

```

1 # Import modules
2 import numpy as np
3 from scipy.integrate import odeint
4
5
6 def a_dot(t, a):
7     '''Solves a_dot in coordinate time'''
8     omega_r0 = 9.4e-5
9     omega_m0 = 0.32
10    omega_c0 = 1 - omega_m0 - omega_r0
11    omega_k0 = 0
12    a0 = 1
13    h = 0.67
14    numsecondsinGy = 1e9 * 3.154e7
15    H0 = (h / 3.086e17) * numsecondsinGy
16    return H0 * a * np.sqrt((omega_r0 * (a0 / a) ** 4 + omega_m0 * (a0 / a) ** 3
+ omega_k0 * (
17        a0 / a) ** 3 + omega_c0))
18
19
20 def H(a):
21     '''Solves for H at a given a using FRW equation'''
22     omega_r0 = 9.4e-5
23     omega_m0 = 0.32
24     omega_c0 = 1 - omega_m0 - omega_r0
25     omega_k0 = 0
26     a0 = 1
27     h = 0.67
28     c = 3e5 # Mpc / s
29     H0 = (100 * h) / c
30     return H0 * np.sqrt((omega_r0 * (a0 / a) ** 4 + omega_m0 * (a0 / a) ** 3 +
31         omega_k0 * (
32             a0 / a) ** 3 + omega_c0))
33
34 # Define t space
35 t_evals = np.linspace(0.001, -13.81964, 1000000)
36
37 # Solve FRW
38 soln_backward = odeint(a_dot, y0=[1, 1], t=t_evals, tfirst=True)
39
40 # Solve for conformal time
41 dt = t_evals[1] - t_evals[0]
42 a_inverse_int = np.trapz(1 / soln_backward, dx=-dt, axis=0)
43 print("Age of Universe in Conformal Time: {:.f}e10 Years".format(a_inverse_int[0
] / 10))
44
45 # Solve for a_eq
46 matter_radiation_time = 0.00006 # GYr
47 t_evals = np.linspace(0.001, -13.81965 + matter_radiation_time, 1000000)
48 dt = t_evals[1] - t_evals[0]
49 soln_backward = odeint(a_dot, y0=[1, 1], t=t_evals, tfirst=True)
50 print("a at MR equality: {:.2e}".format(soln_backward[-1][-1]))
51
52 # Solve for H at various a values
53 print("H(a_eq) = {}".format(H(2.94e-4)))
54 print("H(a = 0.5) = {}".format(H(0.5)))

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