E4

October 17, 2022

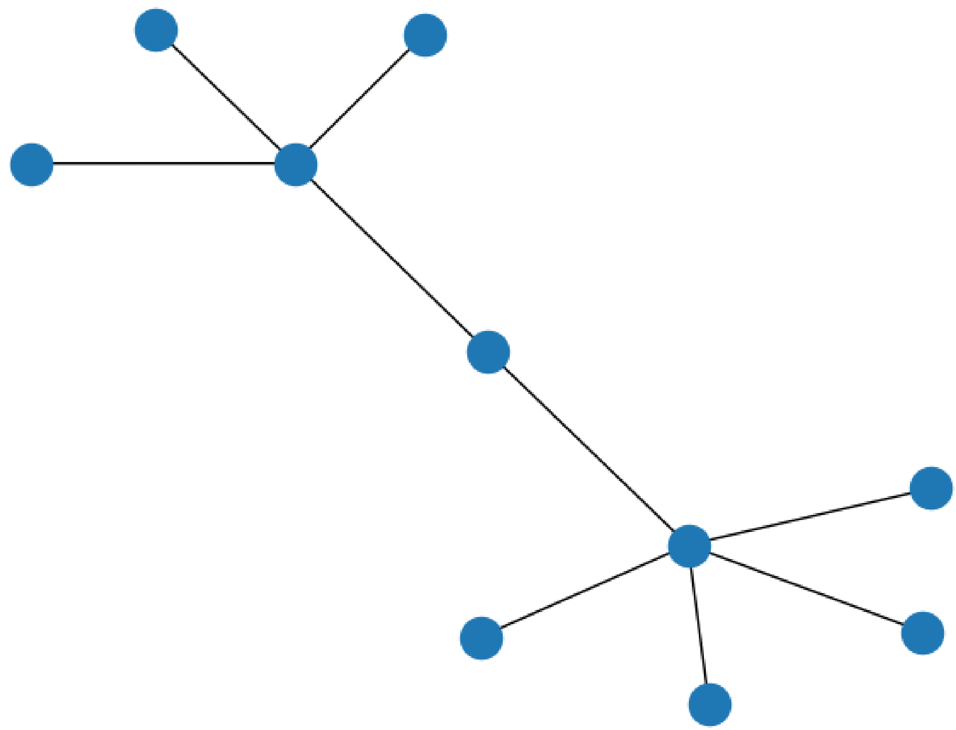
[97]:
```
#import the neccessary packages
import networkx as nx
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
```

1 4.1 & 4.2

We construct a graph such thath there exists a central node (0) which is connected to only two nodes (1,2). The nodes 1 and 2 have much larger degree, which means that the degree centrality is low, however the betweeness centrality is high as node(0) is present in most paths in the network.

[98]:
```
n=10
G=nx.empty_graph(n)
G.add_edge(0, 1)
G.add_edge(0, 2)
G.add_edge(1, 3)
G.add_edge(1, 4)
G.add_edge(1, 5)
G.add_edge(2, 6)
G.add_edge(2, 7)
G.add_edge(2, 8)
G.add_edge(2, 9)

nx.draw(G)

print(nx.degree_centrality(G)[0], nx.betweenness_centrality(G)[0])
```

```
0.2222222222222222 0.5555555555555556
```

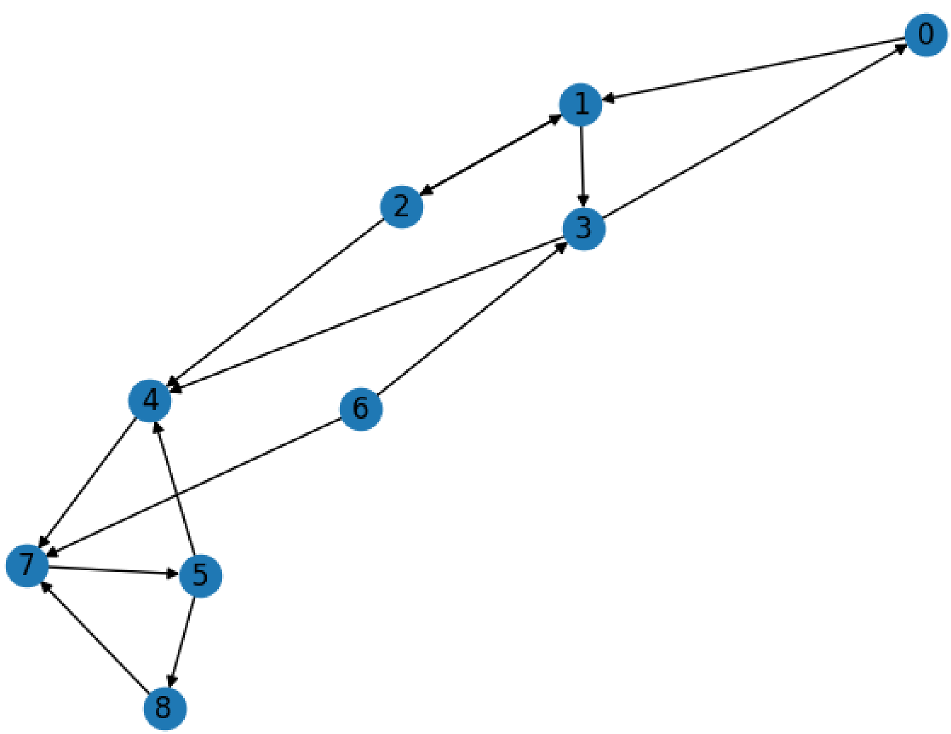


2 4.3

[99]:
```
n=9
F=nx.empty_graph(n, create_using=nx.DiGraph())
F.add_edge(0, 1)

F.add_edge(1, 2)
F.add_edge(1, 3)

F.add_edge(2, 4)
F.add_edge(2, 1)

F.add_edge(3, 0)
F.add_edge(3, 4)

F.add_edge(4, 7)

F.add_edge(5, 4)
F.add_edge(5, 8)
```

```python
F.add_edge(6, 3)
F.add_edge(6, 7)

F.add_edge(7, 5)

F.add_edge(8, 7)

nx.draw(F, with_labels=True)

A=nx.to_numpy_matrix(F)

A
```

[99]:
```
matrix([[0., 1., 0., 0., 0., 0., 0., 0., 0.],
        [0., 0., 1., 1., 0., 0., 0., 0., 0.],
        [0., 1., 0., 0., 1., 0., 0., 0., 0.],
        [1., 0., 0., 0., 1., 0., 0., 0., 0.],
        [0., 0., 0., 0., 0., 0., 0., 1., 0.],
        [0., 0., 0., 0., 1., 0., 0., 0., 1.],
        [0., 0., 0., 1., 0., 0., 0., 1., 0.],
        [0., 0., 0., 0., 0., 1., 0., 0., 0.],
        [0., 0., 0., 0., 0., 0., 0., 1., 0.]])
```



3 4.4

[100]:
```
diag = np.array(list(map(lambda x: x[1], F.out_degree())))

M = np.zeros((n,n))

np.fill_diagonal(M, diag)
M_inv = np.linalg.inv(M)
T = np.matmul(M_inv, A)

print(T)
```

```
[[0.  1.  0.  0.  0.  0.  0.  0.  0. ]
 [0.  0.  0.5 0.5 0.  0.  0.  0.  0. ]
 [0.  0.5 0.  0.  0.5 0.  0.  0.  0. ]
 [0.5 0.  0.  0.  0.5 0.  0.  0.  0. ]
 [0.  0.  0.  0.  0.  0.  0.  1.  0. ]
 [0.  0.  0.  0.  0.5 0.  0.  0.  0.5]
 [0.  0.  0.  0.5 0.  0.  0.  0.5 0. ]
 [0.  0.  0.  0.  0.  1.  0.  0.  0. ]
 [0.  0.  0.  0.  0.  0.  0.  1.  0. ]]
```

[101]:
```
S = np.full((n,n), 1/n)

S[diag > 0, :] = (A / diag[None,:])[diag > 0, :] / diag

S
```

[101]:
```
array([[0.  , 0.25, 0.  , 0.  , 0.  , 0.  , 0.  , 0.  , 0.  ],
       [0.  , 0.  , 0.25, 0.25, 0.  , 0.  , 0.  , 0.  , 0.  ],
       [0.  , 0.25, 0.  , 0.  , 1.  , 0.  , 0.  , 0.  , 0.  ],
       [1.  , 0.  , 0.  , 0.  , 1.  , 0.  , 0.  , 0.  , 0.  ],
       [0.  , 0.  , 0.  , 0.  , 0.  , 0.  , 0.  , 1.  , 0.  ],
       [0.  , 0.  , 0.  , 0.  , 1.  , 0.  , 0.  , 0.  , 1.  ],
       [0.  , 0.  , 0.  , 0.25, 0.  , 0.  , 0.  , 1.  , 0.  ],
       [0.  , 0.  , 0.  , 0.  , 0.  , 0.25, 0.  , 0.  , 0.  ],
       [0.  , 0.  , 0.  , 0.  , 0.  , 0.  , 0.  , 1.  , 0.  ]])
```

4 4.5

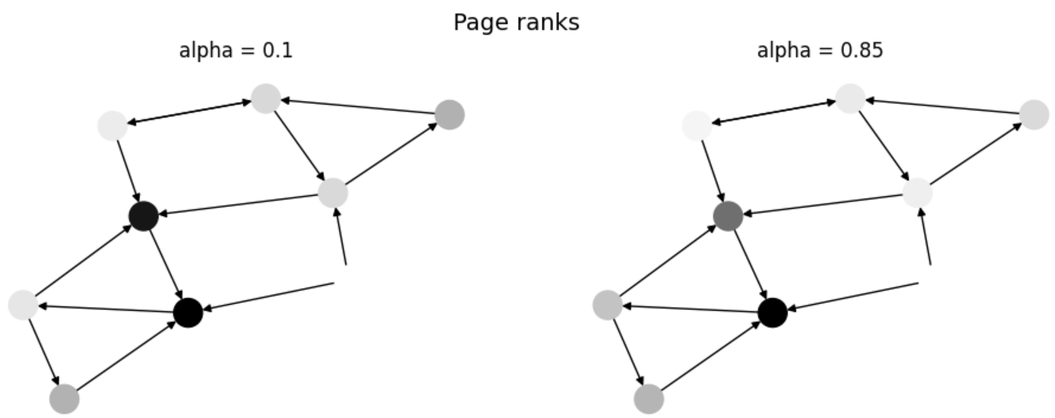
Not sure what to do here...

5 4.6

[102]:
```python
def page_rank(p, alpha, S):
    return  alpha * np.matmul(p, S) + (1-alpha) / n * np.ones(n).T

def compute_ps(alphas, iterations, S, p_inits):
    ps = []
    for alpha in alphas:
        for p_init in p_inits:
            p = p_init.copy()
            for i in range(iterations):
                p = page_rank(p.copy(), alpha, S)

            ps.append(p)

    return ps

ps = compute_ps([0.1, 0.85], 30, S, [np.full(n, 1/n)])
```

[103]:
```python
fig, (ax1, ax2) = plt.subplots(nrows=1, ncols=2, figsize=(10, 4))
fig.suptitle("Page ranks", fontsize=14)
fig.tight_layout()

p1 = nx.draw_kamada_kawai(F, ax=ax1, node_color=ps[0], cmap=plt.cm.binary)

nx.draw_kamada_kawai(F, ax=ax2, node_color=ps[1], cmap=plt.cm.binary)

ax1.set_title("alpha = 0.1")
ax2.set_title("alpha = 0.85")

print(ps[0])
print(ps[1])
```

```
[0.11051332 0.10532866 0.10263322 0.10513322 0.13111201 0.10335362
 0.1        0.13414474 0.11033536]
[0.03924403 0.02989776 0.02301994 0.02656161 0.10401655 0.05318334
 0.01666667 0.17183828 0.06187224]
```

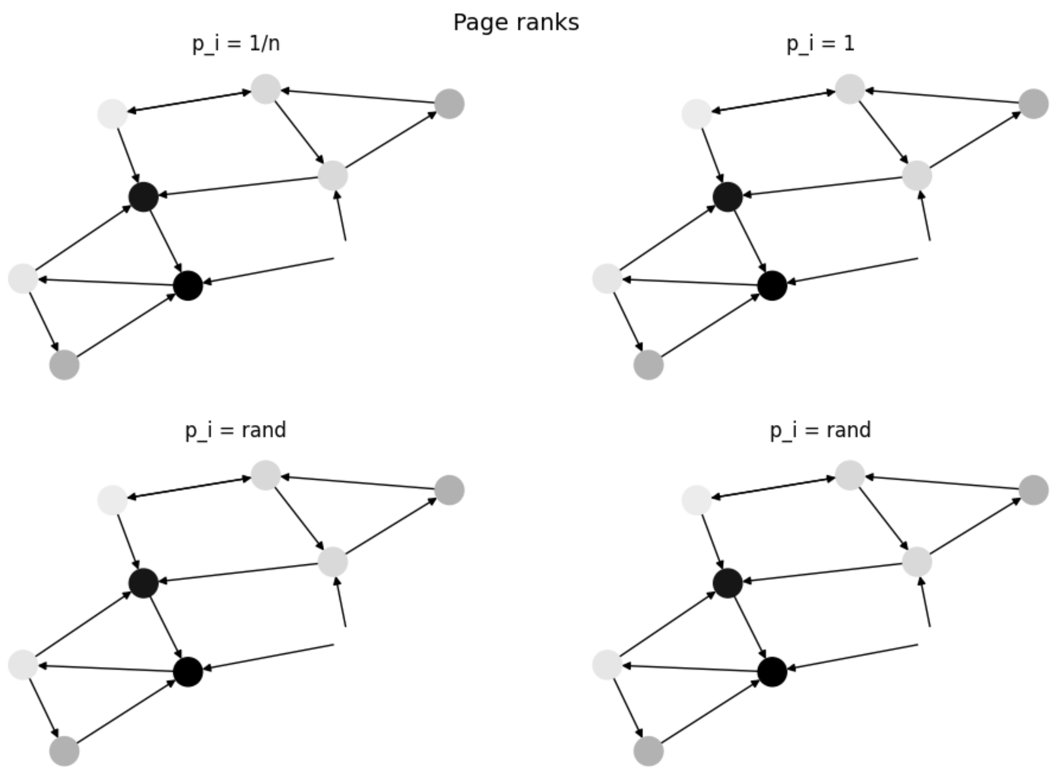


6 4.7

[104]:
```
ps = compute_ps([0.1], 1000, S, [np.full(n, 1/n), np.full(n, 1), np.random.
↪rand(n), np.random.rand(n)])
```

[105]:
```
fig, axs = plt.subplots(nrows=2, ncols=2, figsize=(10, 7))
fig.suptitle("Page ranks", fontsize=14)
fig.tight_layout()

nx.draw_kamada_kawai(F, ax=axs[0,0], node_color=ps[0], cmap=plt.cm.binary)
nx.draw_kamada_kawai(F, ax=axs[0,1], node_color=ps[1], cmap=plt.cm.binary)
nx.draw_kamada_kawai(F, ax=axs[1,0], node_color=ps[2], cmap=plt.cm.binary)
nx.draw_kamada_kawai(F, ax=axs[1,1], node_color=ps[3], cmap=plt.cm.binary)



axs[0,0].set_title("p_i = 1/n")
axs[0,1].set_title("p_i = 1")
axs[1,0].set_title("p_i = rand")
axs[1,1].set_title("p_i = rand")

print(ps[0], '\n', ps[1], '\n', ps[2], '\n', ps[3])
```

```
[0.11051332 0.10532866 0.10263322 0.10513322 0.13111201 0.10335362
 0.1        0.13414474 0.11033536]
 [0.11051332 0.10532866 0.10263322 0.10513322 0.13111201 0.10335362
 0.1        0.13414474 0.11033536]
 [0.11051332 0.10532866 0.10263322 0.10513322 0.13111201 0.10335362
 0.1        0.13414474 0.11033536]
 [0.11051332 0.10532866 0.10263322 0.10513322 0.13111201 0.10335362
 0.1        0.13414474 0.11033536]
```



The final results do not differ at all. It all converges to the leading eigenvector.

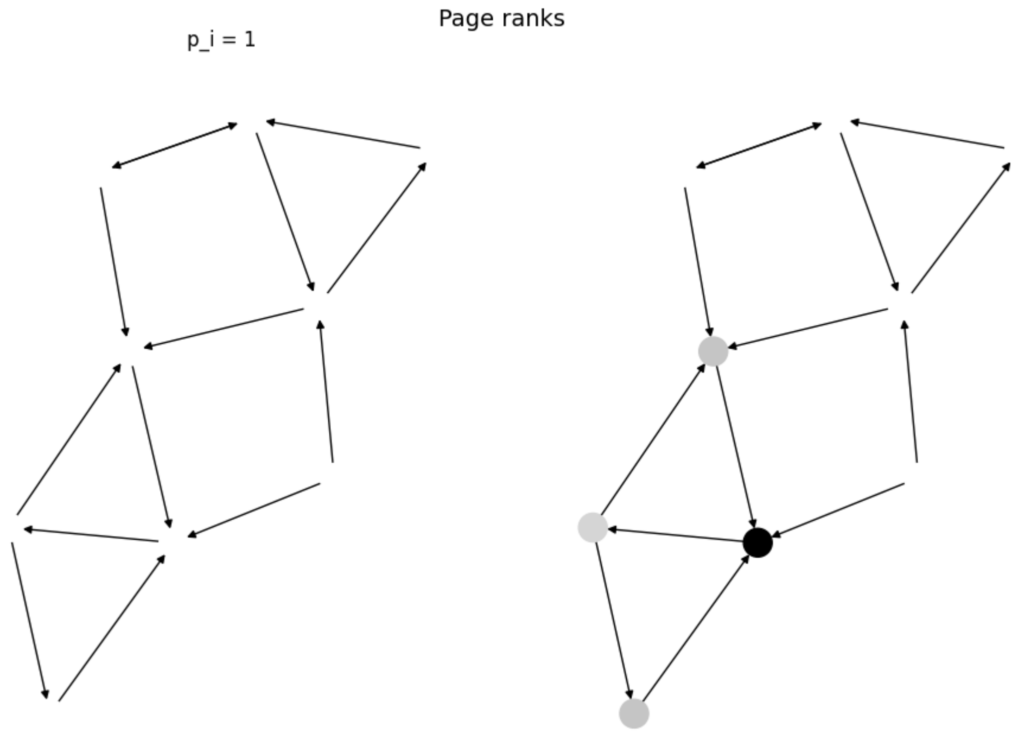
7 4.8

[106]:
```
ps = compute_ps([0,1], 1000, S, [np.random.rand(n)])
```

[107]:
```
fig, axs = plt.subplots(nrows=1, ncols=2, figsize=(10, 7))
fig.suptitle("Page ranks", fontsize=14)
fig.tight_layout()

nx.draw_kamada_kawai(F, ax=axs[0], node_color=ps[0], cmap=plt.cm.binary)
nx.draw_kamada_kawai(F, ax=axs[1], node_color=ps[1], cmap=plt.cm.binary)


axs[0].set_title("p_i = 1/n")
axs[0].set_title("p_i = 1")

print(ps[0], '\n', ps[1])
```

```
[0.11111111 0.11111111 0.11111111 0.11111111 0.11111111 0.11111111
 0.11111111 0.11111111 0.11111111]
 [0.00000000e+000 0.00000000e+000 0.00000000e+000 0.00000000e+000
```

```
3.06621496e-101 2.22603020e-101 0.00000000e+000 1.33518484e-100
3.06621496e-101]
```



For edge case alpha = 0 the distribution converges to (1/n) for every entry. Which makes sense as only the term (1-alpha) / n * np.ones(n).T of the equation has an effect. For the case alha = 1 the distribution is similar but just a lot more extreme.

[ ]:

[ ]:

[ ]: