An improvement of the statistical power of a rating system decreases the potential effects of adverse selection, and, combined with meeting several qualitative standards, decreases the amount of regulatory capital requirements. As a consequence, many banks have to make investment decisions where they have to consider the costs and the potential benefits of improving their rating systems.

In a competitive framework a poor statistical power of a bank's internal rating system will deteriorate the economic performance due to adverse selection, i.e. customers with a better credit quality than assessed by the bank will potentially walk away and leave the bank with a portfolio of customers with a credit quality lower than estimated. Obviously, improving the statistical power of a rating system will have a positive impact on economic performance. The size of this effect depends mainly on the degree of competitivity of the market environment. The counterweight of these potential benefits are the costs of investing into the power of a rating system such as organizational costs, costs of information technology, and costs of collecting and managing the required data.

It is the main objective of this paper to model the decision whether to invest into the quality of a rating system in a rather general framework. Our model is aimed to quantify the benefits of such an investment. The first part of our analysis is focused on the economic value of increasing the statistical power of a bank's internal rating system.

In our model the statistical power of a rating system depends on several parameters such as its accuracy and the rating class structure. We measure the accuracy of forecasting individual default probabilities as the variance of the deviations of the forecasted from the actual default probabilities. In this setup this measure is more closely related to the economic impact than the area-under-the curve measures traditionally used by other researchers.

The cornerstone of our model is the assumption that a bank possesses estimates (not necessarily free of error) of the true individual default probabilities of all its customers. These estimates may be taken from regression-based models which yield individual estimates of default probabilities or from cohort methods where the individual estimated default probabilities are set equal to the average default probability of the cohort.

A bank prices the loans offered to its customers according to this estimated default probability. More specifically, the spread over the risk-free rate has to cover the expected loss and the proportional 'general' costs including operating costs and risk premia related to unexpected losses. For simplicity we assume that the ability to measure unexpected losses is not influenced by the statistical power of the rating system. Note that unexpected losses are likely to be very low for large, well-diversified portfolios.

We model the competitivity of the market environment by parametrizing customer elasticity. Customers are assumed to have some better information about their true credit quality. In a full competitive framework with no transaction costs customers who are offered a too high credit spread will eventually walk away to a bank with a more powerful rating system. As a consequence, the bank is left with the customers who are offered a too low credit spread and know about their worse credit quality. This adverse selection effect deteriorates the economic performance of the bank and may lead to insolvency of the bank in extreme cases. However, the fraction of overpriced customers leaving the bank might not be 100% for several reasons. To account for all these possible effects, we assume that there is a probability that a customer with a better credit quality than assessed does not leave the bank. If this probability is zero we have perfect customer elasticity, if this probability is one there is no competitivity at all.

Our model provides a framework to quantify potential positive effects of an improvement of the rating system. Of course in real-world decisions the costs of investing into the power of a rating system have to be taken into account.

In section 2 we describe the setup of our model. The key ingrediencies are the distribution of individual default probabilities which captures the portfolio structure, the degree of competitivity in the market environment, and the way the accuracy of a rating system is measured.

2 Model Setup

Many banks are expected to base their PD estimation on the observation of empirical default rates within rating classes. This so-called "cohort method" (see e.g. Jarrow et al. (1997) and Lando and Skodeberg (2002)) is the basic object of our analysis. The main alternative, however, the usage of regression-based forecasts of individual PDs can be seen as a special case of our framework where we have one customer per rating class or - put more precisely - one rating class for each different PD because it is possible to observe customers with identical PDs.

In our setup the credit portfolio of a bank is characterized by the number of customers and the actual or “true” probability of default of each customer. We assume that the recovery rates are known for all customers. So, we concentrate on the quality of rating systems with respect to the estimation of PDs. To simplify the analysis we assume that all exposures are of equal size, which is a reasonable approximation for a large, well-diversified portfolio, and that the PDs in the portfolio may be described by a certain ex-ante distribution, which describes the PD distribution of all potential customers for the bank. In our numerical approach the true PD for each customer in the portfolio is drawn from this distribution.

The rating system of the bank will only provide estimates for the true PD of each customer. The difference between the estimated and the true PD will depend on the number and sizes of the rating classes, and the measurement error of the PD. For the purpose of the implementation of our model we will assume that the bank observes a PD of each customer, e.g. by using a logistic regression model, which is not necessarily equal to the true PD and uses this observed PD to slot the customer into a particular rating class. Once the customers are slotted into rating classes the bank estimates the PD of each rating class and uses this PD for pricing and risk management of the customers of this rating class. The estimated PD of each rating class is taken as the expected number of defaults divided by the number of customers. Under the assumption of a stationary PD distribution in the portfolio this expected default rate should on average equal the actual or observed default rate of the past period which is the usual basis for PD calculation.

In the next step we introduce measurement errors for the observed PDs of each customer which are used for slotting into the rating classes.

$$PD_{true} = \frac{1}{1+e^{-credit\ score_{true}}} \Leftrightarrow credit\ score_{true} = ln\left(\frac{1 - PD_{true}}{PD_{true}}\right) \qquad (1)$$

$$PD_{observed} = \frac{1}{1+e^{-(credit\ score_{true}+\varepsilon)}} \qquad \text{with} \qquad \varepsilon \sim N(0,\sigma^2) \qquad (2)$$

The parameter sigma controls the magnitude of the estimation error. Introducing measurement errors means that there will be differences between the true PD and the observed PD for some customers and therefore these customers are potentially slotted into the wrong rating class. Since the estimated PD of each rating class is taken as the expected number of defaults divided by the number of customers higher measurement errors will increase the probability that customers with high PDs are slotted in low risk rating classes and vice versa. Thus the estimated PD for low risk rating classes will be higher and the estimated PD for high risk rating classes will be lower compared to rating systems without measurement errors.

Therefore, banks with measurement errors will have less accurate estimated PDs and will be stronger exposed to adverse selection.

The number and sizes of rating classes and the parameter sigma of the measurement error are under the control of the bank. Investing into the predictive power of a rating system thus means to be able to reduce the measurement error and to use more and better dispersed rating classes.

Since we want to apply a risk-adjusted pricing we first define the loan pricing mechanism. We assume that the bank needs to receive an interest rate *r* to cover all 'general' costs besides credit risk related to the expected loss. The general costs include operating costs and risk premia related to unexpected losses. We assume that the ability to measure unexpected losses is not influenced by the decisions in our model. Credit risk related to expected losses is priced by demanding a credit spread *s*.

The credit spread depends on the estimated PD and the loss-given-default (LGD) of the individual exposures. We assume that the LGD is estimated according to the Basel II definition meaning that *(1-LGD)* is the recovery for the whole loan, i.e. principal and interest including the spread. If no default occurs the bank receives *(1+r+s)*, if default occurs the bank receives *(1+r+s)·(1-LGD)*.

In assuming the LGD to be constant in line with the assumption of the foundation IRB approach the credit spread is a function of PD, LGD, and *r*. In essence, the expected payoff of the loan has to be equal to the risk-free payoff of *1+r*:

$$1 + r = (1 - PD) \times (1 + r + s) + PD \times (1 - LGD) \times (1 + r + s)$$

Solving for the credit spread *s* yields:

$$s = (1 + r) \cdot \frac{PD \cdot LGD}{1 - PD \cdot LGD} \tag{4}$$

Using this pricing mechanism we are now able to introduce our concept of adverse selection.

The rating system of a bank provides the estimated PD for each customer who applies for a loan. Using this PD and the LGD the credit spread which is offered to customers can be calculated by the pricing mechanism. If the PD is overestimated the customer will be offered a credit spread, which is too high compared to her true PD. We will assume that customers, who are offered a too high spread, will leave the bank with a probability which is dependent on the magnitude *m* of the deviation from the spread corresponding to their true PD.

$$m = s_{estimated} - s_{true} = (1 + r) \cdot \frac{PD_{estimated} \cdot LGD}{1 - PD_{estimated} \cdot LGD} - (1 + r) \cdot \frac{PD_{true} \cdot LGD}{1 - PD_{true} \cdot LGD} \tag{5}$$

There are several possible reasons why a customer might not leave the bank in a situation where she is offered a too high spread. Since we do not want to separate among these reasons it suffices to model the outcome of the customers' decisions using a simple probability distribution. The probability to leave the bank is dependent on *m* and on the elasticity of the customer. To model the elasticity of the customer we assume the following functional relation between the probability of leaving the bank and the estimation error in the spread:

$$probability\ to\ leave = 1 - e^{-\alpha \cdot m} \tag{6}$$

where alpha is a elasticity parameter. If alpha is zero, all customers will stay. If alpha goes to infinity, all customers with an overestimated PD will leave the bank. All customers who are offered a spread corresponding to their true PD or a lower spread will stay with the bank.

This probability to leave the bank models the impact of adverse selection. We will analyse the effects using different degrees of elasticity.

Given the customer elasticity the bank observes which customers form the actual portfolio and can now evaluate the return of its portfolio ***r_portfolio*** which is the average over the returns on the individual loans ***r_i***. The return ***r_i*** is dependent on whether the customer ***i*** defaults or not, which will happen with the individual true PD of the customer:

$$1 + r_i = \begin{cases} 1 + r + s & \text{no default of customer } i \\ (1 + r + s) \cdot (1 - LGD) & \text{default of customer } i \end{cases} \quad (7)$$

$$r_{portfolio} = \frac{1}{n} \sum_{i=1}^{n} r_i \quad (8)$$

3 Design of numerical analysis

In the numerical analysis we fix the number of customers at 10,000. This number represents a well diversified portfolio implying that the observed loss is likely to be close to the expected loss. After fixing the number of customers we have to determine their true PDs. As explained in section 2 the true PD for each customer in the portfolio is drawn from a certain ex-ante distribution, which describes the PD distribution of all potential customers for the bank. We choose to use the Beta distribution for this analysis because it is easy to handle and has some attractive properties, e.g. it is defined over a finite interval as PDs are and it allows for extreme skewness as we expect for PD distributions.

Good portfolio: Beta distribution with *p = 0.4* and *q = 19* (median PD = 0.77%)
Average portfolio: Beta distribution with *p = 0.7* and *q = 37.6* (median PD = 1.08%)
Weak portfolio: Beta distribution with *p = 1.4* and *q = 58* (median PD = 1.84%)

In this context the notion of a 'weak' and a 'good' portfolio has a relative meaning.

Given the number of customers and their true PDs we generate the PDs the bank observes for slotting the customers into rating classes. This is achieved by transforming the true PD drawn from the relevant Beta distribution to a true credit score using equation (1) and by adding a simulated measurement error for each customer as described in equation (2). The magnitude of the measurement error of the rating system is controlled by the parameter sigma (see equation 2). The 4 numbers for sigma that we use are calibrated to data of the Austrian Major Loans Register provided by the Austrian National Bank where all banks have to report the sizes of major loans along with the rating of the customers and the documentation of the rating system.

· **Low accuracy (sigma = 2):** The bank has recently started to develop its rating system for estimating PDs. The rating system is not calibrated to default data and is only determined by qualitative judgement.

· **Medium accuracy (sigma = 0.5):** The bank has one or two years of experience and the rating system is calibrated to this short history of default data.

· **High accuracy (sigma = 0.1):** The bank has three to fours years of experience and the rating system has been improved through rating validation.

· **Perfect accuracy (sigma = 0):** The bank has the experience of at least one full economic cycle and the rating system is improved through repeated rating validation (no bank fulfilled these criteria in our data set).

In the next step the bank has to choose the number and sizes of the rating classes. We will consider banks which use one, two, five, ten, and infinitely many rating classes.

The next parameter, which is necessary for the bank to price loans, is the LGD. We will assume that the LGD is equal for all customers and known to the bank.

- **High (75%):** The Basel II IRB foundation approach sets the LGD to 75% for subordinated unsecured loans.
- **Medium (45%):** The Basel II IRB foundation approach sets the LGD to 45% for senior unsecured loans.
- **Low (25%):** This is consistent with typical senior loans which are completely secured by real estate (in the Basel II IRB foundation approach complete securitization by real estate reduces the LGD to 35%) and additional provide some eligible financial collateral.

Having an estimate for PD and LGD the bank can calculate the credit spread for each customer by equation (4) given some **interest rate** *r*, which covers all costs besides credit risk related to the expected loss. We set this interest rate *r* to **3%**, but the results are virtually the same for any other reasonable level of *r*.

The customers decide then whether to accept or to reject the loan offered to them. Every customer, who is offered the spread corresponding to her true PD or a lower credit spread, will accept the loan. All customers offered a higher credit spread will reject the loan with a probability depending on the elasticity parameter _ and on the magnitude *m* of the deviation. In this paper we define three **levels for the customer elasticity (low: alpha = 100, medium: alpha = 500, high: alpha = 10,000)**. We set the values for alpha in a way that it potentially covers most real-world scenarios. Since to our knowledge there is no research published about the empirical relationship between credit spread and customer behavior and we do not have access to empirical data to estimate _, we have to restrict ourselves to choices of 'plausible' values of alpha which might be judged by the probabilities of a customer to leave a bank implied by certain levels of alpha given in table 2 below.

Given the level of elasticity we simulate which customers leave the portfolio and which stay. Through adverse selection even with a low level of elasticity the bank will loose some customers. After determining which customers form the actual portfolio of the bank, we are now ready to calculate the return of this portfolio *r_portfolio* which is the average over the returns on the individual loans ***r_i*** (see equations 7 and 8). To calculate ***r_i*** we simulate which customers in the portfolio actually default using their individual true PDs. The return of the portfolio is the result of one simulation path. For each combination of the parameters we run 100 simulations to estimate the average of the portfolio returns. These average returns are the main results of our numerical analysis. We are able to examine the portfolio return effects for different parameter constellations.

The main task to compare rating systems with different predictive power can now be achieved by simulating their returns in the proposed way.

4 Numerical results

$$\Delta_{medium} = r_{portfolio,\ medium\ accuracy} - r_{portfolio,\ low\ accuracy} \tag{9}$$

$$\Delta_{high} = r_{portfolio,high\ accuracy} - r_{portfolio,\ low\ accuracy} \tag{10}$$

$$\Delta_{perfect} = r_{portfolio,\ perfect\ accuracy} - r_{portfolio,\ low\ accuracy} \tag{11}$$