

1 (a) Note that $\frac{1}{1+e^z} = 1 - \sigma(z)$ and $\frac{\partial \sigma(z)}{\partial z} = \sigma(z)(1 - \sigma(z))$.

$$\begin{aligned}
 \frac{\partial E(w)}{\partial w} &= \\
 & y^T \frac{1}{\sigma(Xw)} \sigma(Xw)(1 - \sigma(Xw))X \\
 & + \frac{1}{1 - \sigma(Xw)} (-\sigma(Xw)(1 - \sigma(Xw)))X - y^T \frac{1}{(1 - \sigma(Xw))} (-\sigma(Xw)(1 - \sigma(Xw)))X \\
 & = y^T X - y^T \sigma(Xw)X - \sigma(Xw)X + y^T \sigma(Xw)X \\
 & = y^T X - \sigma(Xw)X \\
 \frac{\partial^2 E(w)}{\partial w} &= -\sigma(Xw)(1 - \sigma(Xw))XX^T
 \end{aligned}$$