Intrinsic Charm Implementation

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Abstract

In these set of notes I will describe the strategy to include the intrinsic charm (IC) contribution to the FONLL structure functions as implemented in APFEL. I will first consider the massive sector (and its massless limit), where the IC implies the presence of the charm in the initial state with the consequence of additional diagrams to be include in the computation. I will the consider the massless sector where the presence of an IC implies a retratment of the PDF matching conditions at the charm threshold.

Contents

1 Intrinsic Charm Contribution to the Massive Structure Functions

1.1 Order α_s^0 Contributions

Assuming the presence of IC in the proton, the massive structure functions with $N_f = 3$ light flavours acquire a further contribution coming from the presence of a massive charm in the initial state. As a consequence, the massive structure functions get a term that is proportional to a *static* charm PDF, *i.e.* a PDF that, being massive, does not evolve according to the DGLAP equantion. Such a contribution starts already at order α_s^0 and has the novel effect to "allign" the massive scheme to the massless scheme in terms of power counting because, contrary to what happens without IC, the two sectors start at α_s^0.

In order to write explicitly the form of such LO contributions to the DIS structure functions, I consider eq. (2) of [1] where the function Q_1 schould be indentified with the charm PDF. It should be noticed that in the $N_f = 3$ scheme, such PDF does not obey the DGLAP equation because, due to the presence of the mass of the charm m_c, no large collinear logarithms appear in the calculation and thus there is no need to resum them.

From eq. (2) of [1] one reads that the $\mathcal{O}(\alpha_s^0)$ IC contributions to the massive structrure functions are given by:

$$F_1^{\rm FF,IC}(x,Q^2) = \frac{S_+\Sigma_{++} - 2m_1m_2S_-}{2\Delta}c(\chi) \tag{1.1a}$$

$$F_2^{\rm FF,IC}(x,Q^2) = \frac{S_+\Delta}{2Q^2}2xc(\chi) \tag{1.1b}$$

$$xF_3^{\rm FF,IC}(x,Q^2) = 2Rxc(\chi) \tag{1.1c}$$

where m_1 and m_2 are the masses of the incoming and outcoming quarks, respectively, while $\Delta \equiv \Delta(m_1^2, m_2^2, -Q^2)$ with the function Δ defined as:

$$\Delta(a,b,c) = \sqrt{a^2+b^2+c^2-2(ab+ac+bc)} \tag{1.2}$$

and:

$$\Sigma_{\pm\pm} = Q^2 \pm m_2^2 \pm m_1^2 \tag{1.3}$$

$$\chi = \frac{x}{2Q^2}(\Sigma_{+-} + \Delta) \tag{1.4}$$

The quantities $S_\pm$ and $R_\pm$, instead, are linked to the EW couplings and depend on the vector boson that strikes the heavy quark with mass m_1 in the initial state. Notice that in eq. (1.1) the PDF c does not depend on any factorization scale and, as mentioned before, the reason is that it is a static distribution of non-perturbative origin that does not evolve according to the DGLAP equation.

In practice, assuming the presence of IC in the proton, the massive (FF) structure functions become:

$$F_i^{\mathrm{FF}}(x,Q^2) \longrightarrow F_i^{\mathrm{FF}}(x,Q^2) + F_i^{\mathrm{FF,IC}}(x,Q^2) \quad \text{with} \quad i=1,2,3 \tag{1.5}$$

Now, for a purely *electromagnetic* process, where only a γ strikes the charm, one has:

$$S_+ = S_- = e_c^2 \quad \text{and} \quad R = 0 \tag{1.6}$$

Moreover, in this case both the incoming and the outcoming quarks are of the same flavour (charm) therefore we have $m_1 = m_2 = m_c$. Under this conditions one finds:

$$F_1^{\mathrm{FF,IC}}(x,Q^2) = \frac{1}{2\sqrt{1+4\lambda}} e_c^2 c(\chi) \tag{1.7a}$$

$$F_2^{\mathrm{FF,IC}}(x,Q^2) = \left(\sqrt{1+4\lambda}\right) e_c^2 x c(\chi) \tag{1.7b}$$

$$xF_3^{\mathrm{FF,IC}}(x,Q^2) = 0 \tag{1.7c}$$

with:

$$\chi = \frac{x}{2}\left(1+\sqrt{1+4\lambda}\right) = \frac{x}{\eta}\,, \tag{1.8}$$

where I have defined:

$$\eta = \frac{2Q^2}{\Sigma_{+-}+\Delta} = 2\left(1+\sqrt{1+4\lambda}\right)^{-1} = 2\left(1+\sqrt{1+4\lambda}\right)^{-1}\,, \tag{1.9}$$

with $\lambda = \frac{m_c^2}{Q^2}$.

For a *neutral current* process, where all the γ, the Z and the interference γZ contributions are considered, one has:

$$S_+ = S_- = B_c = e_c^2 - 2e_cV_eV_cP_Z + (V_e^2+A_e^2)(V_c^2+A_c^2)P_Z^2 \quad \text{and} \quad R = D_c = -2e_cA_cA_eP_Z + 4V_cA_cV_eA_eP_Z^2 \tag{1.10}$$

with:

$$V_c = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W \quad \text{and} \quad A_c = \frac{1}{2} \tag{1.11}$$

and

$$V_e = -\frac{1}{2} + 2\sin^2\theta_W \quad \text{and} \quad A_e = -\frac{1}{2} \tag{1.12}$$

the vector and the axial coupling of charm and electron to the Z and where:

$$P_Z = \frac{1}{4\sin^2\theta_W(1-\sin^2\theta_W)}\frac{Q^2}{Q^2+M_Z^2} \tag{1.13}$$

Here, exactly as in the electromagnetic case, $m_1 = m_2 = m_c$ so that one ends up with:

$$F_1^{\mathrm{FF,IC}}(x,Q^2) = \frac{1}{2\sqrt{1+4\lambda}} B_c c(\chi)\,, \tag{1.14a}$$

$$F_2^{\mathrm{FF,IC}}(x,Q^2) = \left(\sqrt{1+4\lambda}\right) B_c x c(\chi)\,, \tag{1.14b}$$

$$xF_3^{\mathrm{FF,IC}}(x,Q^2) = 2D_c x c(\chi)\,. \tag{1.14c}$$

Finally, for a *charged current* process, where a charged boson $W^\pm$ strikes the charm, one has:

$$S_+ = S_- = 2|V_{cs}|^2 \quad \text{and} \quad R = |V_{cs}|^2 \tag{1.15}$$

if the outcoming quark is a strange or an anti-strange, and:

$$S_+ = S_- = 2|V_{cd}|^2 \quad \text{and} \quad R = |V_{cd}|^2 \tag{1.16}$$

if the outcoming quark is a down or an anti-down.

In this case $m_1 = m_c$ but $m_2 = 0$ with the consequence that:

$$F_1^{\mathrm{FF,IC}}(x,Q^2) = |V_{cj}|^2 c(x) \tag{1.17a}$$

$$F_2^{\mathrm{FF,IC}}(x,Q^2) = 2\,(1+\lambda)\,|V_{cj}|^2 x c(x) \tag{1.17b}$$

$$xF_3^{\mathrm{FF,IC}}(x,Q^2) = 2|V_{cj}|^2 x c(x) \tag{1.17c}$$

with $j = d, s$. Note that in this case $\eta = 1$ and thus $\chi = x$.

In order to take into account the possible contributions due to intrinsic charm, one has to consider all diagrams contributing to a given process. As far as the neutral current (electromagnetic) case is concerned, one has to consider also the presence of $\overline{c}$ in the proton which, summed to the contribution of the c, gives:

$$F_1^{\mathrm{FF,IC}}(x,Q^2) = \frac{1}{2\sqrt{1+4\lambda}} B_c c^+(\chi) = \frac{1}{2x}\frac{\eta^2}{2-\eta} B_c \chi c^+(\chi) \tag{1.18a}$$

$$F_2^{\mathrm{FF,IC}}(x,Q^2) = \frac{2\sqrt{1+4\lambda}}{1+\sqrt{1+4\lambda}} B_c \chi c^+(\chi) = (2-\eta) B_c \chi c^+(\chi) \tag{1.18b}$$

$$xF_3^{\mathrm{FF,IC}}(x,Q^2) = \frac{4}{1+\sqrt{1+4\lambda}} D_c \chi c^-(\chi) = 2\eta D_c \chi c^-(\chi) \tag{1.18c}$$

where:

$$c^\pm = c \pm \overline{c} \tag{1.19}$$

therefore:

$$F_L^{\mathrm{FF,IC}}(x,Q^2) = F_2^{\mathrm{FF,IC}}(x,Q^2) - 2xF_1^{\mathrm{FF,IC}}(x,Q^2) = 4\frac{1-\eta}{2-\eta} B_c \chi c^+(\chi) \tag{1.20}$$

In the charged current case, instead, one has to distinguish between neutrino and anti-neutrino scattering. The neutrino scattering gives as a result the following structure functions:

$$F_1^{\nu,\mathrm{FF,IC}}(x,Q^2) = (|V_{cd}|^2 + |V_{cs}|^2)\overline{c}(x) \tag{1.21a}$$

$$F_2^{\nu,\mathrm{FF,IC}}(x,Q^2) = 2\,(1+\lambda)\,(|V_{cd}|^2 + |V_{cs}|^2) x\overline{c}(x) \tag{1.21b}$$

$$xF_3^{\nu,\mathrm{FF,IC}}(x,Q^2) = 2(|V_{cd}|^2 + |V_{cs}|^2) x\overline{c}(x) \tag{1.21c}$$

$$F_L^{\nu,\mathrm{FF,IC}}(x,Q^2) = 2\lambda(|V_{cd}|^2 + |V_{cs}|^2) x\overline{c}(x) \tag{1.21d}$$

The anti-neutrino scattering instead gives as a result the following structure functions:

$$F_1^{\overline{\nu},\mathrm{FF,IC}}(x,Q^2) = (|V_{cd}|^2 + |V_{cs}|^2) c(x) \tag{1.22a}$$

$$F_2^{\overline{\nu},\mathrm{FF,IC}}(x,Q^2) = 2\,(1+\lambda)\,(|V_{cd}|^2 + |V_{cs}|^2) x c(x) \tag{1.22b}$$

$$xF_3^{\overline{\nu},\mathrm{FF,IC}}(x,Q^2) = 2(|V_{cd}|^2 + |V_{cs}|^2) x c(x) \tag{1.22c}$$

$$F_L^{\overline{\nu},\mathrm{FF,IC}}(x,Q^2) = 2\lambda(|V_{cd}|^2 + |V_{cs}|^2) x c(x) \tag{1.22d}$$

It should be pointed out that since the charm quark belongs to the sea it is symmetric under isospin symmetry and thus all the above structure functions are the same for proton and neutron.

1.2 Massless Limit

When implementing the FONLL scheme, one also needs to consider the massless limit of the massive structuture functions (FF0). To this end, we just need to take the limit for $m_c \to 0$ of eqs. (1.24), (1.25) and (3.5). Considering that:

$$\eta \underset{m_c \to 0}{\longrightarrow} 1 \quad \Rightarrow \quad \chi \underset{m_c \to 0}{\longrightarrow} x\,, \tag{1.23}$$

one finds :

$$F_1^{\mathrm{FF0,IC}}(x,Q^2) = \frac{1}{2} B_c c^+(x) \tag{1.24a}$$

$$F_2^{\mathrm{FF0,IC}}(x,Q^2) = B_c x c^+(x) \tag{1.24b}$$

$$xF_3^{\mathrm{FF0,IC}}(x,Q^2) = 2D_c x c^-(x) \tag{1.24c}$$
$$F_L^{\mathrm{FF0,IC}}(x,Q^2) = 0 \tag{1.24d}$$

and:

$$F_1^{\nu,\mathrm{FF0,IC}}(x,Q^2) = (|V_{cd}|^2 + |V_{cs}|^2)\overline{c}(x) \tag{1.25a}$$
$$F_2^{\nu,\mathrm{FF0,IC}}(x,Q^2) = 2(|V_{cd}|^2 + |V_{cs}|^2)x\overline{c}(x) \tag{1.25b}$$
$$xF_3^{\nu,\mathrm{FF0,IC}}(x,Q^2) = 2(|V_{cd}|^2 + |V_{cs}|^2)x\overline{c}(x) \tag{1.25c}$$
$$F_L^{\nu,\mathrm{FF0,IC}}(x,Q^2) = 0 \tag{1.25d}$$

and:

$$F_1^{\overline{\nu},\mathrm{FF0,IC}}(x,Q^2) = (|V_{cd}|^2 + |V_{cs}|^2)c(x) \tag{1.26a}$$
$$F_2^{\overline{\nu},\mathrm{FF0,IC}}(x,Q^2) = 2(|V_{cd}|^2 + |V_{cs}|^2)xc(x) \tag{1.26b}$$
$$xF_3^{\overline{\nu},\mathrm{FF0,IC}}(x,Q^2) = 2(|V_{cd}|^2 + |V_{cs}|^2)xc(x) \tag{1.26c}$$
$$F_L^{\overline{\nu},\mathrm{FF0,IC}}(x,Q^2) = 0 \tag{1.26d}$$

2 The FONLL Structure Functions

Once the inclusion of the IC into the massive sectors has been established, one can construct the FONLL structure functions using the usual recipe but now including the additional contributions. Calling F_i^{FONLL} the usual FONLL structure functions without IC and $F_i^{\mathrm{FONLL,IC}}$ the structure function with IC, the relation is:

$$\begin{aligned} F_i^{\mathrm{FONLL,IC}} &= F_i^{\mathrm{FF}} + F_i^{\mathrm{FF,IC}} + D(Q^2)\left[F_i^{\mathrm{ZM}} - F_i^{\mathrm{FF0}} - F_i^{\mathrm{FF0,IC}}\right] \\ &= F_i^{\mathrm{FONLL}} + \left[F_i^{\mathrm{FF,IC}} - D(Q^2)F_i^{\mathrm{FF0,IC}}\right] = F_i^{\mathrm{FONLL}} + \Delta F_i^{\mathrm{FONLL,IC}} \end{aligned} \tag{2.1}$$

where $D(Q^2)$ is a damping factor needed to quench undesired possibly large subleading terms at small energies. In the rest of theses notes I will concentrate on the implementation of the $\Delta F_i^{\mathrm{FONLL,IC}}$ in APFEL.

3 The Implementation

At $\mathcal{O}(\alpha_s^0)$ there is no convolution between PDFs and coefficient functions and the charm PDFs appear directly in the expressions. According to whether one considers CC or NC heavy-quark-initiated processes, PDFs enter either as $xc(x)$, where x is the measured Bjorken variable, or as $\chi c(\chi)$, where χ is the rescaled variable defined in eq. (1.8). Now, in order to achive a proper implementation of the FONLL scheme in APFEL, I need to know all the component of the struncture functions (massive and massless) on the same x-space interpolation grid, defined as $\{x_\alpha\}$, , $\alpha \in= 0,\dots,N_x$. At LO, this essentially means knowing both $xc(x)$ and $\chi c(\chi)$ on the same grid. But choosing to tabulate $xc(x)$, such that in the CC case:

$$F^{\mathrm{CC}}(x_\alpha) \propto x_\alpha c(x_\alpha) = \tilde{c}(x_\alpha) = \sum_{\beta=0}^{N_x} \delta_{\alpha\beta}\tilde{c}(x_\beta)\,, \tag{3.1}$$

in the NC case, using the usual interpolation formula, the structure function can be expanded as:

$$F^{\mathrm{NC}}(x_\alpha) \propto \chi(x_\alpha)c(\chi(x_\alpha)) = \tilde{c}(\chi(x_\alpha)) = \sum_{\beta=0}^{N_x} w_\beta^{(k)}(\chi(x_\alpha))\tilde{c}(x_\beta) = \sum_{\beta=0}^{N_x} w_\beta^{(k)}\left(\frac{x_\alpha}{\eta}\right)\tilde{c}(x_\beta)\,. \tag{3.2}$$

As a consequence, in the APFEL framework, quantities to store are $\delta_{\alpha\beta}$ and $w_\beta^{(k)}(x_\alpha/\eta)$ to be combined in a proper way to the other coefficient functions. First of all, let us compute case by case the quantity $\Delta F_i^{\mathrm{FONLL,IC}}$. In the NC case one has:

$$\Delta F_2^{\mathrm{FONLL,IC}}(x_\alpha) = B_c \sum_{\beta=0}^{N_x}\left[(2-\eta)w_\beta^{(k)}\left(\frac{x_\alpha}{\eta}\right) - D(Q^2)\delta_{\alpha\beta}\right]\tilde{c}^+(x_\beta) \tag{3.3a}$$

$$x_\alpha \Delta F_3^{\rm FONLL,IC}(x_\alpha) \quad = \quad D_c \sum_{\beta=0}^{N_x} \left[2\eta w_\beta^{(k)} \left(\frac{x_\alpha}{\eta} \right) - D(Q^2) 2\delta_{\alpha\beta} \right] \tilde{c}^-(x_\beta) \tag{3.3b}$$

$$\Delta F_L^{\rm FONLL,IC}(x_\alpha) \quad = \quad B_c \sum_{\beta=0}^{N_x} \left[2\frac{1-\eta}{2-\eta} w_\beta^{(k)} \left(\frac{x_\alpha}{\eta} \right) \right] \tilde{c}^+(x_\beta) \tag{3.3c}$$

Finally, the CC case is slightly simpler:

$$\Delta F_2^{\nu,\rm FONLL,IC}(x_\alpha) = 2(|V_{cd}|^2 + |V_{cs}|^2) \left[(1+\lambda) - D(Q^2) \right] \sum_{\beta=0}^{N_x} \delta_{\alpha\beta} \tilde{\bar{c}}(x_\beta) \tag{3.4a}$$

$$x_\alpha \Delta F_3^{\nu,\rm FONLL,IC}(x_\alpha) = 2(|V_{cd}|^2 + |V_{cs}|^2) \left[1 - D(Q^2) \right] \sum_{\beta=0}^{N_x} \delta_{\alpha\beta} \tilde{\bar{c}}(x_\beta) \tag{3.4b}$$

$$\Delta F_L^{\nu,\rm FONLL,IC}(x_\alpha) = 2(|V_{cd}|^2 + |V_{cs}|^2) \lambda \sum_{\beta=0}^{N_x} \delta_{\alpha\beta} \tilde{\bar{c}}(x_\beta) \tag{3.4c}$$

and:

$$\Delta F_2^{\overline{\nu},\rm FONLL,IC}(x_\alpha) = 2(|V_{cd}|^2 + |V_{cs}|^2) \left[(1+\lambda) - D(Q^2) \right] \sum_{\beta=0}^{N_x} \delta_{\alpha\beta} \tilde{c}(x_\beta) \tag{3.5a}$$

$$x_\alpha \Delta F_3^{\overline{\nu},\rm FONLL,IC}(x_\alpha) = 2(|V_{cd}|^2 + |V_{cs}|^2) \left[1 - D(Q^2) \right] \sum_{\beta=0}^{N_x} \delta_{\alpha\beta} \tilde{c}(x_\beta) \tag{3.5b}$$

$$\Delta F_L^{\overline{\nu},\rm FONLL,IC}(x_\alpha) = 2(|V_{cd}|^2 + |V_{cs}|^2) \lambda \sum_{\beta=0}^{N_x} \delta_{\alpha\beta} \tilde{c}(x_\beta) \tag{3.5c}$$

Now, since structure functions in APFEL are expressed in the so-called evolution basis $\{\Sigma, g, V, V_3, \dots\}$, we only need to re-express the charm PDFs in terms of the distributions in the evolution basis. In particular, it is easy to show that:

$$c^+ = \frac{1}{6}\Sigma - \frac{1}{4}T_{15} + \frac{1}{20}T_{24} + \frac{1}{30}T_{35}\,, \tag{3.6}$$

and:

$$c^- = \frac{1}{6}V - \frac{1}{4}V_{15} + \frac{1}{20}V_{24} + \frac{1}{30}V_{35}\,. \tag{3.7}$$

In addition:

$$c = \frac{1}{2}(c^+ + c^-) \quad \text{and} \quad \overline{c} = \frac{1}{2}(c^+ - c^-)\,. \tag{3.8}$$

References

[1] S. Kretzer and I. Schienbein, Phys. Rev. D **58** (1998) 094035 [hep-ph/9805233].