Háskólinn í Reykjavík

Varmafræði

Hópverkefni 2 – jarðhitavirkjun

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1 Introduction

In this project, a geothermal power plant that consists of four boreholes, throttling valves, separator, turbine, condenser and a heat exchanger, is evaluated. The main values are calculated through each step of the system. That is the mass flow rate, temperature, pressure, enthalpy, entropy and the steam quality. From that, the power generated by the turbine is found along with the efficiency of the power plant. Different scenarios are put forward, and we take a look at how that affects results.

2 Question 1

2.1 Numbers

To start off, we will be calculating all of the numbers that we need to do the tasks assigned. We will be looking at the 5 states for the power plant and for each state we will find mass flow, temperature, pressure, enthalpy, entropy and the steam quality.

2.1.1 State 1

We are given that the mass flow in to the system is $\dot{m}_1 = 200\frac{kg}{s}$. From that, we know that the mass flow will remain steady until it enters the separator. We are also given that the initial temperature is $T_1 = 270°C$. From that, we can find the total pressure from the four boreholes from table A-4. That gives us $P_1 = 5503.0$ kPa [1]. From the same table we can find the values for enthalpy and entropy, $h_1 = 1185.1$ and $S_1 = 2.9762$ [1]. The steam quality is $x_1 = 0$ since the water from the boreholes is a saturated liquid.

2.1.2 State 2

Here, we are at the exit from the vales. We are given that the exit pressure is $P_2 =$ 7 bar = 700 kPa. The throttling valves are isenthalpic which gives us that $h_1 = h_2$. To calculate the steam quality out of the throttling valves, we use:

$$x_2 = \frac{h_2 - h_f}{h_{fg}} \tag{1}$$

Using the values from table A5, $h_{f@700kPa} = 697.0\frac{kJ}{kg}$ and $h_{fg@700kPa} = 2065.8\frac{kJ}{kg}$ [1] and equation 1, we obtain that the steam quality in state 2 is $x_2 = 0.236$. The mass flow remains the same as in state 1. From table A-5, we also find the temperature, $T_2 = 164.95°C$ [1]. Finally, we find the entropy S_2 by using the values from table A-5, $S_{f@700kPa} = 1.9918\frac{kJ}{kg}$ and $S_{fg@700kPa} = 4.7153\frac{kJ}{kg}$ [1], and the equation:

$$S_2 = S_f + x_2 * S_{fg} = 3.1046 \tag{2}$$

2.1.3 State 3

Now we are in the separator stage. The separator separates the steam and the liquid. Because of that, the mass flow will get split into two:

$$\dot{m}_3 = \dot{m}_{steam} = x_2 * \dot{m}_2 = 47.2\frac{kg}{s} \tag{3}$$

$$\dot{m}_{water,out} = \dot{m}_2 - \dot{m}_{steam} = 152.8\frac{kg}{s} \tag{4}$$

Note that for question 1, we will only be looking at the steam that comes from the separator. The separator is isobaric, so $P_2 = P_3$ and $T_2 = T_3$. Since we only have steam, the enthalpy and entropy are $h_3 = h_{g@700kPa} = 2762.8$ and $S_3 = S_{g@700kPa} = 6.7071$ [1]. The steam quality is $x_3 = 1$.

2.1.4 State 4

Now we are at the turbine exhaust. We are given that the pressure is $P_4 = 0.1$ bar = 10 kPa. The mass flow remains the same, i.e. $\dot{m}_3 = \dot{m}_4$. To calculate the steam quality, we will be using equation 1, but now for state 4:

$$x_{s,4} = \frac{s_3 - s_{f,4}}{s_{fg,4}} \tag{5}$$

We already know S_3 from state 3 and from table A-5 we obtain $S_{f@10kPa} = 0.6492$ and $S_{fg@10kPa} = 7.4996$ [1]. From these values and equation 5, we get $x_4 = 0.8078$. From the same table, A-5, we get that $T_4 = 45.81°\text{C}$ [1]. For the enthalpy, we will be using the equation:

$$h_{s,4} = h_{f,4} + x_{s,4} * h_{fg,4} \tag{6}$$

From table A-5 we get $h_{f,4} = h_{f@10kPa} = 191.81\frac{kJ}{kg}$ and $h_{fg,4} = h_{fg@10kPa} = 2392.1\frac{kJ}{kg}$. Using that and equation 6 we obtain that $h_{s,4} = 2124.15\frac{kJ}{kg}$. Finally, we find the entropy by using equation 2, but with the numbers for state 4:

$$s_4 = 0.6492\frac{kJ}{kgK} + 0.8078 * 7.4996\frac{kJ}{kgK} = 6.707\frac{kJ}{kgK}$$

2.1.5 State 5

Now we are located at the condenser's exit. We know that $\dot{m}_4 = \dot{m}_5$, $T_4 = T_5$ and $P_4 = P_5$. The state of the water is saturated liquid, so the steam quality is $x_5 = 0$. That also gives us that $h_5 = h_{f,4}$ and $S_5 = S_{f,4}$.

2.1.6 Results from each state

Now we have found all the numbers we need to do the calculations. Table 1 shows the values for each state that the system has.

Table 1: Numerical results from each state of the geothermal installation

State	$\dot{m}$ **[kg/s]**	**T [°C]**	**P [kPa]**	**h [kJ/kg]**	**S [kJ/kgK]**	**x**
1	200	270.00	5503	1185.1	2.9762	0
2	200	164.95	700	1185.1	3.1046	0.236
3	47.2	164.95	700	2762.8	6.7071	1
4	47.2	45.81	10	2124.1	6.707	0.8078
5	47.2	45.81	10	191.81	0.6492	0

2.2 System diagram for the powerplant

The following system diagram was made for the power plant:

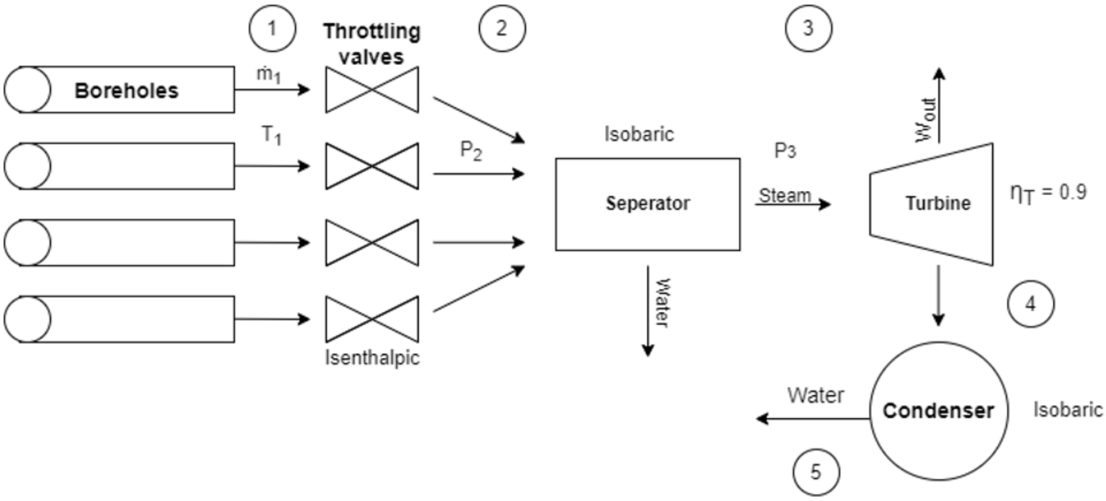

Figure 1: System diagram for the powerplant

The diagram in figure 1 shows the 5 states of the system.

2.3 Power generated by the turbine - P = 7 bar

To calculate the power generated by the turbine, we will be looking at stages 3 and 4 of the diagram. We know that the isentropic efficiency is $n_T = 0.9$ and that the formula for it is:

$$n_T = \frac{h_3 - h_{g,4}}{h_3 - h_{s,4}} = \frac{W_a}{W_s} \tag{7}$$

To calculate the power generated by the turbine, we need to find W_a. That is, the equation we will be using to calculate the power generated is:

$$W_a = n_T * W_s = n_T * (h_3 - h_{s,4}) \tag{8}$$

Now, since we know the value of h_3 from table 1, we can find the value of $h_{s,4}$ by using:

$$h_{s,4} = h_{f,4} + x_{s,4} * h_{fg,4} \tag{9}$$

Since the pressure at stage 4 is $P_4 = 10$ kPa, we obtain from steam table A-5 the values $h_{f,4} = h_{f@10kPa} = 191.81\frac{kJ}{kg}$ and $h_{fg,4} = h_{fg@10kPa} = 2392.1\frac{kJ}{kg}$ [1]. From table 1 we know that $x_s, 4 = x_4 = 0.8078$. Now using these numbers and equation 9 we can calculate:

$$h_{s,4} = 191.81\frac{kJ}{kg} + 0.8078 * 2392.1\frac{kJ}{kg} = 2124.15\frac{kJ}{kg}$$

Finally, we get the value for the power generated by the turbine. By using equation 8 and the and values stated above, we obtain:

$$W_a = (2762.8 - 2124.15)\frac{kJ}{kg} * 0.9 = 574.79\frac{kJ}{kg}$$

And accounting for the mass flow, $m_3 = 47.2\frac{kg}{s}$, we get that the power generated is:

$$\text{Power generated} = 574.79\frac{kJ}{kg} * 47.2\frac{kg}{s} = 27.13 \text{ MW}$$

2.4 What is the thermal efficiency of the power plant?

Now to calculate the thermal efficiency we will be using:

$$n_{th} = \frac{W_{net,out}}{Q_{in}} \tag{10}$$

From section 2.3 we obtained the value of $W_{net,out} = 27.13$ MW. Now to find Q_{in} we use:

$$Q_{in} = \dot{m}_1 * (h_{in} - h_{out}) = \dot{m}_1 * (h_2 - h_{4,f}) \tag{11}$$

Using the values from table 1 for $\dot{m}_1$ and h_2 and the values $h_{4,f} = 191.81\frac{kJ}{kg}$ from table A-5 [1] and equation 11, we obtain that:

$$Q_{in} = 200\frac{kg}{s} * (1185.1 - 191.81)\frac{kJ}{kg} = 198.66 \text{ MW}$$

Now we can calculate the thermal efficiency, by using equation 10 and the values stated above:

$$n_{th} = \frac{27.13 \text{ MW}}{198.66 \text{ MW}} = 0.1366$$

2.5 What would be the answer from b. and c. if the separator pressure were 8 bar?

Now we will do the same as in section 2.3 and 2.4 but with the value $P_2 = P_3 = 800$ kPa. We will have to update the values for stages 2-4 again by using the same methods as stated in section 2.1. From table A-5 we get that [1]:

$$s_3 = s_{g@800kPa} = 6.6616\frac{kJ}{kgK}$$

$$h_3 = h_{g@800kPa} = 2768.3\frac{kJ}{kg}$$

Using that, we can find the new value for $x_{s,4}$ by using equation for 5:

$$x_{s,4} = \frac{6.6616\frac{kJ}{kgK} - 0.6492\frac{kJ}{kgK}}{7.4996\frac{kJ}{kgK}} = 0.8017$$

And using that we can find the updated value for $h_{s,4}$ by using equation 9:

$$h_{s,4} = 191.81\frac{kJ}{kg} + 0.8017 * 2392.1\frac{kJ}{kg} = 2109.56\frac{kJ}{kg}$$

Then we obtain the updated value for W_a by using equation 8:

$$W_a = (2768.3 - 2109.56)\frac{kJ}{kg} * 0.9 = 592.866\frac{kJ}{kg}$$

Now we need to find the updated value for $\dot{m}_3$. First we find in table A-5, $h_{f,2} = h_{f@800kPa} = 720.87\frac{kJ}{kg}$ and $h_{fg,2} = h_{fg@800kPa} = 2047.5\frac{kJ}{kg}$ [1]. Then we can find the updated steam quality in state 2 by using equation 1:

$$x_2 = \frac{1185.1\frac{kJ}{kg} - 720.87\frac{kJ}{kg}}{2047.5\frac{kJ}{kg}} = 0.2267$$

Then we get that the update mass flow in state 3 by using equation 3:

$$\dot{m}_3 = 0.2267 * 200\frac{kg}{s} = 45.34\frac{kg}{s}$$

Finally, we find the updated value for the power generated by multiplying the mass flow with W_a:

$$\text{Power generated} = 592.866\frac{kJ}{kg} * 45.34\frac{kg}{s} = 26.88 \text{ MW}$$

Then we can find the thermal efficiency. We know that $W_{net,out} = 26.88$ MW and that the heat that goes in is the same as before, $Q_{in} = 198.66$ MW. Then we can calculate the thermal efficiency by using equation 10:

$$n_{th} = \frac{26.88 \text{ MW}}{198.66 \text{ MW}} = 0.1353$$

2.6 What is the exergy destruction in the process?

To find the exergy destruction, we will use the equation:

$$X_{dest} = T_0 * S_{gen} \tag{12}$$

We will assume that $T_0 = 10°\text{C} = 283$ K, which is around the average temperature in Iceland [2].
To find S_{gen} we will use:

$$S_{gen} = S_{in} - S_{out} = S_1 - S_5 \tag{13}$$

We know from table 1 that $S_1 = 2.9762\frac{kJ}{kgK}$ and $S_2 = 0.6492\frac{kJ}{kgK}$. Using equation 13 we then obtain:

$$S_{gen} = 2.9762\frac{kJ}{kgK} - 0.6492\frac{kJ}{kgK} = 2.327\frac{kJ}{kgK}$$

Now we can use equation 12 and the values stated above to find the exergy destruction:

$$X_{dest} = 283 \text{ K} * 2.327\frac{kJ}{kgK} = 651.6\frac{kJ}{kg}$$

2.7 Draw the process on a T-s diagram.

Now, using the T and s values for each state from table 1, we can draw a T-s diagram.

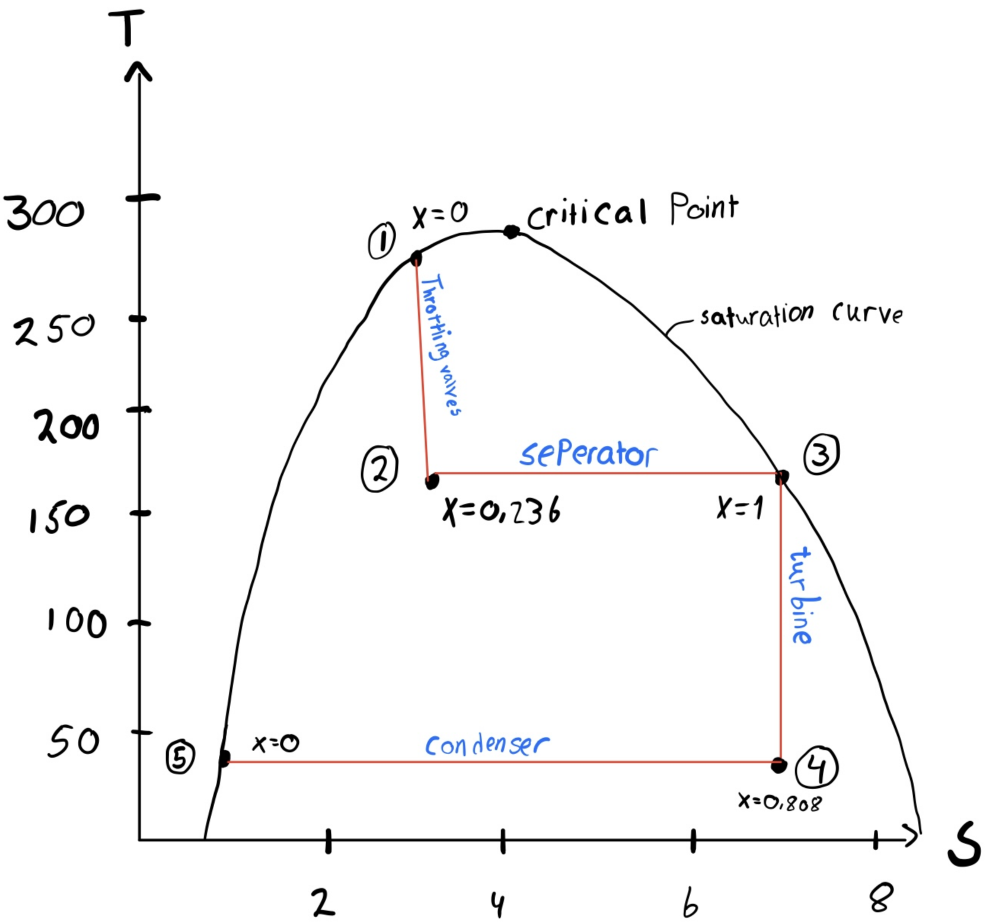

Figure 2: T-s diagram for the powerplant

3 Question 2

Now we will assume that the water that previously went to waste from the separator will be used to heat fresh water from 5°C to 80°C by using a heat exchanger.

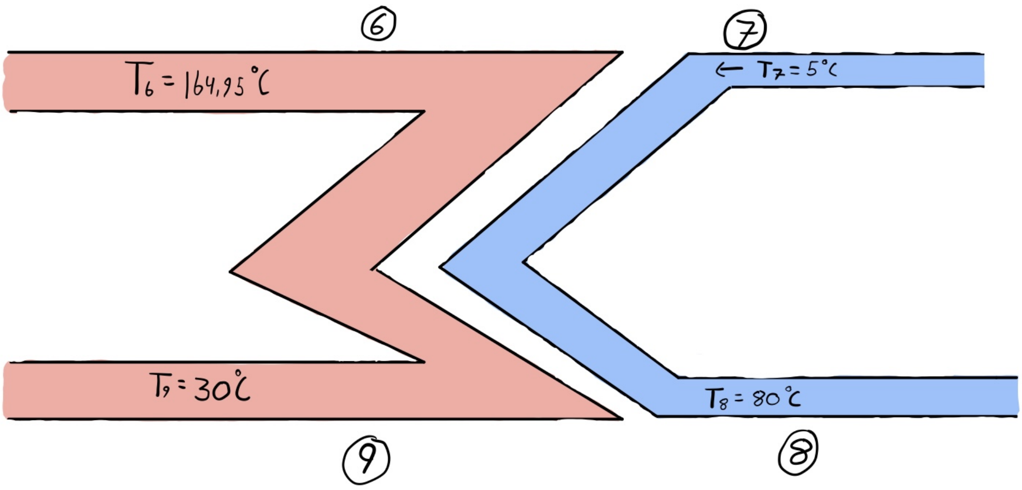

Figure 3: Explanation of the heat exchanger in the system

In figure 3 we can see the new states in the system and the temperature of each state.

3.1 How much freshwater water can be heated to 80°C by the separated liquid?

Now we will be using a heat exchanger to heat the fresh water. We know from equation 4 that the water that comes from the separator is $\dot{m}_{water,out} = m_6 = 152.8\frac{kg}{s}$. Since this is a heat exchanger, we also know that $m_6 = m_9$ and $m_7 = m_8$. To find how much freshwater can be heated, we need to use the energy balance equation for heat exchangers:

$$m_6h_6 + m_7h_7 = m_9h_9 + m_8h_8 \tag{14}$$

Which gives us:

$$m_7 = \frac{m_9h_9 - m_6h_6}{h_7 - h_8} \tag{15}$$

From table A-4 we obain the values [1]:

Table 2: Enthalpy values for the new states

State	**In table**	**h [kJ/kg]**
6	$h_{f@164.95°C}$	697.24
7	$h_{f@5°C}$	21.02
8	$h_{f@80°C}$	335.02
9	$h_{f@30°C}$	125.74

Using equation 15 and the values in table 2 we get how much freshwater can be heated to 80°C by the separated liquid:

$$m_7 = \frac{152.8\frac{kg}{s} * 125.74\frac{kJ}{kg} - 152.8\frac{kg}{s} * 697.24\frac{kJ}{kg}}{21.02\frac{kJ}{kg} - 335.02\frac{kJ}{kg}} = 278.11\frac{kg}{s}$$

3.2 Thermal energy delivered to the residential system by the district heating water

Now to calculate the thermal energy delivered, we will use the equation:

$$\dot{Q} = \dot{m} * c * \Delta T \tag{16}$$

We know that the mass is $\dot{m} = \dot{m}_7 = 278.11\frac{kg}{s}$. For the specific heat, we will be using $c = c_{water} = 4.2\frac{kJ}{kgK}$. The delta temperature is 80°C - 5 °C. Using that, we get:

$$278.11\frac{kg}{s} * 4.2\frac{kJ}{kgK} * (80 + 273 - 5 + 273)K = 87.61 \text{ MW}$$

3.3 New efficiency of the geothermal plant

Now we will find the total efficiency of the geothermal plant. For that, we can use equation 10, but we need to take in account the Q_{out} value. We then get the equation:

$$n_{th} = \frac{W_{net,out} + Q_{out}}{Q_{in}} \tag{17}$$

We know that $W_{net,out} = 27.13$ MW, $Q_{in} = 198.66$ MW and $Q_{out} = 87.61$ MW. Using that and equation 17 we obtain the total efficiency of the power plant:

$$n_{total} = \frac{27.13 \text{ MW} + 87.61 \text{ MW}}{198.67 \text{ MW}} = 0.58$$

References

[1] M. K. Yunus A. Çengel, Michael A. Boles, *Thermodynamics – An Engineering Approach.* McGraw-Hill Education, 2018.

[2] arctic adventures, "Iceland weather," 2022. [Online]. Available: https://adventures.is/information/the-weather-in-iceland/