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Decays of $K^{\pm}$ and KØL in MC

In the Tracking Monte Carlo a new routine TRKADC is implemented, which handles the decays of charged kaons and KØL's. (Remember: decays of KØS's are included in the 4 vector generators.) TRKADC foresees six decay channels for either particle type. All decay channels with more than 1% branching ratio are taken into account according to PDG 1982.

decay No.	decay channel	branching ratio
(1)	$K^0 \to \pi^+ e^- \nu$	19.35%
(2)	$\to \pi^- e^+ \nu$	19.35%
(3)	$\to \pi^+ \mu^- \nu$	13.55%
(4)	$\to \pi^- \mu^+ \nu$	13.55%
(5)	$\to \pi^+ \pi^- \pi^0$	12.6 %
(6)	$\to \pi^+ \pi^-$	21.6 %
		100.0 %
(7)	$K^+ \to \mu^+ \nu$	63.5 %
(8)	$\to \pi^+ \pi^0$	21.2 %
(9)	$\to \pi^+ \pi^+ \pi^-$	5.6 %
(10)	$\to \pi^+ \pi^0 \pi^0$	1.7 %
(11)	$\to \pi^0 \mu^+ \nu$	3.2 %
(12)	$\to \pi^0 e^+ \nu$	4.8 %
		100.0 %

For the antiparticle K^- the charge conjugate final states are taken. The KØL has no antiparticle. Therefore somehow artificially the K^0_{e3} decay and the $K^0_{\mu 3}$ decay are split into two parts to make sure that one finds an equal amount of e^+ and e^- (μ^+ and μ^- respectively) in the final state.

Decay Mechanisms

(6), (7) and (8) two body decay

(5), (9) and (10) relativistic phase space decay

As an example the triangular Dalitz plot for the decay $K^o \rightarrow \pi^o\pi^o\pi^o$ is shown in Fig. 1.

(1) to (4)

(11) and (12) $K_{\ell 3}$ decays using the matric elements

$$f_+^2\,(t) * \{2*(p_K p_\ell)(p_K p_\nu) - m_\ell^2\,(p_K p_\nu) + (\tfrac{1}{4}\,m_\ell^2 - m_K^2)(p_K p_\nu)\}$$
$$+ f_+\,(t)\,f_-\,(t) * m_\ell^2 * \{(p_K p_\nu)\}$$
$$+ f_-^2\,(t) * \tfrac{1}{4}\,m_\ell^2 * (p_\ell p_\nu)$$

m_ℓ and m_K being the masses of the leptons and the kaons respectively, and p_K, p_ℓ, p_ν the 4 vectors of the kaons, leptons and neutrinos respectively. f_+ and f_- are the form factors of the $K_{\ell 3}$ decays. They depend only on $t = (p_K - p_\pi)^2$, the square of the four-momentum transfer to the leptons. Several parametrizations for f_+ and f_- are possible. Here the λ_+, λ_o parametrization is used because of the small correlation between both parameters (see PDG 1982 pp. xii et seq. and pp. 73 + 74).

$$f_+\,(t) = f_+\,(0) * \left[1 + \lambda_+ \frac{t}{m_\pi^2}\right]$$

$$f_-\,(t) = (\lambda_o - \lambda_+) * \frac{(m_K^2 - m_\pi^2)}{m_\pi^2}$$

The values for λ_+ and λ_o are taken from PDG 1982 p. 76.

	λ_+	λ_o
$k_{\mu 3}^{\pm}$	0.032 ± 0.008	0.004 ± 0.007
$k_{\mu 3}^{o}$	0.034 ± 0.005	0.025 ± 0.006
$k_{e3}^{\pm}$	0.0285 ± 0.0043	
k_{e3}^{o}	0.0300 ± 0.0016	

In the k_{e3} decays all terms with m_e^2 are neglected.

Fig. 2 shows the Dalitz plot and the lepton and the pion spectrum for the decay $k^{\pm} \to \pi^0 \mu^{\pm} \nu$.

Performance

General: Only decays inside the inner surface of the lead glass are performed. Beyond the particles are handled by the lead glass routines TRLGL and ENDCLG. Muons, kaons and pions are tracked further by the muon routines which start at the inner surface of the lead glass.

Old scheme for charged kaons: 1) In subroutine TRCDET the routine PIKMUF is called in case of tracking at $k^{\pm}$ or $\pi^{\pm}$. PIKMUF determines the distance STPLEN the particles travel before decaying. STPLEN is exponentially distributed. 2) If the flightpath of a $k^{\pm}$ or $\pi^{\pm}$ becomes bigger than STPLEN the routine PIKDEC (P, PV2, STPLEN) is called with P(1...10) the usual parameters of the mother particle and PV2 (1...10) the equivalent set of parameters for the daughter particle. 3) SVECT1 (PV2, R) is called to store PV2 and R into the 'VECT', 1 bank. R(1...3) is the space point where the decay takes place and hence the starting point of the daughter particle. The old scheme foresees only one decay channel which has only one daughter particle to be tracked further on.

e.g.	$\pi^+ \to$	$\mu^+ \nu$	Only the μ^+ is tracked.
	$k^+ \to$	$\pi^+ \pi^0$	$(\to \pi^+ \gamma\gamma)$ Only the π^+ is tracked, the two γ's are not.

New scheme for charged kaons: Step 1) is unchanged. 2) For the decay of a kaon the routine TRKADC (P,R) is called. 3) A list of the final state particles is set up in TRKADC. Neutrinos are omitted. If there are π^0's in the final state first the decay $\pi^0 \to \gamma\gamma$ is made and the two γ's are put into the list. (The decay $\pi^0 \to \gamma\, e^+ e^-$ is neglected.) For all particles in this list SVECT1 (PV2 , R) is called. In this case the γ's from π^0 decays are tracked.

Old scheme for neutral kaons: The routine JTRNTR 'tracks' kaons to the surface of the lead glass without any interaction. (This routine is called for neutrons as well.) Then the particles are taken over by the lead glass routines TRLGL if they hit the barrel or ENDCLG if they hit the endcap. These routines determine the particles energy loss in the lead glass.

New scheme for neutral kaons: JTRNTR is modified slightly. For each kaon a decay point is determined. If this point is inside the space surrounded by the lead glass the routine TRKADC is called, proceeding like step 3) in the new scheme for charged kaons.

Final remarks

1) In the Tracking Monte Carlo running at the Heidelberg IBM since 1984 the new routines have been used. Unfortunately for the KØL the same decay length as for the $k^{\pm}$ was put in, namely 3709 mm instead of 15540 mm. I studied the consequences on the 4 vector level with the LUND generator. In 20 000 events with 14600 KØL's I found with:

3709 mm decay length	2160 KØL-decays and with
15540 mm decay length	610 KØL-decays in the Inner Detector.

The error is corrected since 20/10/84.

2) The new routines will become standard on F22ELS.JMC.S and F22ELS.JMC.L at the date of appearance of this JADE NOTE.

3) If somebody wants to have kaon decay at the 4-vector level in the LUND generator it is recommended to use the routine F11HEL.LU52.S (LUDECY) instead of F22PET.LUND52.S(LUDECY). At the first call part of some arrays are overwritten in BLOCK DATA LUDATA. These are DECLEN(18), DECLEN(38), IDB(18) IDB(38), CBR(203...214), KDP(809...856). DECLEN $\neq$ Ø. and IDB $\neq$ Ø. give rise to the decays of KØL and $k^{\pm}$ in the LUND generator. Take care that no other data are stored in the above mentioned arrays.

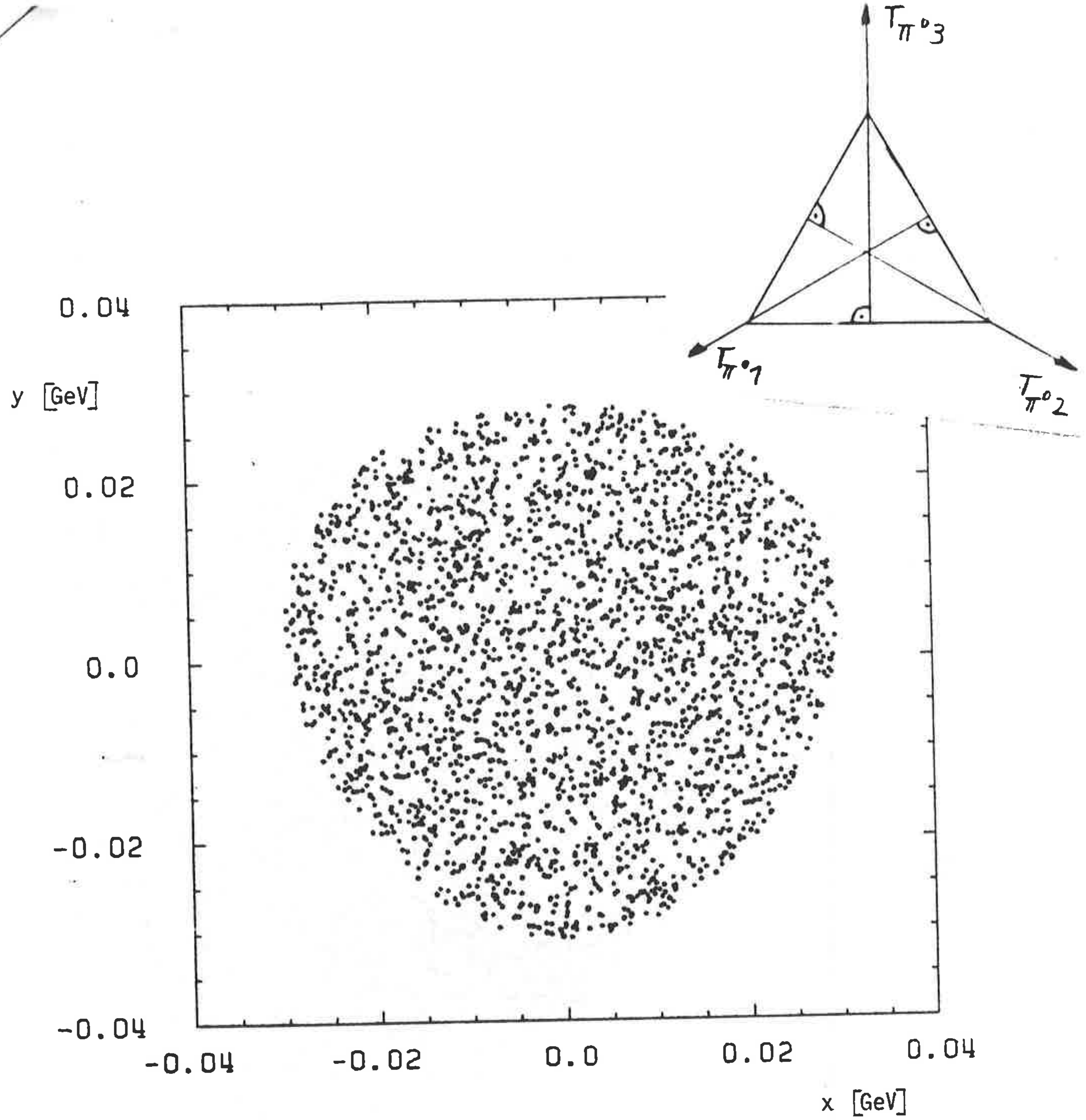

Fig. 1: Triangular Dalitz plot of the decay $K^0 \to \pi^0\pi^0\pi^0$.

$$x = \frac{(T_2 - T_1)}{\sqrt{3}}, \quad y = T_3 - \frac{Q}{3}, \quad Q = T_1 + T_2 + T_3.$$

$T_{1,2,3}$ are the kinetic energies of the π^0's.

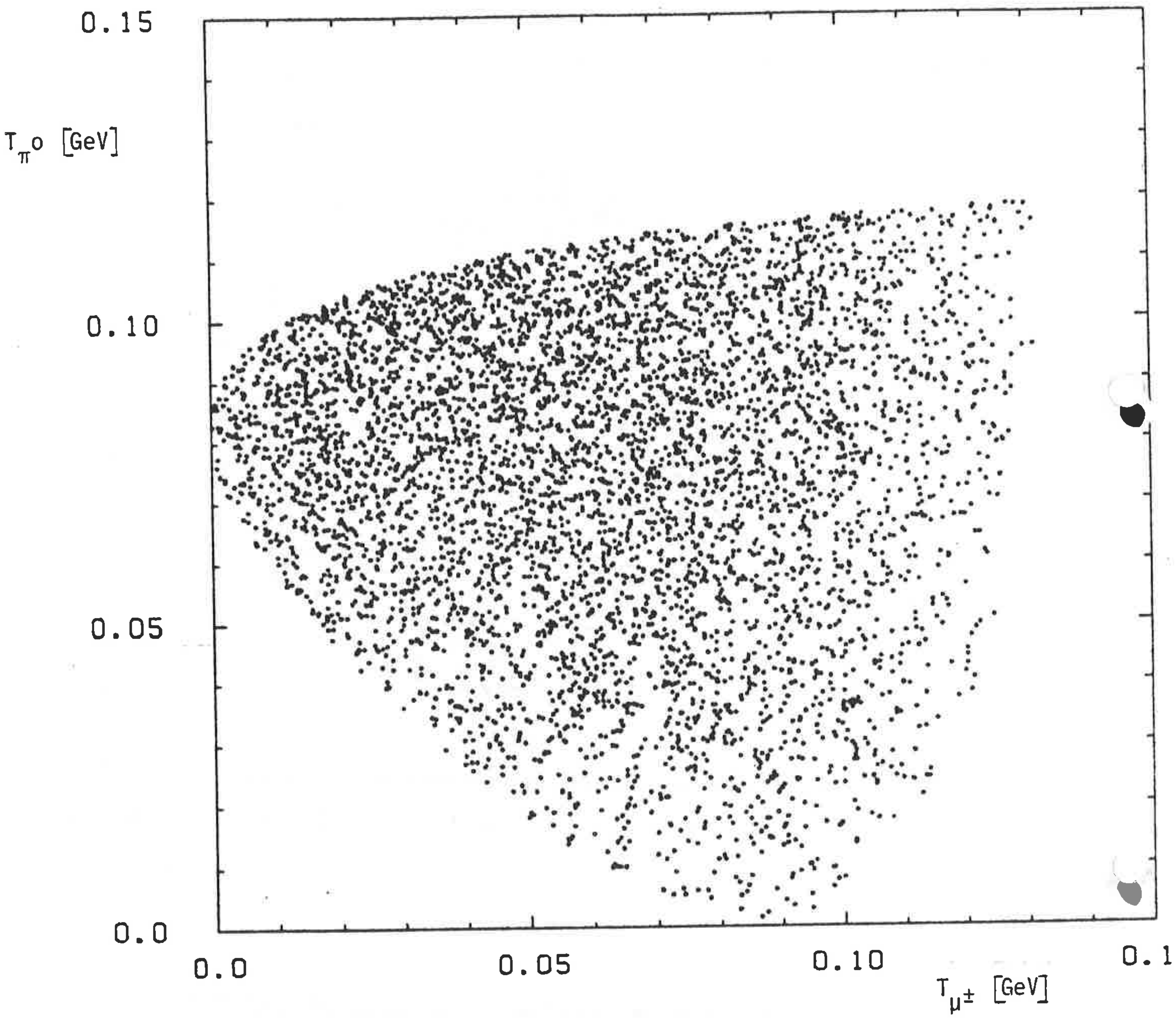

Fig. 2a): Dalitz plot of the decay $K^\pm \to \pi^0 \mu^\pm \nu$.
T_{π^0} and $T_{\mu^\pm}$ are the kinetic energies of the π^0 and the $\mu^\pm$ respectively.

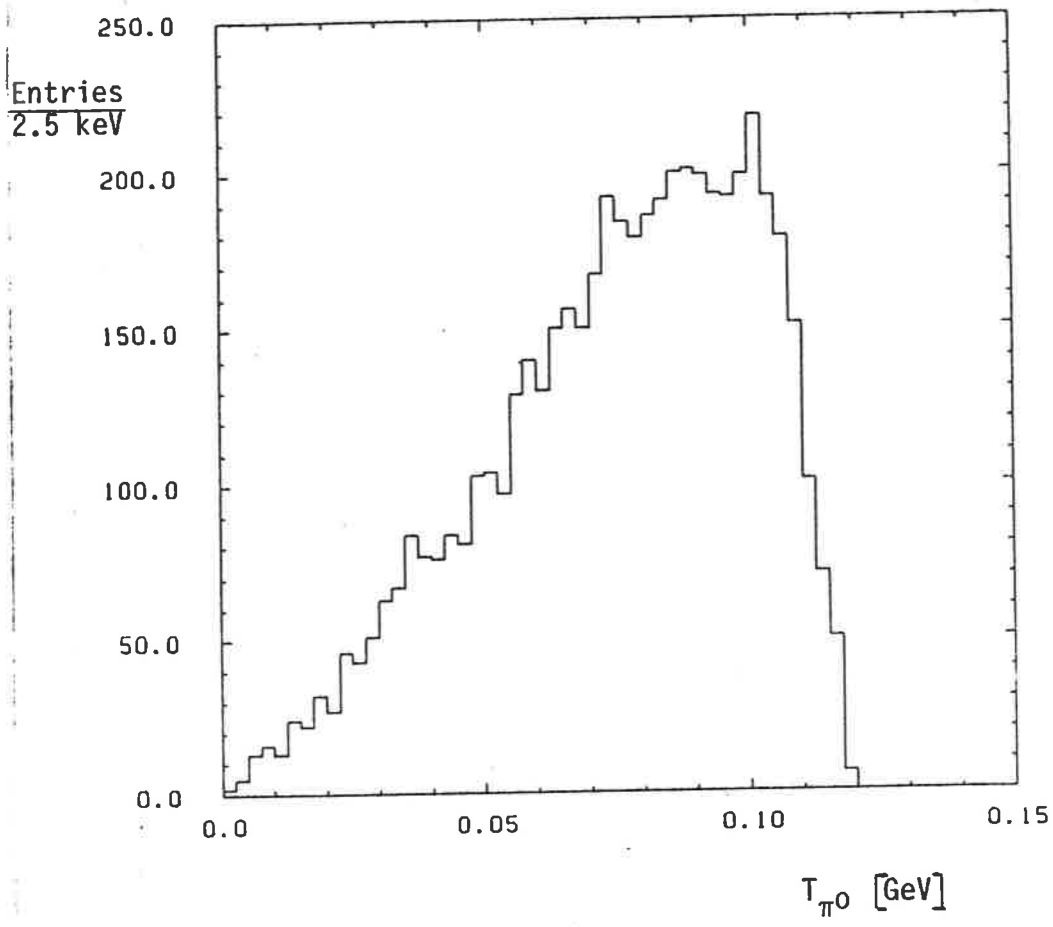

Fig. 2b: Spectrum of kinetic energy of the π^0 in the decay $k^\pm \to \pi^0 \mu^\pm \nu$ (y-projection of Fig. 2a)

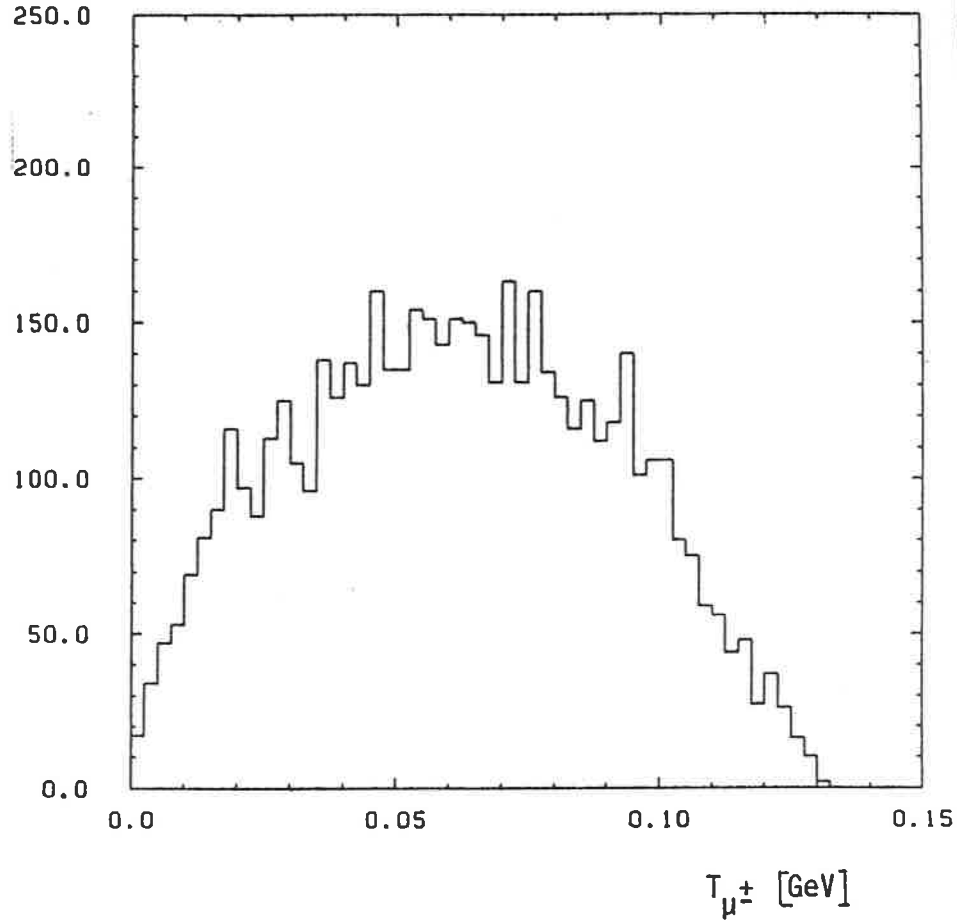

Fig. 2c: Spectrum of kinetic energy of the $\mu^\pm$ in the decay $k^\pm \to \pi^0 \mu^\pm \nu$ (x-projection of Fig. 2a)