# RStan: the R interface to Stan

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#### **Abstract**

In this vignette we present the RStan package **rstan** for using Stan in R. Stan is a package for making Bayesian inferences using the No-U-Turn sampler (a variant of Hamiltonian Monte Carlo) or frequentist inference via optimization. We illustrate the features of RStan through an example in Gelman et al. (2003).

#### 1 Introduction

Stan is a C++ library for Bayesian modeling and inference that primarily uses the No-U-Turn sampler (NUTS) (Hoffman and Gelman 2012) to obtain posterior simulation given user-specified model and data. Alternatively, Stan can utilize the LBFGS optimization algorithm to maximize an objective function, such as a log-likelihood. The R package, **rstan** allows one to conveniently use Stan from R (R Core Team 2014) and to access Stan output, which includes posterior inferences and also intermediate quantities such as evaluation of the log posterior density and its gradients.

The website for Stan and RStan, http://mc-stan.org, provides up-to-date information about how to operate Stan and RStan. For example, "RStan Getting Started" (The Stan Development Team 2014a) has a couple of examples. The present article provides a concise introduction to the functionality of package **rstan** and provides pointers to many functions in **rstan** from the user's perspective.

We start with the prerequisites for using **rstan** (section 1.1) and a typical workflow of using Stan and RStan (section 1.2). In section 2, we illustrate the process of using **rstan** to estimate a Bayesian model. Section 3 presents further details on **rstan**. In section 4, we discuss some functions that **rstan** provides to access the results when Stan is used from the command line.

#### 1.1 Prerequisites

Users need to know how to specify statistical models using the Stan modeling language, which is detailed in the manual of Stan (The Stan Development Team 2014c). We give an example below. To do so, a C++ compiler is required, such

as g++1 or clang++2. There are instructions on how to install a C++ compiler at https://github.com/stan-dev/rstan/wiki/RStan-Getting-Started# prerequisites.

Package **rstan** depends on several other R packages:

- StanHeaders which provides the Stan C++ headers
- BH which provides Boost C++ headers
- **RcppEigen** which provides Eigen C++ headers
- **Rcpp** which facilitates using C++ from R
- inline which compiles C++ for use with R

These package dependencies should be automatically installed if you install the **rstan** package via one of the conventional mechanisms.

#### 1.2 Typical workflow of using RStan

Stan has a modeling language, which is similar to but not identical to that of the Bayesian graphical modeling package BUGS (Lunn et al. 2000). A parser translates the model expressed in the Stan modeling language to C++ code, whereupon it is compiled to an executable program and loaded as a Dynamic Shared Object (DSO) in R and can be called by the user. In summary, the following are typical steps of using Stan for Bayesian inference.

- a. Represent a statistical model by writing its log posterior density (up to an arbitrary normalizing constant that does not depend on the unknown parameters in the model) using the Stan modeling language. We recommend a separate text file for this, although it can be done using a character string within R.
- b. Translate the model coded in Stan modeling language to C++ code using the stanc function (which is called by the stan function)
- c. Compile the C++ code for the model using a C++ compiler to create a DSO (also called a dynamic link library (DLL)) that can be loaded by R (which is called by the stan).
- d. Run the DSO to sample from the posterior distribution using the stan or sampling functions.

http://gcc.gnu.org

<sup>&</sup>lt;sup>2</sup>http://clang.llvm.org

- e. Diagnose non-convergence of the MCMC chains
- f. Conduct inference based on the samples from the posterior distribution

Steps c, d, and e above are all performed implicitly by a single call to stan.

## 2 An example of using rstan

In section 5.5 of Gelman et al. (2003), a hierarchical model is used to model the effect of coaching programs on college admissions tests. The data, shown in Table 1, summarize the results of experiments conducted in eight high schools, with an estimated standard error for each, and these data and model are of historical interest as an example of full Bayesian inference (Rubin 1981). We use this example here for its simplicity and because it represents a nontrivial Markov chain simulation problem in that there is dependence between the parameters of original interest in the study — the effects of coaching in each of the eight schools — and the hyperparameter representing the variation of these effects in the modeled population. Certain implementations of a Gibbs sampler or a Hamiltonian Monte Carlo sampler can be slow to converge in this example. For short, we call this example "eight schools." The statistical model is specified as

$$y_j \sim \text{normal}(\theta_j, \sigma_j), \quad j = 1, \dots, 8$$
 (1)

$$\theta_1, \dots, \theta_8 \sim \text{normal}(\mu, \tau),$$
 (2)

in which each  $\sigma_j$ 's assumed known and a uniform prior density is used,  $p(\mu, \tau) \propto 1$ .

#### 2.1 Express the model in Stan

We first need to express this model in the Stan modeling language. The **rstan** package allows a model to be coded in a text file (typically with suffix .stan) or in a R character vector (of length one). We put the following text into a file called schools.stan:

```
data {
  int<lower=0> J; // number of schools
  real y[J]; // estimated treatment effects
  real<lower=0> sigma[J]; // s.e. of effect estimates
}
parameters {
```

	Estimated	Standard error
	treatment	of effect
School	effect, $y_j$	estimate, $\sigma_j$
A	28	15
В	8	10
C	-3	16
D	7	11
E	-1	9
F	1	11
G	18	10
Н	12	18

Table 1: Observed effects of coaching on college admissions test scores in eight schools. We fit these data using a hierarchical model allowing variation between schools.

```
real mu;
real<lower=0> tau;
vector[J] eta;
}
transformed parameters {
  vector[J] theta;
  theta <- mu + tau * eta;
}
model {
  eta ~ normal(0, 1);
  y ~ normal(theta, sigma);
}</pre>
```

The first section of the above code specifies the data that is conditioned upon by Bayes Rule: the number of schools, J; the vector of estimates,  $y_1, \ldots, y_J$ ; and the standard errors,  $\sigma_1, \ldots \sigma_J$ . Data are labeled as integer or real and can be vectors (or, more generally, arrays) if dimensions are specified. Data can also be constrained; for example, in the above model J has been restricted to be nonnegative and the components of  $\sigma_y$  must all be positive.

The next section of the code defines the parameters whose posterior distribution is sought using Bayes Rule. These are the their mean,  $\mu$ , and standard deviation,  $\tau$ , of the school effects, plus the standardized school-level effects  $\eta$ . In this model, we let the undstandardized school-level effects,  $\theta$ , be a transformed

parameter that uses  $\mu$  and  $\tau$  to shift and scale the standardized effects  $\eta$  instead of directly declaring  $\theta$  as a parameter. By parameterizing the model this way, the sampler runs more efficiently because the resulting multivariate geometry is more amendable to Hamiltonian Monte Carlo (Neal 2011).

Finally, the model block looks similar to standard statistical notation. (Just be careful: the second argument to Stan's normal $(\cdot, \cdot)$  distribution is the standard deviation, not the variance as is usual in statistical notation.) We have written the model in vector notation, which allows Stan to make use of more efficient algorithmic differentiation (AD). It would also be possible — but less efficient — to write the model by replacing y ~ normal (theta, sigma); with a loop over the J schools, for (j in 1:J) y[j] ~ normal (theta[j], sigma[j]);

Stan has versions of many of the most useful R functions for statistical modeling, including probability distributions, matrix operations, and special functions. However, the names of the Stan functions may differ from their R counterparts and more subtly, the parameterizations of probability distributions in Stan may differ from those in R for the same distribution. To mitigate this problem, the <code>lookup</code> function can be passed a R function or character string naming an R function, and rstan will attempt to look up the corresponding Stan function, display its arguments, and give the page number in The Stan Development Team (2014c) where the Stan function is discussed.

```
> lookup("dnorm")
   StanFunction
                                       Arguments ReturnType Page
344
                        (reals mu, reals sigma) real 369
         normal
    normal_log (reals y, reals mu, reals sigma)
                                                      real 369
   SamplingStatement
344
                TRUE
               FALSE
348
> tail(lookup("~")) # looks up all Stan sampling statements
                                                        Arguments Page
    StanFunction
562
     student t
                                 (reals nu, reals mu, reals sigma)
                                                                   372
608
        uniform
                                        (reals alpha, reals beta)
                                                                   393
      von mises
                                          (reals mu, reals kappa)
                                                                   391
618
                                        (reals alpha, reals sigma)
                                                                   383
621
        weibull
                                                                   386
627
         wiener (reals alpha, reals tau, reals beta, reals delta)
629
        wishart
                                           (real nu, matrix Sigma)
                                                                   405
> lookup(dwilcox) # no corresponding Stan function
```

```
[1] "no matching Stan functions"
```

If the lookup function fails to find an R function that corresponds to a Stan function, it will treat its argument as a regular expression and attempt to find matches with the names of Stan functions.

#### 2.2 User-defined Stan functions

Stan permits users to define their own functions in a functions block of a Stan program. The functions block is optional but if it exists, it must come before any other block. This mechanism allows users to implement statistical distributions or other functionality that is not currently available in Stan. However, even if the user's function merely wraps calls to existing Stan functions, the code in the model block can be much more readible if several lines of Stan code that accomplish one (or perhaps two) task(s) are replaced by a call to a user-defined function.

Another reason to utilize user-defined functions is that **rstan** provides an expose\_stan\_functions function that exports such functions to the R global environment so that they can be tested in R to ensure that they are working properly. For example,

```
> model_code <-
+ '
+ functions {
+    real standard_normal_rng() {
+        return normal_rng(0,1);
+    }
+ }
+ model {}
+ '
> expose_stan_functions(stanc(model_code = model_code))
> standard_normal_rng(seed = 1)
[1] -0.9529876
```

#### 2.3 Preparing the data

The stan function in **rstan** accepts data as a list or an environment. Alternatively the data argument can be omitted and R will search for objects that have the same names as in the data block of a Stan program. To prepare the data in R, we create a list as follows.

```
> schools_data <-
+ list(J=8,
+ y=c(28, 8, -3, 7, -1, 1, 18, 12),
+ sigma=c(15, 10, 16, 11, 9, 11, 10, 18))</pre>
```

It would also be possible (indeed, encouraged) to read in the data from a file rather than to directly enter the numbers in the R script.

#### 2.4 Sample from the posterior distribution

Next, we can call the stan function to draw posterior samples:

Function stan wraps the following three steps:

- a. Translate a model in Stan code to C++ code
- b. Compile the C++ code to a dynamic shared object (DSO) and load the DSO
- c. Sample given some user-specified data and other settings

A single call to stan performs all three steps, but they can also be executed one by one, which can be useful for debugging. In addition, Stan saves the DSO so that when the same model is fit again (possibly with new data), function stan can be called so that only the third step is performed, thus saving compile time.

Function stan returns an object of S4<sup>3</sup> class stanfit. If no error occurs, the returned stanfit object includes the samples drawn from the posterior distribution for the model parameters and other quantities defined in the model. If there is an error (for example, when we have syntax error in our Stan code), stan will either quit or return a stanfit object that contains no samples. Including the DSO as part of a stanfit object allows it to be reused so that compiling the same model could be avoided when we want to sample again with the same or different input of data and other settings. Also if an error happens after the model is

<sup>&</sup>lt;sup>3</sup>For those who are not familiar with the concept of class and S4 class in R, refer to Chambers (2008). A S4 class consists of some attributes (data) to model an object and some methods to model the behavior of the object. From a user's perspective, once a stanfit object is created, we are mainly concerned about what methods are defined for the stanfit.

compiled but before sampling (for example, problems with input such as data and initial values), we can reuse the previous compiled model. For class stanfit, many methods such as print and plot are defined to work with the samples and conduct model inference. For example, the following shows a summary of the parameters for our example using function print.

```
> print(fit1, pars=c("theta", "mu", "tau", "lp___"),
      probs=c(.1,.5,.9))
Inference for Stan model: schools.
4 chains, each with iter=2000; warmup=1000; thin=1;
post-warmup draws per chain=1000, total post-warmup draws=4000.
        mean se_mean sd
                         10%
                               50%
                                    90% n eff Rhat
theta[1] 11.60 0.20 8.31 2.49 10.30 22.58 1678
theta[2] 7.94
               0.11 6.35 0.34
                             7.89 15.83
                                        3364
theta[3] 6.18 0.16 7.44 -2.73 6.63 14.65 2223
                                                1
theta[4] 7.54 0.13 6.81 -0.80 7.60 15.89 2765
                                                1
theta[5] 5.15 0.12 6.41 -3.15 5.57 12.78 2928
theta[6] 6.06 0.13 6.79 -2.60 6.39 14.12 2876
                                               1
theta[8] 8.36 0.16 7.86 -0.64 8.15 17.52 2464
                                               1
             0.14 5.03 1.59 7.80 14.20 1274
        7.87
tau
       6.56 0.18 5.35 1.04 5.40 13.59 927
                                                1
lp
      -4.81 0.09 2.57 -8.10 -4.58 -1.73
                                          906
```

Samples were drawn using NUTS(diag\_e) at Tue Jan 5 11:16:44 2016. For each parameter, n\_eff is a crude measure of effective sample size, and Rhat is the potential scale reduction factor on split chains (at convergence, Rhat=1).

The last line of this output, 1p\_\_\_, is the logarithm of the (unnormalized) posterior density as calculated by Stan. This log density can be used in various ways for model evaluation and comparison (see, e.g., Vehtari and Ojanen 2012).

#### 3 Advanced features

In this section, we discuss more details and other advanced features of **rstan**. The details pertain to the optional arguments of the stan function, data preprocessing, and methods for the S4 class stanfit. In addition, we discuss optimization, which can be used to obtain a point estimates via Stan.

#### **3.1 Arguments to the stan function**

The primary arguments for sampling (in function stan and sampling) include data, initial values, and the options of the sampler such as chains, iter, and warmup. In particular, warmup specifies the number of iterations that are used by NUTS sampler for the adaptation phase before sampling begins. After the warmup, the sampler turns off adaptation and continues until a total of iter iterations have been completed. There is no theoretical guarantee that the samples are drawn from the posterior distribution during warmup, so the warmup samples should only be used for diagnosis and not for inference. The summaries for the parameters shown by the print method are calculated using only the samples after warmup.

For function stan, argument init is used for specifying the initial values. There are several options for init and the details can be found in the documentation of the stan function. The vast majority of the time, it is adequate to allow Stan to generate its own initial values randomly. However, sometimes it is better to specify the initial values for at least a subset of the objects declared in the parameters block of a Stan program.

Stan uses a random number generator (RNG) that supports parallelism. The initialization of the RNG is determined by arguments seed and chain\_id. Even if we are running multiple chains from one call to the stan, function we only need to specify one seed, which is randomly generated by R if not specified.

#### 3.2 Data preprocessing and passing

The data passed to stan will go through a preprocessing procedure. The details of this preprocessing are documented in the help for function stan. Here we stress a few important steps. First, **rstan** allows the user to specify more than what is declared in the data block and anything beyond that is silently omitted. In general, an element in the input R list should be numeric data and its dimension should match the declaration in the data block of the model. So for example, factor type in R is not supported as data element for RStan and must be converted to integer codes via as.integer(). The Stan modeling language distinguishes between integers and doubles (type int and real in Stan modeling language, respectively). The stan function will convert some R data (which is double-precision usually) to integers if possible.

In Stan, we have scalars and other types that are a set of scalars, such as vectors and matrices. As R does not have scalars, **rstan** treats vectors of length

one as scalars. However, we might have a model with data block defined as in Figure 1, in which N can be 1 as a special case. So if we know that N is always larger than 1, we can use a vector of length N in R as the data input for y (for example, a vector created by "y <- rnorm(N)"). If we want to prevent **rstan** from treating the input data for y as a scalar when N=1, we need to explicitly make it an array as the following R code shows.

```
> y <- as.array(y)

data {
   int<lower=0> N;
   real y[N];
}
```

Figure 1: Data block of an example model in Stan code

As Stan cannot handle missing values in data automatically, so no element of the data can contain NA in R. An important step in **rstan**'s data preprocessing is to check missing values and issue an error if any. To model missing values using Stan, you should create binary indicators of whether a data point is observed or missing and then change the NA values in R to valid numbers before calling stan.

#### **3.3 Methods for the** stanfit **class**

For the fitted object that is an instance of the S4 class stanfit, we have defined methods such as print, summary, plot, pairs, and traceplot. We can use these methods to assess the convergence of the Markov chains by looking at the trace plots and calculating the split  $\hat{R}$ . The print method outputs the mean, standard deviation, quantiles of interest, split  $\hat{R}$ , and effective sample size for each unknown quantity over all the chains combined.

The plot method provides an overview of the output, while the pairs method shows two-dimensional density plots for each pair of unknown quantities that are stratified according to the condition argument. The traceplot method plots the traces of all chains for the specified parameters. If we include the warmup draws by setting inc\_warmup=TRUE (the default), the background color of the warmup area is different from the post-warmup phase.

<sup>&</sup>lt;sup>4</sup>Split  $\hat{R}$  is an updated version of  $\hat{R}$  statistic proposed in Gelman and Rubin (1992) that is based on splitting each chain into two halves. See the Stan manual for more details.

Figure 2 presents the plot of the eight schools example. In this plot, credible intervals (by default 80%) for all the parameters as well as lp\_\_ (the log of posterior density function up to an additive constant), and the median of each chain are displayed. In addition, under the lines representing intervals, small colored areas are used to indicate which range the value of the split  $\hat{R}$  statistic is in. Figure 3 shows the traceplot for the  $\tau$  parameter.

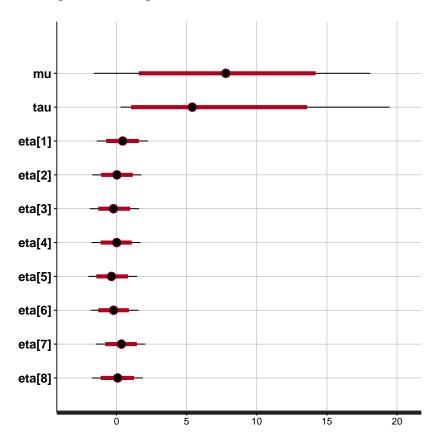


Figure 2: An overview plot for eight schools example

The stanfit class has a set of methods to work with the samples drawn from the posterior distribution. First, the extract method provides different ways to access the samples. If the argument permuted is TRUE, then the samples after warmup are returned in an permuted order as a list, each element of which are the samples for a parameter. Here by "one parameter", we mean a scalar/vector/array parameter as a whole defined in the parameters block, transformed parameters block, or generated quantities block of our Stan program. In the eight schools

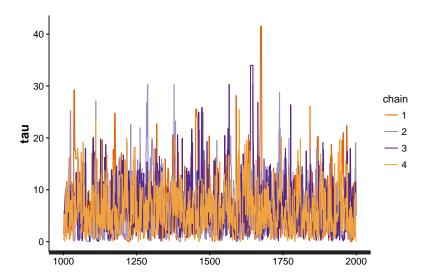


Figure 3: Trace plots of  $\tau$  in the eight schools model

example,  $\theta$  is one parameter though it is an array of length J.

If permuted=FALSE, the result depends on the inc\_warmup argument. In either case, the returned object is an array with the first dimension indicating iterations, the second indicating chains, and the third indicating parameters. If inc\_warmup=TRUE, all iterations are included and if inc\_warmup=FALSE, only the post-warmup iterations are included. The latter is appropriate for inference, while the former may be useful for diagnosis. In the returned array, each vector/array parameter is "flattened" and are included as the third dimension of the array. In our eight schools examples, the third dimension is theta[1], ..., theta[8].

```
> s <- extract(fit1, pars = c("theta", "mu"), permuted = TRUE)
> names(s)

[1] "theta" "mu"
> dim(s$theta)

[1] 4000 8
> dim(s$mu)

[1] 4000
```

In addition, the as.array, as.matrix, and as.data.frame methods are defined for stanfit object. These method return the draws of samples in forms of a 3-dimension array, matrix (rbinding the chains), or data.frame (that is coerced from a matrix). There are also dimnames and names methods for stanfit objects.

A stanfit object keeps all the information regarding the sampling procedure, for example, the model in Stan code, the initial values for all parameters, the seed for the RNG, and parameters used for the sampler (for example, the step size for NUTS) The following methods

```
    get_seed
    get_inits
    get_adaptation_info
    get_sampler_params
```

for shown in Table 2 along with other methods defined for the stanfit class.

Last, a common feature for many methods that are defined for the stanfit class is that the pars argument can be specified to indicate a subset of the parameters. This feature is helpful when there are too many parameters in the model or when we need to reduce memory usage. For instance, in the eight schools example, we have parameter  $\theta$  defined as "real theta[J]". So we can specify pars="theta" or pars="theta[1]". However, specifying part of  $\theta$ 

(i.e., pars="theta[1:2]") as in R is not allowed — a workaround for this is to specify pars=c("theta[1]", "theta[2]"). The stan function allows the user to specify pars so that only part of the samples are returned, which might be problematic from the perspective of diagnosing MCMC convergence since we would apply our diagnostic criterion to a subset of our parameters. To mitigate this loss of diagnostics information, we can use the get\_posterior\_mean function, which returns the posterior mean of all parameters for each chain and all chains combined (excluding warmup samples). Another alternative is to write the samples to external files using the sample\_file argument of the stan function and then conduct diagnostics with the external files.

#### 3.4 Sampling difficulties

The best way to visualize the output of a model is through the **shinyStan** package, which is currently only available from https://github.com/stan-dev/shinystan. The **shinyStan** package facilitates both the visualization of parameter distributions and diagnosing problems with a sampler.

However, it is also possible to diagnose problems with a sampler directly via the get\_sampler\_params function.

```
> # all chains combined
> summary(do.call(rbind, args = get_sampler_params(fit1, inc_warmup = TRUE)),
        digits = 2)
accept_stat__
             stepsize___
                            treedepth__ n_leapfrog__
                                                     n divergent
Min. :0.00 Min. : 0.031 Min. :1 Min. : 1.0
                                                      Min. :0.000
1st Qu.: 0.74   1st Qu.: 0.367   1st Qu.: 3   1st Qu.: 7.0   1st Qu.: 0.000
Median: 0.93 Median: 0.439 Median: 3 Median: 7.0
                                                      Median : 0.000
                                        Mean : 8.5
Mean :0.81 Mean : 0.462
                            Mean :3
                                                      Mean :0.011
3rd Qu.:0.99
             3rd Qu.: 0.450
                            3rd Qu.:3 3rd Qu.: 7.0
                                                      3rd Qu.:0.000
Max. :1.00
             Max. :14.386 Max. :7
                                        Max. :127.0
                                                      Max. :1.000
> # each chain separately
> lapply(get_sampler_params(fit1, inc_warmup = TRUE), summary, digits = 2)
[[1]]
accept_stat___
             stepsize___
                            treedepth__ n_leapfrog__ n_divergent__
Min. :0.00
             Min. :0.048
                            Min. :1 Min. : 1.0 Min. :0.000
                            1st Qu.:3 1st Qu.: 7.0 1st Qu.:0.000
1st Qu.:0.69
             1st Qu.:0.431
Median :0.92 Median :0.439
                            Median: 3 Median: 7.0 Median: 0.000
Mean :0.78
             Mean :0.463
                            Mean :3 Mean :8.2 Mean :0.009
                            3rd Qu.:3 3rd Qu.: 7.0 3rd Qu.:0.000
3rd Qu.:0.98
             3rd Qu.:0.439
```

```
:1.00
                       :9.155
                                        : 6
                                                     :63.0
                                                                     :1.000
Max.
                Max.
                                Max.
                                              Max.
                                                              Max.
[[2]]
accept_stat___
                                   treedepth___
                                                 n_leapfrog___
                                                                n_divergent__
                  stepsize___
       :0.00
Min.
                Min. : 0.048
                                 Min.
                                         :1.0
                                                Min.
                                                      : 1.0
                                                                Min.
                                                                       :0.000
1st Ou.:0.81
                1st Ou.: 0.367
                                  1st Qu.:3.0
                                                1st Ou.: 7.0
                                                                1st Ou.:0.000
Median :0.95
                Median : 0.367
                                 Median :3.0
                                                Median: 7.0
                                                                Median : 0.000
                                                                       :0.012
Mean
      :0.83
                Mean
                     : 0.437
                                  Mean :3.2
                                                Mean : 9.4
                                                                Mean
3rd Qu.:0.99
                3rd Qu.: 0.439
                                  3rd Qu.:4.0
                                                3rd Qu.:15.0
                                                                3rd Qu.:0.000
Max. :1.00
                Max. :14.386
                                  Max.
                                        :6.0
                                                Max.
                                                       :63.0
                                                                Max.
                                                                       :1.000
[[3]]
accept_stat__
                  stepsize___
                                 treedepth__ n_leapfrog__
                                                               n_divergent__
Min.
       :0.00
                Min.
                      :0.038
                                 Min.
                                        :1
                                              Min.
                                                         1.0
                                                               Min.
                                                                      :0.000
1st Qu.:0.73
                1st Qu.:0.447
                                                               1st Qu.:0.000
                                 1st Qu.:3
                                              1st Qu.:
                                                         7.0
Median :0.93
                Median : 0.450
                                 Median :3
                                              Median :
                                                         7.0
                                                               Median : 0.000
Mean
       :0.80
                Mean
                       :0.470
                                 Mean
                                        :3
                                              Mean
                                                     :
                                                         8.4
                                                               Mean
                                                                      :0.011
                                                         7.0
3rd Ou.:0.99
                3rd Ou.:0.450
                                 3rd Ou.:3
                                              3rd Ou.:
                                                               3rd Ou.:0.000
       :1.00
                Max.
                       :6.325
                                 Max.
                                        :7
                                              Max.
                                                     :127.0
                                                               Max.
                                                                      :1.000
Max.
[[4]]
accept_stat__
                  stepsize___
                                 treedepth__ n_leapfrog__
                                                              n_divergent__
       :0.00
                      :0.031
                                        :1
                                              Min.
                                                     : 1.0
                                                              Min.
                                                                     :0.00
Min.
                Min.
                                 Min.
1st Ou.:0.75
                1st Ou.:0.432
                                 1st Ou.:3
                                              1st Ou.: 7.0
                                                              1st Ou.:0.00
Median :0.93
                Median :0.432
                                              Median: 7.0
                                 Median :3
                                                              Median :0.00
                                        :3
Mean
       :0.81
                Mean
                       :0.478
                                Mean
                                              Mean
                                                      : 7.9
                                                              Mean
                                                                     :0.01
3rd Qu.:0.99
                3rd Qu.:0.477
                                 3rd Qu.:3
                                              3rd Qu.: 7.0
                                                              3rd Qu.:0.00
Max.
       :1.00
                Max.
                       :7.078
                                 Max.
                                        :7
                                              Max.
                                                     :79.0
                                                              Max.
                                                                     :1.00
```

Here we see that there are a small number of divergent transitions, which are identified by n\_divergent\_\_ being 1. Ideally, there should be no divergent transitions after the warmup phase. The best way to try to eliminate divergent transitions is by increasing the target acceptance probability, which by default is 0.8. Here we see that the mean of accept\_stat\_\_ is close to 0.8 for all chains, but has a very skewed distribution because the median is near 0.95. We could go back and call stan again and specify the optional argument control = list(adapt\_delta = 0.9) to eliminate the divergent transitions. However, sometimes when the target acceptance rate is high, the stepsize is very small and the sampler hits its limit on the number of leapfrog steps it can take per iteration. In this case, it is a non-issue because each chain has a treedepth\_\_ of at most 7 and the default is 10. But if any treedepth\_\_ were 11, then it would be wise to increase the limit by passing control = list(max\_treedepth = 12) (for example) to stan.

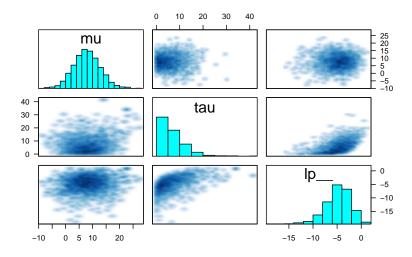


Figure 4: Pairs plots of the common parameters in the eight schools model

Figure 4 gives a graphical representation of the same information. The marginal distribution of each indicated parameter is included as a histogram. By default, draws with below-median accept\_stat\_\_ are plotted below the diagonal and those with above-median accept\_stat\_\_ are plotted above the diagonal. Each off-diagonal square represents a bivariate distribution of the draws for the intersection of the row-variable and the column-variable. Ideally, the below-diagonal intersection and the above-diagonal intersection of the same two variables should have distributions that are mirror images of each other. Any yellow points would indicate transitions where the maximum treedepth\_\_ was hit, and the red points indicate a transition where n\_divergent\_ = 1. Thus, the pairs plot should be used to get a sense of whether any sampling difficulties are occurring in the tails or near the mode.

#### 3.5 The log posterior function and its gradient

Essentially, we define the log of the probability density function of a posterior distribution up to an unknown additive constant. In Stan, we use  $lp_{\underline{}}$  to represent the realizations of this log kernel at each iteration. In **rstan**,  $lp_{\underline{}}$  is treated as an unknown in the summary and the calculation of split  $\hat{R}$  and effective sample size.

A nice feature of **rstan** is that functions for calculating lp\_\_ and its gradients

for a stanfit object are exposed. They are defined for a stanfit object, since we need data to create a model instance. These two functions are  $log_prob$  and  $grad_log_prob$  respectively. Both take parameters on the *unconstrained* space, even if the support of a parameter is not the whole real line. See The Stan Development Team (2014c) for more details about transformations from the entire real line to some subspace of it. Also the number of unconstrained parameters might be less than the number of parameters. For example, when a parameter is a simplex of length K, the number of unconstrained parameters are K-1 due to the constraint that all elements of a simplex must be nonnegative and sum to one. The  $get_num_upars$  method is provided to get the number of unconstrained parameters, while the unconstrained\_pars and constrained\_pars methods can be used to unconstrain or constrain parameters respectively. The former takes a list of parameters as input and transforms it to an unconstrained vector, and the latter does the opposite. Using these functions, we can implement other algorithms such as maximum a posteriori estimation of Bayesian models.

#### 3.6 Optimization in Stan

RStan also provides an interface to Stan's optimizers, which can be used to obtain a point estimate by maximizing the (perhaps penalized) likelihood function defined by a Stan program. We illustrate the feature using a very simple example, estimating the mean from samples assumed to be drawn from normal distribution with known standard deviation. That is, we assume

```
y_1, \ldots, y_n \sim \text{normal}(\mu, 1).
```

By specifying prior of  $\mu$  with  $p(\mu) \propto 1$ , the maximum a posteriori estimator for  $\mu$  is just the sample mean. The following R code shows how to use Stan's optimizers in **rstan**; we first create a stanmodel object of **rstan** and then use its optimizing method, to which data and other arguments can be fed.

```
> ocode <- "
+    data {
+        int<lower=1> N;
+        real y[N];
+    }
+    parameters {
+        real mu;
+    }
```

Name	Function
print	print the summary for parameters obtained using all chains
summary	summarize the sample from all chains and individual chains for parameters
plot	plot the inferences (intervals, medians, split $\hat{R}$ ) for parameters
traceplot	plot the traces of chains
pairs	make a matrix of scatter plots for the samples of parameters
extract	extract samples of parameters
get_stancode	extract the model code in Stan modeling lan- guage
get_stanmodel	extract the stanmodel object
get_seed	get the seed used for sampling
get_inits	get the initial values used for sampling
get_posterior_mean	get the posterior mean for all parameters
get_logposterior	get the log posterior (that is, 1p)
<pre>get_sampler_params</pre>	get parameters used by the sampler such as treedepth of NUTS
<pre>get_adaptation_info</pre>	get adaptation information of the sampler
get_num_upars	get the number of parameters on unconstrained space
unconstrain_pars	transform parameter to unconstrained space
constrain_pars	transform parameter from unconstrained space to its defined space
log_prob	evaluate the log posterior for parameter on un- constrained space
grad_log_prob	evaluate the gradient of the log posterior for parameter on unconstrained space
as.array	extract the samples excluding warmup to a three
as.matrix	dimension array, matrix, data.frame
as.data.frame	•
dimnames	obtain the dimension names of the object in its array representation
names	obtain the "flattened" parameter names

Table 2: Methods for the S4 class stanfit

```
model {
      y \sim normal(mu, 1);
> sm <- stan_model(model_code = ocode)</pre>
> y2 <- rnorm(20)
> mean(y2)
[1] -0.1228876
> op <- ifelse(grepl('SunOS', Sys.info()['sysname']),</pre>
    "This may not work on Solaris",
    optimizing(sm, data = list(y = y2, N = length(y2)), hessian = TRUE))
STAN OPTIMIZATION COMMAND (LBFGS)
init = random
save\_iterations = 1
init_alpha = 0.001
tol_obj = 1e-12
tol_grad = 1e-08
tol_param = 1e-08
tol_rel_obj = 10000
tol_rel_grad = 1e+07
history_size = 5
seed = 1478894135
initial log joint probability = -33.8126
Optimization terminated normally:
  Convergence detected: gradient norm is below tolerance
> print(op)
[[1]]
        mu
-0.1228876
```

## 3.7 Model compiling in rstan

In RStan, for every model, we use function stanc to translate the model from Stan modeling language code to C++ code and then compile the C++ code to dynamic shared object (DSO), which is loaded by R and executed to draw sample. The process of compiling C++ code to DSO, sometimes, takes a while. When the model is the same, we could reuse the DSO from previous run. In function stan, if parameter fit is specified with a previous fitted object, the compiled model is

reused. When reusing a previous fitted model, we can specify different data and other parameters for function stan.

In addition, if fitted models (objects in our working space of R) are saved, for example, by R function save and save.image, **rstan** is able to save the DSO for models, so that they can be used across R sessions. To (not) save the DSO, specify the save\_dso argument, which is TRUE by default, in the stan function.

If the user executes rstan\_options (auto\_write = TRUE), then a serialized version of the compiled model will be automatically saved to the hard disk in the same directory as the .stan file or in R's temporary directory if the Stan program is expressed as a character string. Although this option is not enabled by default due to CRAN policy, it should ordinarily be specified by users in order to eliminate redundant compilation.

Stan runs much faster when the code is compiled at the maximum level of optimization, which is -03 on most C++ compilers. However, the default value is -02 in R, which is appropriate for most R packages but entails a slight slowdown for Stan. You can change this default locally by following the instructions at http://cran.r-project.org/doc/manuals/r-release/R-admin.html#Customizing-package-compilation. However, you should be advised that setting CXXFLAGS = -03 may cause adverse side effects for other R packages.

#### 3.8 Run multiple chains in parallel

For function stan, we can specify the number of chains using the chains argument. By default, the chains are executed serially (i.e., one at a time) using the parent R process. There is a cores argument to stan and sampling that can be set to the number of chains (if the hardware has sufficient processors and RAM), which is appropriate on most laptops. We ordinarily recommend first calling options (mc.cores = parallel::detectCores()) once per R session so that stan and sampling can utilize all available cores.

If you are using another parallelization scheme (perhaps with a remote cluster) **rstan** provides a function called sflist2stanfit that consolidates a list of multiple stanfit objects (sampled from one model with the same number of warmup and iteration) into one stanfit object. It is important to specify the same seed for all the chains and equally important to use a different chain ID (argument chain\_id). This ensures that the random numbers generated in Stan for all chains are essentially independent. This part is handled automatically by

## 4 Working with CmdStan

RStan provides some functions to help use Stan from the command line, CmdStan. First, when Stan reads data or initial values, it supports a subset of the syntax of R dump data formats. So if we use dump function in R to prepare data, Stan might not be able to read the data. The stan\_rdump function in **rstan** dumps the data from R to a format that is supported by Stan with symantics that are very similar to the dump function in R.

Second, the read\_stan\_csv function in **rstan** creates a stanfit object from reading the comma separated files (CSV) generated by CmdStan. As a result, we can use any methods defined for the stanfit class to diagnose and analyze the samples.

## 5 Summary

In this vignette, we have described the main functionality of RStan from a user's perspective. The help pages with the **rstan** package provide more details for all exposed **rstan** functions. The Stan manual (The Stan Development Team 2014c) provides many details and includes a variety of model examples, many of which are can be executed via function <code>stan\_demo</code> in the **rstan** package. Finally, the **loo** package, which is on CRAN, is very useful for model comparison using <code>stanfit</code> objects.

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