# Methods Lectures: Financial Econometrics

# Linear Factor Models and Event Studies

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# Outline

# Outline

1. Linear Factor Models
   - Motivation
   - Time-series approach
   - Cross-sectional approach
   - Comparing approaches
   - Odds and ends

2. Event Studies
   - Motivation
   - Basic methodology
   - Bells and whistles

# Linear factor pricing models

- Many asset pricing models can be expressed in linear factor form

$$\mathrm{E}[r_{t+1} - r_0] = \beta' \lambda$$

  where $\beta$ are regression coefficients and $\lambda$ are factor risk premia.

- The most prominent examples are the CAPM

$$\mathrm{E}[r_{i,t+1} - r_0] = \beta_{i,m} \mathrm{E}[r_{m,t+1} - r_0]$$

  and the consumption-based CCAPM

$$\mathrm{E}[r_{i,t+1} - r_0] \simeq \beta_{i,\Delta c_{t+1}} \underbrace{\gamma \mathrm{Var}[\Delta c_{t+1}]}_{\lambda}$$

- Note that in the CAPM the factor is an excess return that itself must be priced by the model $\Rightarrow$ an extra testable restriction.

# Econometric approaches

- Given the popularity of linear factor models, there is a large literature on estimating and testing these models.
- These techniques can be roughly categorized as
  - Time-series regression based intercept tests for models in which the factors are excess returns.
  - Cross-sectional regression based residual tests for models in which the factors are not excess returns or for when the alternative being considered includes stock characteristics.
- The aim of this first part of the lecture is to survey and put into perspective these econometric approaches.

# Outline

# Black, Jensen, and Scholes (1972)

- Consider a single factor (everything generalizes to $k$ factors).
- When this factor is an excess return that itself must be priced by the model, the factor risk premium is identified through

$$\hat{\lambda} = \hat{\mathrm{E}}_T[f_{t+1}].$$

- With the factor risk premium fixed, the model implies that the intercepts of the following time-series regressions must be zero

$$r_{i,t+1} - r_0 \equiv r^e_{i,t+1} = \alpha_i + \beta_i f_{t+1} + \epsilon_{i,t+1}$$

  so that $\mathrm{E}[r^e_{i,t+1}] = \beta_i \lambda$.
- This restriction can be tested asset-by-asset with standard $t$-tests.

# Joint intercepts test

- The model implies, of course, that all intercepts are jointly zero, which we test with standard Wald test

$$\hat{\alpha}' \left(\text{Var}[\hat{\alpha}]\right)^{-1} \hat{\alpha} \sim \chi^2_N.$$

- To evaluate $\text{Var}[\hat{\alpha}]$ requires further distributions assumptions

- With iid residuals $\epsilon_{t+1}$, standard OLS results apply

$$\text{Var}\begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = \frac{1}{T}\left( \begin{bmatrix} 1 & \hat{E}_T[f_{t+1}] \\ \hat{E}_T[f_{t+1}] & \hat{E}_T[f^2_{t+1}]^2 \end{bmatrix}^{-1} \otimes \Sigma_\epsilon \right)$$

which implies

$$\text{Var}[\hat{\alpha}] = \frac{1}{T}\left[ 1 + \left( \frac{\hat{\text{E}}_T[f_{t+1}]}{\hat{\sigma}_T[f_{t+1}]} \right)^2 \right] \hat{\Sigma}_\epsilon.$$

# Gibbons, Ross, and Shanken (1989)

- Assuming further that the residuals are iid multivariate normal, the Wald test has a finite sample $F$ distribution

$$\frac{(T-N-1)}{N}\left[1+\left(\frac{\hat{\mathrm{E}}_T[f_{t+1}]}{\hat{\sigma}_T[f_{t+1}]}\right)^2\right]^{-1}\hat{\alpha}'\hat{\Sigma}_\epsilon^{-1}\hat{\alpha}\ \sim\ F_{N,T-N-1}.$$

- In the single-factor case, this test can be further rewritten as

$$\frac{(T-N-1)}{N}\left(\frac{\left(\frac{\hat{\mathrm{E}}_T[r^e_{q,t+1}]}{\hat{\sigma}_T[r^e_{q,t+1}]}\right)^2-\left(\frac{\hat{\mathrm{E}}_T[r^e_{m,t+1}]}{\hat{\sigma}_T[r^e_{m,t+1}]}\right)^2}{1+\left(\frac{\hat{\mathrm{E}}_T[r^e_{m,t+1}]}{\hat{\sigma}_T[r^e_{m,t+1}]}\right)^2}\right),$$

where $q$ is the ex-post mean-variance portfolio and $m$ is the ex-ante mean variance portfolio (i.e., the market portfolio).

# MacKinlay and Richardson (1991)

- When the residuals are heteroskedastic and autocorrelated, we can obtain $\text{Var}[\hat{\alpha}]$ by reformulating the set of $N$ regressions as a GMM problem with moments

$$g_T(\theta) = \hat{\mathrm{E}}_T \left[ \begin{array}{c} r_{t+1} - \alpha - \beta f_{t+1} \\ (r_{t+1} - \alpha - \beta f_{t+1}) f_{t+1} \end{array} \right] = \hat{\mathrm{E}}_T \left[ \begin{array}{c} \epsilon_{t+1} \\ \epsilon_{t+1} f_{t+1} \end{array} \right] = 0$$

- Standard GMM results apply

$$\text{Var} \left[ \begin{array}{c} \hat{\alpha} \\ \hat{\beta} \end{array} \right] = \frac{1}{T} \left[ D S^{-1} D' \right]^{-1}$$

with

$$D = \frac{\partial g_T(\theta)}{\partial \theta} = - \left[ \begin{array}{cc} 1 & \hat{\mathrm{E}}_T[f_{t+1}] \\ \hat{\mathrm{E}}_T[f_{t+1}] & \hat{\mathrm{E}}_T[f_{t+1}^2] \end{array} \right] \otimes I_N$$

$$S = \sum_{j=-\infty}^{\infty} \mathrm{E} \left[ \left[ \begin{array}{c} \epsilon_t \\ \epsilon_t f_t \end{array} \right] \left[ \begin{array}{c} \epsilon_{t-j} \\ \epsilon_{t-j} f_{t-j} \end{array} \right]' \right].$$

# Outline

# Cross-sectional regressions

- When the factor is not an excess return that itself must be priced by the model, the model does not imply zero time-series intercepts.
- Instead one can test cross-sectionally that expected returns are proportional to the time-series betas in two steps
  1. Estimate asset-by-asset $\beta_i$ through time-series regressions
  $$r^e_{t+1} = a + \beta_i f_{t+1} + \epsilon_{t+1}.$$
  2. Estimate the factor risk premium $\lambda$ with a cross-sectional regression of average excess returns on these betas
  $$\bar{r}^e_i = \hat{\mathrm{E}}_T[r^e_i] = \lambda \hat{\beta}_i + \alpha_i.$$
- Ignore for now the "generated regressor" problem.

# OLS estimates

- The cross-sectional OLS estimates are

$$\hat{\lambda} = (\hat{\beta}'\hat{\beta})^{-1}(\hat{\beta}\bar{r}_i^e)$$
$$\hat{\alpha} = \bar{r}_i^e - \hat{\lambda}\hat{\beta}.$$

- A Wald test for the residuals is then

$$\hat{\alpha}\text{Var}[\hat{\alpha}]^{-1}\hat{\alpha} \sim \chi^2_{N-1}$$

with
$$\text{Var}[\hat{\alpha}] = \left(I - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\beta'\right)\frac{1}{T}\Sigma_\epsilon\left(I - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\beta'\right)$$

- Two important notes
  - $\text{Var}[\hat{\alpha}]$ is singular, so $\text{Var}[\hat{\alpha}]^{-1}$ is a generalized inverse and the $\chi^2$ distribution has only $N-1$ degrees of freedom.
  - Testing the size of the residuals is bizarre, but here we have additional information from the time-series regression (in red).

# GLS estimates

- Since the residuals of the second-stage cross-sectional regression are likely correlated, it makes sense to instead use GLS.
- The relevant GLS expressions are

$$\hat{\lambda}_{\text{GLS}} = (\hat{\beta}' \Sigma_\epsilon^{-1} \hat{\beta})^{-1} (\hat{\beta} \Sigma_\epsilon^{-1} \bar{r}_i^e)$$

and

$$\text{Var}[\hat{\alpha}_{\text{GLS}}] = \frac{1}{T} \left( \Sigma_\epsilon - \hat{\beta}(\hat{\beta}' \Sigma_\epsilon^{-1} \hat{\beta})^{-1} \beta' \right).$$

# Shanken (1992)

- The generated regressor problem, the fact that $\beta$ is estimated in the first stage time-series regression, cannot be ignored.
- Corrected estimates are given by

$$\text{Var}[\hat{\alpha}_{\text{OLS}}] = \left(I - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\beta'\right)\frac{1}{T}\Sigma_\epsilon\left(I - \hat{\beta}(\hat{\beta}'\hat{\beta})^{-1}\beta'\right) \times \left(1 + \frac{\lambda^2}{\sigma_f^2}\right)$$

$$\text{Var}[\hat{\alpha}_{\text{GLS}}] = \frac{1}{T}\left(\Sigma_\epsilon - \hat{\beta}(\hat{\beta}'\Sigma_\epsilon^{-1}\hat{\beta})^{-1}\beta'\right) \times \left(1 + \frac{\lambda^2}{\sigma_f^2}\right)$$

- In the case of the CAPM, for example, the Shanken correction for annual data is roughly

$$1 + \left(\frac{\hat{\text{E}}_T[r_{\text{mkt},t+1}]}{\hat{\sigma}_T[r_{\text{mkt},t+1}]}\right)^2 = 1 + \left(\frac{0.06}{0.15}\right)^2 = 1.16$$

# Cross-sectional regressions in GMM

- Formulating cross-sectional regressions as GMM delivers an automatic "Shanken correction" and allows for non-iid residuals.
- The GMM moments are

$$g_T(\theta) = \mathrm{E}_T[r_{t+1} \left[ \begin{array}{c} r^e_{t+1} - a - \beta f_{t+1} \\ (r^e_{t+1} - a - \beta f_{t+1}) f_{t+1} \\ r_{t+1} - \beta\lambda \end{array} \right] = 0.$$

- The first two rows are the first-stage time-series regression.
- The third row over-identifies $\lambda$
  - If those moments are weighted by $\beta'$ we get the first order conditions of the cross-sectional OLS regression.
  - If those moments are instead weighted by $\beta'\Sigma_\epsilon^{-1}$ we get the first order conditions of the cross-sectional GLS regression.
- Everything else is standard GMM.

# Linear factor models in SDF form

- Linear factor models can be rexpressed in SDF form

$$\mathrm{E}[m_{t+1} r_{t+1}^e] = 0 \quad \text{with} \quad m_{t+1} = 1 - b f_{t+1}.$$

- To see that a linear SDF implies a linear factor model, substitute in

$$0 = \mathrm{E}[(1 - b f_{t+1}) r_{t+1}^e] = \mathrm{E}[r_{t+1}^e] - b \underbrace{\mathrm{E}[r_{t+1}^e f_{t+1}]}_{\mathrm{Cov}[r_{t+1}^e, f_{t+1}] + \mathrm{E}[r_{t+1}^e]\mathrm{E}[f_{t+1}]}$$

and solve for

$$\begin{aligned} \mathrm{E}[r_{t+1}^e] &= \frac{b}{1 - b\mathrm{E}[f_{t+1}]} \mathrm{Cov}[r_{t+1}^e, f_{t+1}] \\ &= \underbrace{\frac{\mathrm{Cov}[r_{t+1}^e, f_{t+1}]}{\mathrm{Var}[f_{t+1}]}}_{\beta} \underbrace{\frac{b\mathrm{Var}[f_{t+1}]}{1 - b\mathrm{E}[f_{t+1}]}}_{\lambda}. \end{aligned}$$

# GMM on SDF pricing errors

- This SDF view suggests the following GMM approach

$$g_T(b) = \hat{\mathrm{E}}_T\left[(1 + bf_{t+1})r^e_{t+1}\right] = \underbrace{\hat{\mathrm{E}}_T[r^e_{t+1}] - b\hat{\mathrm{E}}_T[r^e_{t+1}f_{t+1}]}_{\text{pricing errors } = \pi} = 0.$$

- Standard GMM techniques can be used to estimate $b$ and test whether the pricing errors are jointly zero in population.
- GMM estimation of $b$ is equivalent to
  - Cross-sectional OLS regression of average returns on the second moments of returns with factors (instead of betas) when the GMM weighting matrix is an identity.
  - Cross-sectional GLS regression of average returns on the second moments of returns with factors when the GMM weighting matrix is a residual covariance matrix.

# Fama and MacBeth (1973)

- The Fama-MacBeth procedure is one of the original variants of cross-sectional regressions consisting of three steps
  1. Estimate $\beta_i$ from stock or portfolio level rolling or full sample time-series regressions.
  2. Run a cross-sectional regression

     $$r^e_{t+1} = \lambda_{t+1}\hat{\beta} + \alpha_{i,t+1}$$

     for every time period $t$ resulting in a time-series

     $$\{\hat{\lambda}_t, \hat{\alpha}_{i,t}\}_{t=1}^{T}.$$

  3. Perform statistical tests on the time-series averages

     $$\hat{\alpha}_i = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}_{i,t} \quad \text{and} \quad \hat{\lambda}_i = \tfrac{1}{T}\sum_{t=1}^{T}\hat{\lambda}_{i,t}.$$

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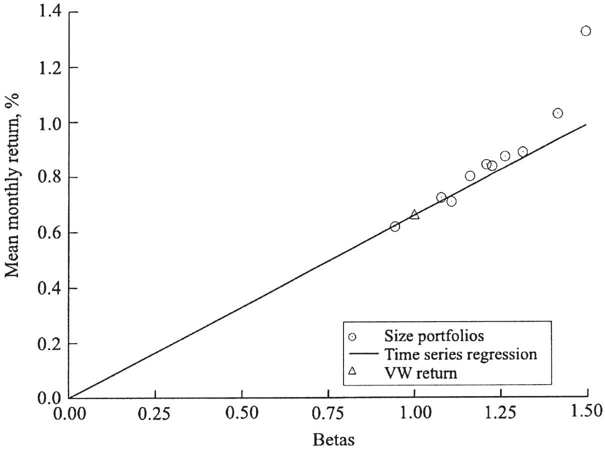
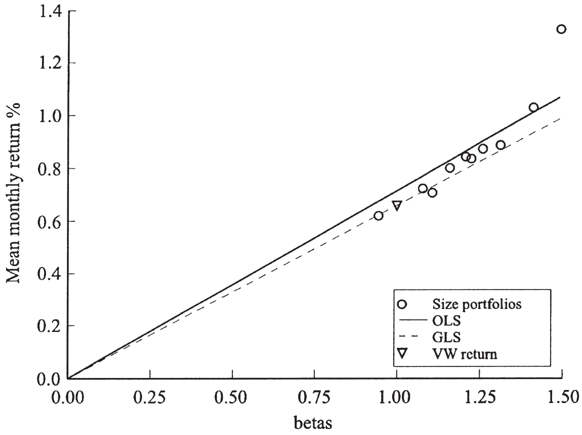
# Time-series regressions



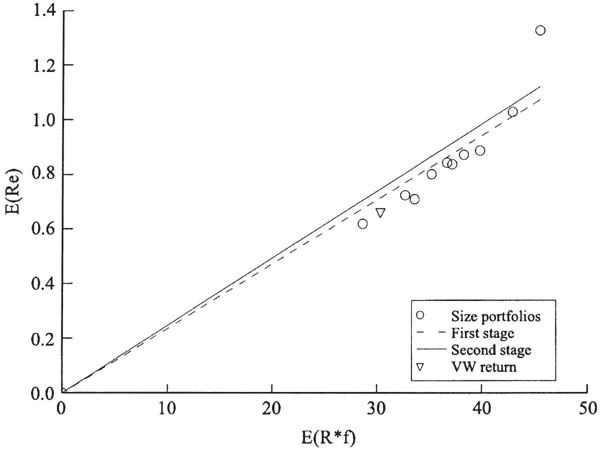

Source: Cochrane (2001)
Avg excess returns versus betas on CRSP size portfolios, 1926-1998.

# Cross-sectional regressions




Source: Cochrane (2001)
Avg excess returns versus betas on CRSP size portfolios, 1926-1998.

# SDF regressions




Source: Cochrane (2001)
Avg excess returns versus predicted value on CRSP size portfolios, 1926-1998.

# Estimates and tests

| | Beta model $\lambda$ | | | GMM/DF $b$ | | |
|---|---|---|---|---|---|---|
| | Time-Series | Cross-section OLS | Cross-section GLS | 1st stage | 2nd stage *Estimate* | 2nd stage Standard Error |
| *Estimate* | *0.66* | *0.71* | *0.66* | *2.35* | | |
| i.i.d. | 0.18 (3.67) | 0.20 (3.55) | 0.18 (3.67) | | | |
| 0 lags | 0.18 (3.67) | 0.19 (3.74) | 0.18 (3.67) | 0.63 (3.73) | *2.46* | 0.61 (4.03) |
| 3 lags, NW | 0.20 (3.30) | 0.21 (3.38) | 0.20 (3.30) | 0.69 (3.41) | *2.39* | 0.64 (3.73) |
| 24 lags | 0.16 (4.13) | 0.16 (4.44) | 0.16 (4.13) | 1.00 (2.35) | *2.15* | 0.69 (3.12) |

| | Time series | | Cross section | | GMM/DF | |
|---|---|---|---|---|---|---|
| | $\chi^2_{(10)}$ | % p-value | $\chi^2_{(9)}$ | % p-value | $\chi^2_{(9)}$ | % p-value |
| i.i.d. | 8.5 | 58 | 8.5 | 49 | | |
| GRS F | 0.8 | 59 | | | | |
| 0 lags | 10.5 | 40 | 10.6 | 31 | 10.5 | 31 |
| 3 lags NW | 11.0 | 36 | 11.1 | 27 | 11.1 | 27 |
| 24 lags | −432 | −100 | 7.6 | 57 | 7.7 | 57 |

Source: Cochrane (2001)
CRSP size portfolios, 1926-1998.

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# Firm characteristics

- So far

$$H_0 : \alpha_i = 0 \quad i = 1, 2, ..., n$$
$$H_A : \alpha_i \neq 0 \quad \text{for some } i$$

- We might, typically on the basis of preliminary portfolio sorts, a more specific alternative in mind

$$H_0 : \alpha_i = 0 \quad i = 1, 2, ..., n$$
$$H_A : \alpha_i = \gamma' x_i$$

  for some firm level characteristics $x_i$.
- We can test against this more specific alternative by including the characteristics in the cross-sectional regression

$$\bar{r}_i^e == \lambda \hat{\beta}_i + \gamma' x_i + \alpha_i.$$

# Portfolios

- Stock level time-series regressions are problematic
  - Way too noisy.
  - Betas may be time-varying.

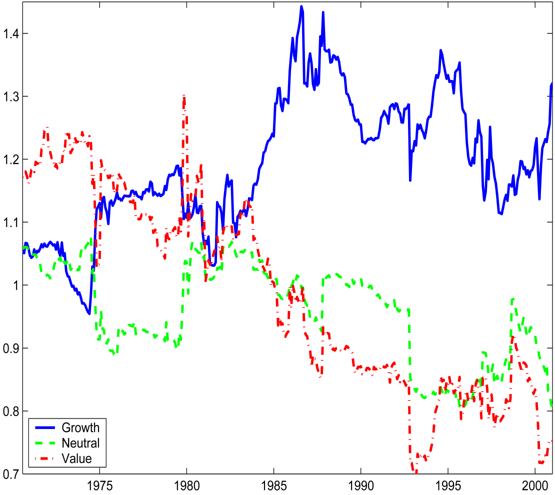
- Fama-MacBeth solve this problem by forming portfolios
  1. Each portfolio formation period (say annually), obtain a noisy estimate of firm-level betas through stock-by-stock time series regressions over a backward looking estimating period.
  2. Sort stocks into "beta portfolios" on the basis of their noisy but unbiased "pre-formation beta".

- The resulting portfolios should be fairly homogeneous in their beta, should have relatively constant portfolio betas through time, and should have a nice beta spread in the cross-section.

# Characteristic sorts

- Somewhere along the way sorting on pre-formation factor exposures got replaced by sorting on firm characteristics $x_i$.
- Sorting on characteristics is like sorting on ex-ante alphas (the residuals) insead of betas (the regressor).
- It may have unintended consequence
  1. Characteristic sorted portfolios may not have constant betas.
  2. Characteristic sorted portfolios could have no cross-sectional variation in betas and hence no power to identify $\lambda$.

# Characteristic sorts (cont)




Source: Ang and Liui (2004)

# Conditional information

- In a conditional asset pricing model expectations, factor exposures, and factor risk premia all have a time-$t$ subscripts

$$\mathrm{E}_t[r_{t+1} - r_0] = \beta_t' \lambda_t.$$

- This makes the model fundamentally untestable.

- Nevertheless, there are at least two ways to proceed
  1. Time-series modeling of the moments, such as

$$\beta_t = \beta_0 - \kappa(\beta_{t-1} - \beta_0) + \eta_t.$$

  2. Model the moments as (linear) functions of observables

$$\beta_t = \gamma_\beta' Z_t \quad \text{or} \quad \lambda_t = \gamma_\lambda' Z_t.$$

- Approach #2 is by far the more popular.

# Conditional SDF approach

- In the context of SDF regressions

$$E_t[m_{t+1} r^e_{t+1}] = 0 \quad \text{with} \quad m_{t+1} = 1 - b_t f_{t+1}.$$

- Assume $b_t = \theta_0 + \theta_1 Z_t$ and substitute in

$$E_t \left[(1 - (\theta_0 + \theta_1 Z_t) f_{t+1}) \, r^e_{t+1}\right] = 0.$$

- Condition down

$$E\left[E_t[(1 - (\theta_0 + \theta_1 Z_t) f_{t+1}) r^e_{t+1}]\right] = E\left[(1 - (\theta_0 + \theta_1 Z_t) f_{t+1}) r^e_{t+1}\right] = 0.$$

- Finally rearrange as unconditional multifactor model

$$E\left[(1 - \theta_0 f_{1,t+1} - \theta_1 f_{2,t+1}) r^e_{t+1}\right] = 0,$$

where $f_{1,t+1} = f_{t+1}$ and $f_{2,t+1} = z_t f_{t+1}$.

# Outline

# What is an event study?

- An event study measures the immediate and short-horizon delayed impact of a specific event on the value of a security.
- Event studies traditionally answer the question
  - Does this event matter? (e.g., index additions)

  but are increasingly used to also answer the question
  - Is the relevant information impounded into prices immediately or with delay? (e.g., post earnings announcement drift)
- Event studies are popular not only in accounting and finance, but also in economics and law to examine the role of policy and regulation or determine damages in legal liability cases.

# Short- versus long-horizon event studies

- Short-horizon event studies examine a short time window, ranging from hours to weeks, surrounding the event in question.
- Since even weekly expected returns are small in magnitude, this allows us to focus on the information being released and (largely) abstract from modeling discount rates and changes therein.
- There is relatively little controversy about the methodology and statistical properties of short-horizon event studies.
- However event study methodology is increasingly being applied to longer-horizon questions. (e.g., do IPOs under-perform?)
- Long-horizon event studies are problematic because the results are sensitive to the modeling assumption for expected returns.
- The following discussion focuses on short-horizon event studies.

# Outline

# Setup

- Basic event study methodology is essentially unchanged since Ball and Brown (1968) and Fama et al. (1969).
- Anatomy of an event study
  1. Event Definition and security selection.
  2. Specification and estimation of the reference model characterizing "normal" returns (e.g., market model).
  3. Computation and aggregation of "abnormal" returns.
  4. Hypothesis testing and interpretation.
- #1 and the interpretation of the results in #4 are the real economic meat of an event study – the rest is fairly mechanical.

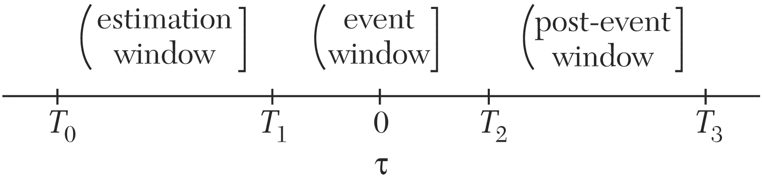
# Reference models

- Common choices of models to characterize "normal" returns
  - Constant expected returns
    $$r_{i,t+1} = \mu_i + \epsilon_{i,t+1}$$
  - Market model
    $$r_{i,t+1} = \alpha_i + \beta_i r_{m,t+1} + \epsilon_{i,t+1}$$
  - Linear factor models
    $$r_{i,t+1} = \alpha_i + \beta_i' f_{t+1} + \epsilon_{i,t+1}$$

  (Notice the intercepts.)
- Short-horizon event study results tend to be relatively insensitive to the choice of reference models, so keeping it simple is fine.

# Time-line and estimation

- A typical event study time-line is




Source: MacKinlay (1997)

- The parameters of the reference model are usually estimated over the estimation window and held fixed over the event window.

# Abnormal returns

- Using the market model as reference model, define

$$\hat{AR}_{i,\tau} = r_{i,\tau} - \hat{\alpha}_i - \hat{\beta}_i r_{m,\tau} \quad \tau = T_1 + 1, ..., T2.$$

- Assuming iid returns

$$\text{Var}\left[\hat{AR}_{i,\tau}\right] = \sigma^2_{\epsilon_i} + \underbrace{\frac{1}{T_1 - T_0}\left[1 + \left(\frac{(r_{m,\tau} - \hat{\mu}_m)}{\hat{\sigma}_m}\right)^2\right]}_{\text{due to estimation error}}.$$

- Estimation error also introduces persistence and cross-correlation in the measured abnormal returns, even when true returns are iid, which is an issue particularly for short estimation windows.
- Assume for what follows that estimation error can be ignored.

# Aggregating abnormal returns

- Since the abnormal return on a single stock is extremely noisy, returns are typically aggregated in two dimensions.
  1. Average abnormal returns across all firms undergoing the same event lined up in event time

$$\bar{AR}_\tau = \frac{1}{N} \sum_{i=1}^{N} \hat{AR}_{i,\tau}.$$

  2. Cumulate abnormal returns over the event window

$$C\hat{A}R_i(T_1, T_2) = \sum_{\tau=T_1+1}^{T_2} \hat{AR}_{i,\tau}$$

or

$$C\bar{A}R(T_1, T_2) = \sum_{\tau=T_1+1}^{T_2} \bar{AR}_\tau.$$

# Hypothesis testing

- Basic hypothesis testing requires expressions for the variance of $\bar{AR}_\tau$, $\hat{CAR}_i(T_1, T_2)$, and $\bar{CAR}(T_1, T_2)$.

- If stock level residuals $\epsilon_{i,\tau}$ are iid through time

$$\text{Var}\left[\hat{CAR}_i(T_1, T_2)\right] = (T_2 - T_1)\sigma^2_{\epsilon_i}.$$

- The other two expressions are more problematic because they depend on the cross-sectional correlation of event returns.

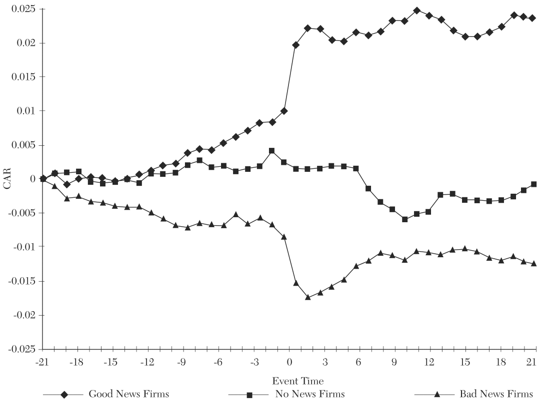
- If, in addition, event windows are non-overlapping across stocks

$$\text{Var}\left[\bar{AR}_\tau\right] = \frac{1}{N^2}\sum_{i=1}^{N} \sigma^2_{\epsilon_i}$$

$$\text{Var}\left[\bar{CAR}(T_1, T_2)\right] = \sum_{\tau=T_1+1}^{T_2} \text{Var}\left[\bar{AR}_\tau\right].$$

# Example

- Earnings announcements




Source: MacKinlay (1997)

# Outline

# Dependence

- If event windows overlap across stocks, the abnormal returns are correlated contemporaneously or at lags.
- One solution is Brown and Warner's (1980) "crude adjustment"
    - Form a portfolio of firms experiencing an event in a given month or quarter (still lined up in event time, of course).
    - Compute the average abnormal return of this portfolio.
    - Normalize this abnormal return by the standard deviation of the portfolio's abnormal returns over the estimation period.

# Heteroskedasticity

- Two forms of heteroskedasticity to deal with in event studies
  1. Cross-sectional differences in $\sigma_{\epsilon_i}$.
     - Standardize $AR_{i,\tau}$ by $\sigma_{\epsilon_i}$ before aggregating to $\bar{AR}_\tau$.
     - Jaffe (1974).
  2. Event driven changes in $\sigma_{\epsilon_i,t}$.
     - Cross-sectional estimate of $\text{Var}\left[AR_{i,\tau}\right]$ over the event window.
     - Boehmer et al. (1991).

# Regression based approach

- An event study can alternatively be run as multivariate regression

$$r_{1,t+1} = \alpha_1 + \beta_1 r_{m,t+1} + \sum_{\tau=0}^{L} \gamma_{1,\tau} D_{1,t+1,\tau} + u_{1,t+1}$$

$$r_{2,t+1} = \alpha_2 + \beta_2 r_{m,t+1} + \sum_{\tau=0}^{L} \gamma_{2,\tau} D_{2,t+1,\tau} + u_{2,t+1}$$

$$\ldots$$

$$r_{n,t+1} = \alpha_n + \beta_n r_{m,t+1} + \sum_{\tau=0}^{L} \gamma_{n,\tau} D_{n,t+1,\tau} + u_{n,t+1}$$

  where $D_{i,t,\tau} = 1$ if firm $i$ had an event at date $t - \tau$ ($= 0$ otherwise).

- In this case, $\gamma_{i,\tau}$ takes the place of the abnormal return $AR_{i,\tau}$.