(·,1) holds the corresponding quantities for the common fit.

The options IOPT $\rightarrow$ IOPT+8 are also valid; when used, the single track fitting routine ZSRFT1 creates a bank 'ZSPD' with certain hit data for the graphics program (only if the fit is successful and there is enough space in /BCS/). If such banks are already present, you may delete them to allow for creating new ones, on LEVEL 2. The new graphics command ZFIT has been provided to look at the single track $s - z$ fit results together with the hit coordinates with symbols indicating the measurement error and whether or not the hit was included in the fit.

The cut parameters mentioned in the explanation of the IOPT-parameter for selecting tracks for the common fits are in COMMON /CCMZCT/ DIMPCT, ZCUTV, ZCUTVV with the values 15.0, 800.0, 1600.0, respectively and hence can easily be changed. The track selection should in fact be carefully designed for the different applications. In particular, the 15 mm cut on the impact in $r - \varphi$ might be too lose in many cases (the corresponding cut in the old routine FITEVR is 25 mm), and the z cuts should be chosen appropriately depending on the track quality requirements employed in the analysis. We encourage you to consider changing the parameters and even the code, as far as the selection is concerned, for your actual needs. (The routine is written in SHELTRAN but you need not know more about the language than you see around the code you are going to modify.) A cut on $z - \bar{z}|_{\text{common } x,y}$ is coded on the lines starting with 'COMIT' and may be activated by removing the comments.

Fit parameters in the 'PATR' bank.

The new $s-z$ fits are associated with fit type 2. The two parameters stored are: $dz/ds = \cot\vartheta$ and z at the closest point on the circle to the origin in $r-\varphi$ (ϑ is the polar angle measured from the $+z$ axis). The meaning of these parameters is identical to that approximated by the $r-z$ (type 1) fit parameters dz/dr and $z(r=0)$, when the curvature is small and the particle motion is radial in $r-\varphi$. ($dz/ds = dz/dr \cdot dr/ds$; while the product is invariant, both factors vary along the track if it does not lie in the radial direction even when the curvature is small.) The corresponding helix is displayed by Jan Olsson's old routine HELDIS. The locations of the parameters are given in JCN 12. The quantities connected with the last and first fitted points are, of course, updated too.

Please check the routine you use for calculating the momentum as to whether it checks on the fit type ! The same code as for type 1 should be used. Subroutine MOMENT has already been changed.

The covariance matrix (see JCN 12 and also JCN 94 Suppl. 1) is provided if the number of hits used in the fit is at least 4 and the length of the track area in the 'PATR' bank is at least 59. The length can be extended (prior to calling ZSRFTV) by using the routine EXPATR as described in JCN 94 Suppl. 1.

The results from the single track fit routine ZSRFT1 are overwritten in ZSRFTV if the track is used in a subsequent common fit, except for the covariance matrix.

Hit selection, fit procedure.

The "P. Dittmann's" z coordinates are used with *amplitude dependent weights* normalized, for the different periods, such that the average weight is close to 1. The z dependence of the weight has been modified for the FADC. If one of the amplitudes is zero or the reconstructed $|z| > 1250$ mm or, in case of FADC, the wire has another hit within ≈ 3.6 mm, the hit is marked not to be used for z fit.

The routines JFETCH and its version for the new ID calibration JFTNEW have been modified in the following two aspects:

1 The z coordinates, hit classification and weight are determined in a new routine AMPS2Z (IP, NPJETC, Z, W, IFLAG), where NPJETC=IDATA(IBLN('JETC')) is the pointer to the calibrated 'JETC' bank and IP points to the (I*4) hit data in it. The Dittmann's z coordinates and weights are set if the corresponding z calibration has previously been requested by calling ZSFIT, otherwise one gets the "standard" z and weight 1. AMPS2Z is called now also in Dittmann's package.

2 The meaning of the input flag INDEX for selecting the coordinate system has been slightly modified. The same role inside the routine is now played by the variable INDX which gets the value of INDEX when it is 1, 2 or 3 (the only valid values until now) and is set to 1 (coordinates in real space) when INDEX has the new value 4. In this latter case the hit data passed in /CWORK/ are different from those set if INDEX < 4 and no track data is stored.

In the single track fitting routine ZSRFT1, it is first checked whether the very wide ± 600 mm road around the original fit contains 75% of the available (marked as usable) hits. In the small fraction of cases when it does not, starting values are searched for in the following way. Fit parameters are calculated from hit pairs taken in the following sequence: $(1,n)$, $(1,n-1)$, $(2,n)$, || $(2,n-1)$, $(2,n-2)$, $(3,n-1)$, || .... (The hits are ordered in s so that the hits 1 and n are at the two ends of the track. The sequence does not include all possible pairs.) The search is terminated if the "75% test" is satisfied for some hit pair. If that never happens, the parameters taken correspond to that hit pair for which the largest number of hits is included in the road (and for which the unweighted χ^2 is the smallest in case of equal number of hits.)

A sequence of straight line fits are then performed. From the $k+1$-th fit, k hits having the largest weighted residuals to the k-th fit are excluded. These hits, if their residuals happen to decrease, may be reincluded later on. On the other hand, the hits which, in any step, are further than 600 mm from the fit, will not be reconsidered. The termination of the sequence is based on two quantities, the (weighted) χ^2/dof and its rate of decrease to the next fit, similarily to the procedure implemented for the $r-\varphi$ fit in XYRFT1. The number of hits k allowed to be discarded (the number of iterations, more precisely) is limited to 25% of the available hits when the iteration is started.

For the common z fits in ZSRFTV those and only those hits are taken which were included in the fit in ZSRFT1.

Resolution in the fit parameters.

For the statistical error of the fit parameters $z_0 \equiv z(r \approx 0)$ and $\cot\vartheta = dz/ds$, one can easily derive the following formulae:

$$\sigma_{int}^2(z_0) = \frac{\sigma_{hit}^2}{n}\left[1 + \left(\frac{r_{mid}}{d}\right)^2 \frac{12}{n^2-1}\right] \tag{1a}$$

$$\sigma_{int}^2(\cot\vartheta) = \frac{\sigma_{hit}^2}{n}\,\frac{12}{d^2(n^2-1)}, \tag{1b}$$

where n is the number hits measured with the (same) error σ_{hit} at equal distances d from each other in r, and r_{mid} is the radius of the middle of the track. Both errors are independent of the track position and orientation if one neglects the z-dependence of σ_{hit}.

The resolutions can be measured experimentally by making use of the back-to-back property of μ-pairs (or Bhabha's). While the width of the z_0 plot includes the beam spread, the quantity $(z_{01} - z_{02})/\sqrt{2}$ is not affected by the uncertainty in the interaction point and its width $\sigma_{int}(z_0)$ is a direct measure of the intrinsic extrapolation error at the beam line. From comparing the two quantities, the spread in z of the interaction point can be extracted as well as from the comparison of $\sigma(z_0)$'s in fits with and without common z constraint. (See Table 1 for more details.)

For determining the resolution in $\cot\vartheta$, the quantity $(\cot\vartheta_1 + \cot\vartheta_2)/\sqrt{2}$ can be plotted. (The sum of $\cos\vartheta$, for example, would have an error depending on ϑ and one would mix different resolutions in a plot if ϑ is left unconstrained.) The contribution of the effect of initial

		DL8	FADC
$\sigma(z_0) = \sqrt{\sigma_{int}^2(z_0) + \sigma_{beam}^2(z_0)}$		17.8 mm	27.6 mm
$\sigma(z_0)\vert_{\text{common } z} = \sqrt{\frac{1}{2}\sigma_{int}^2(z_0) + \sigma_{beam}^2(z_0)}$	ZSRFTV FITEVR	14.9 mm 15.5 mm	20.3 mm 23.5 mm
$\sigma\left(\frac{z_{01} - z_{02}}{\sqrt{2}}\right) = \sigma_{int}(z_0)$		**12.8 mm**	**25.4 mm**
$\sigma_{beam}(z_0)$ from $\sigma(z_0)$ and $\sigma\left(\frac{z_{01} - z_{02}}{\sqrt{2}}\right)$		12.4 mm	10.8 mm
$\sigma_{beam}(z_0)$ from $\sigma(z_0)$ and $\sigma(z_0)\vert_{\text{common } z}$		11.3 mm	7.9 mm
$\sigma\left(\frac{\cot\vartheta_1 + \cot\vartheta_2}{\sqrt{2}}\right) = \sqrt{\sigma_{int}^2(\cot\vartheta) + \sigma_{beam}^2(\cot\vartheta)} =$		0.028	0.048
$= \sigma\left(\frac{\cot\vartheta_1 + \cot\vartheta_2}{\sqrt{2}}\right)\Big\vert_{\text{common } z}$	ZSRFTV FITEVR	0.028 0.030	0.048 0.056
$\sigma_{int}(\cot\vartheta)$		**0.023**	**0.045**
σ_{hit}^{eff}		19.1 mm	37.9 mm

Table 1. Measured and derived resolutions for μ-pairs

state radiation to the intrinsic resolution in that plot can not be removed that easily as the spread of the interaction point for, on one hand, the $\cot\vartheta$ distribution of single tracks gives no information and, on the other hand, the improvement by an approximate factor of $\sqrt{2}$ in $\cot\vartheta$ in a common z fit has no effect on $(\cot\vartheta_1 + \cot\vartheta_2)/\sqrt{2}$: the sum does not improve due to the correlation introduced by the common z constraint itself. The same correlation makes that the difference of $\cot\vartheta$'s has almost no statistical error and this is what roughly enters into an invariant mass! The effects can, nevertheless, be separated by using the fact that the relative change in $\sigma_{int}(\cot\vartheta)$ has to be the same as that in $\sigma_{int}(z_0)$ when going from DL8 to FADC. One obtains $\sigma_{beam}(\cot\vartheta) = 0.016$ that can be translated into $\sigma_p/p|_{beam} = 0.016$, where σ_p denotes the fluctuation of the total longitudinal momentum in the initial state. (The average transverse momentum of the muons in the sample was taken to be $p_{beam}/\sqrt{2}$.)

From these measurements, one may estimate an effective (hits are used with weights, far hits are discarded) single hit resolution in the following way. From the consistency of the equations in (1), one obtains the relation

$$r_{mid} = \sqrt{\left(\frac{\sigma_{int}(z_0)}{\sigma_{int}(\cot\vartheta)}\right)^2 - d^2\frac{n^2 - 1}{12}}. \quad (2)$$

The experimental ratio of the σ's, by our construction, is the same for DL8 and FADC. Quite independently from the value of n wich we take to be 44, Eq.(2) gives $r_{mid} = 550\ mm$ (instead of the half radius of 496 mm of the sensitive part of the jet chamber). With these values, the single hit resolution can then be estimated by using Eq.(1).

The performance of three single track fitting routines have been compared: the standard ZSFIT routine, its new version ZSFIT$^{(M)}$ with modified parameters for the FADC and the new routine ZSRFT1 used by ZSRFTV. (The routine ZRFIT in the pattern recognition package uses unweighted hits and gives much worse, upto factor 2, results.) All three routines do equally well in simple topological configurations such as μ- or Bhabha-pairs where the tracks have many hits and are little disturbed by other tracks. Their common results are listed in Table 1 for DL8 and FADC μ-pairs.

The single hit resolutions derived are in agreement with the results obtained during the calibration of the z-coordinate in late 86. The resolutions as function of the amplitude for DL8 and FADC data are given in Fig.7. At the working point (average amplitude 550 cnts.) the single hit resolutions are in rough agreement with the values in Table 1, indicating that systematic effects possibly left in the calibration do not have a dominant contribution to the track resolution.

Fit results with common z constraint are also given in the Table. The old routine FITEVR is slightly worse than our ZSRFTV because unweighted hits are used and, in addition, cut parameters have not been tuned to the worse resolution of the FADC resulting in rejection of too many hits (Fig.3).

In the simple case of two track events, the track resolution is limited by the single hit resolution and depends neither on the method, nor on the cuts applied to fit the tracks. In a multiparticle environment, however, the hit cleaning technique and starting value search may have some importance, and it is this application where the three routines can be distiguished. Fig.4a shows the distributions of the track intersection with the beam line in FADC multihadron events. The width of this distribution includes the effects of interactions in the beam pipe and the resolution. ZSRFT1 gives a slightly smaller width of this distribution than the other routines and also a smaller tail as seen on the logarithmic display in Fig.4b. The quantity

$$\chi_n^2 \equiv \frac{1}{n-1} \sum_{k=1}^{n} (z_{0k} - \bar{z}_0)^2$$

constructed on an event with n tracks, would follow a known distribution under certain conditions which do not satisfy for multihadronic events. (They do for the μ-pairs and the same resolution can be extracted as from the method described above.) Still, the distribution of χ^2 measuring the spread of the track intersections at the beam line within events, is a good indicator of the quality of the different fits. Fig.5 gives a comparison of $\log \chi^2$ for the fits considered (all n included). The optimized ZSFIT$^{(M)}$ gives a better result than the old standard ZSFIT, ZSRFT1 is still better. Fig.6 indicates that ZSRFT1 rejects the least number of hits, while the old ZSFIT clearly rejects too many hits. This does not apply to the DL8 data and correspondingly we do not observe a similar improvement there.

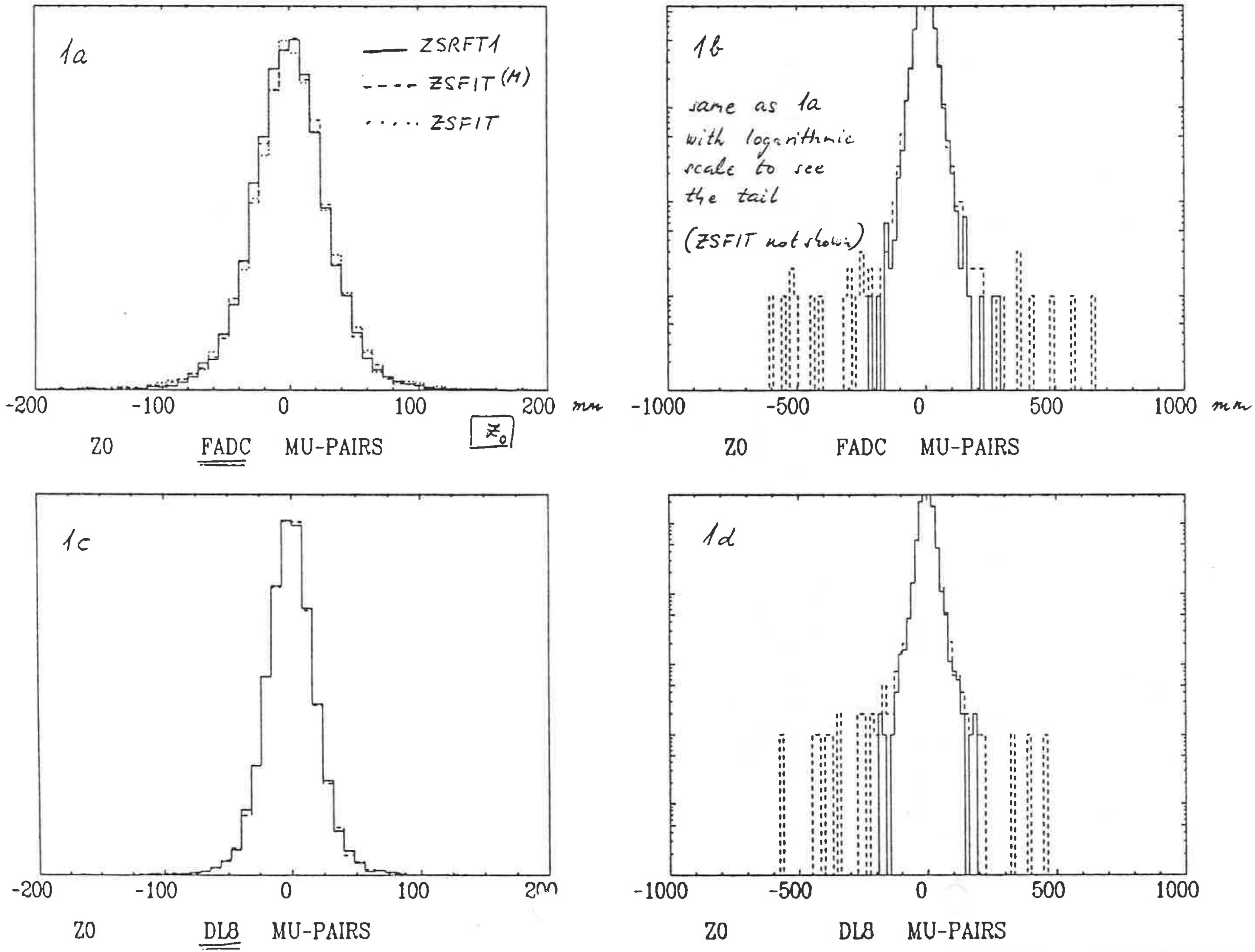
Addendum.

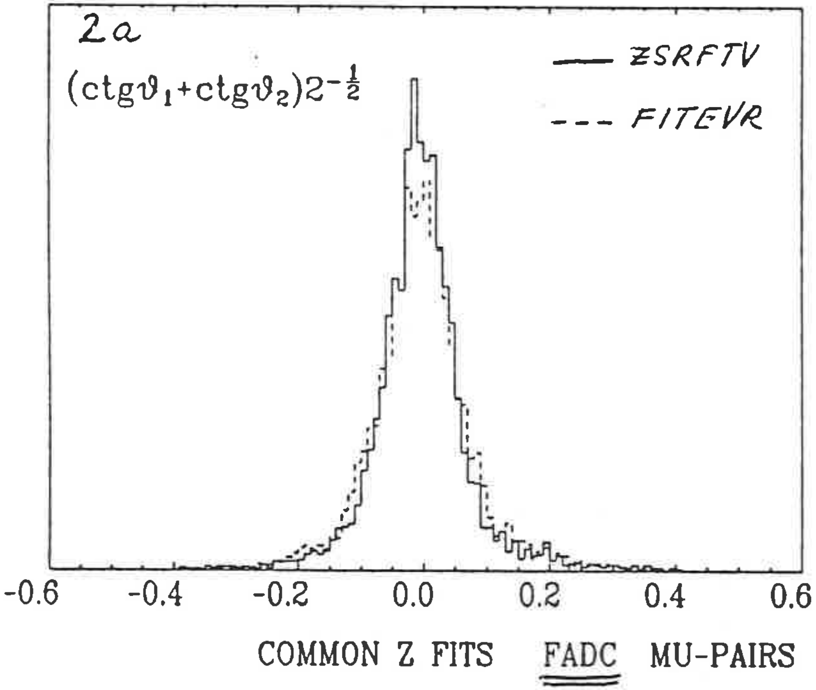
When a common z fit is requested (IOPT= 2 or 4), you may ask for having a constraint on the common z to $z = 0$ with a width of 10 mm. In the case of FADC μ-pairs this additional constraint improves the width of the $(\cot\vartheta_1 + \cot\vartheta_2)/\sqrt{2}$ plot by 40%. Plots with and without this "vertex constraint" (common z constraint in both cases) are shown in Fig.10. In case of DL8 μ-pairs, where the resolution is better, the corresponding improvement is 9% only. The distributions of the track intersections with the beam line in FADC multihadronic events when common z fits with and without the vertex constraint are performed, are shown in Fig.11 that can be compared with the corresponding distribution in uncostrained fits in Fig.4a.

The common in ZSRFTV, /CCMZCT/, described in this note, has been extended by the flags IZVCST(5) which are initialized in BLOCK DATA to /5*0/. For IOPT=2, IZVCST(1)=1 has to be set to get the new constraint and for IOPT=4, the new constraint will be applied to the K-th track set if IZVCST(K)=1 is set. (No combined fit and no new constraint is applied if IOPT=4 is used for refitting a single track.)

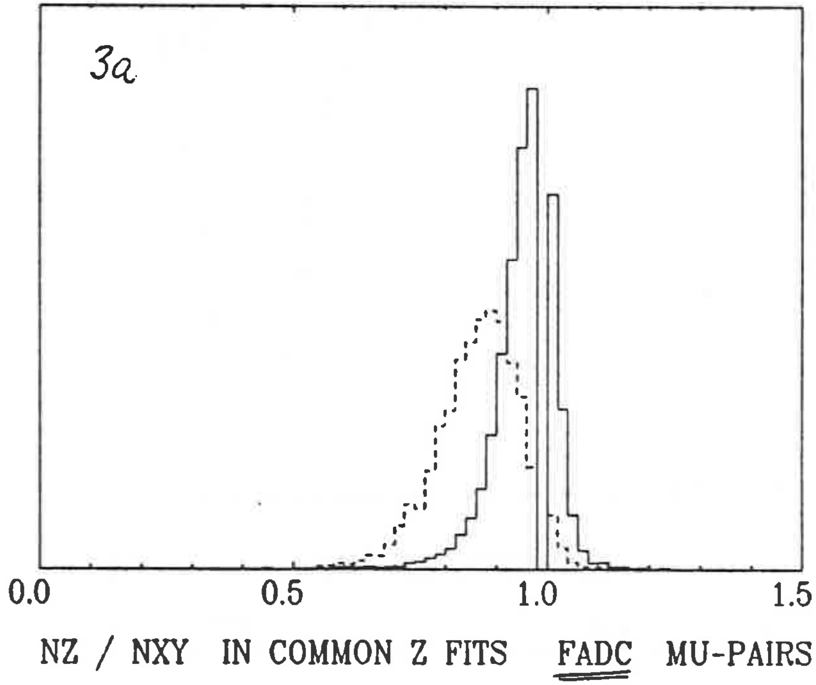
Warning:

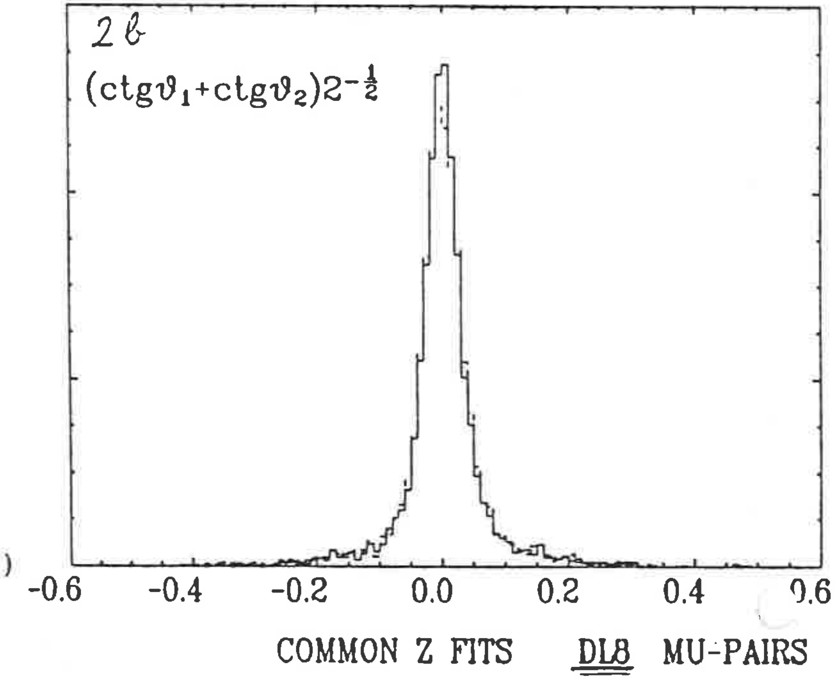
The current test version of the dE/dx program in member (DEDXCN) on the standard library calls ZSFIT with MODE=0. Besides updating the JETC and JHTL banks, the z fit results are overwritten in the PATR bank and hence earlier fit results are lost. This side effect will be avoided in a new version of the program which will be released soon.

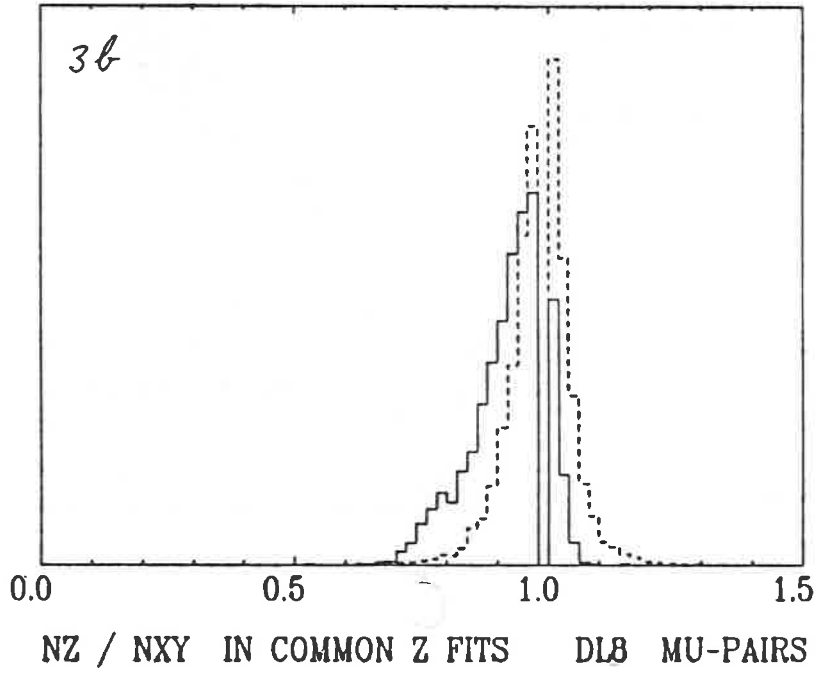

1a
ZSRFT1
ZSFIT (M)
ZSFIT
-200
-100
0
100
200
mm
Z0 FADC MU-PAIRS
z_0
1b
same as 1a
with logarithmic
scale to see
the tail
(ZSFIT not shown)
-1000
-500
0
500
1000
mm
Z0 FADC MU-PAIRS
1c
-200
-100
0
100
200
Z0 DL8 MU-PAIRS
1d
-1000
-500
0
500
1000
Z0 DL8 MU-PAIRS

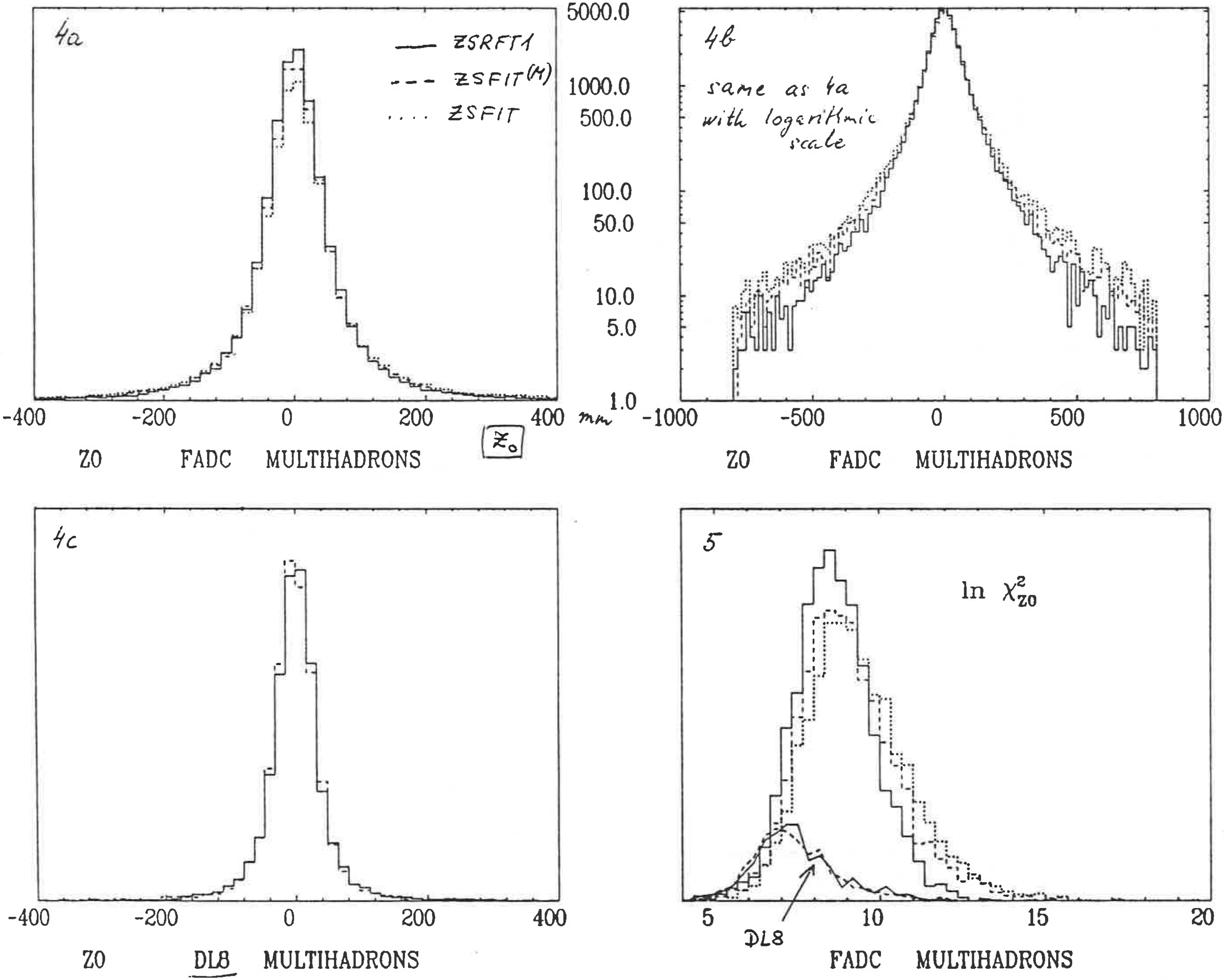


Hit rejection

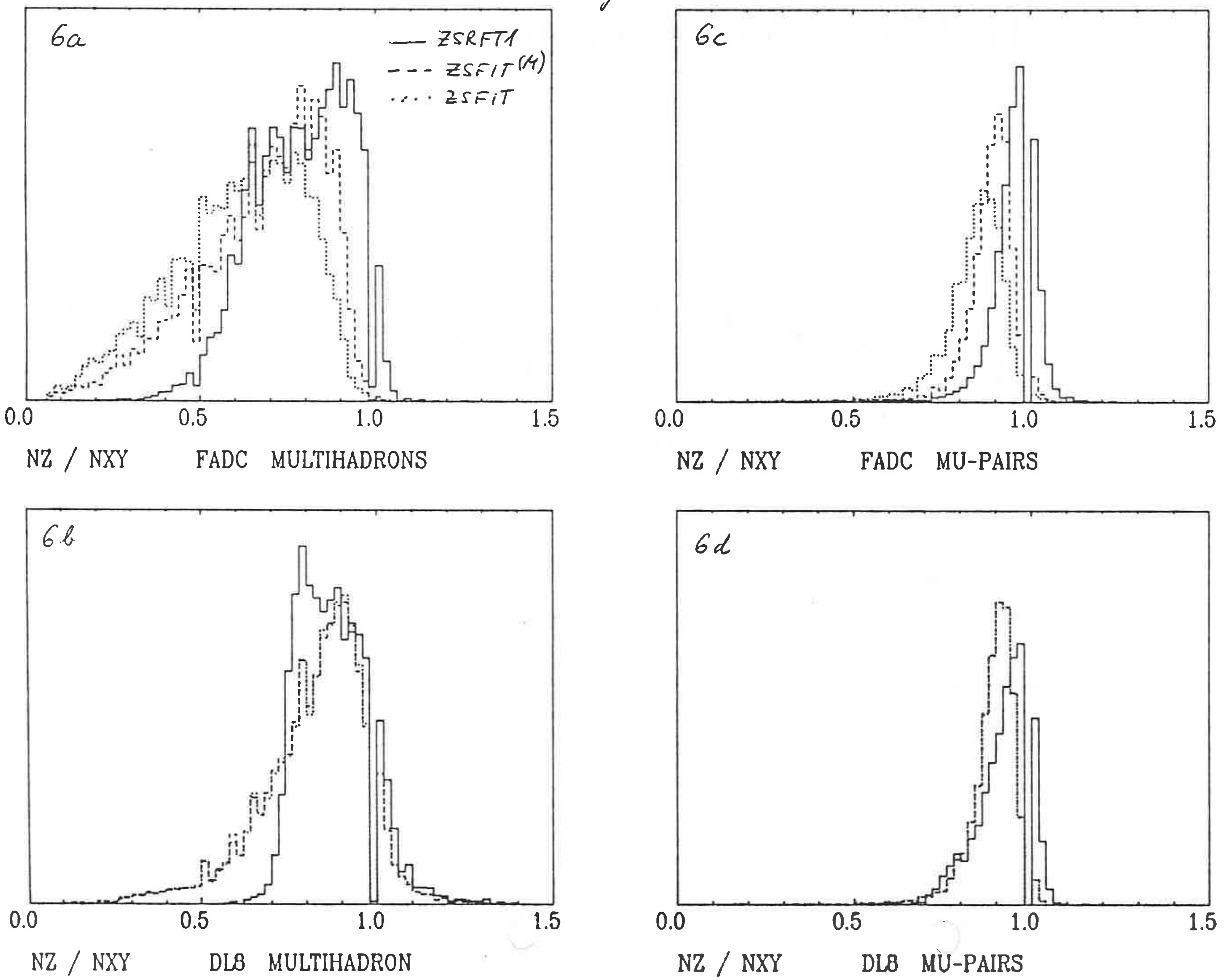







4a
ZSRFT1
ZSFIT(M)
ZSFIT
-400
-200
0
200
400
mm
z_0
ZO FADC MULTIHADRONS
4b
same as 4a with logarithmic scale
5000.0
1000.0
500.0
100.0
50.0
10.0
5.0
1.0
-1000
-500
0
500
1000
ZO FADC MULTIHADRONS
4c
-400
-200
0
200
400
ZO DL8 MULTIHADRONS
5
ln χ^2_{ZO}
DL8
5
10
15
20
FADC MULTIHADRONS

Hit rejection



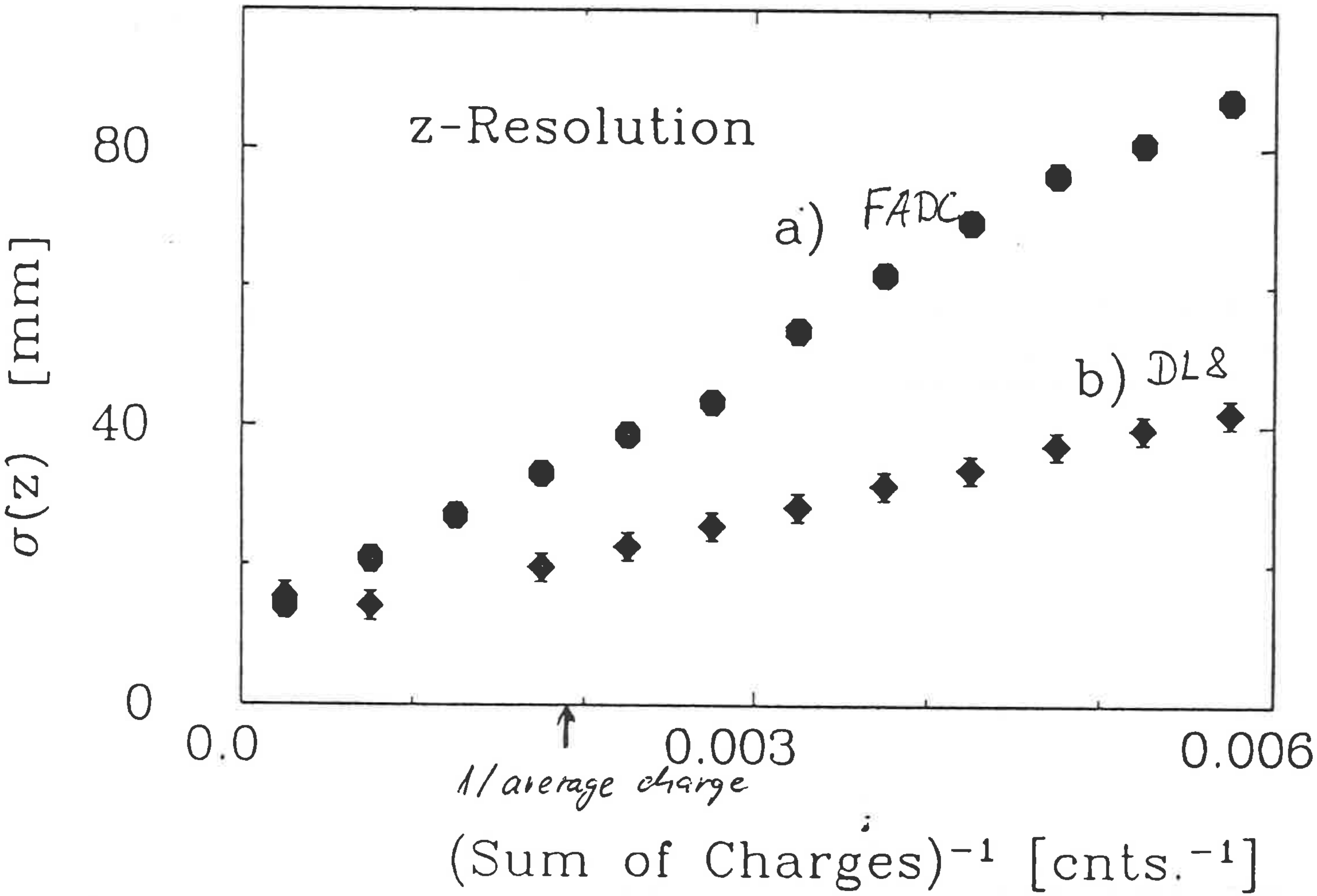

z-Resolution
a) FADC
b) DL8
80
40
0
0.0
0.003
0.006
1/average charge
σ(z) [mm]
(Sum of Charges)⁻¹ [cnts.⁻¹]

Fig. 7

DSN F11SPI.BHABH826

27600 2632 1
IDHITS 1042
ELGTOT 11875
MUHITS 15
LGCYL 9081
LGCAPS 525 2269
FWCAPS 0 313

JADE

Z-X SECTI(
ROTATE MODE

BEAM 17.501 GEV FIELD -4.844 KG TALC C879 DATE 04/06/87 TIME 09.30.21
T1A 1AC T1P .01 T2C 8089 CAMAC TIME 27.26.19 22/ 7/1986

BANK PATR 9 NR OF TRACKS 19

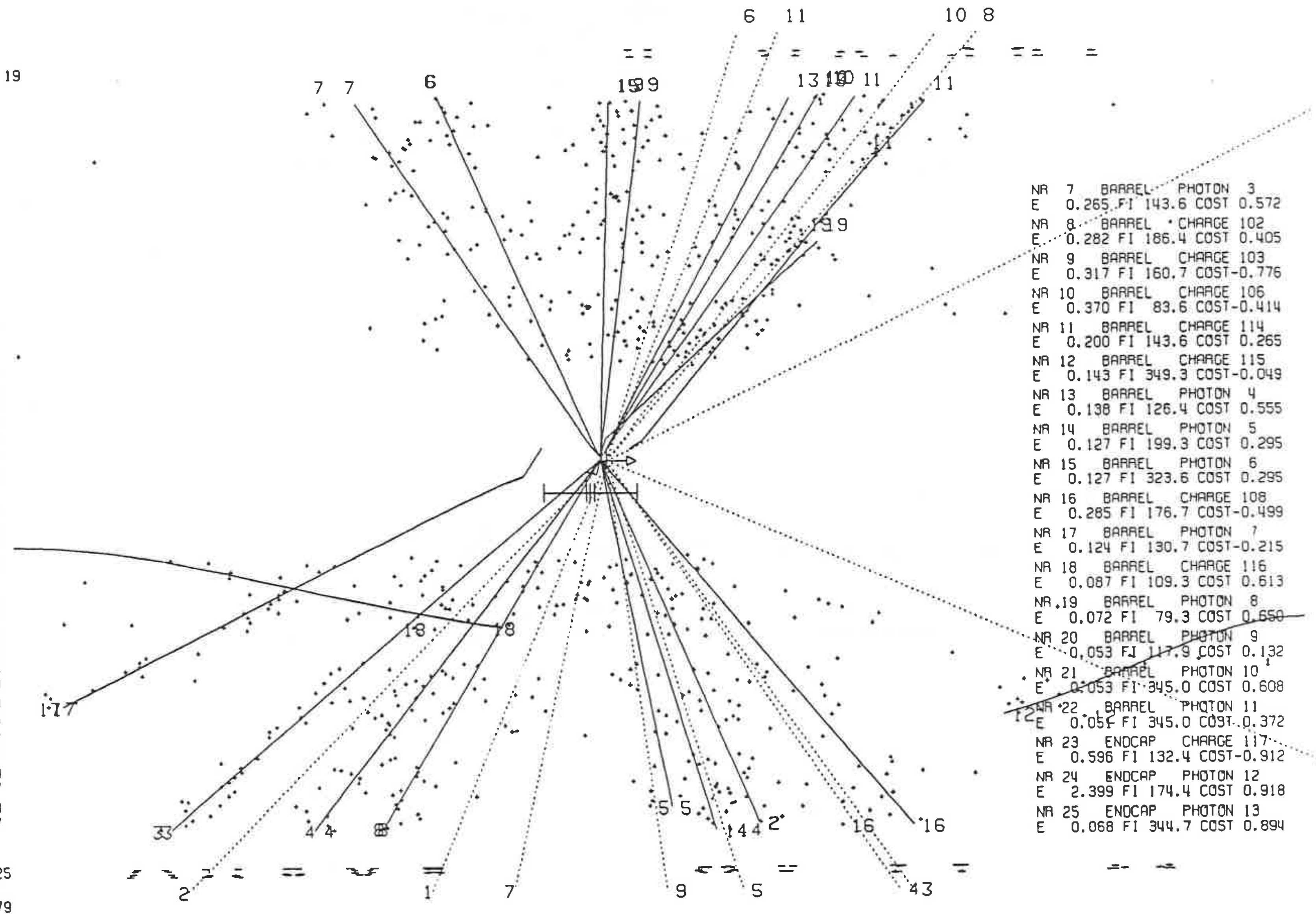
NR +-	RMSRFI	RMSRZ/HIT	PHI
PTOT	PLONG	PTRANS	COSTHE
MUQUAL	CHIP	MUPR	PIPR
1 +	0.45/48	44.1/32	333.2
3.797	2.167	3.119	0.571
2 +	0.41/46	41.6/40	178.7
0.458	0.180	0.421	0.393
3 +	0.17/48	44.5/46	161.9
6.344	-4.812	4.134	-0.759
2	0.0	1.00	0.017
4 -	0.37/46	40.6/33	133.8
1.646	-1.011	1.299	-0.614
5 +	0.32/38	47.2/28	117.9
0.612	0.119	0.601	0.194
6 -	0.56/47	43.7/37	89.2
0.719	-0.287	0.660	-0.399
7 -	0.70/49	43.1/40	74.6
0.337	-0.187	0.280	-0.556
8 -	0.38/46	42.5/40	177.7
2.093	-1.067	1.801	-0.510
9 -	0.35/46	40.2/44	310.7
1.121	0.131	1.113	0.117
10 +	0.52/48	44.2/36	334.1
2.742	1.397	2.360	0.509
11 -	0.58/52	34.8/50	57.4
0.128	0.079	0.100	0.616
12 -	0.79/14	53.2/11	122.4
0.044	-0.040	0.017	-0.921
13 +	1.00/48	52.6/19	340.5
2.374	1.091	2.108	0.460
14 +	0.66/44	41.6/27	136.4
0.410	0.119	0.392	0.291
15 -	0.32/48	44.8/47	358.8
0.307	0.009	0.307	0.030
16 -	0.74/47	60.2/30	121.0
0.423	0.273	0.324	0.645
17 -	0.72/29	30.7/18	141.7
0.320	-0.285	0.145	-0.891
18 -	0.83/14	85.3/10	283.9
0.071	0.069	0.015	0.977
19 +	0.97/20	30.0/15	64.8
0.110	0.079	0.076	0.717

BANK LGCL 1 NR OF CLUSTERS 25

NR 1 BARREL PHOTON 1
E 1.403 FI 135.0 COST-0.379
NR 2 BARREL CHARGE 110
E 2.612 FI 329.9 COST 0.404
NR 3 BARREL PHOTON 2
E 1.919 FI 200.7 COST-0.698
NR 4 BARREL CHARGE 104
E 0.545 FI 128.7 COST-0.581
NR 5 BARREL CHARGE 109
E 0.547 FI 307.6 COST 0.201
NR 6 BARREL CHARGE 101
E 0.698 FI 338.2 COST 0.504

X

Z



NR 7 BARREL PHOTON 3
E 0.265 FI 143.6 COST 0.572
NR 8 BARREL CHARGE 102
E 0.282 FI 186.4 COST 0.405
NR 9 BARREL CHARGE 103
E 0.317 FI 160.7 COST-0.776
NR 10 BARREL CHARGE 106
E 0.370 FI 83.6 COST-0.414
NR 11 BARREL CHARGE 114
E 0.200 FI 143.6 COST 0.265
NR 12 BARREL CHARGE 115
E 0.143 FI 349.3 COST-0.049
NR 13 BARREL PHOTON 4
E 0.138 FI 126.4 COST 0.555
NR 14 BARREL PHOTON 5
E 0.127 FI 199.3 COST 0.295
NR 15 BARREL PHOTON 6
E 0.127 FI 323.6 COST 0.295
NR 16 BARREL CHARGE 108
E 0.285 FI 176.7 COST-0.499
NR 17 BARREL PHOTON 7
E 0.124 FI 130.7 COST-0.215
NR 18 BARREL CHARGE 116
E 0.087 FI 109.3 COST 0.613
NR 19 BARREL PHOTON 8
E 0.072 FI 79.3 COST 0.650
NR 20 BARREL PHOTON 9
E 0.053 FI 117.9 COST 0.132
NR 21 BARREL PHOTON 10
E 0.053 FI 345.0 COST 0.608
NR 22 BARREL PHOTON 11
E 0.055 FI 345.0 COST 0.372
NR 23 ENDCAP CHARGE 117
E 0.596 FI 132.4 COST-0.912
NR 24 ENDCAP PHOTON 12
E 2.399 FI 174.4 COST 0.918
NR 25 ENDCAP PHOTON 13
E 0.068 FI 344.7 COST 0.894

Fig. 8 Example for common z fit

*** SUMS (GEV) *** PTOT 24.055 PTRANS 19.271 PL 13.402 CHARGE -3
TOTAL CLUSTER ENERGY 13.481 PHOTON ENERGY 6.801 NR PHOTONS 13

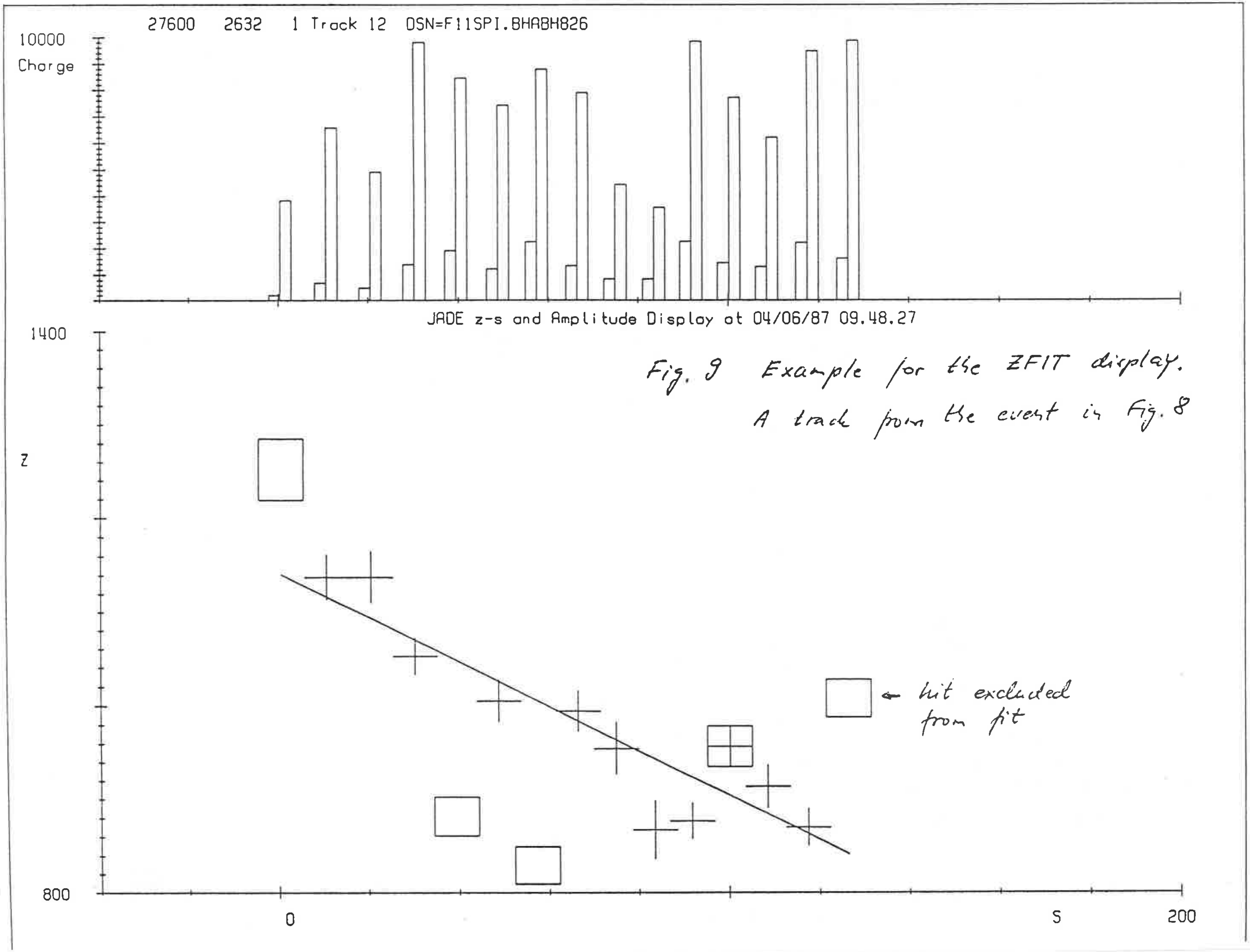


Fig. 9 Example for the ZFIT display.
A track from the event in Fig. 8

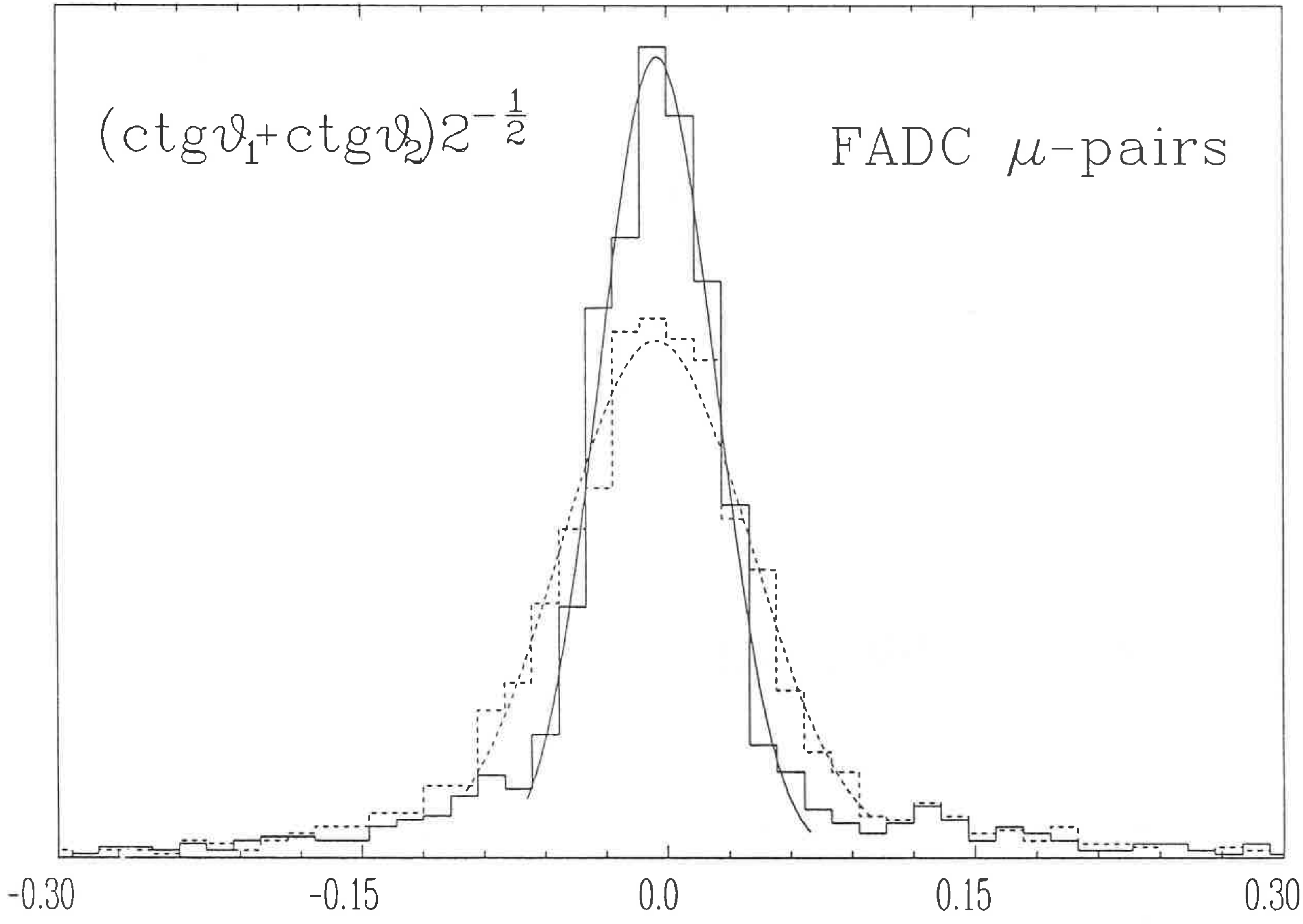


Fig.10. Common z fits w/ and w/o constraint to z≈0 (ZSRFTV)

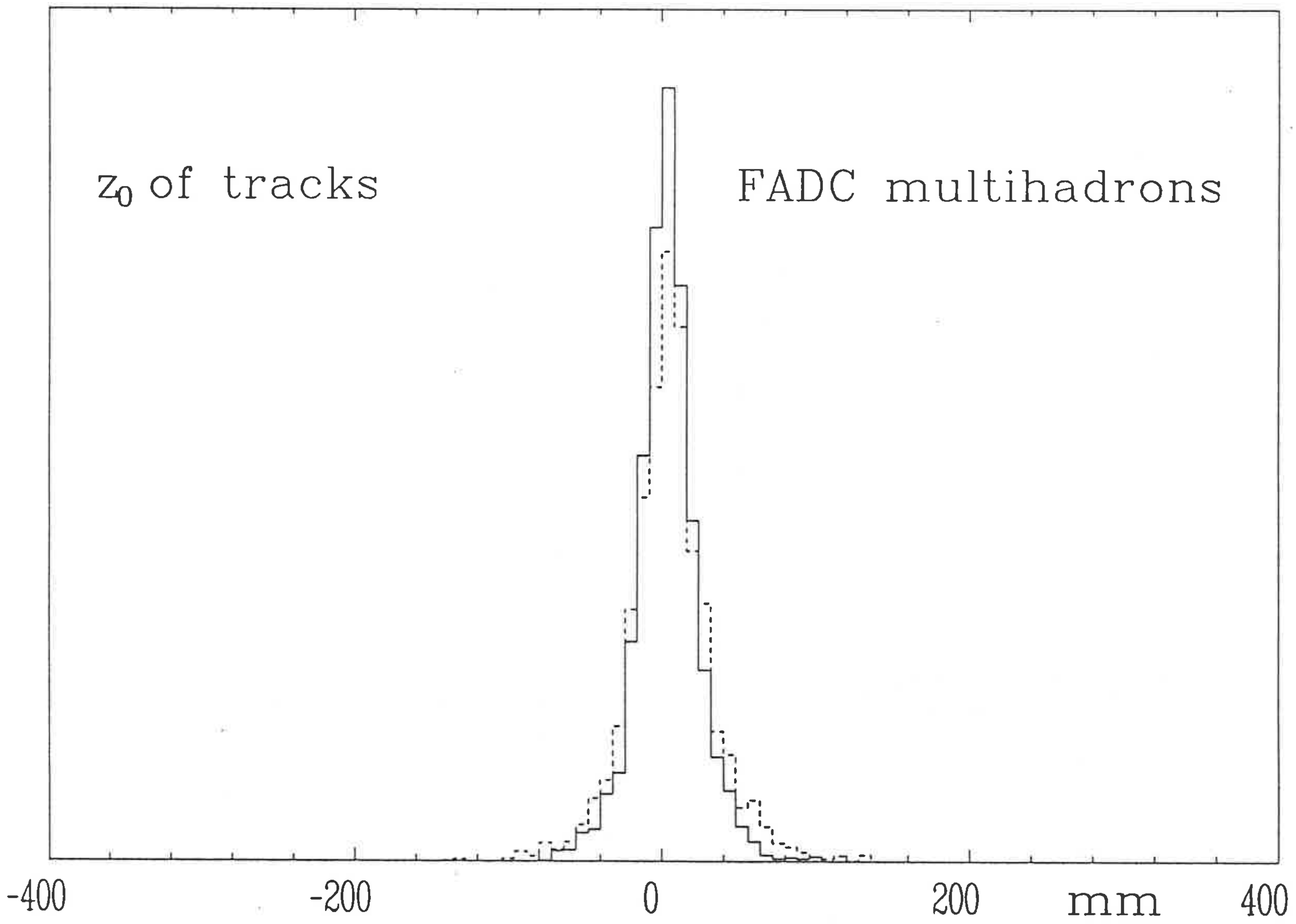


Fig.11. Common z fits w/ and w/o constraint to $z \approx 0$ (ZSRFTV)