Chapter 9 Decision Analysis

- 9.1 Context and basic steps (most important part)
- 9.2 Example
- 9.3 Multistage decision analysis (example)
- 9.4 Hierarchical decision analysis (example)
- 9.5 Personal vs. institutional decision analysis

Bayesian decision theory

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  - or actions a

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- Expected utility $\mathrm{E}[U(x) \mid d] = \int U(x)p(x \mid d)dx$
- Choose decision d^*, which maximizes the expected utility

$$d^* = \arg\max_d \mathrm{E}[U(x) \mid d]$$

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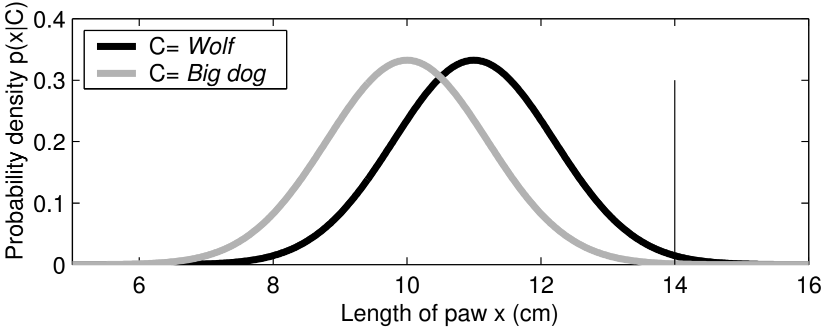
- Helen is going to pick mushrooms in a forest, while she notices a paw print which could made by a dog or a wolf
- Helen measures that the length of the paw print is 14 cm and goes home to Google how big paws dogs and wolves have, and tries then to infer which animal has made the paw print



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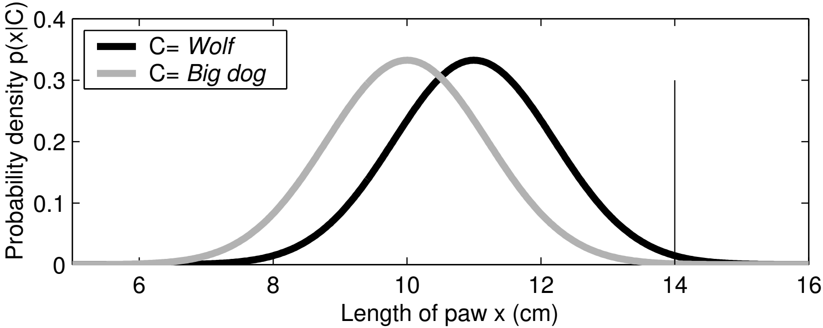

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- Likelihood of wolf is 0.92 (alternative being dog)

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Wolf	0.92	0.10
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- Posterior probability of wolf is 10%

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Decision d	Wolf	Dog
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Utility matrix $U(x)$

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Decision d	Animal Wolf	Animal Dog
Stay home	0	0
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Utility matrix $U(x)$

Action d	Expected utility $\mathrm{E}[U(x) \mid d]$
Stay home	0
Go to the forest	-100+0.9

Utilities for different actions

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- Example illustrates that the uncertainties (probabilities) related to all consequences need to be carried on until final decision making

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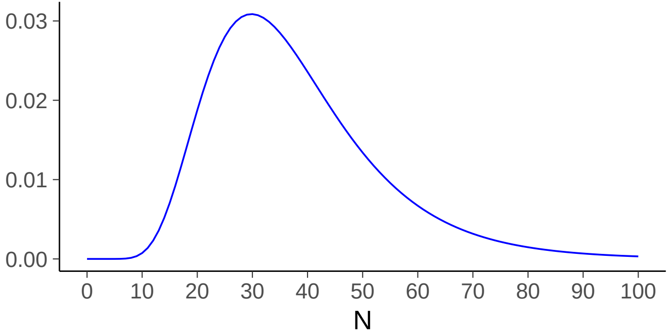
  - You ask your friends how many they used to sell and estimate a distribution for how many you might sell



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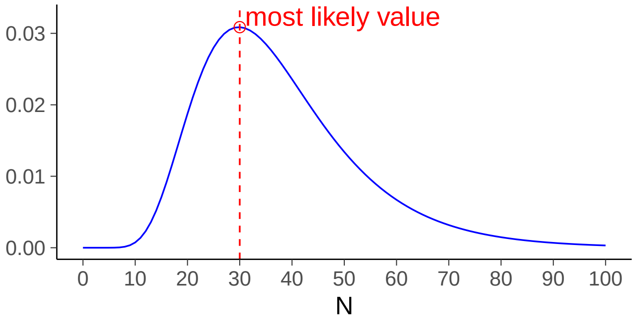

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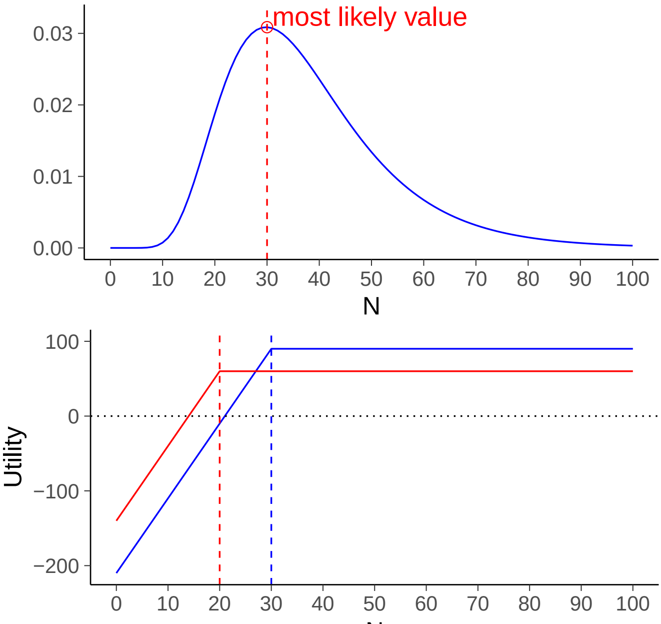

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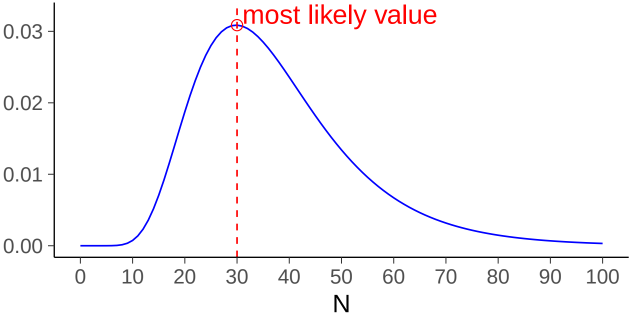

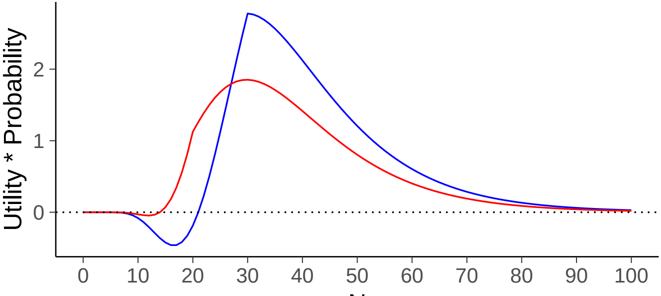

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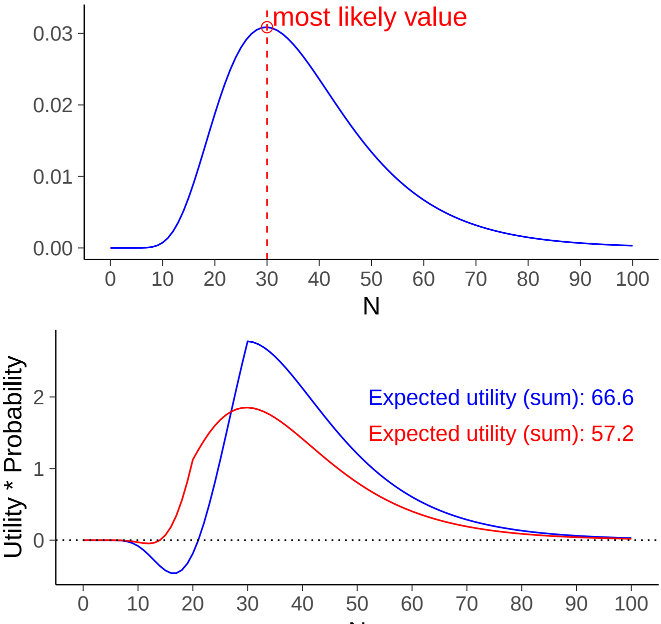

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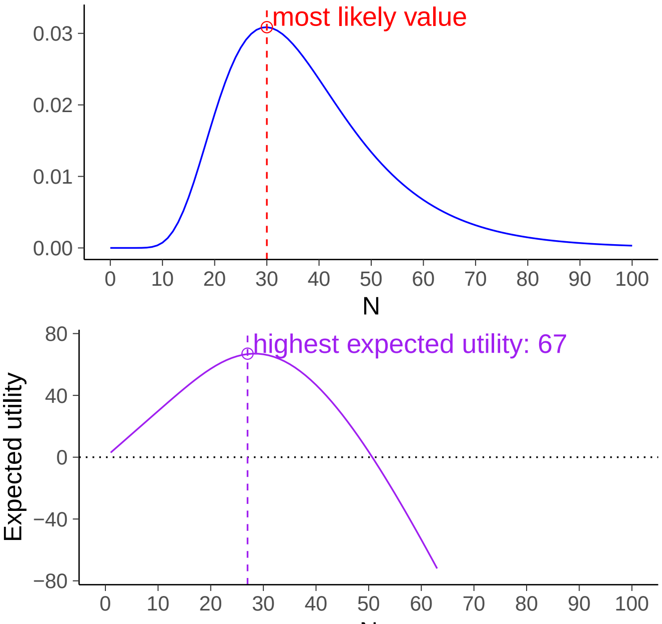

Decision making in sales

- Common task in commerce and restaurants

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- What is the cost of human life?
- Multiple parties having different utilities

Model selection as decision problem

- Choose the model that maximizes the expected utility of using the model to make predictions / decisions in the future

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  - Surgery: $0.35*0 + 0.65*(0.9*20.3 + 0.1*34.8 - 1) = 13.5$mo
  - No treatment: $0.9*5.6 + 0.1*34.8 = 8.5$mo

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- Example 1
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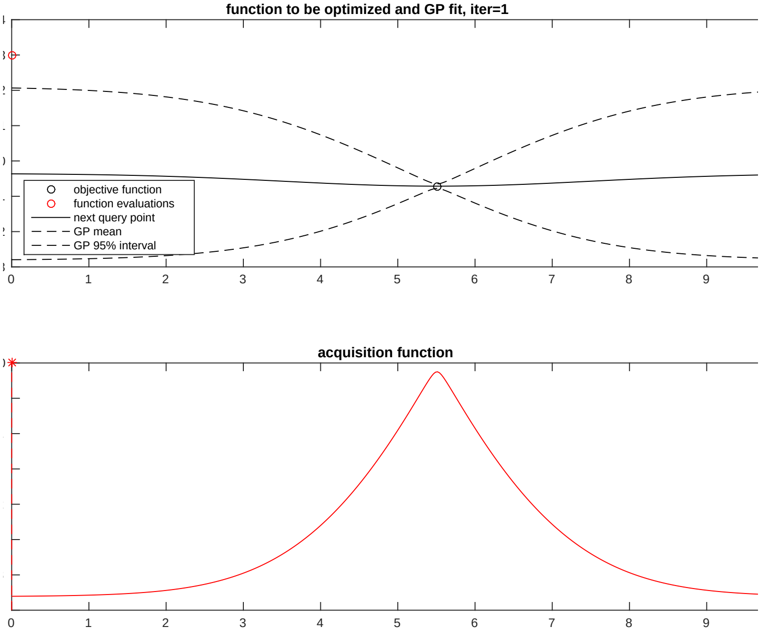
- Example 2
  - Imagine that in bioassay the posterior uncertainty of LD50 is too large
  - which dose should be used in the next experiment to reduce the variance of LD50 as much as possible ?
    - this way less experiments need to be made (and less animals need to be killed)

Bayesian optimization

- Design of experiment
- Used to optimize, for example,
  - machine learning / deep learning model structures, regularization, and learning algorithm parameters
  - material science
  - engines
  - drug testing
  - part of Bayesian inference for stochastic simulators

Bayesian optimization



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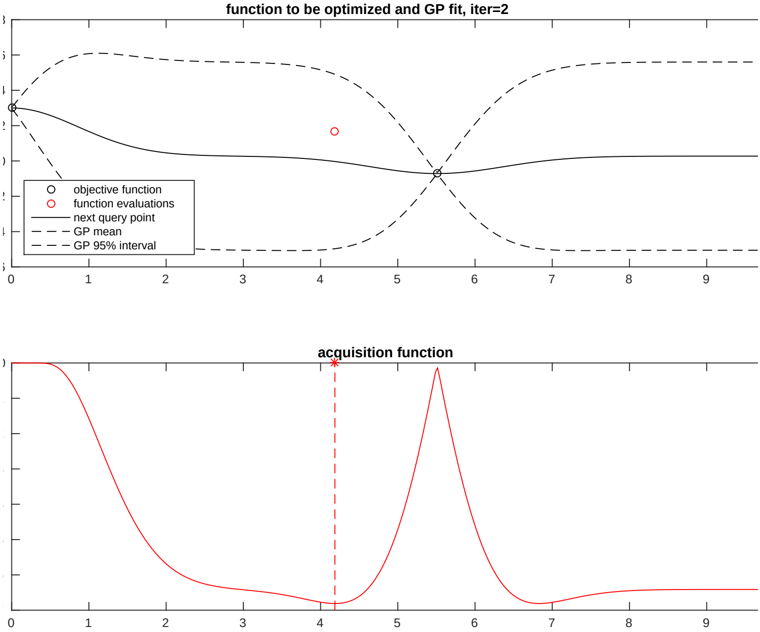

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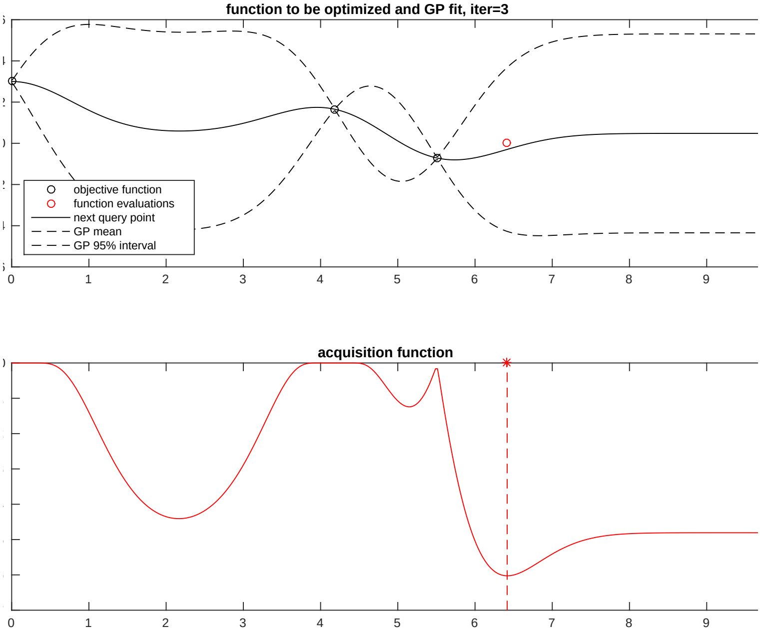

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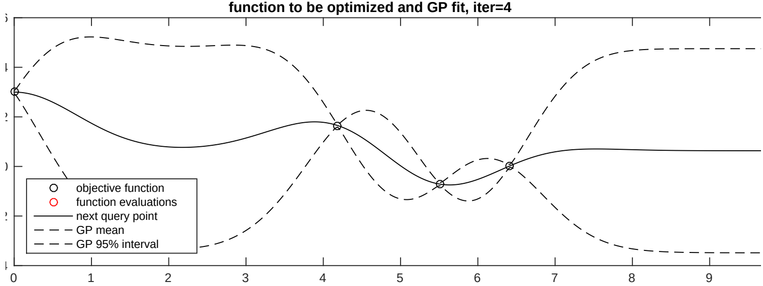

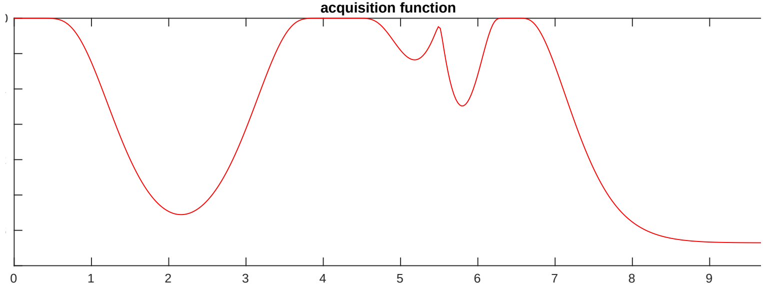

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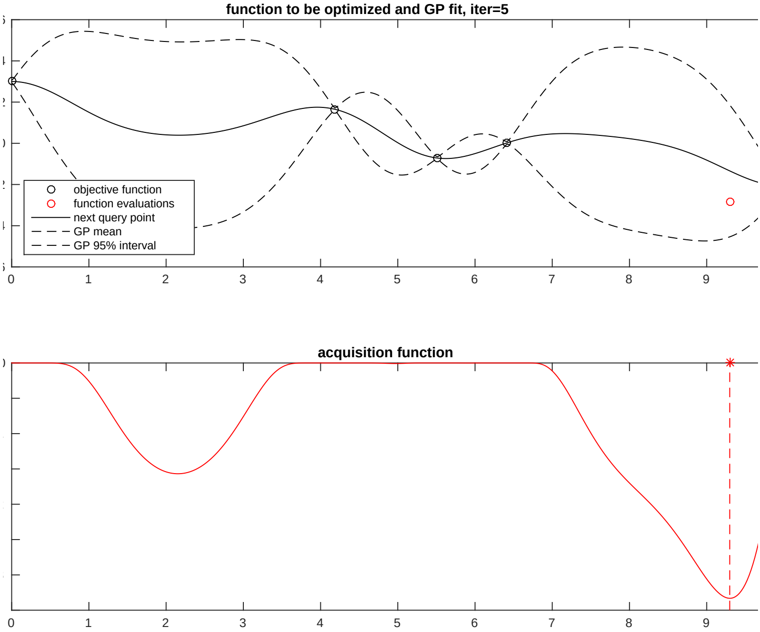

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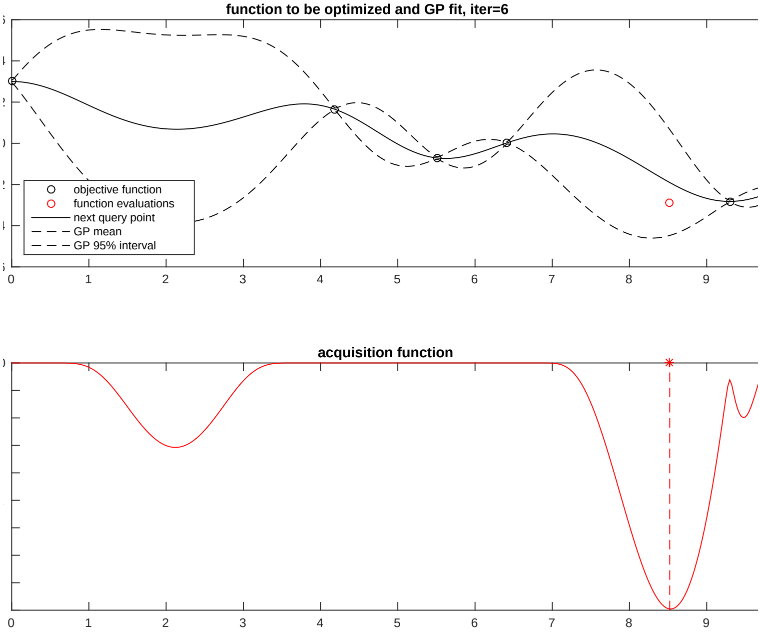

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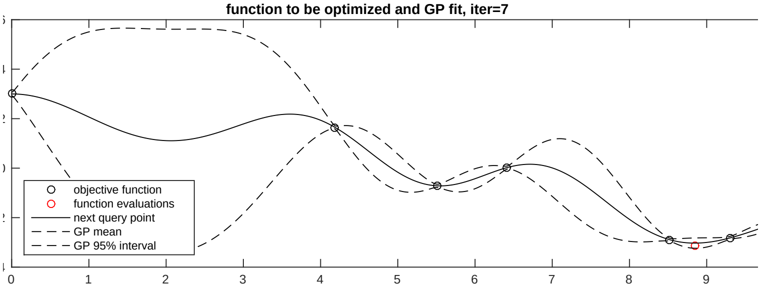

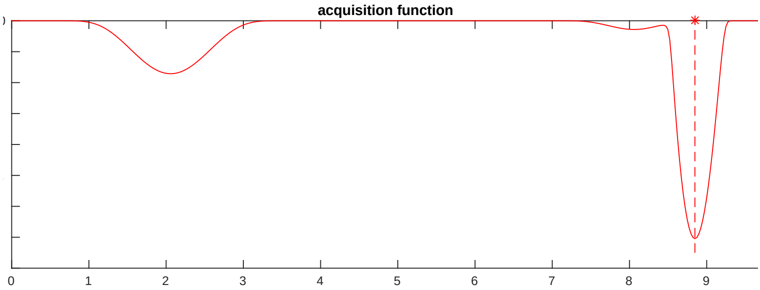

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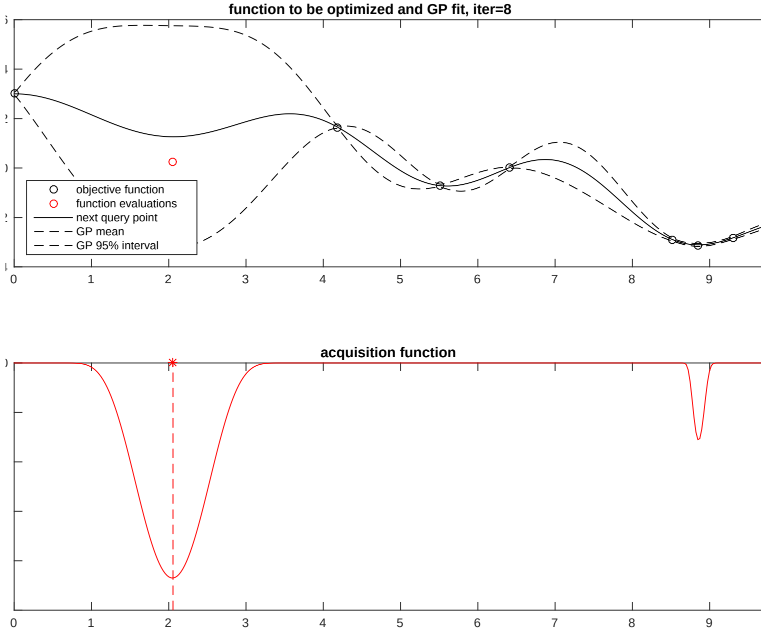

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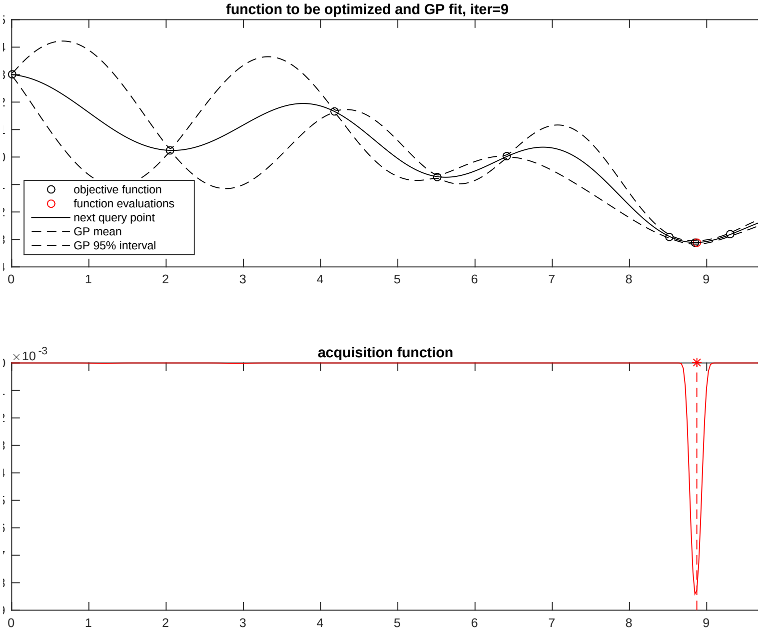

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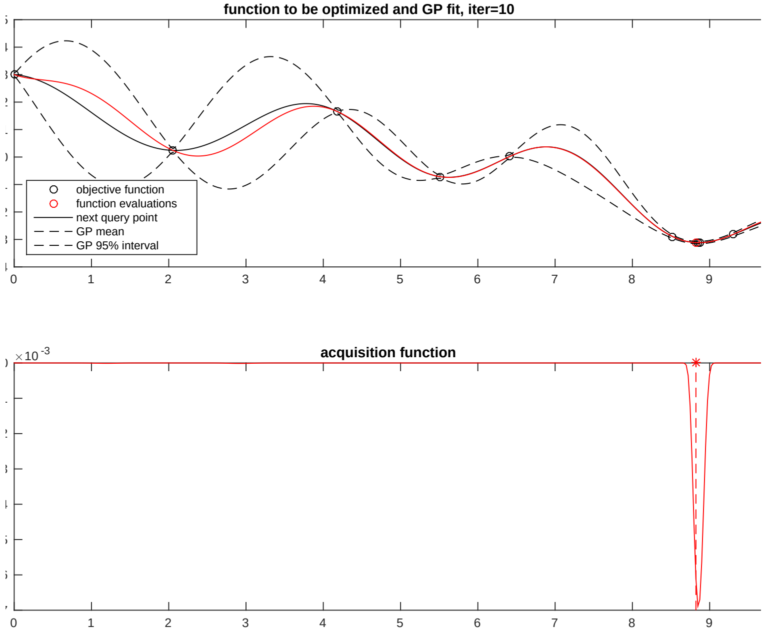

Examples of big Bayesian decision making success stories

- Bayesian optimization of ML algorithms
- A/B testing
- Customer retention / satisfaction
- Marketing