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SUBROUTINE GGFIT(EMAS,PROB)

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PROGRAM DESCRIPTION

The subroutine GGFIT was written to do a kinematic fit to the hypothesis that two observed photons were produced from the decay of a particle of fixed mass, e.g. π^0. The main purpose of the fit is to test consistency with the mass hypothesis so there is no calculation of the fitted particle directions. The fit uses the kinematics of massless particles for the photons. For this case the constraint in any Lorenz frame can be written:

$$M^2 = 4\ E_1\ E_2\ \sin^2(\delta/2)$$

where δ is the space angle between the two photons and M is the parent mass. The variables for the fit are the two energies and $\sin^2(\delta/2)$. in the laboratory frame. No attempt is made to deal with correlations in the input data.

The program has been tested on a large sample of events. If it fails to find a fit within 30 iterations it reports failure. (Typical events converge in two to six iteraticns.) It is safe to regard such rare failures as though the chi-squared were very large.

After setting the values of the input kinematic variables in the common GGFITC (available in the MACRO 'F32BOB.JGAMS(¢GGFIT)), the program call is simply

CALL GGFIT (EMAS,PROB)

where EMAS is the value of the mass and PROB is the returned value of the probability of chi-squared (1 degree of freedom).

Input variables in GGFITC are:

XXM(1-3) - e1,e2,a; measured energies and $\sin^2(\delta/2)$
VAR(1-3) - variance of the measured values (VAR=σ^2)
NDIM = number of variables allowed in the fit (2 to fit only energies, 3 to also fit the angle) - preset to 3
LITER = maximum number of iterations - preset to 30

Output variables are:

XXF(1-3) - E1,E2,A, fitted values of the measured quantities
CHI = value of chi-squared
NST = number of steps to converge (-1 means solution not found in LITER iterations)

MATHEMATICAL DESCRIPTION

The standard technique for adjusting the values of a set of measured quantities to be consistent with some set of constraints is the method of least squares. One writes the sum of squares of the discrepancies between the data and the (unknown) values of the adjusted quantities and adds to this the products of the Lagrange multipliers and the constraint functions. Minimizing this sum then provides the necessary condition for a local minimum which also satisfies the constraints. In the current case we have only one constraint so the modified least squares sum can be written as:

$$M=\sum_j\Big((X_j-x_j)/\sigma_j\Big)^2 - \lambda(X_1X_2X_3 - \mu)$$

where X_1 and X_2 are the fitted energies, X_3 is the fitted angle variable ($\sin^2(\delta/2)$). The x_j are the corresponding measured values and $\mu=(\mathrm{EMAS})^2/4$. Minimizing this expression with respect to the X_j and λ leads to the four equations (skipping some of the algebra):

$$(1)\quad X_1 = \frac{1+(1/\gamma)W}{1-W^2}\,x_1$$

$$(2)\quad X_2 = \frac{1+\gamma W}{1-W^2}\,x_2$$

$$(3)\quad X_3{}^3 = x_3X_3{}^2+x_3{}^3\alpha W$$

$$(4)\quad (1-W^2)^2 = \rho(X_3/x_3)\Big((1+W^2)+\beta W\Big)$$

Several abbreviations have been introduced:

$$W = \lambda\sigma_1\sigma_2X_3$$

$$\gamma = (\sigma_2/\sigma_1)(x_1/x_2)$$