NIPoPoWs under Velvet Fork

1   Introduction

Since the release of Bitcoin about a decade ago, the interest in cryptocurrencies has increased tremendously, while a number of other "altcoins" have been constructed in the meantime. Given that cryptocurrencies are starting to be considered a generally accepted means of payment and are used for everyday transactions, the issue of efficiently handling cryptocurrencies by light clients, such as smartphones, has become of great importance.

In this work, we consider the problem of optimizing light clients, or "SPV clients" as described in the original Bitcoin paper[4]. As blockchains are ever growing, the main setback for efficient light client applications is the processing of data amount linear to the size of the blockchain, e.g. for syncrhronization purposes.

Our work is based on the construction of Non-Interactive Proofs of Proof of Work[1] that achieves SPV proofs of polylogarithmic portion of the blockchain size. The NIPoPoWs construction suggests a protocol update, that could be possibly implemented by a soft or a hard fork. Given the reluctancy of the community to proceed to such forks, we consider the case of a velvet fork[1][2].

Under this scope, our contributions come as follows:

- We describe an attack, that we name *Chainsewing Attack*, against NIPoPoWs suffix proof construction, which is used for the light client's synchronization

- We suggest a patch to the NIPoPoWs protocol that eliminates the *Chainsewing Attack* and prove its security

2   Suffix Proofs

NIPoPoWs suffix proofs are used to prove predicates that pertain to the suffix of the blockchain. For example, this is the case of light client synchronization to the longest valid chain. [...]

3   Security of NIPoPoWs Suffix Proofs

In this section we provide the full security proof for the NIPoPoWs suffix proof protocol[1]. Apart from the proof itself (Theorem 2), we describe the definitions and lemmas being used. We try to give intuition for arguments and conclusions in each step.

Assume t adversarial out of n total parties, each with q PoW random oracle queries per round. We define $p = \frac{T}{2^\kappa}$ the probability of a successful Random Oracle query. We will call a query to the RO μ-*successful* if the RO returns a value h such that $h \leq 2^{-\mu}T$.

We define the boolean random variables X_r^μ, Y_r^μ, Z_r^μ. Fix some round r, query index j and adversarial party index k (out of t). If at round r an honest party obtains a PoW with $id < 2^{-\mu}T$, set $X_r^\mu = 1$, otherwise $X_r^\mu = 0$. If at round r exactly one honest party obtains $id < 2^{-\mu}T$, set $Y_r^\mu = 1$, otherwise

$Y_r^\mu = 0$. If at round r the j-th query of the k-th corrupted party is μ-successful, set $Z_{rjk}^\mu = 1$, otherwise $Z_{rjk}^\mu = 0$. Let $Z_r^\mu = \sum_{k=1}^t \sum_{j=1}^q Z_{rjk}^\mu$. For a set of rounds S, let $X^\mu(S) = \sum_{r \in S} X_r^\mu$ and similarly define Y_S^μ, Z_S^μ.

Definition 1 (Typical Execution). *An execution of the protocol is (ϵ, η)-typical if:*

Block counts don't deviate. *For all $\mu \geq 0$ and any set S of consecutive rounds with $|S| \geq 2^\mu \eta k$, we have:*

- *$(1-\epsilon)E[X^\mu(S)] < X^\mu(S) < (1+\epsilon)E[X^\mu(S)]$ and $(1-\epsilon)E[Y^\mu(S)] < Y^\mu(S)$*

- *$Z^\mu(S) < (1 + \epsilon)E[Z^\mu(S)]$*

Round count doesn't deviate. *Let S be a set of consecutive rounds such that $Z^\mu(S) \geq k$ for some security parameter k. Then $|S| \geq (1 - \epsilon)2^\mu \frac{k}{pqt}$ with overwhelming probability.*

Chain regularity. *No insertions, no copies and no predictions [3] have occurred.*

Theorem 1 (Typicality). *Executions are (ϵ, η)-typical with overwhelming probability in κ.*

Proof. **Block counts and regularity.** We refer to [3] for the full proof.

Round count. First, observe that for a specific round r we have $Z_{rjk} \sim Bern(p)$, so for the μ-level superblocks $Z_{rjk}^\mu \sim Bern(2^{-\mu}p)$ and these are jointly independent. Therefore, since for $|S|$ rounds we have $tq|S|$ adversarial RO queries, we have that $Z_S^\mu \sim \text{Bin}(tq|S|, 2^{-\mu}p)$. So $tq|S| \sim \text{NB}(Z_S^\mu, 2^{-\mu}p)$. Negative Binomial distribution is defined as $\text{NB}(r, p')$ and, for our purposes, expresses the number of trials in a sequence of independent and identically distributed Bernoulli trials before a specified (r) number of successes occurs. The expected total number of trials of a negative binomial distribution with parameters (r, p') is r/p'. To see this, imagine an experiment simulating the negative binomial performed many times, that is a set of trials is performed until r successes occur. Consider you perform n experiments of total N trials. Now we would expect $Np' = nr$, so $N/n = r/p'$. See that N/n is just the average number of trials per experiment. So we have $E[tq|S|] = \frac{Z_S^\mu}{2^{-\mu}p} \Rightarrow E[|S|] = 2^\mu \frac{Z_S^\mu}{tqp}$. So if $Z^\mu(S) \geq k$ then $E[|S|] \geq 2^\mu \frac{k}{tqp}$. Applying a tail bound to the negative binomial distribution, we obtain that $\Pr[|S| < (1 - \epsilon)E(|S|)] \in \Omega(\epsilon^2 m)$.

Lemma 1. *Suppose S is a set of consecutive rounds $r_1...r_2$ and C_B is a chain adopted by an honest party at round r_2 of a typical execution. Let $C_S^B = \{ b \in C_B : b$ was generated during $S\}$. Let $\mu_A, \mu_B \in \mathbb{N}$. Suppose $C_S^B \uparrow^{\mu_B}$ is good. Suppose C_A' is a μ_A-superchain containing only adversarially generated blocks generated during S and suppose that $|C_A'| \geq k$. Then $2^{\mu_A}|C_A'| < 2_B^\mu|C_S^B \uparrow^{\mu_B}|$.*

Proof. From $|C_A'| \geq k$ we have that $|Z_S^{\mu_A}| \geq k$. Applying Theorem 1, we conclude that $|S| \geq (1 - \epsilon')2^{\mu_A} \frac{|C_A'|}{pqt}$. Applying the Chain Growth theorem [3] we obtain $|C_S^B| \geq (1 - \epsilon)f|S|$. But from the goodness of $C_B^S \uparrow^{\mu_B}$, we know that $|C_B^S \uparrow^{\mu_B}| \geq (1 - \delta)2^{-\mu_B}|C_B^S|$. So we have $|C_B^S \uparrow^{\mu_B}| \geq (1 - \delta)2^{-\mu_B}(1 - \epsilon)f|S|$ and follows that $|C_B^S \uparrow^{\mu_B}| \geq (1 - \delta)2^{-\mu_B}(1 - \epsilon)f(1 - \epsilon')2^{\mu_A}\frac{|C_A'|}{pqt}$. Consequently

we have that $2^{\mu_A}|C'_A| \leq \frac{pqt}{(1-\delta)(1-\epsilon)(1-\epsilon')f}2^{\mu_B}|C_B^S \uparrow^{\mu_B}|$.

So, according to the above equation we have that $2^{\mu_A}|C'_A| < 2_B^\mu|C_S^B \uparrow^{\mu_B}|$ considering that honest majority assumption holds, specifically considering that $\frac{pqt}{f} \approx \frac{t}{n-t} \leq 1$.

Definition 2 (Adequate level of honest proof). *Let π be an honestly generated proof constructed upon some adopted chain C and let $b \in \pi$. Then μ' is defined as $\mu' = \max\{\mu : |\pi\{b :\} \uparrow^\mu | \geq \max(m+1, (1-\delta)2^{-\mu}|\pi\{b :\} \uparrow\downarrow |)\}$. We call μ' the adequate level of proof π with respect to block b with security parameters δ and m. Note that the adequate level of a proof is a function of both the proof π and the chosen block b.*

Intuitively, adequate is the level μ' of a proof π for a block b if there are at least m blocks after b in π under the condition that there is good chain quality for this level, meaning that there are at least so many blocks at this level as expected considering the number of 0-level blocks.
NOTE: adequate level is mostly useful for Claim 1a of the Security Proof (Theorem 2).

Lemma 2. *Let π be some honest proof generated with security parameters δ, m. Let C be the underlying chain, $b \in C$ be any block and μ' be the adequate level of the proof with respect to b and the same security parameters.*
Then $C\{b :\} \uparrow^{\mu'} = \pi\{b :\} \uparrow^{\mu'}$.

Proof. $\pi\{b :\} \uparrow^{\mu'} \subseteq C\{b :\} \uparrow^{\mu'}$ is trivial. For the converse, we have that in the iteration of the *Prove for loop*[1] with $\mu = \mu^*$, the block stored in variable B precedes b in C.

Note that the Prover's for loop iterates over all levels in the interlink structure, and places in the proof all of the blocks that are of the corresponding level and succeed B in C.

Suppose $\mu = \mu^*$ is the first *for* iteration during which the property is violated. This cannot be the first iteration since $B = C[0]$ and Genesis precedes all blocks. By induction hypothesis we see that during the iteration $\mu = mu^* + 1$, B preceded b. From the definition of μ' we know that μ' is the highest level for which $|\pi\{b :\} \uparrow^\mu | \geq \max(m, (1-\delta)2^{-\mu}|\pi\{b :\} \uparrow\downarrow |)$.

Hence, this property cannot hold for $\mu^* > \mu$ and therefore $|\pi\{b :\} \uparrow^\mu | < m$ or $\neg$local-good$_\delta(\pi\{b :\} \uparrow \mu^*[1 :], C, \mu^*)$.

In case local-good is violated, variable B remains unmodified and the induction step holds. If local-good is not violated, then $|\pi\{b :\} \uparrow^{\mu^*} [1 :]| < m$ and so $\pi \uparrow^{\mu^*} [-m]$, which is the updated value of B at the end of μ^* iteration, precedes b.

Lemma 3. *Suppose the verifier evaluates $\pi_A \geq \pi_B$ in a protocol interaction where B is honest and assume during the comparison that the compared level of the honest party is μ_B. Let $b = LCA(\pi_A, \pi_B)$ and let μ'_B be the adequate level of π_B with respect to b. Then $\mu'_B \geq \mu_B$.*

Proof. Because μ_B is the compared level of the honest party, from the definition of the $\geq_m$ operator, we have $2^{\mu_B}|\pi\{b :\} \uparrow^{\mu_B} | > 2^{\mu'_B}|\pi\{b :\} \uparrow^{\mu'_B} |$. This is true, otherwise the Verifier would have chosen level μ'_B as level of compar-

ison. The proof is by contradiction. Suppose $\mu'_B < \mu_B$. By definition, μ'_B is the maximum level such that $|\pi_B\{b :\} \uparrow^\mu [1 :]| \geq max(m, (1-\delta)2^{-\mu}|\pi_B\{b :\} \uparrow^\mu [1 :] \downarrow|)$, therefore μ_B does not satisfy this condition. But we know that $|\pi_B\{b :\} \uparrow^\mu [1 :]| > m$ because μ_B was selected by the Verifier. Therefore $2^{\mu_B}|\pi\{b :\} \uparrow^{\mu_B}| < (1-\delta)|C\{b :\}|$.

But also μ'_B satisfies goodness, so $2^{\mu'_B}|\pi\{b :\} \uparrow^{\mu'_B}| > (1-\delta)|C\{b :\}|$.

From the last two equations we obtain $2^{\mu_B}|\pi\{b :\} \uparrow^{\mu_B}| < 2^{\mu'_B}|\pi\{b :\} \uparrow^{\mu'_B}|$ which contradicts the initial equation.

@To Be Discussed: would the verifier ever choose a non-adequate level for proof comparison?

Intuitively the above Lemma says: the comparison level chosen by the Verifier can be no other than the adequate level in respect to block b ($LCA(\pi_A, \pi_B)$), since any other choice would be a level of non-good quality, because of the definition of the adequate level. A level of non-good quality would contain less PoW than that of the adequate level for the range of interest $C\{b :\}$.

Theorem 2. **(Security)** *Assuming honest majority, the non-interactive proofs-of-proof-of-work construction for computable κ-stable monotonic suffix-sensitive predicates is secure with overwhelming probability in κ.*

By contradiction. Let Q be a $\kappa-$stable monotonic suffix-sensitive chain predicate. Assume NIPoPoWs on Q is insecure. Then, during an execution at some round r_3, $Q(C)$ is defined and the verifier V disagrees with some honest participant. Assume the execution is typical. V communicates with adversary A and honest prover B. The verifier receives proofs π_A, π_B. Because B is honest, π_B is a proof constructed based on underlying blockchain C_B (with $\pi_B \subseteq C_B$), which B has adopted during round r_3 at which π_B was generated. Furthermore, π_A was generated at round $r'_3 \leq r_3$.

The verifier outputs $\neg Q(C_B)$. Thus it is necessary that $\pi_A \geq \pi_B$. We show that $\pi_A \geq \pi_B$ is a negligible event.

Let $b = LCA(\pi_A, \pi_B)$. Let $b^\star$ be the most recently honestly generated block in C_B preceding b. Note that $b^\star$ necessarily exists because Genesis is honestly generated. Let the levels of comparison decided by the verifier be μ_A and μ_B respectively. Let μ'_B be the adequate level of proof π_B with respect to block b. Call $\alpha_A = \pi_A \uparrow^{\mu_A} \{b :\}$, $\alpha'_B = \pi_B \uparrow^{\mu'_B} \{b :\}$.

Note that we consider the parts of the proofs succeeding block b the decisive ones for the verifier's choice. This is to adversary's advantage, since the parts preceding this block demonstrate the proof-of-work contained in common (sub)chain of A and B. Thus, considering good chain quality for the proof of any honest player, the adversary could only include equal or less proof-of-work in her proof for this part of the chain.

We will now show three successive claims: First, α_A and $\alpha'_B \downarrow$ are mostly disjoint. Second, a_A contains mostly adversarially generated blocks. And third, the adversary is able to produce this a_A with negligible probability.

Let $\alpha_A = k_1 + k_2 + k_3$ and let k_1, k_2, k_3 be as defined in the following Claims.

Claim 1: $\alpha_A, \alpha'_B \downarrow$ are mostly disjoint. We show this by taking the two possible cases for the relation of μ_A, μ'_B.

Claim 1a: If $\mu'_B \leq \mu_A$ then they are completely disjoint. In such a case of

inequality, every block in α_A would also be of lower level μ'_B. Applying Lemma 2 to $C\{b:\} \uparrow^{\mu'_B}$ we see that $C\{b:\} \uparrow^{\mu'_B} = \pi\{b:\} \uparrow^{\mu'_B}$. Subsequently, any block in $\pi_A \uparrow^{\mu_A} \{b:\}[1:]$ would also be included in proof α'_B, but $b = LCA(\pi_A, \pi_B)$ so there can be no succeeding block common in α_A, α'_B.

Claim 1b: If $\mu'_B > \mu_A$ then $|\alpha_A[1:] \cap \alpha'_B \downarrow [1:]| = k_1 \leq 2^{\mu'_B - \mu_A}$.
First observe that because the adversary is winning $2^{\mu_A}|\alpha_A| > 2^{\mu'_B}|\alpha'_B| \geq 2^{\mu'_B}m \Rightarrow |\alpha_A| > 2^{\mu'_B - \mu_A}m$. Let's call b_1 the first block in α'_B after block b. Suppose for contradiction that $k_1 > 2^{\mu'_B - \mu_A}$. Since $C_B^{\mu'_B}$ is of good chain quality, this would mean that block b_1, of level μ'_B, also exists in α_A since it is of level μ_A too. But b_1 cannot exist in both α_A, α'_B since $\alpha_A \cap \alpha'_B = \emptyset$ by definition.

From Claim 1a and Claim 1b, we conclude that there are $|\alpha_A| - k_1$ blocks after block b in α_A which do not exist in $\alpha_B \downarrow$. We now set $b_2 = LCA(C_B, \alpha_A)$. This makes b_2 the last block before the fork point at the 0-level chain included in the adversary's proof.

Intuition: in this case the common blocks of $\alpha_A, \alpha'_B \downarrow$ may only be blocks of level μ_A which precede the first μ'_B block appearing in α'_B. If this block of level μ'_B was common, it could also be included in α_A. If it is included this would be the LCA of α_A, α'_B. If it is not, then the adversary could no more include blocks from the common part of chain C_B in her proof since they no longer form a valid chain in α_A. The quantity $2^{\mu'_B - \mu_A}$ means: in the range between two consequent μ'_B-level blocks, we have $n = 2^{\mu'_B}$ 0-level blocks and, thus, $2^{-\mu_A}n = 2^{\mu'_B - \mu_A}$ blocks of μ_A-level.

Claim 2: At least k_3 superblocks of α_A are adversarially generated. We show this by showing that $\alpha_A[k_1 + k_2 + 1:]$ contains no honestly generated blocks. Suppose for contradiction that the block $\alpha_A[i]$ for some $i \geq k_1 + k_2 + 1$ was honestly generated. This means that an honest party adopted the chain $\alpha_A[:i-1] \downarrow$ at some round $r_2 \leq r_3$. Because of the way honest parties adopt chains, the superchain $\alpha_A[:i-1]$ has an underlying properly constructed 0-level anchored chain C_A such that $\alpha_A[:i-1] \subseteq C_A$. Let j be the index of block b_2 within α_A, $j_\downarrow$ be the index of block b_2 within C_A and $k_{2\downarrow} = |\alpha_A[j:j+k_2] \downarrow|$. See Figure 1 for a demonstration. Observe that $|C_A[:\{\alpha_A[i-1]\}]| \geq |C_A[:j_\downarrow + k_{2\downarrow}]|$, while $C_A[j_\downarrow : j_\downarrow + k_{2\downarrow}] \npreceq C_B$ as proved in Claim 1. But C_A was adopted by an honest party at round r_2, which is prior to round r_3 during which C_B was adopted by an honest party B. This contradicts the Common Prefix[3] with parameter $k_{2\downarrow}$. It follows that with overwhelming probability in $k_{2\downarrow}$, the $k_3 = |\alpha_A| - k_2 - k_1$ last blocks of the adversarial proof have been adversarially generated.

Intuitively: Because of Common Prefix on $k_{2\downarrow}$ parameter, where $k_{2\downarrow} = |\alpha_A[j:j+k_2] \downarrow|$, where $E[k_{2\downarrow}] = 2^{\mu_A}k_2$, there can be no honest party adopting C_A at any round $i \geq k_1 + k_2 + 1$.

From these two Claims we have that k_1 blocks in α_A are blocks of the common zero-level chain, while k_2 blocks are blocks after the fork point at the zero-level chain. Subsequently, k_2 is subject to the Common Prefix $\kappa-$parameter limitations as described in the Backbone paper[3].
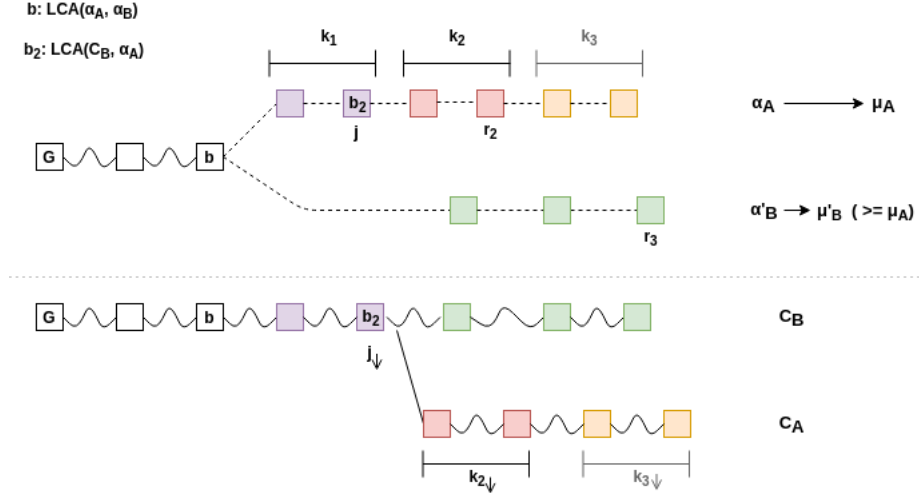
Figure 1: *Two competing proofs at different levels. At the bottom the corresponding 0-level chains are represented.*

Claim 3: Adversary A is able to produce α_A that wins against α_B with negligible probability.

Let b' be the latest honestly generated block in a_A, or $b' = b^*$ if no such block exists in a_A. Let r_1 be the round when b' was generated. Consider the set S of consecutive rounds $r_1..r_3$. Every block in $\alpha_A[-k_3 :]$ has been adversarially generated during S and $|\alpha_A[-k_3 :]| = |\alpha_A\{b' :\}| = k_3$. C_B is a chain adopted by an honest party at round r_3 and filtering the blocks by the rounds during which they were generated to obtain C_B^S, we see that if b'' is the most recently generated block in α_B in a round $r \leq r_1$, then $C_B^S = C_B\{b'' :\}$. But $C_B^S \uparrow^{\mu'_B}$ is good with respect to C_B^S. Applying Lemma 1, we obtain that with overwhelming probability $2^{\mu_A}|\alpha_A\{b' :\}| < 2^{\mu'_B}|C_B^S \uparrow^{\mu'_B}|$, which is equal to

$$2^{\mu_A}|\alpha_A\{b' :\}| < 2^{\mu'_B}|\alpha'_B\{b'' :\}| \tag{1}$$

since α'_B contains all the μ'_B-level blocks in C_B^S.

In order to complete the proof, let us know consider $\alpha_A^{k_1}$, $\alpha_A^{k_2}$, $\alpha_A^{k_3}$ the parts of α_A where the k_1, k_2, k_3 blocks reside and $\alpha_B^{k_1}$, $\alpha_B^{k_2}$, $\alpha_B^{k_3}$ the parts of α_B containing blocks generated in the corresponding round sets as illustrated in Figure 2.

Subsequently to the above Claims we have that:

Because of the common underlying chain in the first round set:

$$2^{\mu_A}|\alpha_A^{k_1}| \leq 2^{\mu'_B}|\alpha'^{k_1}_B| \tag{2}$$

Because of the adoption by an honest party of chain C_B at a later round r_3, we have for the second round set:

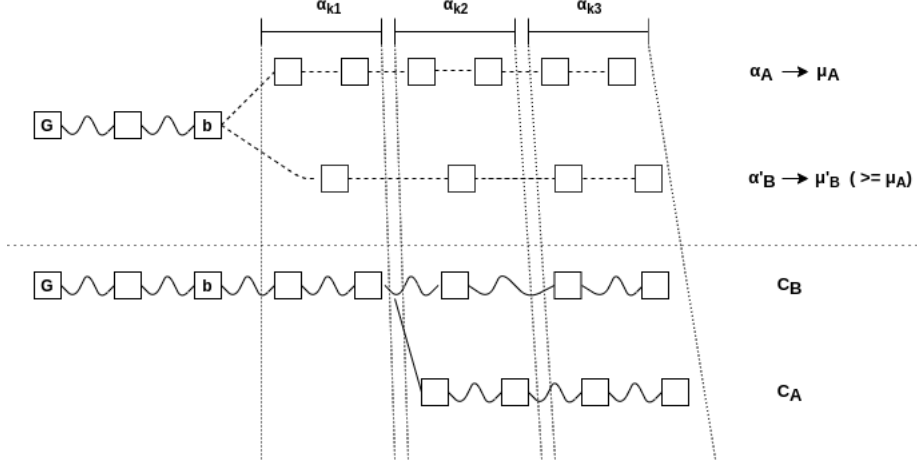$$2^{\mu_A}|\alpha_A^{k_2}| \leq 2^{\mu'_B}|\alpha'^{k_2}_B| \tag{3}$$

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Figure 2: *The three round sets in two competing proofs at different levels. The vertical dashed lines denote the area of interest, across proofs and chains, corresponding to each round set. At the bottom the corresponding 0-level chains are represented.*

Because of Equation (1), we have for the third round set:

$$2^{\mu_A}|\alpha_A^{k_3}| < 2^{\mu'_B}|\alpha'^{k_3}_B| \qquad (4)$$

So we have

$$2^{\mu_A}(|\alpha_A^{k_1}| + |\alpha_A^{k_2}| + |\alpha_A^{k_3}|) < 2^{\mu'_B}(|\alpha'^{k_1}_B| + |\alpha'^{k_2}_B| + |\alpha'^{k_3}_B|)$$

and finally

$$2^{\mu_A}|\alpha_A| < 2^{\mu'_B}|\alpha'_B| \qquad (5)$$

Therefore we have proven that $2^{\mu'_B}|\pi_B \uparrow^{\mu'_B} | > 2^{\mu_A}|\pi_A^{\mu_A}|$. From the definition of μ_B, we know that $2^{\mu_B}|\pi_B \uparrow^{\mu_B} | > 2^{\mu'_B}|\pi_B \uparrow^{\mu'_B} |$ because it was chosen μ_B as level of comparison by the Verifier. So we conclude that $2^{\mu_B}|\pi_B \uparrow^{\mu_B} | > 2^{\mu_A}|\pi_A \uparrow^{\mu_A} |$.

$\square$

It remains to compute the security parameter m that guarantee that all the above hold true in every implementation. It suffices to compute the security parameter values for each set of rounds k_1, k_2, k_3, so that the proof equations 2, 3, 4 hold and then sum these values to obtain parameter m.

In the first set of rounds, for the first k_1 blocks in α_A, we only need 1 block included in α_B for the part of the proof described in Equation 2. In the second set of rounds we need $2^{-\mu_B}\kappa$ blocks for the part of the proof described in Equation 3, just as it directly results from the Common Prefix property. In order to make m independent of any specific level it suffices to consider the upper bound of κ blocks for this set of rounds. In the last set of rounds we need at least κ adversarially generated blocks in $\alpha_A^{k_3}$ so that Lemma 1 is applicable.

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Since we assume honest majority, obliging to at least κ blocks for this set of rounds suffices to guarantee for Equation 4.

So, we finally conclude to the following upper bound for the value of security parameter:

$$m = 2\kappa + 1 \qquad (6)$$

TO BE DISCUSSED: however it seems like this parameter value covers only the α_a, α_B parts of the proofs. What about the rest of it? From Genesis block up to block b? The prover does not know the index of last common block b in order to assert that he sends a proof of valid length.

4 NIPoPoWs under Velvet Fork

5 The Chainsewing Attack

We will now describe an explicit attack against the NIPoPoW suffix proof construction under a velvet fork. Note that since the protocol is implemented under a velvet fork, any adversarial block that is mined in the proper way except containing false interlink data structure will be accepted as valid. A false interlink may contain invalid pointers, for example pointers to superblocks of a fork chain, as shown in Figure 3. Taking advantage of this fact, an adversary maintaining a fork chain could produce suffix proofs that claim blocks of the chain adopted by an honest player as her own. The attack is described in detail in the following.
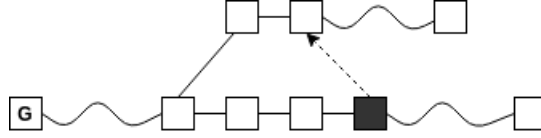

Figure 3: *Example of false interlink structure of an adversarial block, coloured black, in an honest player's chain. The dashed arrow is a pointer to a fork chain superblock included in the interlink.*

Assume that chain C_B was adopted by an honest player B and chain C_A, a fork of C_B, maintained by an adversary A. Assume the adversary wants to produce a suffix proof in order to attack an honest light client to have him adopt chain C_A. In order to achieve this, the adversary needs to include a greater amount of PoW in her suffix proof, π_A, in comparison to the honest player's proof, π_B, so as to achieve $\pi_A \geq_m \pi_B$. For this she works as follows. She produces some blocks in chains C_A and C_B containing false interlink pointers which will allow for claiming blocks of chain C_B as of chain C_A in her suffix proof. She acts as described below. As shown in Figure 4 she first generates a superblock, say a', in chain C_B which contains an interlink pointer to a superblock, say b', in chain C_A. As argued earlier, the block will be accepted as valid in chain C_B despite the false pointers in the interlink. Then, the adversary may mine on chain C_A or C_B. At some point she produces a block, say a, in C_A containing an interlink pointer to a block, say b, in honest player's chain C_B. At this point

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the adversary will produce a suffix proof for chain C_A containing the subchain $C\{ab\} \cup C\{b : a'\} \cup C\{a' : b'\}$. Notice that following the interlink pointers constructed in such a way, a light client perceives $C\{ab\} \cup C\{b : a'\} \cup C\{a' : b'\}$ as a valid chain.
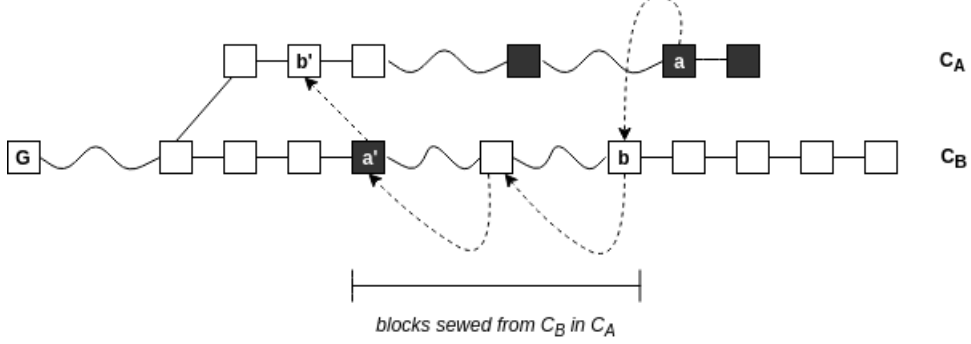

Figure 4: *Chainsewing Attack. The chain at the bottom represents the chain of an honest player, C_B, while the above one is the adversarial fork, C_A. Blocks generated by the adversary are coloured black. Dashed arrows represent interlink pointers included in the suffix proof by the adversary. Wavy lines imply one or more blocks. Firm lines imply the previousId relationship between two sequential blocks.*

In this attack the adversary uses false interlink pointers, under a velvet fork, to "sew" portions of the chain adopted by an honest player to her own fork. This remark justifies the name given.

Note that in order to make this attack successful, the adversary has to produce only a few superblocks which let her arrogate an arbitrary large number of blocks of an honest player's chain, while she can mine for her own fork chain. Thus intuitively we expect this attack to succeed with overwhelming probability.

@TODO

Needs proof via simulation. It is not probable that the attacker will succeed in high probability, since the most important adversarially generated blocks , a and a', set a limit to the adversarial blocks produced in parallel to the honest blocks of subchain $C\{ab\} \cup C\{b : a'\} \cup C\{a' : b'\}$ and can take part in the suffix proof.

6 Protocol Update

In order to eliminate the Chainsewing Attack we propose an update to the NIPoPoWs protocol under velvet fork. The core problem is that in her suffix proof the adversary is able to claim not only blocks of the fork chain, which are in majority adversarially generated because of Common Prefix property, but also an arbitrarily large number of blocks generated by honest miners. The vulnerability that makes this attack possible is the acceptance of blocks containing false interlink pointers. Since we operate under a velvet fork we cannot

eliminate blocks containing false interlinks, we need, however, to restrict the adversary from being able to claim portion of another chain as part of her fork chain.

The key observation on the Chainsewing Attack is that the adversary needs at least two adversarially generated blocks, one in the fork (block a) and one in the honest player's chain (block a'), which are connected in a superblock path formed by interlink pointers. This path may contain both honest or adversarial blocks.

In case it contains only adversarial blocks, the attack cannot harm security since we assume an attacker of less than 50% hashing power. So in order to be successful, the attacker needs to "sew" honestly generated blocks. Thus there will be at least one honest block in the superblock path connecting blocks a and a', which points to an adversarial block containing false interlink or, by induction, pointing to a block containing false interlink. The idea is to ban all blocks generated by honest players from participating in this superblock path. In this way the adversary could not misuse hashing power of the honest players and the sewed blocks could only be adversarially generated, thus the attack would never succeed.

We describe a protocol patch that operates as follows. The NIPoPoW protocol under velvet fork works as usual but each miner constructs a block's interlink excluding the blocks with false interlink (except the pointers of level 0). In this way, blocks containing false interlink pointers are integrated in the chain but are not taken into consideration when updating the interlink structure for the next block to be mined. No honest block could now point to an adversarial superblock that may act as the passing point to the fork chain in an adversarial suffix proof.

After this protocol update the adversary is only able to inject adversarially generated blocks from the chain adopted by an honest party to her own fork. This is illustrated in Figure 5. Since an updated honest miner will not include interlink pointers to adversarially generated blocks with incorrect interlinks, these adversarially generated blocks cannot participate in an honestly generated suffix proof, while they may participate in an adversarially suffix proof for the adversary's fork chain. Consequently, as far as the hashing power included in a suffix proof is concerned, these blocks can be conceived as belonging in the adversary's fork chain.

Hence, the following two Remarks come as results of the suggested protocol update.

Remark 1. A suffix proof constructed by an honest player cannot contain any adversarially generated block with an incorrect interlink structure.

Remark 2. Consider C_B, C_A the 0-level chains adopted by an honest player and the adversary at some round r. Let block $b = LCA(C_A, C_B)$. A suffix proof π_A that contains blocks of C_A constructed by an adversary is subject to the following: i) for the blocks included by the common subchain $C_B\{: b\}$ either they are honestly generated or once a block with incorrect interlink is included no other honestly generated block can be included in the proof, ii) for the blocks in the diverse subchain $C_B\{b :\}$ only adversarially generated blocks with incorrect interlink structure may be included in π_A.

Figure 5 illustrates the above concluding Remarks, while suggesting an equivalent conception of the competing chains considering the suffix proofs. This equivalent forming of the chains will be used in the security proof follow-

ing in the next section.

Figure 5: *Adversarially generated blocks sewed from an honest player's chain into the adversary's suffix proof. The chain at the bottom represents the adversarial fork chain, C_A, while the above one is the chain of an honest player, C_B. Blocks generated by the adversary are coloured black. Dashed arrows represent interlink pointers. Wavy lines imply one or more blocks. **I:** The real picture of the chains. **II:** Equivalent picture considering the hashing power included in the corresponding suffix proof of each chain.*

7 Security Proof

Our security proof is based on the security proof of NIPoPoWs Suffix Security Proof under a soft or hard fork. Consequently, most of the Definitions and Lemmas presented in Section 3 will be referenced and applied for the security proof under velvet fork conditions.

Our working hypothesis is that we operate under honest majority considering only the updated players. This is formally described in the following definition.

Honest Majority Assumption under Velvet Fork Conditions. A number of t' out of n' upgraded parties are corrupted such that $t' \leq (1-\delta)(n' - t')$, where parameter δ is defined as in the Bitcoin Backbone paper [3].

Theorem 3. (Security under velvet fork) *Assuming honest majority under velvet fork conditions, the non-interactive proofs-of-proof-of-work construction for computable κ-stable monotonic suffix-sensitive predicates under velvet fork conditions is secure with overwhelming probability in κ.*

By contradiction. We follow the construction of Theorem 2 proof and extend it. Let Q be a $\kappa-$stable monotonic suffix-sensitive chain predicate. Assume NIPoPoWs under velvet fork on Q is insecure. Then, during an execution at some round r_3, $Q(C)$ is defined and the verifier V disagrees with some honest participant. Assume the execution is typical. V communicates with adversary A and honest prover B. The verifier receives proofs π_A, π_B. Because B is honest, π_B is a proof constructed based on underlying blockchain C_B (with $\pi_B \subseteq C_B$), which B has adopted during round r_3 at which π_B was generated. Consider C_A the chain containing at least some of the blocks in π_A, while the remaining blocks π_A must belong in the C_B.

The verifier outputs $\neg Q(C_B)$. Thus it is necessary that $\pi_A \geq \pi_B$. We show that $\pi_A \geq \pi_B$ is a negligible event.

Let $b = LCA(\pi_A, \pi_B)$. Let the levels of comparison decided by the verifier be μ_A and μ_B respectively. Let μ'_B be the adequate level of proof π_B with respect to block b. Call $\alpha_A = \pi_A \uparrow^{\mu_A} \{b :\}$, $\alpha'_B = \pi_B \uparrow^{\mu'_B} \{b :\}$.

The above are illustrated, among other, in Parts I, II of Figure 6.

We will now show three successive claims under velvet fork conditions: First, $\alpha_A \downarrow\uparrow^{\mu_A}$ and $\alpha'_B \downarrow$ are mostly disjoint. Second, a_A contains mostly adversarially generated blocks. And third, the adversary is able to produce this a_A with negligible probability. Note that the notation $\alpha_A \downarrow\uparrow^{\mu_A}$ denotes the blocks of α_A which belong in the 0-level chain C_A, thus are not sewed in the adversary's proof by C_B.

Let $\alpha_A = k_1 + k_2 + k_3$ and let k_1, k_2, k_3 be as defined in the following Claims.

Let round r_1 be the round when block b is generated and round r_2 when block $b_2 = LCA(\alpha_A \downarrow\uparrow^{\mu_A}, \alpha'_B \downarrow)$ is generated.

Claim 1: $\alpha_A \downarrow\uparrow^{\mu_A}, \alpha'_B \downarrow$ are mostly disjoint. Following the proof of Theorem 2 we conclude that $|\alpha_A \downarrow\uparrow^{\mu_A} [1 :] \cap \alpha'_B \downarrow [1 :]| \leq k_1 = 2^{\mu'_B - \mu_A}$. In order to see this under the velvet fork conditions consider the following two claims. First let block $b_2 = LCA(\alpha_A \downarrow\uparrow^{\mu_A}, \alpha'_B \downarrow)$. This means that b_2 is the LCA block considering the adversary's proof and honest player's 0-level chain excluding any adversarially generated block included in α_A after the 0-level fork point .

Claim 1a: If no adversarially generated block of chain C_B with incorrect interlink is included in α_A until block b_2 then the claim's proof is the same to that of Theorem 2, thus we have $k_1 \leq 2^{\mu'_B - \mu_A}$.

Claim 1b: If a block $b' = \{b' \in C_B\{b : b_2\} \cap \ b' \ contains \ incorrect \ interlink\}$ is included in α_A then we have that $|\alpha_A \downarrow\uparrow^{\mu_A} [1 :] \cap \alpha'_B \downarrow [1 :]| \leq k_1 = k_{1a} + k_{1b}$, where $k_{1a} = 2^{\mu'_B - \mu_A}$.

Note that after block b' no more honestly generated blocks of the common subchain $C_B\{b' : b_2\}$ are included in α_A because of Remark 2. Blocks in $\alpha_A\{b' : b_2\}$ are all adversarially generated of C_B. Thus from that point on it is equivalent considering two disjoint chains, one with honestly and another with adversarially generated blocks, competing each other. Let these adversarially generated blocks count to k_{1b}, where $k_{1b} \leq C_B\{b' : b_2\} \uparrow^{\mu_A}$.

(In the end, consider the rounds when these blocks were generated and apply Lemma 1 to get a desirable inequality)
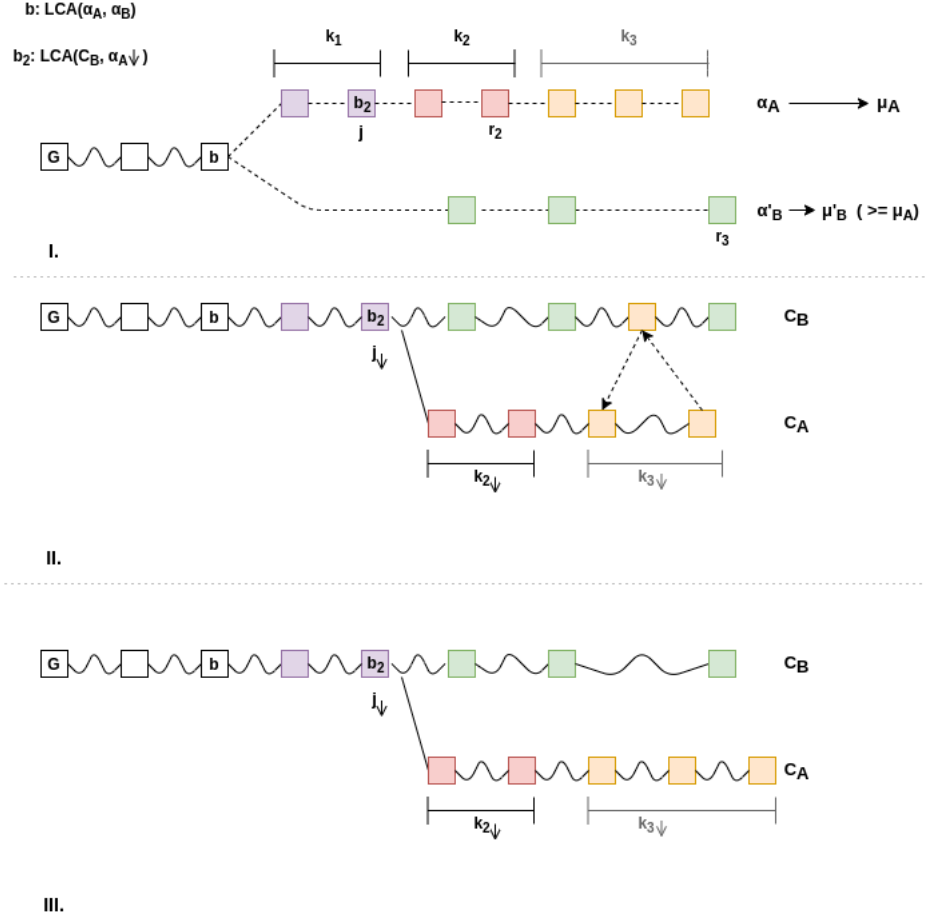
Figure 6: *Wavy lines imply one or more blocks. Dashed lines and arrows imply interlink pointers to superblocks. **I**: the three round sets in two competing proofs at different levels, **II**: the corresponding 0-level chains, **III**: the reformed chains C_B, C_A so that adversarially generated blocks with false interlink participating in cyclic dependency with C_A are removed from C_B and added to C_A.*

From all the above, we conclude that there are $|\alpha_A| - k_1$ blocks after block b in α_A which are not honestly generated blocks existing in $\alpha_B \downarrow$. In other words, there are $|\alpha_A| - k_1$ blocks after block b in α_A, which are either adversarially generated existing in $\alpha_B \downarrow$ either don't belong in $\alpha_B \downarrow$. This makes b_2 the last block before the 0-level fork point included in the adversary's proof.

Claim 2. At least k_3 superblocks of α_A are adversarially generated. Just as the proof of Theorem 2 and using a similar notation, because of the Common Prefix property on parameter $k_{2a\downarrow}$, $\alpha_A \downarrow\uparrow [k_{1a} + k_{2a} :]$ could contain no honestly generated blocks. In case there are no sewed blocks, the above holds for α_A too. In order to generalize to velvet fork conditions, let the sewed blocks count to k_{2b} and the sum $k_2 = k_{2a} + k_{2b}$. We conclude that $\alpha_A[k_1 + k_2 :]$ could contain

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no honestly generated blocks.

Note that for every sewed subchain the adversary includes in her proof, it may be that some other blocks generated in the adversary's chain are left out of the proof. This is due to the way blocks are generated in order to form a valid chain. So, in a case where a number of C_B blocks are sewed in C_A between blocks b_{s_1} and b_{s_2}, say at rounds s_1, s_2 accordingly, any block generated in C_A in the round set $s_1..s_2$ would be left out of the proof. This is illustrated in Figure 7.
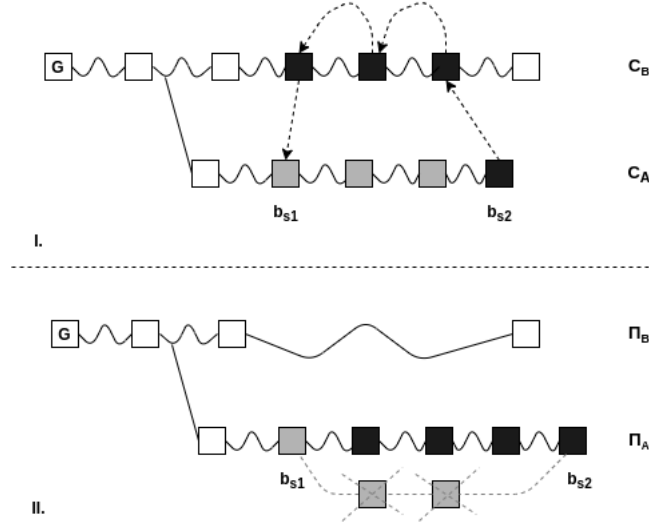

Figure 7: *Wavy lines imply one or more blocks. Dashed arrows imply interlink pointers to superblocks. Adversarially generated blocks are colored black. Grey colored blocks may be honestly or adversarially generated.* **I:** *the 0-level chains,* **II:** *the corresponding proof chains; some blocks generated in C_A are excluded from proof π_A in favor of the sewed blocks from C_B.*

In the following we consider the reformed chains C_A, C_B for blocks in the k_3 region of the proofs. This reforming suggests the following: all adversarially generated blocks with false interlink in subchain $C_B\{b_2 :\}$ which participate in cyclic dependency with fork chain C_A, are removed from C_B and added in C_A just as the interlink pointers forming the cyclic dependency point out. This reforming is helpful for the purposes of our proof construction and does not change the facts of the problem, or put it in another way, gives us an equivalent problem to solve. This is true, because of Remark 1, that such blocks cannot be included in an honest NIPoPoW proof while can be included in the proof constructed by the adversary, thus from the NIPoPoW suffix proofs perspective these block can be perceived as belonging in the adversary's fork chain. This remark is considered in the following Claim.

Claim 3. We reform the chains C_A, C_B as illustrated in Figure 5 (I, II) and Figure 2 (II, III). After this reforming we conclude to an equivalent set of

compared subchains. Note that the problems before and after the reforming are equivalent because of Remarks 1 and 2. Then this Claim is exactly the same as that of Theorem 2.

In order to complete the proof we work in the same way as in Theorem 2, by considering the three parts of α_A proof ....

From all the above Claims we have that:
In the first round set, because of the common underlying chain:

$$2^{\mu_A}|\alpha_A^{k_{1a}}| \leq 2^{\mu'_B}|\alpha'^{k_{1a}}_B|$$

and because of Lemma 1 we obtain with high probability:

$$2^{\mu_A}|\alpha_A^{k_{1b}}| \leq 2^{\mu'_B}|\alpha'^{k_{1b}}_B|$$

So finally:

$$2^{\mu_A}|\alpha_A^{k_1}| \leq 2^{\mu'_B}|\alpha'^{k_1}_B| \tag{7}$$

In the second round set:
(To be completed.)

In the third round set, considering the equivalent problem after reforming the chains because of Honest Majority Assumption under Velvet Fork and Lemma 1 we have:

$$2^{\mu_A}|\alpha_A^{k_3}| < 2^{\mu'_B}|\alpha'^{k_3}_B| \tag{8}$$

8 Infix Proofs

References

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