SIO 112 Final Project
Seismology and MCMC

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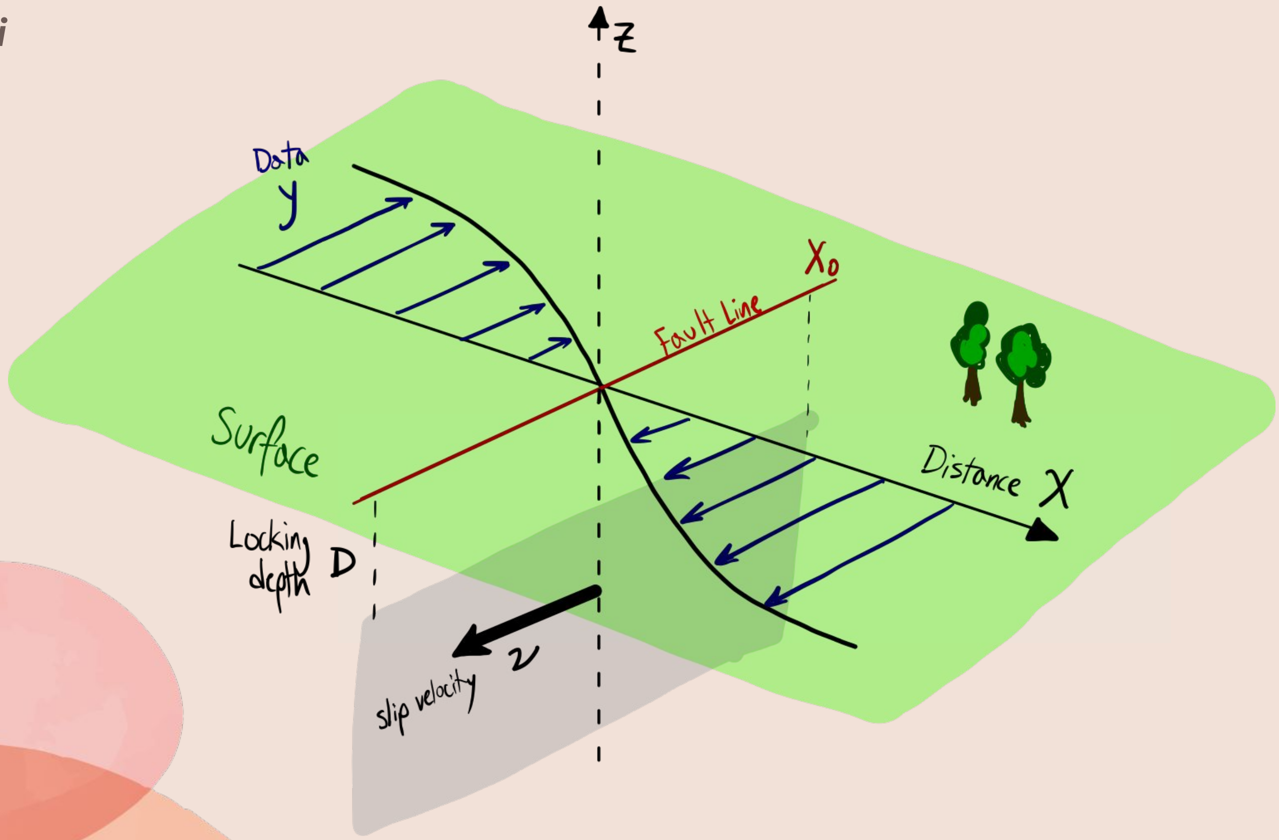
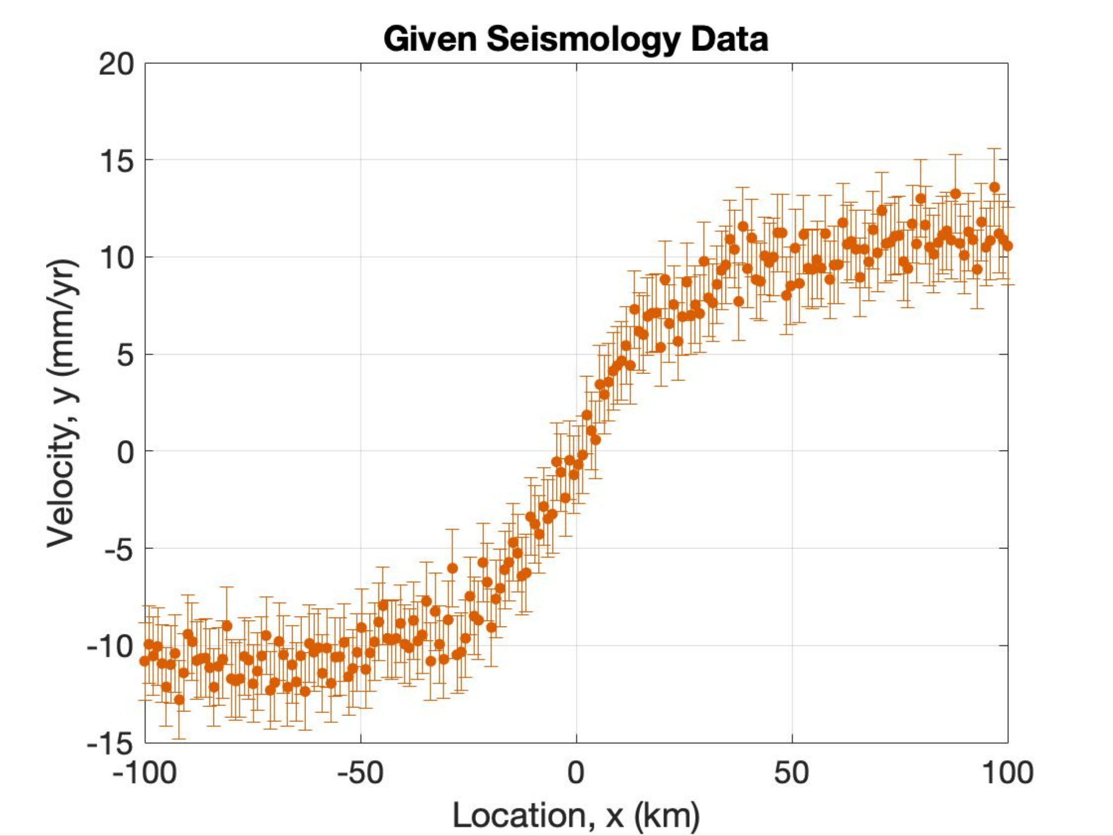
Presentation Outline

Seismology and Given Data	by Kayli
MCMC and Process	by Niya
RMSE and Fit to Data	by Andrina
Posterior Mean and Variance	by Tanisha
Triangle Plot and Troubleshooting	by Ashton

Seismology with MCMC: Strike-Slip Fault

Figure by Ashton Domi





Given Data

$n_y = 200$

$y \sim n_y \times 1$ vector

$s \sim n_y \times 1$ vector

$x \sim 1 \times n_y$ vector

$s_i = 1$ for $i = 1,\ldots,n_y$

Error bars = $2*s_i$

$$m(v, D, x_0) = \frac{v}{\pi} arctan\left(\frac{x - x_0}{D}\right)$$

Seismic Model

SeimoEx.mat (MAT-file)

Name	Value
s	200x1 double
x	1x200 double
y	200x1 double

SeismoEx.mat

Prior Information about Parameters

$$v \in [0, 80]$$

Initial Velocity, mm/yr

$$D \in [0, 80]$$

Locking Depth, km

$$x_0 \in [-50, 50]$$

Location of Fault, km

Translation to MCMC

Posterior pdf

$$\exp\left(-\frac{1}{2}\sum\left(\frac{y-y_M}{s_y}\right)^2\right)$$

posterior pdf ∝ prior pdf * likelihood pdf

prior pdf:

- they are uniformly distributed, thus is a constant

$$f(x)=\begin{cases}\dfrac{1}{b-a} & a\le x\le b\\ 0 & elsewhere\end{cases}$$

likelihood pdf

- yM: aka, m(v, D, x_0)
- sy: standard deviation of y, = 1

$$X' = X_k + \sigma * \eta \qquad \eta \sim \mathcal{N}(0, I)$$

Proposal

$$\alpha = \min(1, \text{posterior pdf(proposed x)}/\text{posterior pdf(previous x)})$$

$X' = X_{k+1}$ with probability α

Acceptance rate

k: 1	2	3	4
initial	k = 2		

v = 10
D = 10
x0 = 5

X' = X1 + step size * randn(3,1)

X': proposed v, D, x0

If α = min(1, posterior pdf(proposed x)/posterior pdf(previous x)))
X2 = X' with probability α

How sig (step size) affects the result

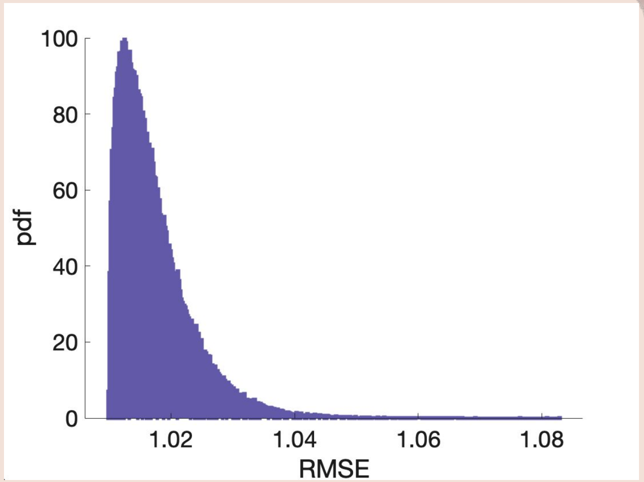
Step Size	Acceptance Rate
(1,1,1)	0.090
(0.5,0.5,0.5)	0.271
(0.5, 1, 1)	0.240
(1, 0.5, 1)	0.108
(1, 1, 0.5)	0.131
(0.8, 0.3, 0.3)	0.234
(0,35, 0.8, 0.8)	0.227

RMSE

Definition of RMSE:

RMSE is Root Mean Square Error. It shows how far predictions fall from measured true values using Euclidean distance.

$$\mathrm{RMSE}(y, \hat{y}) = \sqrt{\frac{\sum_{i=0}^{N-1} \left(\frac{y_i - \hat{y}_i}{\sigma}\right)^2}{N}}$$



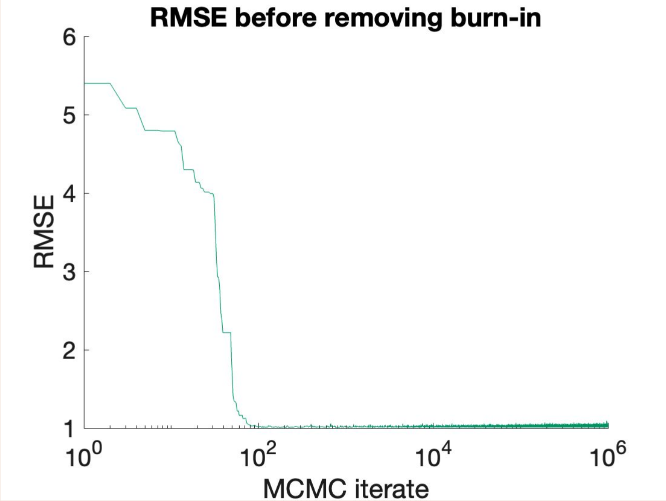
RMSE is a good way to evaluate the model.

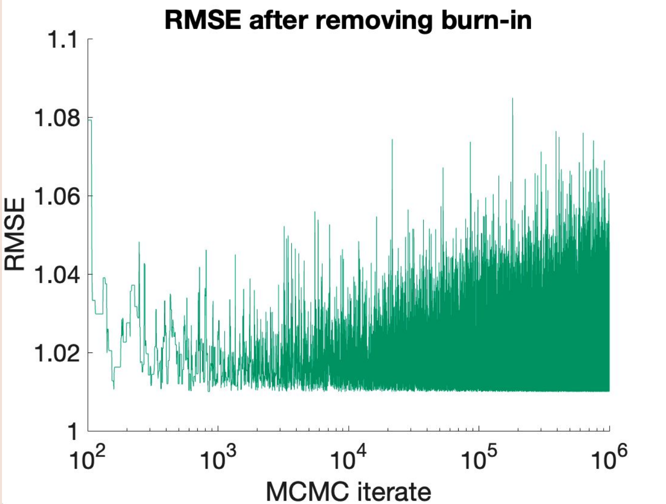
Our expected RMSE is 1. Thus, in this case, if RMSE is approaching 1, we are able to know that we get a good model

Analysis on RMSE before burn-in

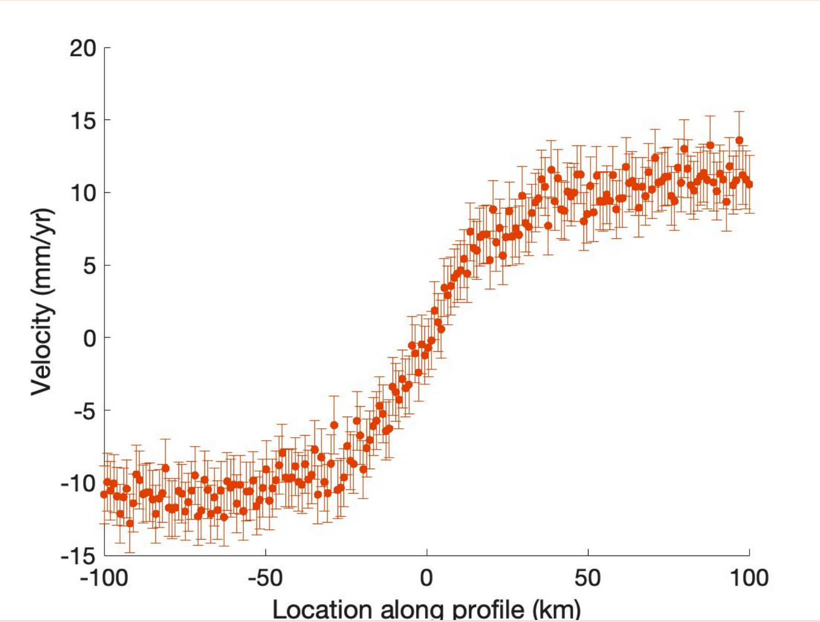
RMSE decreases until about 100 iterations where it starts to converge to 1.

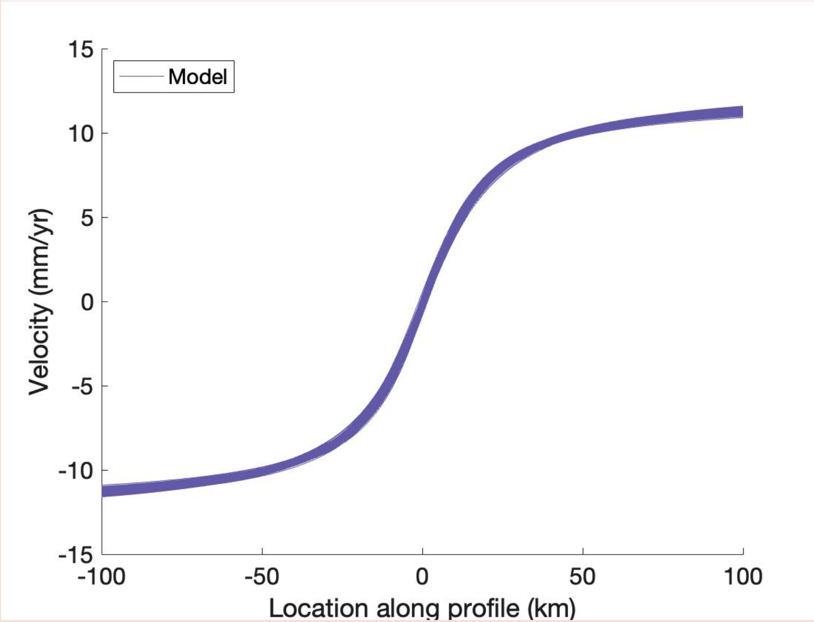
So we can do a burn-in at about 100 iterations then to get a better look at the RMSE.



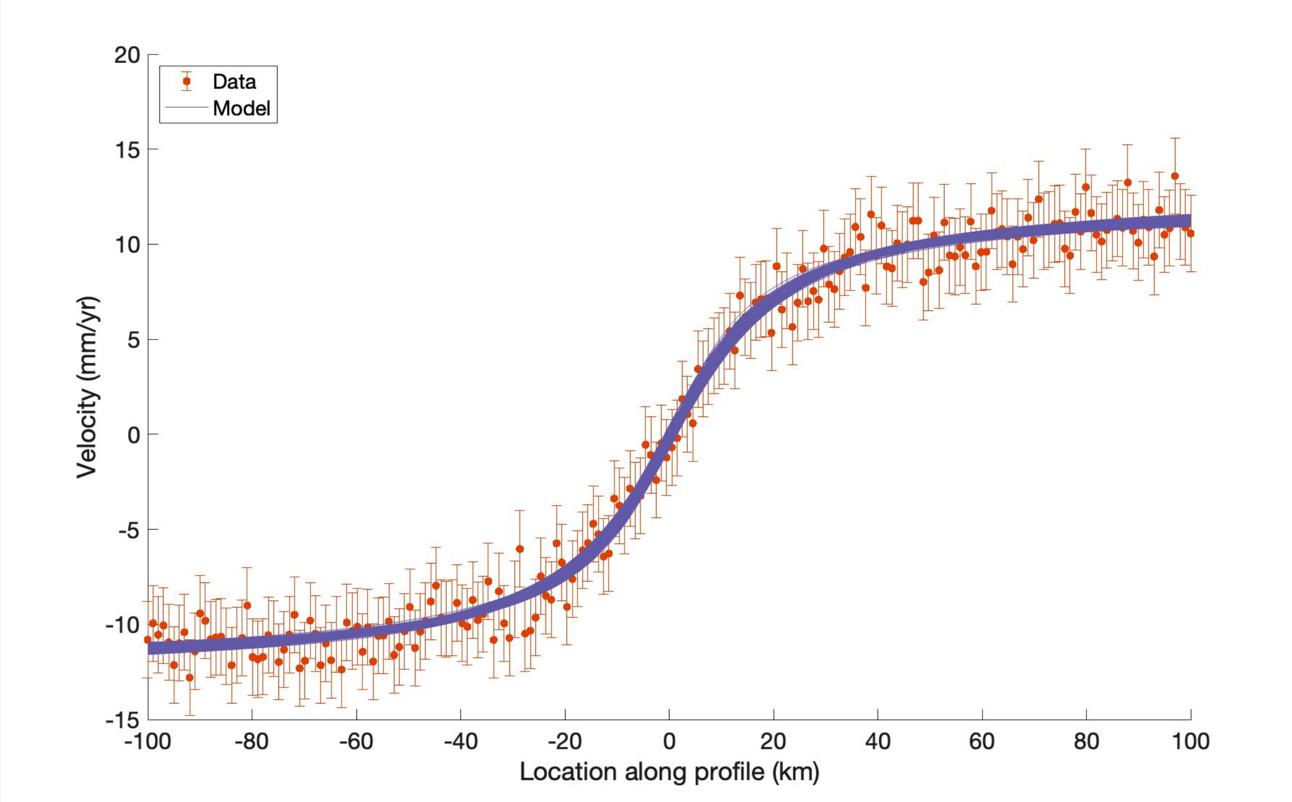


Fit to the Data





Fit to the Data



Posterior Mean and Standard Deviation

Posterior mean of locking depth (km): 15.670
Posterior std. of locking depth (km): 0.825

Posterior mean of velocity (mm/yr): 24.954
Posterior std. of velocity (mm/yr): 0.353

Posterior mean of location (km): 0.453
Posterior std. of location (km): 0.400

	Posterior Mean			Posterior Standard Deviation			σ	Acceptance Ratio
	v	D	x_0	v	D	x_0		
T. D.	24.933	15.6401	0.459	0.349	0.817	0.397	[2.897, 0.305, -0.922]	0.2353
A. D.	24.958	15.674	0.450	0.356	0.833	0.401	[.31; .86; .79]	0.2341
K. M.	24.958	15.675	0.452	0.353	0.826	0.400	[0.516; 0.516; 0.661]	0.2336
N. S.	24.962	15.682	0.454	0.351	0.822	0.399	[0.8; 0.3; 0.3]	0.2344
A. Z.	24.960	15.678	0.449	0.354	0.828	0.400	[0.576; 0.6; 0.575]	0.2233
Average	**24.954**	**15.670**	**0.453**	**0.353**	**0.825**	**0.400**		

Triangle Plot

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function TrianglePlot(samples,numbins)
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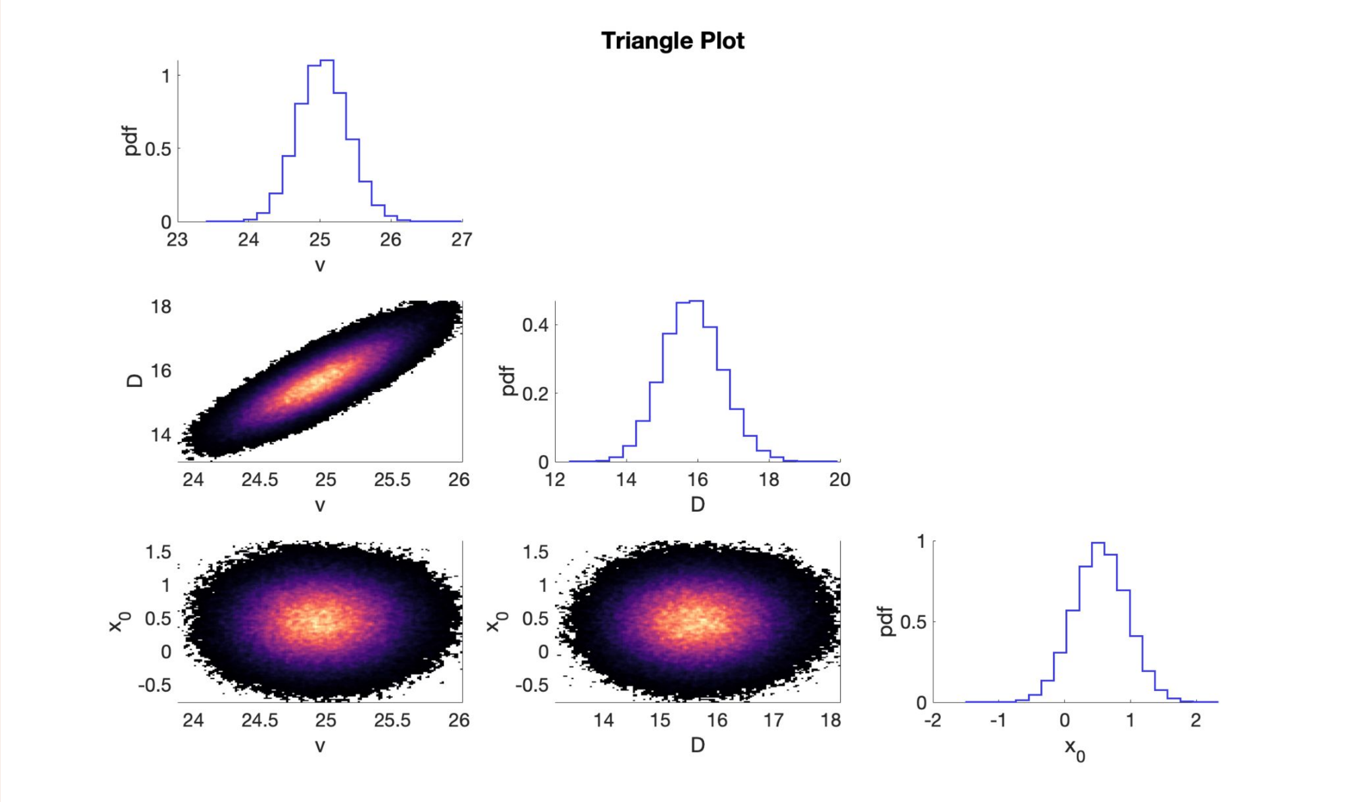
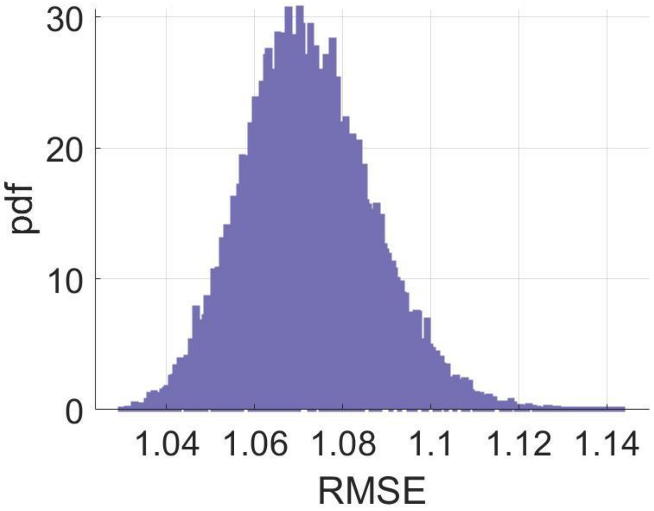
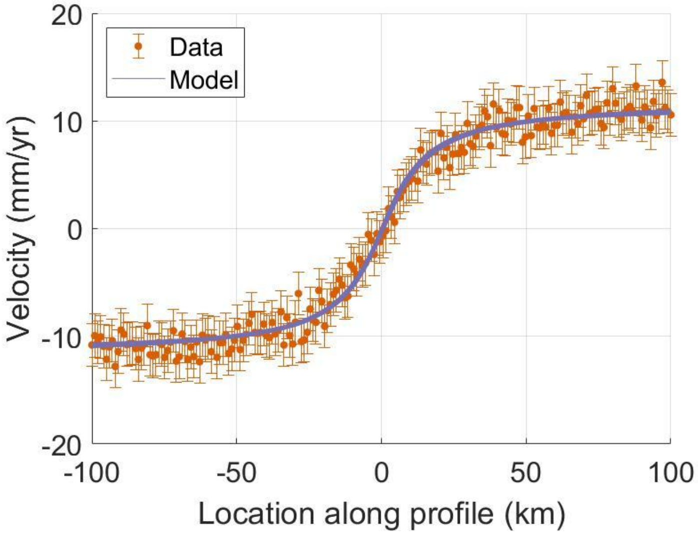


When things go slightly wrong

Adding prior distribution to posterior pdf gets you here

$$\exp\left(-\frac{1}{2}\sum\left(\frac{y-yM}{s_y}\right)^2-\frac{1}{2}(X)^2\right)$$





Thank you for listening!