ex2

September 30, 2023

[ ]:
```python
import numpy as np

from qiskit import QuantumCircuit
from qiskit.quantum_info import Statevector, Operator
```

0.1 1. Controlled rotational gates

Find the circuit decompisition for $CR_x(\theta)$. You can use the code below to check whether your decomposition is correct (in which case there should be no "not equal" prints).

[ ]:
```python
a = 0.62
```

[ ]:
```python
circ1 = QuantumCircuit(2) # your quantum circuit

circ1.ry(-a / 2, 1)
circ1.cx(0,1)
circ1.rz(np.pi / 2, 1)
circ1.cx(0,1)
circ1.ry(a / 2, 1)

print(circ1.draw())
print(Operator(circ1).data)

state = Statevector(circ1)
state.draw('latex')
```

```
q_0:

q_1:  Ry(-0.31)   X   Rz( /2)   X   Ry(0.31)

[[0.70710678-0.67340153j 0.        +0.j         0.        -0.21570903j
  0.        +0.j        ]
 [0.        +0.j         0.70710678+0.67340153j 0.        +0.j
  0.        +0.21570903j]
 [0.        -0.21570903j 0.        +0.j         0.70710678+0.67340153j
  0.        +0.j        ]
```

```
 [0.        +0.j          0.        +0.21570903j 0.        +0.j
  0.70710678-0.67340153j]]
```

[ ]:

$$(0.7071067812 - 0.6734015252i)|00\rangle - 0.2157090305i|10\rangle$$

[ ]:
```
circ2 = QuantumCircuit(2)
circ2.crx(a, 0, 1)
print(circ2.draw())

print(Operator(circ2).data)
```

```
q_0:

q_1:   Rx(0.62)

[[1.        +0.j          0.        +0.j          0.        +0.j
  0.        +0.j        ]
 [0.        +0.j          0.95233357+0.j          0.        +0.j
  0.        -0.30505864j]
 [0.        +0.j          0.        +0.j          1.        +0.j
  0.        +0.j        ]
 [0.        +0.j          0.        -0.30505864j 0.        +0.j
  0.95233357+0.j        ]]
```

[ ]:
```
Operator(circ2).data == Operator(circ1).data
```

[ ]:
```
array([[False,  True, False,  True],
       [ True, False,  True, False],
       [False,  True, False,  True],
       [ True, False,  True, False]])
```

[ ]:
```
for i in range(4):
    for j in range(4):
        if Operator(circ1).data[i][j] - Operator(circ2).data[i][j] > 1e-15:
            print("not equal")
```

0.2 2. Measuring composite operators

Consider the following quantum circuits.

[ ]:
```
from qiskit import QuantumRegister, ClassicalRegister, QuantumCircuit
from numpy import pi

qreg_q = QuantumRegister(2, 'q')
circuit = QuantumCircuit(qreg_q)
```

```
circuit.h(qreg_q[0])
circuit.ry(pi / 6, qreg_q[1])
circuit.cx(qreg_q[0], qreg_q[1])

circuit.draw()
```

[ ]:

```
q_0:    H

q_1:   Ry( /6)   X

c: 4/
```

This circuit acts on two qubits and produce a quantum state which is a reasonable ground state approxiamtion of the two-site (ferromagnetic) quantum transverse field Ising Hamiltonian at the critical point, defined as

$$H = -ZZ - XI - IX$$

1. Find the matrix representation of the Hamiltonian H.

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$ZZ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad XI = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad IX = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

thus:

$$H = -ZZ - XI - IX = \begin{bmatrix} -1 & -1 & -1 & 0 \\ -1 & 1 & 0 & -1 \\ -1 & 0 & 1 & -1 \\ 0 & -1 & -1 & 1 \end{bmatrix}$$

2. Find the statevector represantion $|\psi\rangle$ of the wavefunction generated by the above circuit (acting on the $|00\rangle$ state)

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \times H = \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix} \quad = -|00\rangle - |01\rangle - |10\rangle$$

3. Compute the expectation value of the energy, exactly: $\langle\psi|H|\psi\rangle$

$$\hat{H} = \langle\psi|H|\psi\rangle = \begin{bmatrix}\psi_0^* & \psi_1^* & \psi_2^* & \psi_3^*\end{bmatrix} \times \begin{bmatrix}-1 & -1 & -1 & 0 \\ -1 & 1 & 0 & -1 \\ -1 & 0 & 1 & -1 \\ 0 & -1 & -1 & 1\end{bmatrix} \times \begin{bmatrix}\psi_0 \\ \psi_1 \\ \psi_2 \\ \psi_3\end{bmatrix}$$

$$\begin{bmatrix}\psi_0^* & \psi_1^* & \psi_2^* & \psi_3^*\end{bmatrix} H \begin{bmatrix}\psi_0^* \\ \psi_1^* \\ \psi_2^* \\ \psi_3^*\end{bmatrix}$$

4. Find a procedure to compute the expectation value of the energy using the quantum measurements.

We can measure each of the operators ZZ, XI and IX and then compute the expectation value of the energy as:

$$\langle\psi|H|\psi\rangle = -\langle ZZ\rangle - \langle XI\rangle - \langle IX\rangle$$

In this case ZZ can be measured by measuring the parity of the redout string, as described in the script.

XI cannot be measured directly, we need to find an unitary transformation U such that $U^\dagger XIU = XI$ and then measure the parity of the redout string after applying U to the state.

5. bonus: what's the minimal number of circuits needed in this case