Heterogeneity Shortfalls in IRB Credit Risk Models

Risk-Weighted Assets Impact Analysis

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Model Heterogeneity

- A typical step in building credit risk models is discretizing the model output into ratings, pools, or buckets.
- Practitioners generally follow established principles for this discretization.
- These principles result in specific characteristics, some of which are mandatory, while others vary by model and are desirable but not essential.
- Monotonicity and heterogeneity are typically regarded as mandatory characteristics.
- In this context, heterogeneity refers to adequate differentiation in risk profiles across ratings, pools, or buckets. It is commonly tested in Probability of Default (PD), Loss Given Default (LGD), and Exposure at Default (EAD) models, often using tests like the two-proportion test and t-test.
- Heterogeneity is usually monitored over time, and practitioners are often challenged to assess the impact of potential heterogeneity shortfalls for thorough model validation.
- The following slides present a simplified simulation design for measuring the impact of heterogeneity shortfalls in the PD rating scale on Risk-Weighted Assets (RWA). Practitioners are encouraged to adjust the simulation setup to reflect specific assumptions and to combine the effect of the heterogeneity shortfall with the potential impact of the model's lack of predictive ability.

Simulation Setup

The following steps outline the simulation design for assessing the impact of a heterogeneity shortfall, measured by the change in RWA:

1. Select the model output (rating scale, pools, or buckets) for a specific exposure type.
2. Test the heterogeneity of adjacent ratings, pools, or buckets.
3. Identify adjacent pairs where heterogeneity testing fails.
4. Locate the pair with the closest risk profiles, indicating a lack of heterogeneity.
5. Merge the adjacent pair identified in step 4 into a single group.
6. After merging the pairs identified in step 5, calculate the weighted average calibrated PD for the merged ratings, pools, or buckets and aggregate any additional elements needed to reassess heterogeneity.
7. Reassess heterogeneity.
8. Repeat steps 3 to 6 as needed.
9. Calculate the RWA for the original calibrated PD (RWA_i) and the weighted PD from step 6 (RWA_s).
10. Calculate the RWA change as: $\Delta\text{RWA} = \frac{\text{RWA}_s - \text{RWA}_i}{\text{RWA}_i}$.

The final step is ideally to compare the RWA change against a specified threshold to assess the significance of the heterogeneity shortfall.

The following slides present simulation results, assuming heterogeneity is tested in the PD rating model using a two-proportion test for revolving retail exposures.

Simulation Results

1. PD rating scale for the revolving retail exposure (`no` - number of observations, `nb` - number of defaults, `pd` - calibrated PD, `odr` - observed default rate):

```
##   rating  no  nb     pd    odr
## 1    R01 170   3 0.0241 0.0176
## 2    R02 118  10 0.0937 0.0847
## 3    R03 274  47 0.1786 0.1715
## 4    R04 100  45 0.3194 0.4500
## 5    R05  91  43 0.4822 0.4725
## 6    R06 196 122 0.6277 0.6224
## 7    R07  51  44 0.8704 0.8627
```

2. Heterogeneity testing (`p-value` - p-value from the two-proportion test for adjacent ratings, `significance level` - selected test significance level, `test results` - test outcome):

```
##   rating p-value significance level           test results
## 1    R01      NA                0.05                   <NA>
## 2    R02  0.0035                0.05  H1: DR(R02) > DR(R01)
## 3    R03  0.0127                0.05  H1: DR(R03) > DR(R02)
## 4    R04  0.0000                0.05  H1: DR(R04) > DR(R03)
## 5    R05  0.3775                0.05 H0: DR(R05) <= DR(R04)
## 6    R06  0.0084                0.05  H1: DR(R06) > DR(R05)
## 7    R07  0.0006                0.05  H1: DR(R07) > DR(R06)
```

Simulation Results cont.

3. Heterogeneity testing failed for the pair R04 - R05.
4. The only pair that failed is the one with the closest risk profiles.
5. Merge ratings R04 and R05 into a single rating R05.
6. Recalculate elements needed for reassessing heterogeneity.
7. Retest heterogeneity on the rating scale with the merged ratings R04 and R05:

```
##   rating      p-value significance level          test results
## 1    R01           NA                0.05                  <NA>
## 2    R02 3.494655e-03                0.05 H1: DR(R02) > DR(R01)
## 3    R03 1.267879e-02                0.05 H1: DR(R03) > DR(R02)
## 4    R05 6.939438e-12                0.05 H1: DR(R05) > DR(R03)
## 6    R06 7.047683e-04                0.05 H1: DR(R06) > DR(R05)
## 7    R07 5.645600e-04                0.05 H1: DR(R07) > DR(R06)
```

Simulation Results cont.

8. The heterogeneity test passes for all adjacent pairs.

9. Adjust the initial rating scale to address the failed heterogeneity for the `R04 - R05` pair and add the weighted average calibrated PD (`pd.w`):

```
##   rating  no  nb     pd    odr rating.m   pd.w
## 1    R01 170   3 0.0241 0.0176      R01 0.0241
## 2    R02 118  10 0.0937 0.0847      R02 0.0937
## 3    R03 274  47 0.1786 0.1715      R03 0.1786
## 4    R04 100  45 0.3194 0.4500      R05 0.3970
## 5    R05  91  43 0.4822 0.4725      R05 0.3970
## 6    R06 196 122 0.6277 0.6224      R06 0.6277
## 7    R07  51  44 0.8704 0.8627      R07 0.8704
```

For the revolving portfolio type, assuming a fixed `LGD` value of 75% and an equal `EAD` of 100 for each rating, the RWA is calculated using the following formulas:

$$R = 0.04$$

$$K = LGD \cdot N\left[\frac{G(PD)}{\sqrt{1-R}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right] - PD \cdot \text{LGD}$$

$$RWA = K \cdot 12.5 \cdot EAD$$

with results of RWA_i of 1105.3 and RWA_s of 1115.47

10. The RWA change is recorded as an increase of 0.92% of the RWA_i.