Margin of Conservatism Type C in PD Modeling

Central Tendency Uncertainty in the Presence of Autocorrelation

Andrija Djurovic

www.linkedin.com/in/andrija-djurovic

Margin of Conservatism in PD modeling

- After completing the Probability of Default (PD) model development, practitioners usually address overall model uncertainties by defining the so-called Margin of Conservatism (MoC).
- The European Banking Authority (EBA) defines three types (categories) for distinct sources of uncertainty in the estimated parameters:
  1. Type A: covers the uncertainty related to the data and methodology deficiencies;
  2. Type B: covers the uncertainty related to changes in risk appetite and internal processes;
  3. Type C: covers the statistical uncertainty from the estimated parameter.
- One of the most commonly used MoC Type C is the uncertainty related to the estimation of the default rate central tendency.

Central Tendency Uncertainty in Practice

The central tendency (CT), defined as the mean of default rates over a specific period, inherently involves variability and uncertainty in its calculation.

Various methods can be employed to manage the variability associated with calculating the CT of default rates. The straightforward and most frequently used method in practice is the one that relies on the standard error of the mean, defined as:

$$\frac{\sigma}{\sqrt{n}}$$

with σ being the standard deviation of the observed default rates and n the number of observations in the observed period.

The direct application of the above method assumes that default rates between observed periods are independent. However, how realistic is this assumption in practice, considering the possibility of calculating the CT from non-overlapping and overlapping time frames?

How does autocorrelation affect the estimation of the standard error of the CT?

The following slides provide a simulation framework and compare results between independent and autocorrelated observations of default rates.

Simulation Setup

The following steps outline a Monte Carlo simulation to compare the standard error of the CT under assumptions of independence and correlated observations over the observation period. To align with the IRB framework, the observed default rates are simulated from the Vasicek distribution for parameters $PD = 0.05$ and $\rho = 0.10$, while the autoregressive assumption is implemented in developing the systemic factor z.

1. Select the sample size n and the autoregressive structure.
2. For the selected n, simulate independent observations from the Vasicek distribution with parameters $PD = 0.05$ and $\rho = 0.10$.
3. For the selected n and autoregressive structure (ϕ), first simulate the values of the systemic factor z:

   $$z_t = \phi z_{t-1} + \sqrt{1-\phi^2}\epsilon_t$$

   with ϕ is the autoregressive coefficient of order 1 and ϵ is the error term from the standard normal distribution.Then, simulate the observed default rates from the Vasicek distribution given parameters $PD = 0.05$ and $\rho = 0.10$, and the autocorrelated values of the systemic factor z.
4. Calculate the mean of the simulated observed default rates from steps 2 and 3.
5. Repeat the steps 2 to 4, $N = 10,000$ times.
6. Compare the distributions and standard deviations of the simulated means under the assumptions of independence and correlated observations.

Simulation Results

```
##     pd rho phi  n sd correlated sd independent
##   0.05 0.1 0.5 10        0.0172         0.0111
##   0.05 0.1 0.5 15        0.0146         0.0090
##   0.05 0.1 0.5 20        0.0127         0.0078
##   0.05 0.1 0.5 25        0.0114         0.0070
##   0.05 0.1 0.5 30        0.0105         0.0063
##   0.05 0.1 0.7 10        0.0219         0.0111
##   0.05 0.1 0.7 15        0.0187         0.0089
##   0.05 0.1 0.7 20        0.0166         0.0078
##   0.05 0.1 0.7 25        0.0152         0.0069
##   0.05 0.1 0.7 30        0.0140         0.0063
##   0.05 0.1 0.9 10        0.0291         0.0110
##   0.05 0.1 0.9 15        0.0270         0.0090
##   0.05 0.1 0.9 20        0.0257         0.0079
##   0.05 0.1 0.9 25        0.0240         0.0069
##   0.05 0.1 0.9 30        0.0223         0.0064
```

Simulation Results cont.