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1 # Import modules
2 import numpy as np
3 import matplotlib.pyplot as plt
4 from scipy.integrate import odeint
5
6
7 def system(current_state, t):
8     '''
9     System of equations that solves the coupled system of ODEs for delta and phi
10    '''
11
12    phi, delta, dphi, ddelta = current_state
13    k = 1
14    ddphi = -(3 / t) * dphi - k ** 2 * phi
15    dddelta = 3 * ddphi - k ** 2 * phi - (1 / (2 * t)) * (ddelta - 3 * dphi)
16
17    return [dphi, ddelta, ddphi, dddelta]
18
19
20 # Solve System
21 initial_state = [-3 / 4, -3 / 4, 0, 0]
22 ktau_vals = np.arange(1e-2, 1e2, 0.001)
23 soln = odeint(system, initial_state, ktau_vals)
24
25 # Obtain solutions and plot
26 delta = soln[:, 1]
27 phi = soln[:, 0]
28
29 plt.plot(ktau_vals, delta, 'k-')
30 plt.title(r"$\delta_c$ Behaviour", fontsize=16)
31 plt.ylabel(r'$\frac{\delta_c}{\mathcal{R}_k}$', fontsize=16, rotation=0, labelpad=
10)
32 plt.xlabel(r'$k\tau$', fontsize=13)
33 plt.xscale('log')
34 plt.legend(loc='best')
35 plt.savefig('delta_behaviour', dpi=300)
36 plt.show()
37
38 plt.plot(ktau_vals, phi, 'k-')
39 plt.title(r"$\Phi$ Oscillations", fontsize=16)
40 plt.ylabel(r'$\frac{\Phi}{\mathcal{R}_k}$', rotation=0, fontsize=16)
41 plt.xlabel(r'$k\tau$', fontsize=13)
42 plt.xscale('log')
43 plt.savefig('phi_oscillations', dpi=300)
44 plt.show()
45
46 ref = lambda ktau: 1 / (ktau - 0.75) ** (1.5)
47 ref_minus = lambda ktau: -1 / (ktau - 1) ** (1.5)
48
49 plt.plot(ktau_vals, [ref(ktau) for ktau in ktau_vals], 'k--')
50 plt.plot(ktau_vals, [ref_minus(ktau) for ktau in ktau_vals], 'k--')
51 plt.plot(ktau_vals, phi, 'k-')
52 plt.ylim(-0.15, 0.15)
53 plt.xlim(1, 100)
54 plt.title(r"$\Phi$ Oscillations", fontsize=16)
55 plt.ylabel(r'$\frac{\Phi}{\mathcal{R}_k}$', rotation=0, fontsize=16)
56 plt.xlabel(r'$k\tau$', fontsize=13)

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57 plt.xscale('log')
58 plt.legend(loc='best')
59 plt.savefig('phi_oscillations_zoomed', dpi=300)
60 plt.show()
61
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