Nama : Afifah Khoirunnisa
Kelas : TI - 2F
No/NIM: 04 / 2341720250

No.
Date: 26 Nov. 2024

INTEGRASI NUMERIK

1. $\int_0^1 \frac{1}{(2x-1)^3}\,dx \Rightarrow \int_0^1 (2x-1)^{-3}\,dx$

⇒ Metode Analitik

Misal : $u = 2x-1$

$Du = 2dx$

$\frac{Du}{2} = \frac{2dx}{2}$

$\frac{Du}{2} = dx$

$\int u^3 \frac{Du}{2}\,dx = \frac{1}{2}\int \frac{u^{-3+1}}{-3+1}$

$= \frac{1}{2}\int \frac{u^{-2}}{-2}$

$= -\frac{1}{4}u^{-2}$

$= -\frac{1}{4}\cdot\frac{1}{(2x-1)^2}$

$= -\frac{1}{4(2x-1)^2}$

$\int_0^1 \frac{1}{(2x-1)^3}\,dx = -\frac{1}{4(2(1)-1)^2} - \left(-\frac{1}{4(2(0)-1)^2}\right)$

$= -\frac{1}{4} + \frac{1}{4}$

$= 0$

⇒ Metode Numerik

$f(x) = \frac{1}{(2x-1)^3}$

Interval = [0, 1]

N = 10

$\Delta x = \frac{1-0}{10} = 0,1$

Iterasi	x	f(x)
1	0	-1
2	0,1	-1,953
3	0,2	-4,629
4	0,3	-15,625
5	0,4	-125
6	0,5	0 (Tidak Terdefinisi)
7	0,6	125
8	0,7	15,625
9	0,8	4,629
10	0,9	1,953
11	1	1

⇒ $x = 0 \rightsquigarrow \frac{1}{(2(0)-1)^3} = \frac{1}{-1} = -1$

⇒ $x = 1 \rightsquigarrow \frac{1}{(2(0,1)-1)^3} = \frac{1}{-0,512} = -1,953$

⇒ $x = 2 \rightsquigarrow \frac{1}{(2(0,2)-1)^3} = \frac{1}{-0,216} = -4,629$

⇒ x selanjutnya menyesuaikan

- Metode Reimann

$\sum f(x) \times \Delta x$ = tak terdefinisi × 0,1

= tak terdefinisi

- Metode Trapezoid

$\frac{\Delta x}{2}\left[f(x)_0 + 2\sum f(x) + f(x)_{n-1}\right]$

$= \frac{0,1}{2}\left[-1 + \text{tak terdefinisi} + 1\right]$

= tak terdefinisi

⇒ Metode Simpson 1/3

$\frac{\Delta x}{3}\left[f(x_0) + 4(fx_{ganjil}) + 2(fx_{genap}) \ldots + f(x_n)\right] =$

$\frac{0,1}{3}\left[-1 + \text{tak terdefinisi} + (-1,777) + 1\right] = \text{tak terdefinisi}$

⇒ Metode Simpson 3/8

$\frac{3\Delta x}{8}\left[f(x_0) + 3\sum(fx_{non\text{-}kel.3}) + 2\sum f(x_{kel})_3 + \ldots + f(x)_n\right]$

$\Rightarrow \frac{3(0,1)}{8}\left[-1 + \text{tak terdefinisi} + 390,625 + 1\right]$

$\Rightarrow \frac{0,3}{8}\left[\text{tak terdefinisi}\right]$

⇒ tak terdefinisi

3. $\int_1^3 (5x^2 - 6x + \frac{12}{x^2})\,dx = \int_1^3 (5x^2 - 6x + 12x^{-2})\,dx$

⇒ Metode Analitik

$\int 5x^2 - 6x + 12x^{-2}\,dx = \frac{1}{3}\int 5x^3 - \frac{1}{2}\int 6x^2 - \int 12x^{-1}$

$= \frac{5}{3}x^3 - 3x^2 - 12x^{-1}$

$= \frac{5}{3}x^3 - 3x^2 - \frac{12}{x}$

$\int_1^3 5x^2 - 6x + 12x^{-2}\,dx = \left(\frac{5}{3}(3)^3 - 3(3)^2 - \frac{12}{3}\right) - \left(\frac{5}{3}(1)^3 - 3(1)^2 - \frac{12}{1}\right)$

$= \left(\frac{5}{3}(27) - 3(9) - 4\right) - \left(\frac{5}{3} - 3 - 12\right)$

$= (45 - 27 - 4) - \left(\frac{5}{3} - 3 - 12\right) =$

= 14 - (-13,333)

= 27,333

→ Metode Numerik

$f(x) = 5x^2 - 6x + \frac{12}{x^2}$

N = 10

$\Delta x = \frac{3-1}{10} = \frac{2}{10} = 0,2$

Iterasi	x	f(x)
1	1	11
2	1,2	8,333
3	1,4	7,502
4	1,6	7,88
5	1,8	9,10
6	2	11
7	2,2	13,47
8	2,4	16,48
9	2,6	19,9
10	2,8	23,9
11	3	28,3

→ $x = 1 \rightsquigarrow 5(1)^2 - 6(1) + \frac{12}{(1)^2} = 5 - 6 + 12 = 11$

→ $x = 1,2 \rightsquigarrow 5(1,2)^2 - 6(1,2) + \frac{12}{(1,2)^2} = 7,2 - 7,2 + 8,33 = 8,33$

→ × Selanjutnya menyesuaikan.

No.

Date:

⇒ Metode Reimann

$\Delta x \cdot \Sigma f(x_n) = 0,2 \cdot 157,0488 = 31,4$

Error = |Metode Reimann - Metode Analitik|

= |31,4 - 27,3|

= 4,1

⇒ Metode Trapezoid

$\frac{\Delta x}{2}\left[f(x_0) + 2f(x_n) + f x_{n-1}\right]$

$\frac{0,2}{2}\left[11 + 2(117,7154) + 28,3\right]$

$0,1\left[274,7642\right] = 27,47642$

Error = |Metode Trapezoid - Metode Analitik|

= |27,47642 - 27,3333|

= 0,14312

⇒ Metode Simpson 1/3

$\frac{\Delta x}{3}\left[f(x_0) + 4\Sigma f(x_{ganjil}) + 2\Sigma f(x_{genap}) + f(x_{n-1})\right]$

$\Rightarrow \frac{0,2}{3}\left[11 + 270,5391 + 100,1613 + 28,333\right]$

$\Rightarrow 0,06\left[410,0337\right]$

$\Rightarrow 27,33558$

Error = |Metode Simpson 1/3 - Metode Analitik|

= |27,33558 - 27,33333|

= 0,00225

o) Metode Simpson 3/8

$$\frac{3\Delta x}{8}\left[f(x_0) + 3\Sigma f(x_{non\text{-}kel\,3}) + 2\Sigma f(x_{kel\,3}) + f(x_{n-1})\right]$$

$$\Rightarrow \frac{3(0,2)}{8}\left[11 + 217,2539 + 42,73368 + 28,333\right]$$

$$\Rightarrow \frac{0,6}{8}\left[347,1821\right]$$

$$\Rightarrow 0,075\ \left[347,1821\right]$$

$$\Rightarrow 26,03866$$

Error = |Metode Simpson 3/8 − Metode Analitik|

= |26,03866 − 27,33333|

= 1,29467.

4. $\int_{-1}^{2}(2x^2-3)^2\,dx = \int_{-1}^{2}(2x^2-3)(2x^2-3)\,dx = \int_{-1}^{2}(4x^4-12x^2+9)\,dx$

a) Metode Analitik

$$\int 4x^4-12x^2+9\,dx = \frac{4x^5}{5} - 4x^3 + 9x$$

$$\int_{-1}^{2}(2x^2-3)^2\,dx = \left(\frac{4(2)^5}{5} - 4(2)^3 + 9(2)\right) - \left(\frac{4(-1)^5}{5} - 4(-1)^3 + 9(-1)\right)$$

$$= \left(\frac{4(32)}{5} - 4(8) + 18\right) - \left(\frac{4(-1)}{5} - 4(-1) - 9\right)$$

$$= \left(\frac{128}{5} - 32 + 18\right) - \left(\frac{(-4)}{5} - (-4) - 9\right)$$

$$= \left(\frac{128}{5} - 14\right) - \left(\frac{-4}{5} - 5\right)$$

$$= \left(\frac{128}{5} - \frac{70}{5}\right) - \left(\frac{-4}{5} - \frac{25}{5}\right)$$

$$= \frac{58}{5} - \left(\frac{-29}{5}\right) = \frac{87}{5} = 17,4$$

No.

Date:

⇒ Metode Numerik

$f(x) = (2x^2 - 3)^2$

$N = 9$

$\Delta x = \frac{2 - (-1)}{9} = \frac{3}{9} = \frac{1}{3} = 0,3$

Iterasi	x	f(x)
1	-1	1
2	-0,7	4,0804
3	-0,4	7,1824
4	-0,1	8,8804
5	0,2	8,5264
6	0,5	6,25
7	0,8	2,9584
8	1,1	0,3364
9	1,4	0,8464
10	1,7	7,7284
11	2	25

$x = -1 \rightsquigarrow (2(-1)^2 - 3)^2 = (2 - 3)^2 = (-1)^2 = 1$

$x = 2 \rightsquigarrow (2(2)^2 - 3)^2 = (2(4) - 3)^2 = (8 - 3)^2$

$= 5^2 = 25$

X Lainnya menyesuaikan.

⇒ Metode Reimann

$\Delta x \cdot \Sigma f(x_n) = 0,3 \cdot 72,7892 = 21,83676$

Error = |Metode Reimann - Metode Analitik|

= |21,83676 - 17,4|

= 4,43676

No.

Date:

- Metode Trapezoid

$$\frac{\Delta x}{2}\left[f(x_0) + 2\Sigma f(x_n) + f(x_{n-1})\right] =$$

$$\frac{0,3}{2}\left[1 + 93,5784 + 25\right] =$$

$$0,15\left[119,5784\right] \Rightarrow 17,93676$$

Error = |Metode Trapezoid − Metode Analitik|

= |17,93676 − 17,4|

= 0,53676

- Metode Simpson 1/3

$$\frac{\Delta x}{3}\left[f(x_0) + 4f(x_{ganjil}) + 2\Sigma f(x_{genap}) + f(x_{n-1})\right] =$$

$$\frac{0,3}{3}\left[1 + 109,1024 + 39,0272 + 25\right] =$$

$$0,1\left[174,1296\right] \Rightarrow 17,41296$$

Error = |Metode Simpson 1/3 − Metode Analitik|

= |17,41296 − 17,4|

= 0,01296

- Metode Simpson 3/8

$$\frac{3\Delta x}{8}\left[f(x_0) + 3\Sigma f(x_{non\text{-}kel\,3}) + 2\Sigma f(x_{kel\,3}) + f(x_{n-1})\right]$$

$$\Rightarrow \frac{3(0,3)}{8}\left[1 + 81,666 + 39,1344 + 25\right]$$

$$\Rightarrow \frac{0,9}{8}\left[146,8004\right]$$

$$= 0,1125\left[146,8004\right]$$

$$= 16,51504$$

No.

Date:

Error = | Metode Simpson 3/8 - Metode Analitik |

= | 16,51504 - 17,4 |

= 0,88405