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Article

Time Series Analysis by Fuzzy Logic Methods

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Abstract: Discrete Mathematical Analysis (DMA) is a data analysis method that uses fuzzy mathematics and fuzzy logic. The paper is devoted to the DMA study of the morphology of time series by translating it into the language of fuzzy mathematics: the morphological properties of the time series (background, slopes, vertices, etc.) are treated as fuzzy sets on the domain of its definition. It becomes possible to use fuzzy logic in the study of the morphology of the time series. The results obtained are used to search for its anomalies.

Keywords: DMA; time series; fuzzy logic; magnetic storm; morphological analysis

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1. Introduction

Discrete Mathematical Analysis (DMA) is a discrete data analysis method that uses scenarios of classical continuous mathematics, in which the fundamentals are replaced by fuzzy models of their discrete analogs. From a practical point of view, DMA is a new approach to data analysis, that is focused on an expert and occupying an intermediate position between rigorous mathematical methods and soft combinatorial ones [1,2].

The solution of the problem within the DMA framework consists of two parts. The first is informal. It analyzes the logic of the expert, introduces the necessary concepts, explains the scheme and principles of the solution. The second part has a formal character: with the help of the DMA apparatus, all concepts receive a rigorous definition within the framework of fuzzy mathematics (FM) and fuzzy logic (FL), and schemes and principles become algorithms.

This is how the DMA study of records (time series) has began, the informal basis of which was the logic of an expert looking for anomalies on them [3,4]. Let us recall it: the expert slides over the record, estimating the manifestation degree of the property of interest to him in identical small fragments of the record by positive numbers. The expert assigns these estimates to the centers of the fragments. So from the original record, he goes to a non-negative function, which is called straightening the record with respect to the property of interest to the expert. The mountains on the straightening correspond to already non-local fragments on the record, which are of the most interest to the expert and he considers them to be anomalies. Thus, the expert operates on two levels: local – straightening and global – searching for a mountain on straightening.

The formalization of this logic became the main one at the initial stage of the DMA study of records. The DRAS and FCARS algorithms created at that time adequately competed with the classical spectral analysis algorithms in the search for anomalies [3,4]. DMA anomaly search algorithms leave the expert free to understand (interpret) them through the choice of straightening. This gives them great flexibility.

The mountains on straightening have stochastic nature: there are many large values on them, but there are also small ones. Only vertical considerations in their search are not enough, there must be horizontal ones, and the latter must be non-local. It is also needed to be able to combine into a single whole mountains that are close to each other on straightening, since behind them on the record one large (long) signal is hidden, consisting of a sequence of short ones. Therefore, the search for mountains on non-negative reliefs is an important task in DMA. To date, its best solution is to use the DPS algorithm twice, but even this gives only the main part of the mountain on the straightening (respectively, anomalies on the record) without precise boundaries, initial and final stages, explicit indication of slopes and peaks [1,5]. In other words, in the present there is only recognition of mountains (anomalies) without their morphology.

Modern analysis of anomalies involves not only their recognition, but also their morphological analysis for subsequent classifications and encoding.

The foregoing allows us to formulate the aim of this work – the creation of a morphological analysis of records within the framework of DMA using FL and FM.

In conclusion, let's give a clarifying example of morphological analysis magnetic storm record. In Figures 1–7, the two upper levels are identical: the record x and its straightening "length" L_x (local length of the graph Γ_x of record x at a particular node).

Six geometric properties are fixed: "background", "beginning of a mountain", "left slope of a mountain", "top of a mountain", "right slope of a mountain", "end of a mountain", and fuzzy measures of their manifestation are built in the scale [–1,1] on the relief L_x at one node or another. This is the third graph on the figures. Areas of strong manifestation of the corresponding property are indicated in red (measure value is greater than or equal to 0.5).

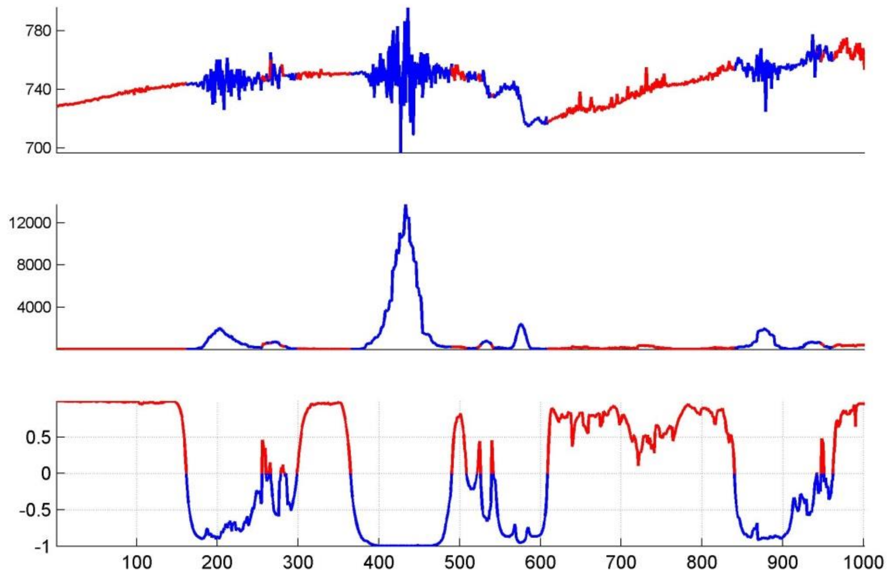


Figure 1. Measure "background".

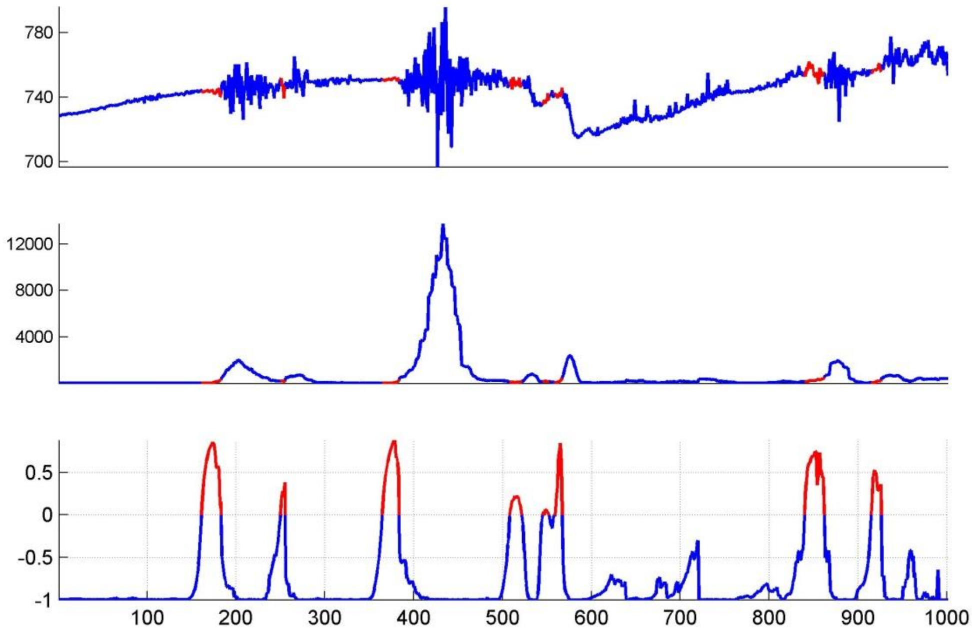


Figure 2. Measure "beginning of a mountain".

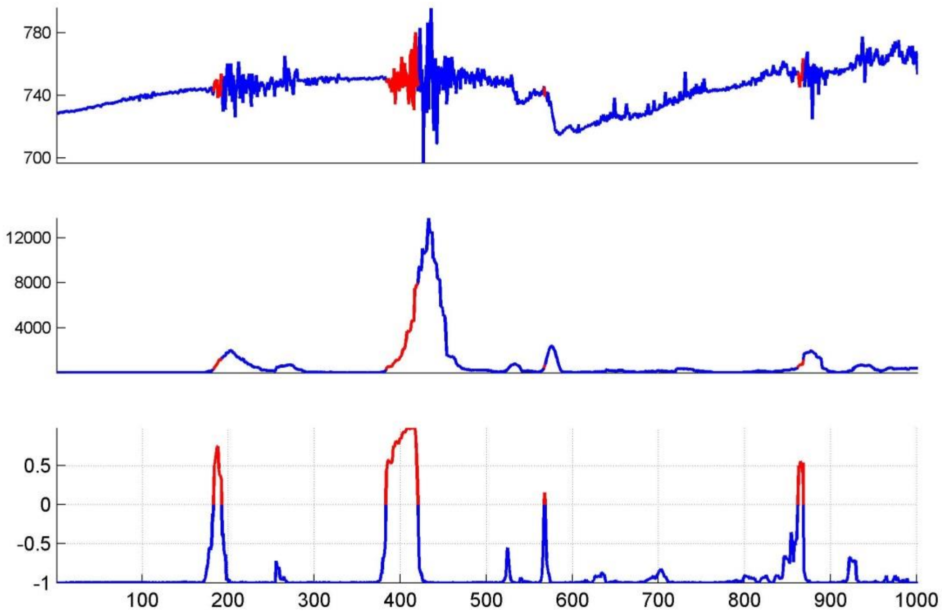


Figure 3. Measure "left slope".

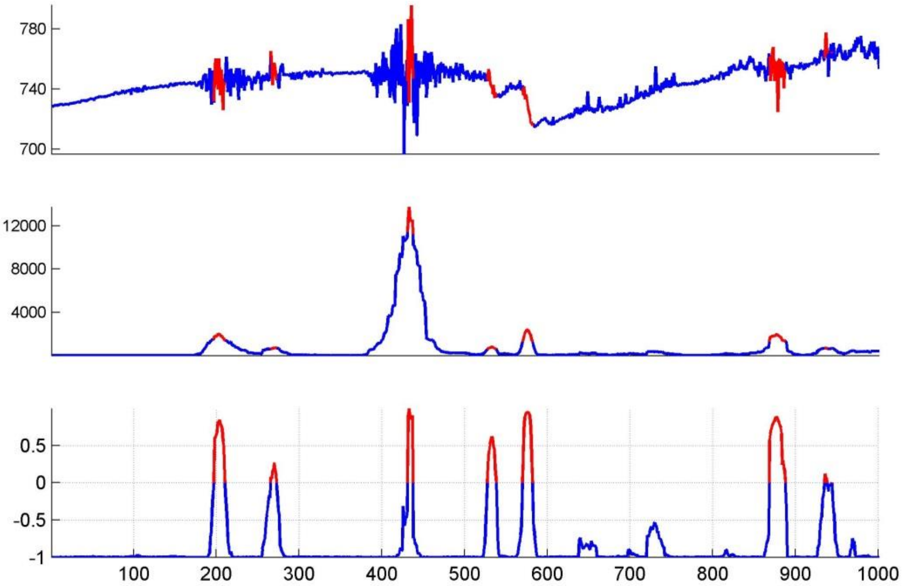


Figure 4. Measure “top of a mountain”.

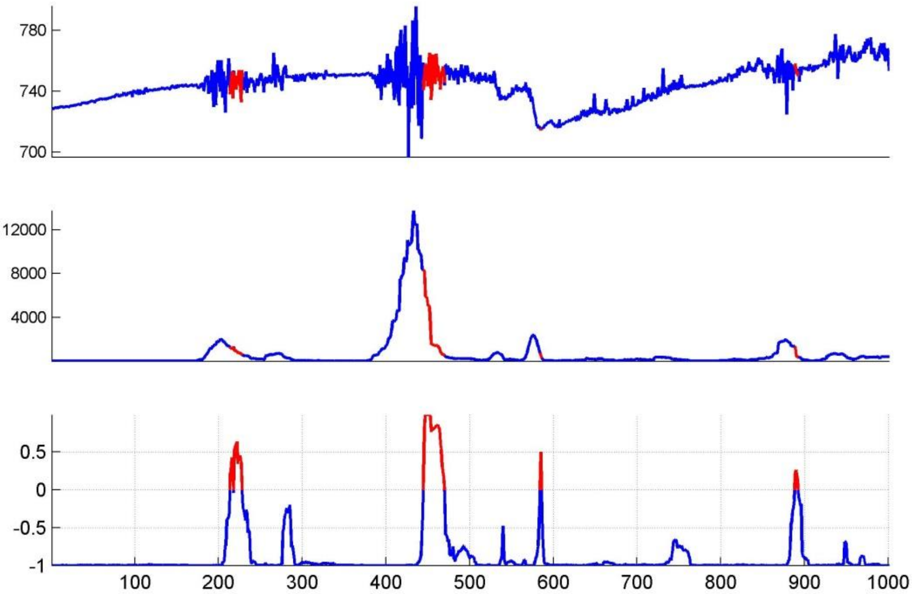


Figure 5. Measure “right slope”.

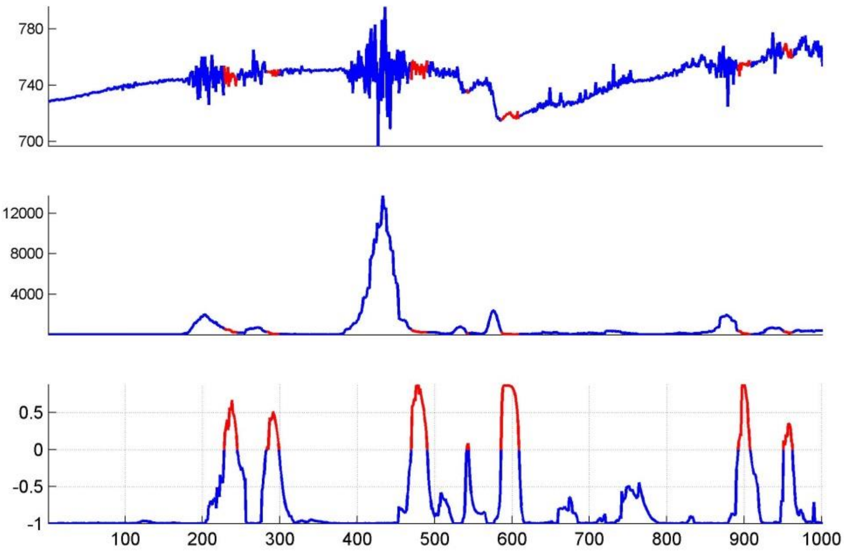


Figure 6. Measure "end of a mountain".

In the last Figure 7 the morphological result of the first six ones is summed up: each property is assigned its own color, and each node is colored in the color of the property that is most manifested. This is the third graph in Figure 7. The corresponding coloring of the nodes rises first to the straightening L_x, then to the original record x.

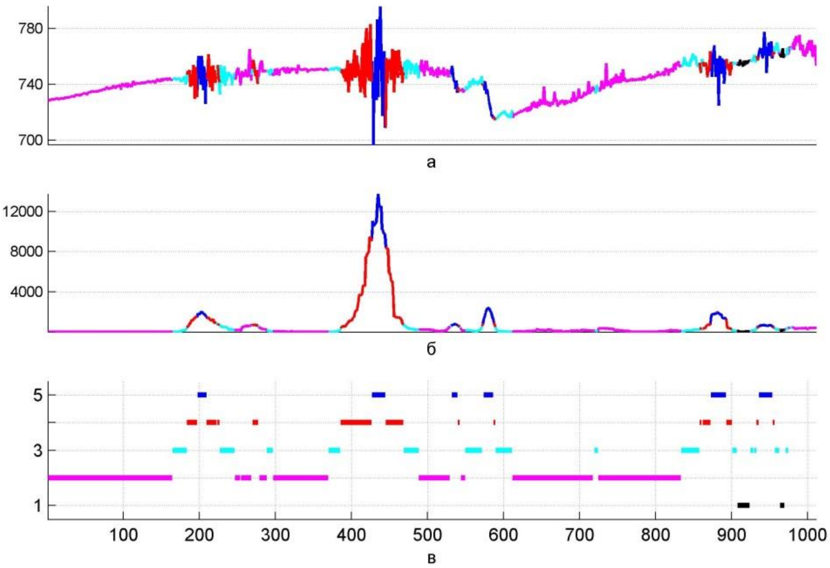


Figure 7. Morphological analysis of the record.

Formalization of everything said and shown is the aim of this paper.

2. Materials and Methods

2.1. DMA – morphological analysis: nonformal logic

The definition domain T of a record x is a finite regular set of nodes with the discretization parameter h:

$T = \{t_1 < \cdots < t_N\},\ t_{i+1} - t_i = h,\ i = 1, \dots, N-1$

Let $F(T)$ denote the space of all records on T

2.1.1. Elementary Measures

Suppose that for any record $x \in F(T)$ at each node $t \in T$ in the scale of the segment [-1,1], we can answer with the help of fuzzy structures, which we will call elementary measures and denote by $\mu_x^{ul}, \mu_x^{dl}, \mu_x^{ur}, \mu_x^{dr}$, to the following four questions:

μ_x^{ul} / μ_x^{ur} $\leftrightarrow$ To what extent, being on the graph Γ_x of a record x at the node t and looking up, we feel our "smallness", respectively $\leftrightarrow$ on the left / on the right from t to T

(1)

μ_x^{dl} / μ_x^{dr} $\leftrightarrow$ To what extent, being on the graph Γ_x of a record x at node t and looking down, we feel our "significance" respectively $\leftrightarrow$ on the left / on the right from t to T

Elementary measures must have an additional normalization: if μ_x^* is any of them ($* \in \{ul, ur, dl, dr\}$), then the value of μ_x^* must be close to 1 (–1) if the property corresponding to μ_x^* at node t is satisfied to record x strongly (weakly).

2.1.2. Morphological measures

On the basis of four elementary measures and their fuzzy negations, using the fuzzy conjunction $\wedge$, sixteen measures are constructed that express more complex morphological aspects (features) of the original record x and allow us to understand its geometry. Therefore, such measures are called morphological or geometric.

Let's define them as follows. Let's call index a four-dimensional Boolean vector $i = \{i(*)$, where $*$ runs through the situations described in (1) and $i(*) = \pm 1\}$. In other words, $i = (i_{ul}, i_{dl}, i_{ur}, i_{dr})$ with coordinates ± 1.

Using the fuzzy conjunction $\wedge$, each index i defines a fuzzy measure (structure) μ_x^i on T:

$$\mu_x^i(t) = \wedge\left((-1)^{i(*)}\mu_x^*(t)\right) \tag{2}$$

Let's present them more explicitly:

$$(-1,-1,-1,-1) \to \mu_x^{(-1,-1,-1,-1)} = \wedge(-\mu_x^{ul}, -\mu_x^{dl}, -\mu_x^{ur}, -\mu_x^{dr})$$

$$(1,-1,-1,-1) \to \mu_x^{(1,-1,-1,-1)} = \wedge(\mu_x^{ul}, -\mu_x^{dl}, -\mu_x^{ur}, -\mu_x^{dr})$$

$$\vdots$$

$$(1,1,1,1) \to \mu_x^{(1,1,1,1)} = \wedge(\mu_x^{ul}, \mu_x^{dl}, \mu_x^{ur}, \mu_x^{dr})$$

2.1.3. Conversion of the geometry of one-dimensional relief into the language of fuzzy logic

Let's first make two important remarks for what follows.

It is difficult to say how geometrically looks a record x in the region of the node t with the condition $\mu_x^*(t) \approx 1$, but the situation $\mu_x^*(t) \approx -1$ is clear: this is "evenness" in the region t on x proximity to a value $x(t)$ on record x near node t in context *.

A large value of any measure (2) is equivalent to large values of all the terms included in its $\wedge$-conjunction.

Numerous studies, and this will be shown below, have given grounds for the conclusion that for any morphological measure μ_x^i, the relation $\mu_x^i(t) \approx 1$, as a rule, is closely related to the well-defined local geometry P of the record x in the region of the node t.

Thus, the conversion $P \to \mu_x^{i(P)}$ of the geometry of one-dimensional relief into the language of fuzzy logic means that if there is a geometry P near the node t on record x, then the corresponding measure $\mu_x^{i(P)}$ at node t is close to 1.

Diagram in the Figure 8 explains how such a conversion works: a graphical model $M(P)$, is assigned to the geometric property P, then its Boolean perception $B(P)$ with the help of squares: a full (empty) square $\leftrightarrow$ many (few) of the graph Γ_x in the corresponding square with the center in its point $(t, x(t))$ in context $*$. $B(P)$ helps to form the Boolean index $i(P)$, and from it the morphological measure $\mu_x(P) \leftrightarrow \mu_x^{i(P)}$, representing property P.

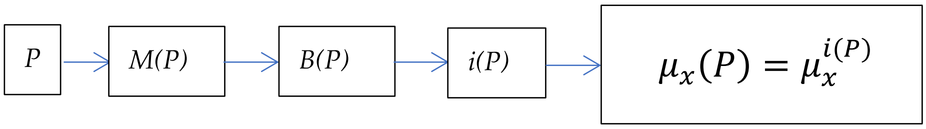
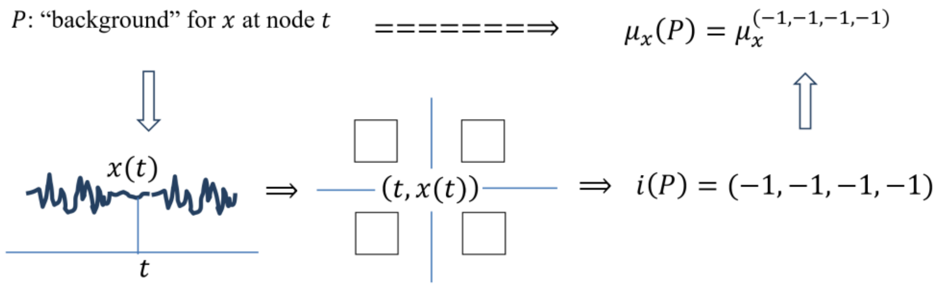


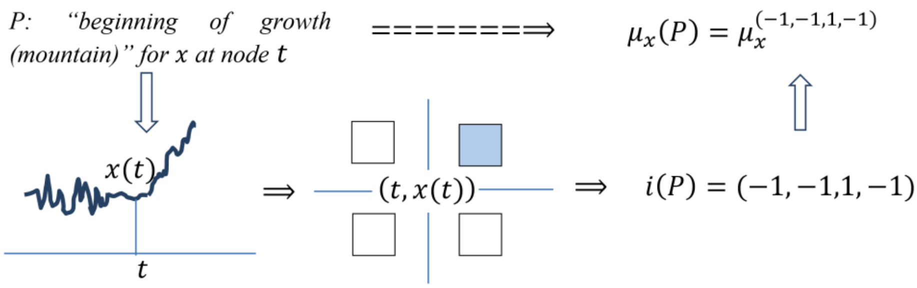
Figure 8. Scheme for conversion of one-dimensional relief into the language of fuzzy logic.

2.1.3.1. P: "background" for x at node t



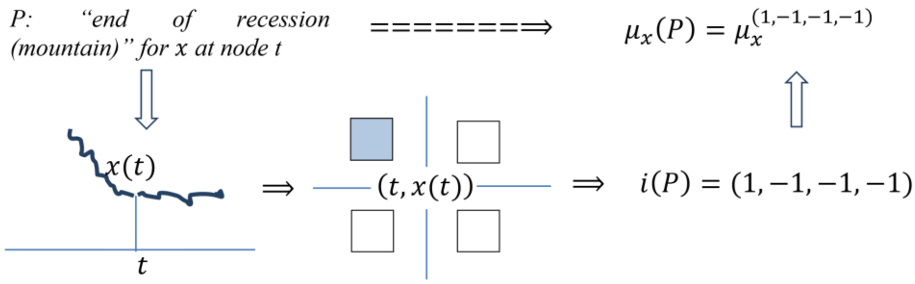
Taking into account the above remarks, a large value of the measure $\mu_x(P)$ at a node t means that all elementary measures μ_x^*, are weakly expressed in it, therefore, all of them must be small.

2.1.3.2. P: "beginning of growth (mountain)" for x at node t



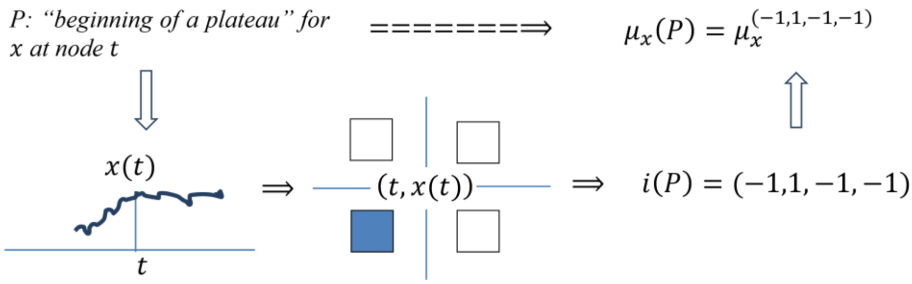
In t for $x(t)$, minimality (ul) and maximality (dl) are weakly expressed on the left, while strongly expressed minimality (ur) and weakly expressed maximality (dr) on the right. Therefore, all elementary measures except μ_x^{ur} in t must be small, while it is large.

2.1.3.3. P: "end of recession (mountain)" for x at node t



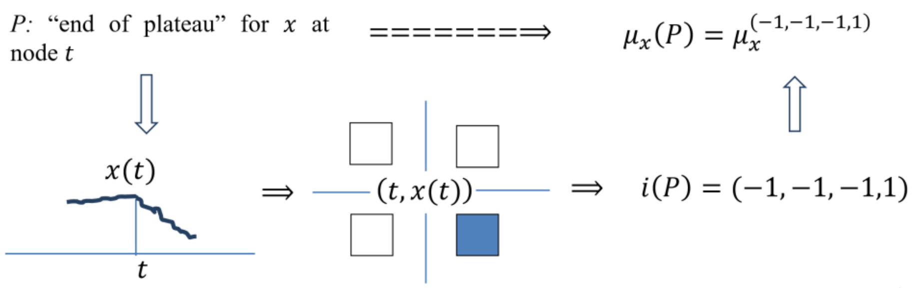
In t for $x(t)$ the minimality (ur) and maximality (dr) are weakly expressed on the right, while the minimality (ul) is strongly expressed and maximality (dl) is weakly expressed on the left. Therefore, all elementary measures except μ_x^{ul} in t must be small, while it is large.

2.1.3.4. P: “beginning of a plateau” for x at node t



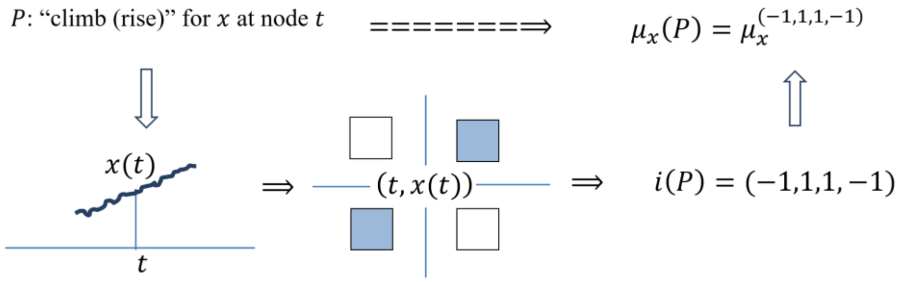
In t for $x(t)$, is weakly expressed minimality (ul) and strongly maximality (dl), on the left, while minimality (ur) and maximality (dr) are weakly expressed on the right. Therefore, all elementary measures except μ_x^{dl} in t must be small, while it is large.

2.1.3.5. P: “end of plateau” for x at node t



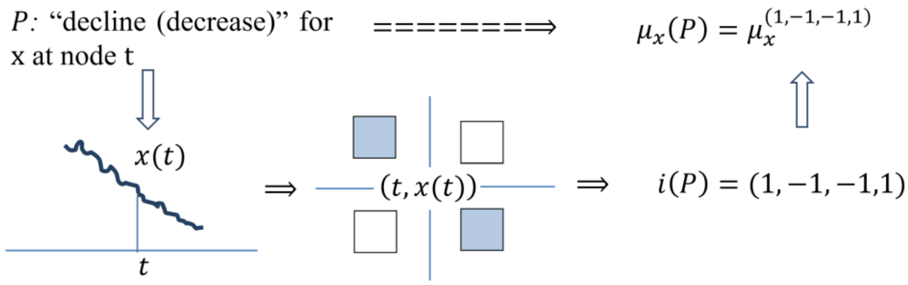
In t for $x(t)$ minimality (ul) and maximality (dl), are weakly expressed on the left, while minimality (ur) is weakly expressed and maximality (dr) is strongly expressed on the right. Therefore, all elementary measures except μ_x^{dr} in t must be small, while it is large.

2.1.3.6. *P*: “climb (rise)” for x at node t



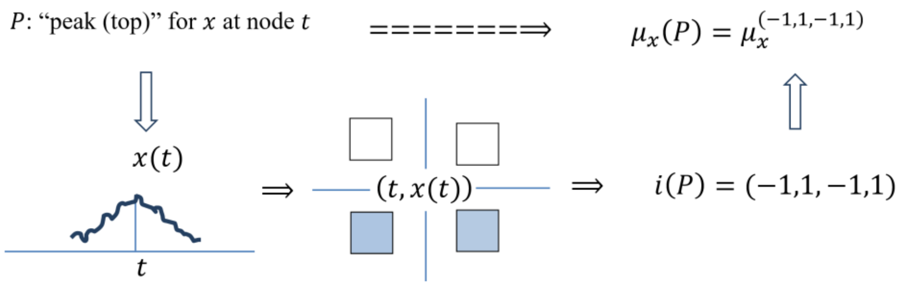
In t, for $x(t)$, minimality (ul) is weakly expressed and maximality (dl) strongly is expressed on the left, while the opposite is true on the right: maximality (dr) is weakly expressed and minimality (ur) is strongly expressed. Therefore, the elementary measures μ_x^{ul} and μ_x^{dr} in t must be small, while the measures μ_x^{dl} and μ_x^{ur} are large.

2.1.3.7. P: "decline (decrease)" for x at node t



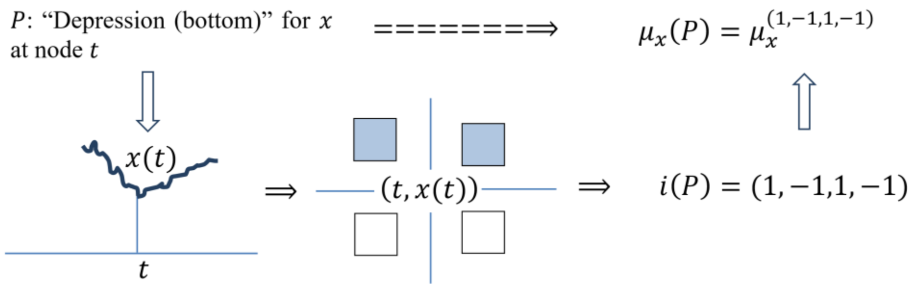
In t for $x(t)$, maximality (dl) is weakly expressed and minimality (ul) is strongly expressed, on the left, while the opposite is true on the right: minimality (ur) is weakly expressed and maximality (dr) is strongly expressed,. Therefore, the elementary measures μ_x^{ul} and μ_x^{dr} in t must be large, while the measures μ_x^{dl} and μ_x^{ur} small.

2.1.3.8. P: "peak (top)" for x at node t



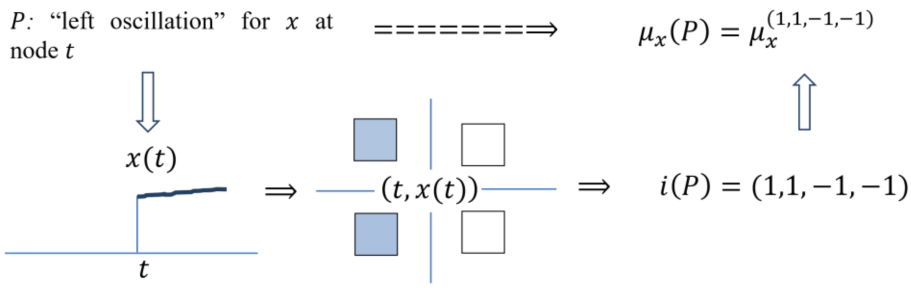
In t for $x(t)$ on the left and on the right, minimality (ul) is weakly expressed and maximality (dl) is strongly expressed. Therefore, the elementary measures μ_x^{ul} and μ_x^{ur} in t must be small, while the measures μ_x^{dl} and μ_x^{dr} are large.

2.1.3.9. P: "depression (bottom)" for x at node t



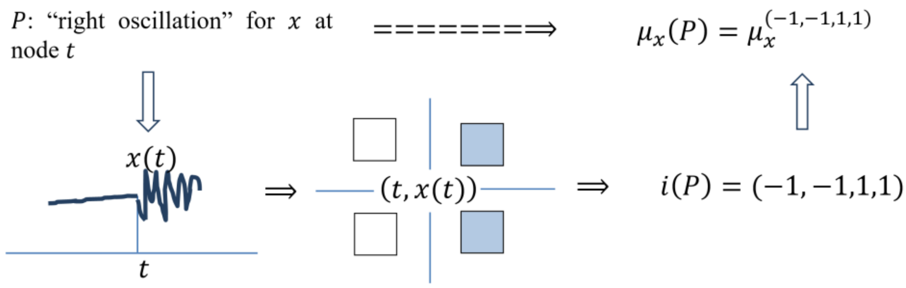
In t for $x(t)$ on the left and on the right, is weakly expressed maximality while strongly expressed minimality. Therefore, the elementary measures μ_x^{dl} and μ_x^{dr} in t must be small, while the measures μ_x^{ul} and μ_x^{ur} are large.

2.1.3.10. P: "left oscillation" for x at node t



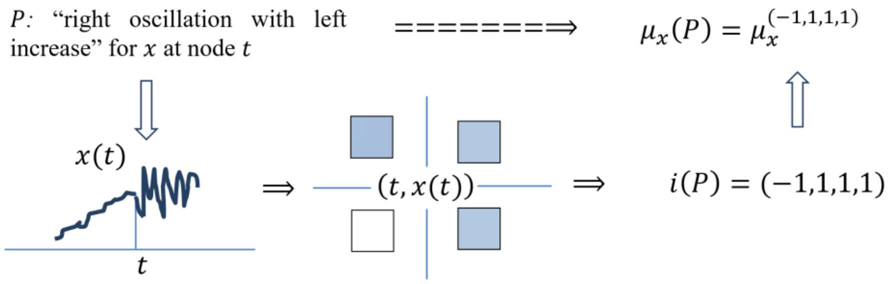
There is a significant oscillation to the left of t, therefore the maximality (dl) and minimality (ul) with respect to $x(t)$ are strongly expressed, while weakly to the right. Therefore, the elementary measures μ_x^{ul} and μ_x^{dl} at t must be large, while the measures μ_x^{ur} and μ_x^{dr} are small.

2.1.3.11. P: "right oscillation" for x at node t



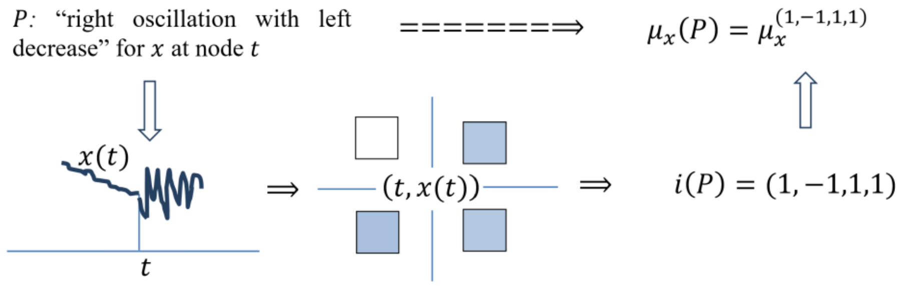
There is a significant oscillation to the right of t, therefore the maximum (dr) and minimum (ur) with respect to $x(t)$ are strongly expressed, while weakly on the left. Therefore, the elementary measures μ_x^{ur} and μ_x^{dr} in t must be large, while the measures μ_x^{ul} and μ_x^{dl} are small.

2.1.3.12. P: "right oscillation with left increase" for x at node t



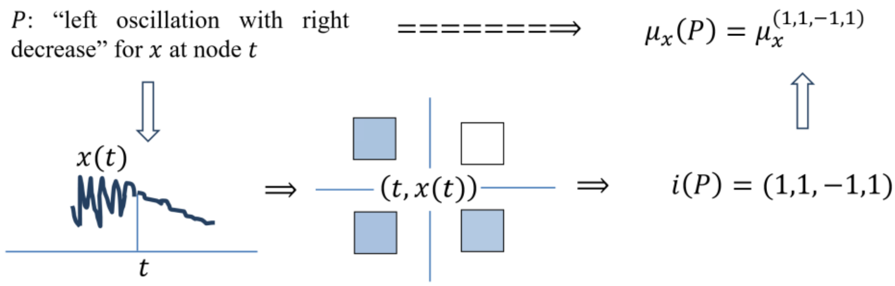
To the left of t, the minimality (ul) for $x(t)$ is weakly expressed, and the maximality (dl) is strongly expressed, while on the right, both minimality (ur) and maximality (dr) for $x(t)$ are strongly expressed. Therefore, all measures except μ_x^{ul} in t are large, and it is small.

2.1.3.13. P: "right oscillation with left decrease" for x at node t



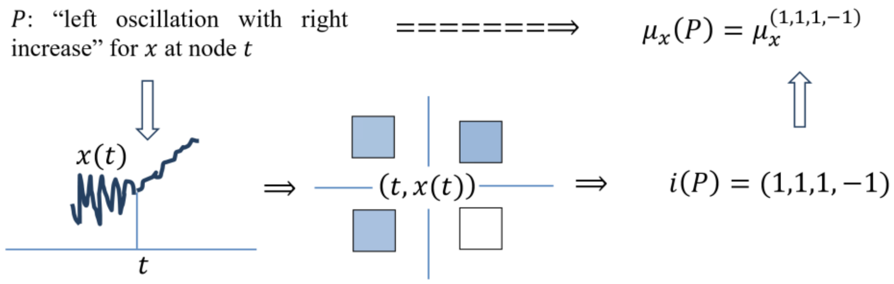
To the left of t, minimality (ul) for $x(t)$ is strongly expressed, and maximality (dl) is weakly expressed, while on the right, both minimality (ur) and maximality (dr) for $x(t)$ are weakly expressed. Therefore, all measures except μ_x^{dl} in t must be large, while it is small.

2.1.3.14. P: "left oscillation with right decrease" for x at node t



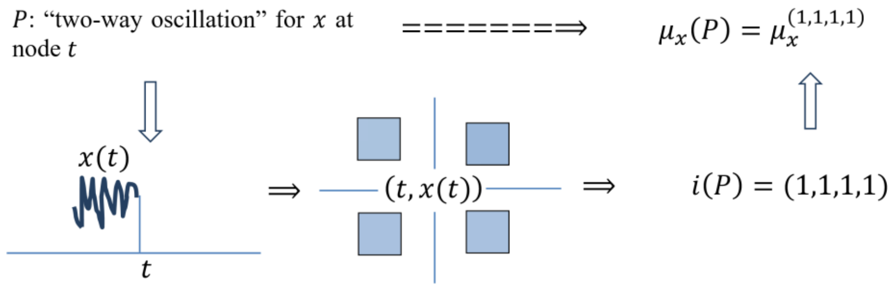
To the left of t, the maximality (dl) and minimality (ul) for $x(t)$ are strongly expressed, while on the right, the minimality (ur) is weakly expressed and the maximality (dr) is strongly expressed. Therefore, all measures except μ_x^{ur} in t must be large, while it is small.

2.1.3.15. P: "left oscillation with right increase" for x at node t



To the left of t, the maximality (dl) and minimality (ul) for $x(t)$ are strongly expressed, while on the right, the minimality (ur) is strongly expressed, and the maximality (dr) is weakly expressed. Therefore, all measures except μ_x^{dr} in t must be large, while it is small.

2.1.3.16. P: "two-way oscillation" for x at node t



In t, minimality and maximality are strongly expressed, both on the left and on the right. Therefore, all elementary measures must be large.

2.2. Next steps

The next steps appear to be in three parts

the first of them consists in any activity that allows to extract knowledge about the record based on the construction of morphological measures for it according to the scenario 2.1.3;

the second part consists in any activity that helps the quality of conversion according to the scenario 2.1.3 of the geometry of one-dimensional relief into the language of fuzzy logic;

the third part ideologically coincides with the first, but technically differs from it in more complex scenarios for the use of morphological measures, in which they are combined with other approaches to the study of the original record.

Let's start with the first one: let's denote by I the set of all four-dimensional Boolean indices ($|I| = 16$), and by $\mathfrak{M}_x$ the set of all morphological measures. Its value $\mathfrak{M}_x(t) = \{\mu_x^i(t),\ i \in I\}$ at each node t gives in the language of fuzzy sets a representation of the fulfillment in t for x of the geometric properties corresponding to morphological measures.

The multivalued mapping $\mathfrak{M}_x: T \to Fuzzy(I)$, where $Fuzzy(I)$ is understood as the set of values $\{\{\mu_x^i(t),\ i \in I\},\ t \in T\}$, is complete, but complex. Therefore, it is necessary to start with the analysis of the measures μ_x^i or their simple but important logical combinations. The analysis of the measures μ_x^i is certainly important, but it is appropriate in connection with one or another of their specific implementation and refers rather to the second part of further actions, which will be discussed below. Much more interesting is the conversation about logical combinations of morphological measures, the analysis of which would give much knowledge about the record x.

The first such combination, according to the authors, is the classical fuzzy disjunction

$$\mu_x^I(t) = \max_{i \in I} \mu_x^i(t) \tag{4}$$

It is always non-negative, since at each node t there is at least one morphological measure that manifests itself non-negatively. Moreover, if the numerical set $\mathfrak{M}_x(t)$ does not contain zero, then it contains a unique positive value $\mu_x^{i_x(t)}(t)$, which naturally coincides with $\mu_x^I(t)$. The geometric property corresponding to the measure $\mu_x^{i_x(t)}$ will be defining for x in t.

Thus, on the support $Supp(\mu_x^I) = \{t \in T: \mu_x^I(t) > 0\}$ of the measure μ_x^I, a morphological encoding $i_x: t \to i_x(t)$ of the record x arises, which seems to be very interesting due to two reasons.

- Reason one. The kernel $Ker(\mu_x^I) = \{t \in T: \mu_x^I(t) > 0\}$ of the measure μ_x^I consists of exactly those nodes where at least one of the elementary measures μ_x^*. is equal to zero. In this way,

$$Ker(\mu_x^I) = \cup Ker(\mu_x^*): * \in \{ul, dl, ur, dr\}.$$

- Second reason. In the general case, taking into account the large number of nodes ($T \gg 1$), and the stochasticity of x, we can conclude that the kernels $Ker(\mu_x^*)$ are small ("measure zero") in T, and therefore their union $Ker(\mu_x^I)$ is small in T.

The support $Supp(\mu_x^I)$ "almost everywhere" coincides with T, and therefore all sorts of statistical characteristics of the encoding i_x and the measure μ_x^I itself are weighty characteristics of the record x and can serve as the basis for its comparison, in particular, correlation with other records.

The very first and interesting characteristic of this kind $\leftrightarrow$ the histogram of morphological coding i_x on 16 features.

The second part of further actions is related to the specific implementation of scenario 2.1.3: analysis of the work of certain μ_x^I, their stability, dependence on parameters, as well as measures (techniques) that improve the quality of translation. The latter definitely include pre-smoothing of record x.

The third part of the further actions is based on the second: knowing the possibilities of a particular implementation, it is combined with other approaches to the record x. This allows us to obtain for it new results of a more general, in particular, non-local nature. So in the next paragraph 2.3, a specific implementation of 2.1 will be built and with the help of its measures of the beginning and end of the mountain $\mu_x^{(-1,-1,1,-1)}$ and $\mu_x^{(1,-1,-1,-1)}$, as well as the measures of the peak $\mu_x^{(-1,1,-1,1)}$ in fourth part will be found and non-locally morphologically analysed elevations on non-negative "smooth" reliefs and, as a result, very important results in terms of anomalies on the record x by DMA methods (interpreter logic).

2.3. DMA-morphological analysis: formalization

DMA methods make it possible to construct different variants of elementary measures (1), and therefore, taking into account (2), also morphological ones. One of them was implemented in [6].

In what follows, we will need three types of localization at the node t:

two-sided: $U(t,\Delta) = \{\bar{t} \in T: |\bar{t} - t| \le \Delta\}$;

left: $U^l(t,\Delta) = \{\bar{t} \in T: t - \bar{t} \le \Delta\}$; (5)

right: $U^r(t,\Delta) = \{\bar{t} \in T: \bar{t} - t \le \Delta\}$,

where $\Delta \ll |T|$ is local view parameter.

Let's present a construction Q, expressing in an elementary context $*$ the deviation of the record x from its value $x(t)$.

$$x^{ul}(t) \leftrightarrow Q_x^{ul}(t) = \frac{\sum\big(x(t^+) - x(t)\big): \big(t^+ \in U^l(t,\Delta)\big) \wedge \big(x(t^+) > x(t)\big)}{|U^l(t,\Delta)|}$$

$$x^{dl}(t) \leftrightarrow Q_x^{dl}(t) = \frac{\sum\left(x(t)-x(t^-)\right):\left(t^- \in U^l(t,\Delta)\right)\wedge\left(x(t^-)<x(t)\right)}{|U^l(t,\Delta)|} \quad (6)$$

$$x^{ur}(t) \leftrightarrow Q_x^{ur}(t) = \frac{\sum\left(x(t^+)-x(t)\right):\left(t^+ \in U^r(t,\Delta)\right)\wedge\left(x(t^+)>x(t)\right)}{|U^r(t,\Delta)|}$$

$$x^{dr}(t) \leftrightarrow Q_x^{dr}(t) = \frac{\sum\left(x(t)-x(t^-)\right):\left(t^- \in U^r(t,\Delta)\right)\wedge\left(x(t^-)<x(t)\right)}{|U^r(t,\Delta)|}$$

The series $x^*(t)$, $* \in \{ul, dl, ur, dr\}$, quantifies the $*$-deviation of the record x from its value $x(t)$ at node t and is only the first half in the definition of the elementary measure $\mu_x^*(t)$ at node t. The second half is the answer to the question: "to what extent can $*$-deviation $x^*(t)$ be considered large?". It is obtained by comparing $x^*(t)$ with the values $x^*(\bar{t})$ at the remaining nodes $\bar{t} \in T$, that is, with the pattern $x^*(T)$. In DMA, it is called the maximum measure of the series $x^*(t)$ and is denoted as $\mu esmax\, x^*(t)$:

$$\mu_x^*(t) = \mu esmax\, x^*(t) \quad (7)$$

There are several of its designs. Here is one of them (σ-construction):

$$\mu_x^*(t) = \frac{\sigma_-^*(x,t)-\sigma_+^*(x,t)}{\sigma_-^*(x,t)+\sigma_+^*(x,t)} \quad (8)$$

Where

$$\sigma_-^*(x,t) = \sum\left(x^*(t)-x^*(t^-)\right):(t^- \in T)\wedge\left(x^*(t^-)<x^*(t)\right)$$

$$\sigma_+^*(x,t) = \sum\left(x^*(t^+)-x^*(t)\right):(t^+ \in T)\wedge\left(x^*(t^+)>x^*(t)\right)$$

Figures 9-24 below show model examples of the implementation of scenario (5)-(8) on model examples for the situations described in paragraphs 2.1.3.1-2.1.3.16.

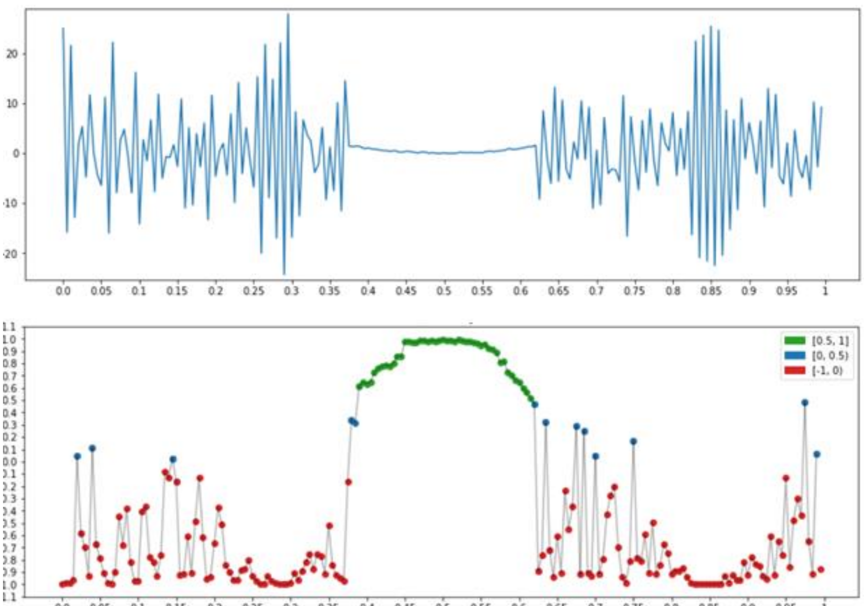


Figure 8. "Background": $\mu_x(P) = \mu_x^{(-1,-1,-1,-1)}$.

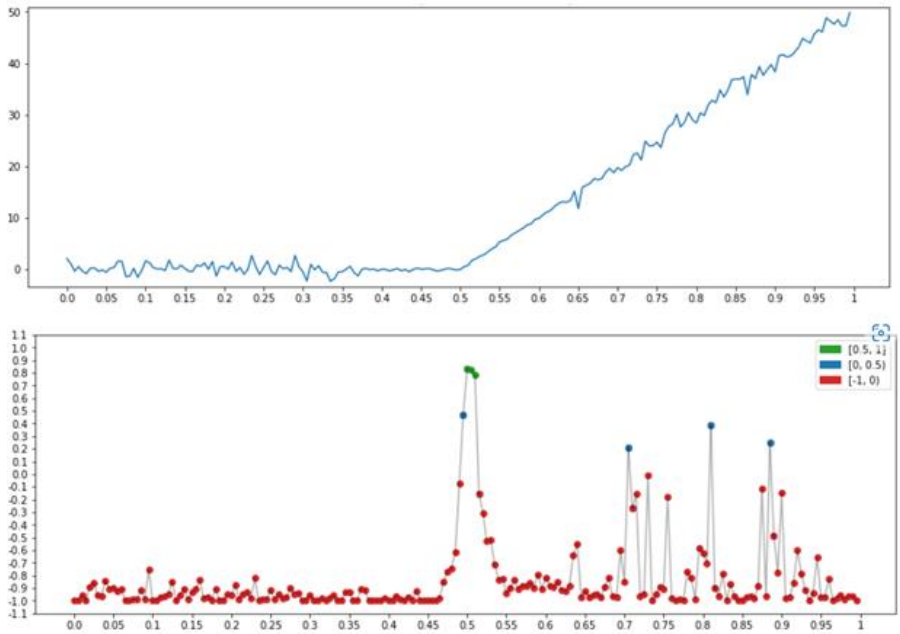


Figure 9. "Beginning of growth (mountain)": $\mu_x(P) = \mu_x^{(-1,-1,1,-1)}$.

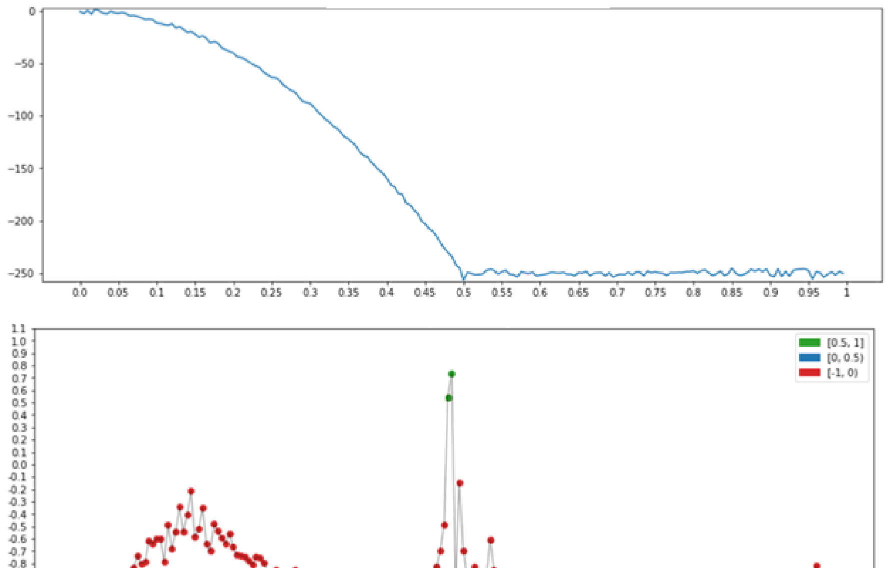


Figure 10. "End of descend (mountain)": $\mu_x(P) = \mu_x^{(1,-1,-1,-1)}$.

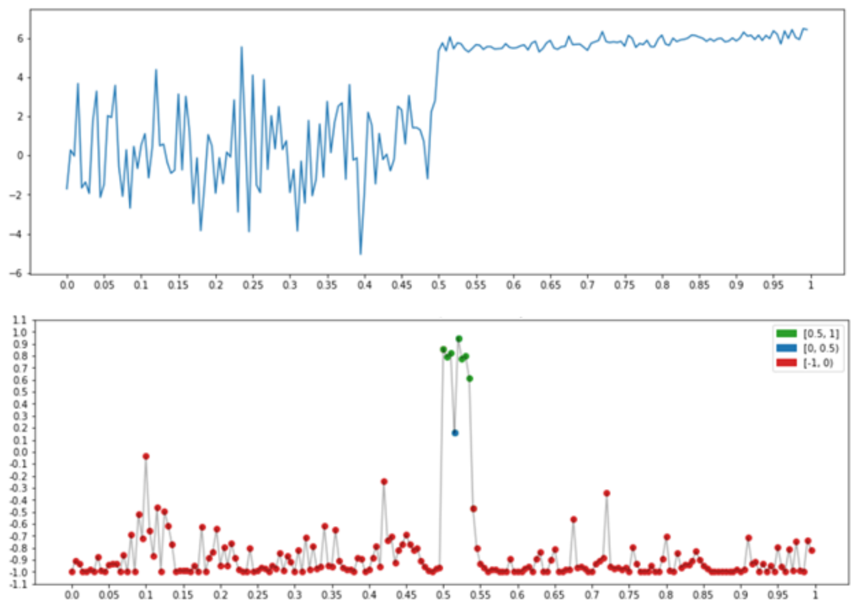


Figure 11. "Beginning of plateau": $\mu_x(P) = \mu_x^{(-1,1,-1,-1)}$.

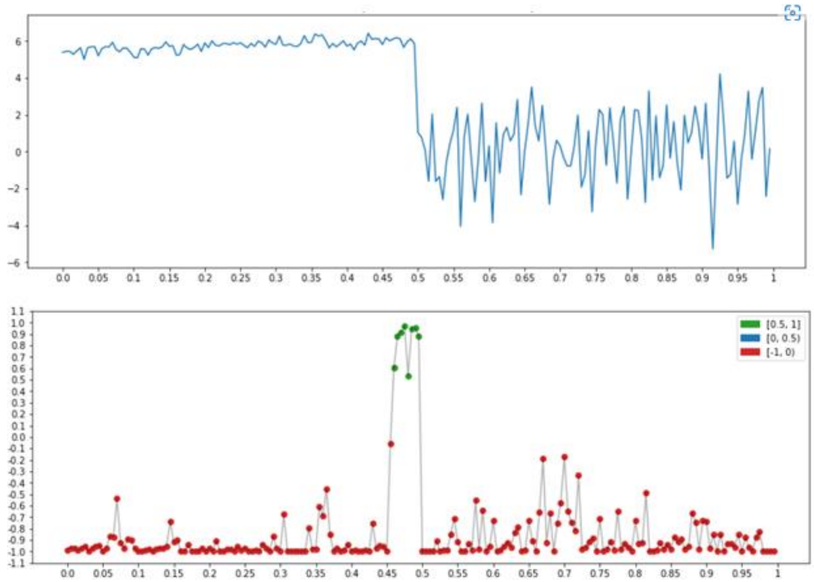


Figure 12. "End of plateau": $\mu_x(P) = \mu_x^{(-1,-1,-1,1)}$.

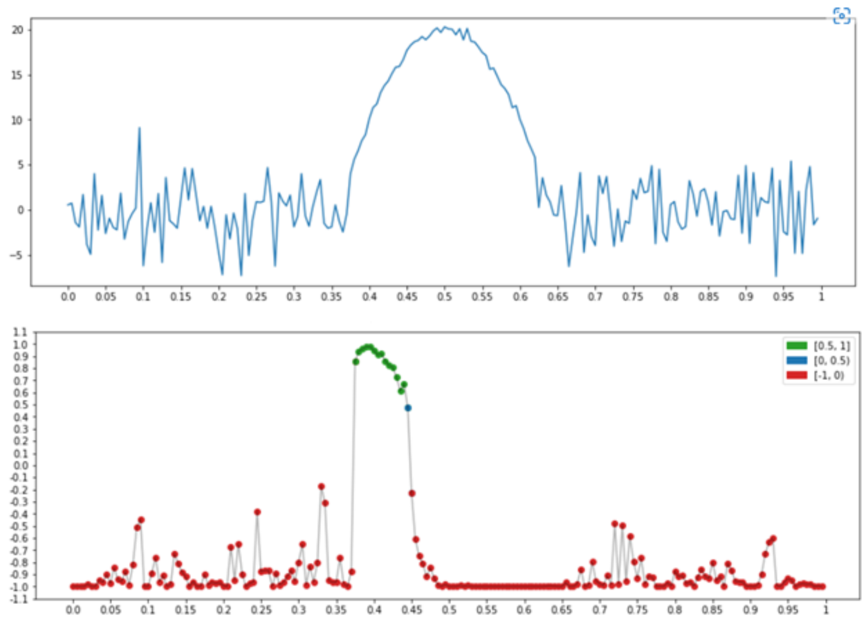


Figure 13. "Climb (growth)": $\mu_x(P) = \mu_x^{(-1,1,1,-1)}$.

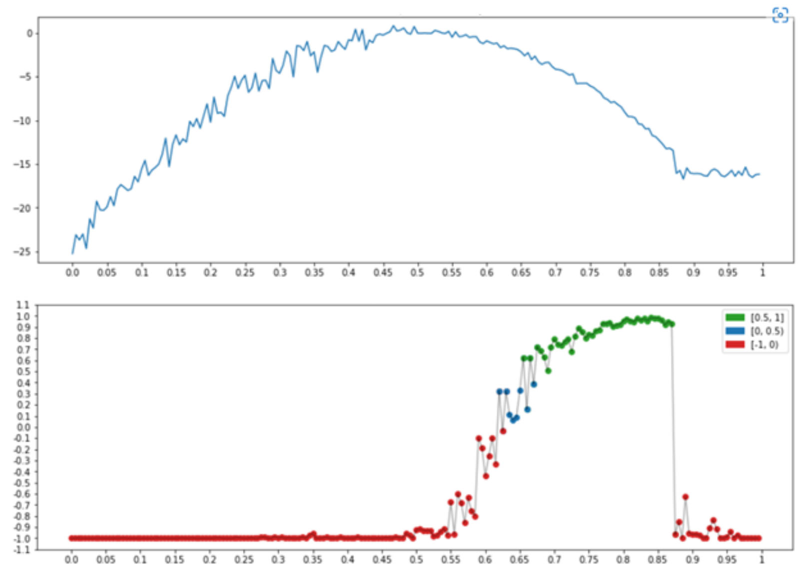


Figure 14. "Descending (decreasing)": $\mu_x(P) = \mu_x^{(1,-1,-1,1)}$.

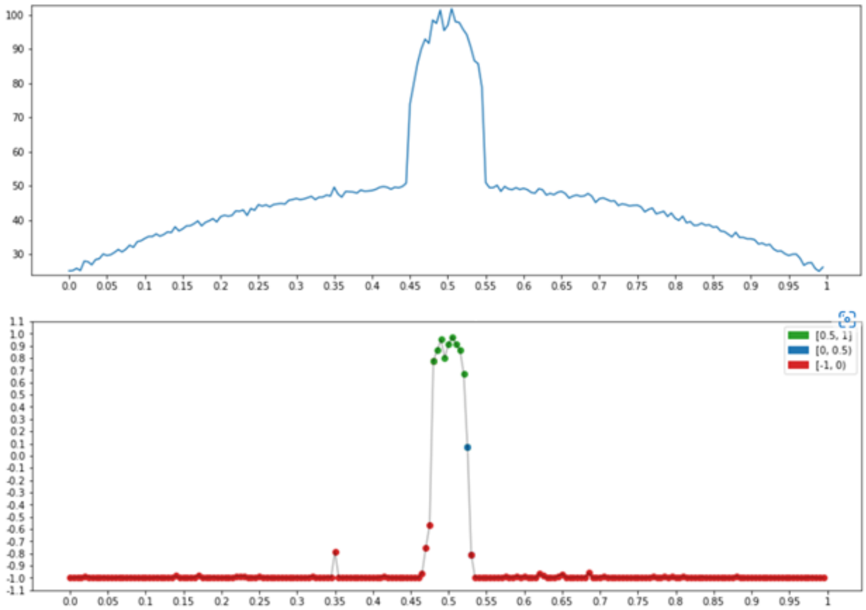


Figure 15. "Peak (top)": $\mu_x(P) = \mu_x^{(-1,1,-1,1)}$.

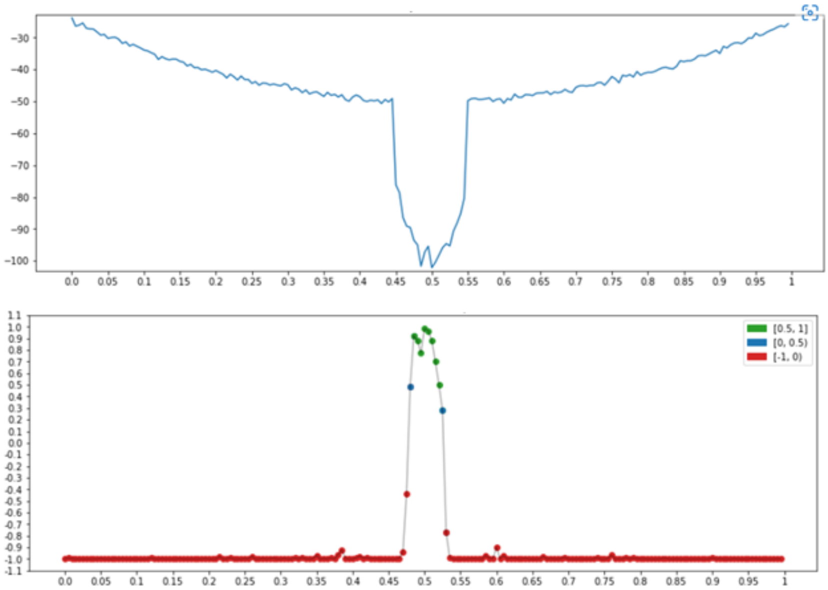


Figure 16. "Depression (bottom)": $\mu_x(P) = \mu_x^{(1,-1,1,-1)}$.

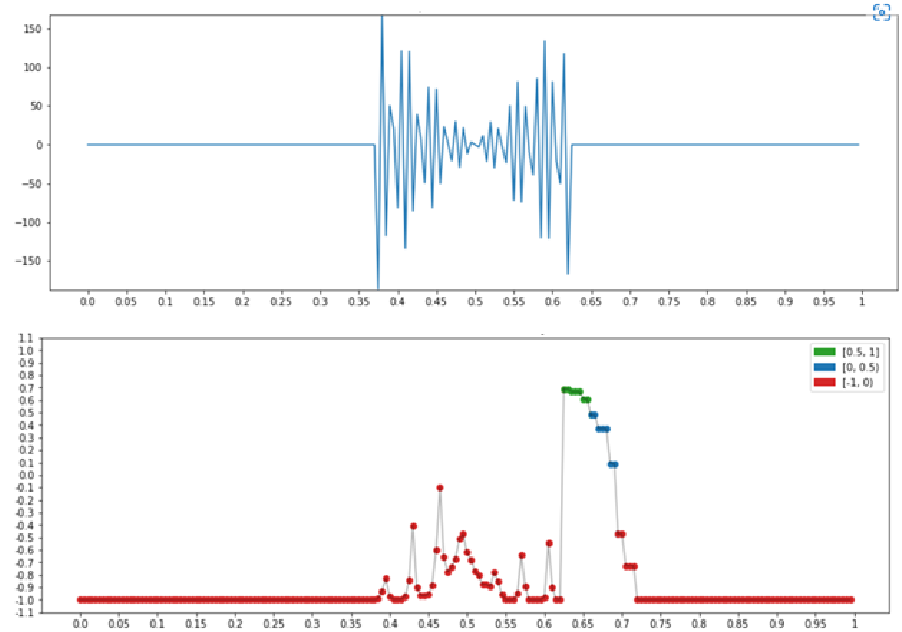


Figure 17. "Left oscillation": $\mu_x(P) = \mu_x^{(1,1,-1,-1)}$.

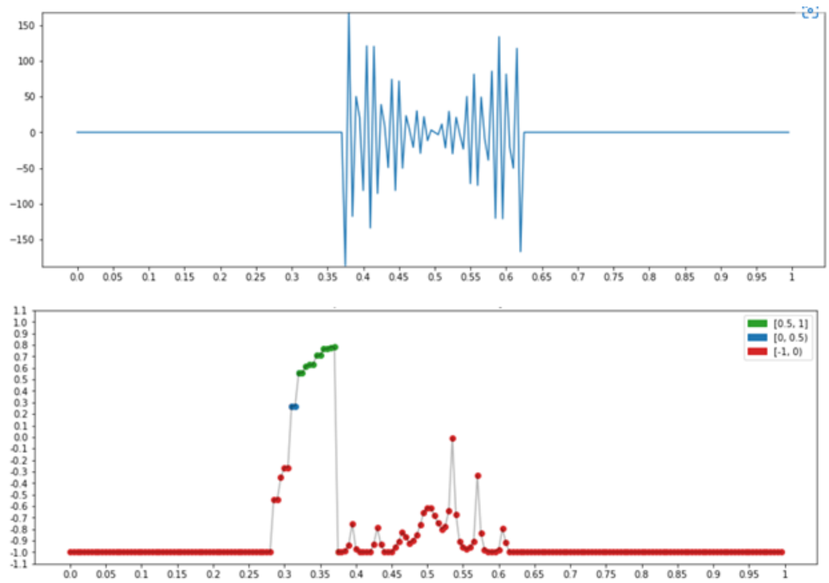


Figure 18. "Right oscillation": $\mu_x(P) = \mu_x^{(-1,-1,1,1)}$.

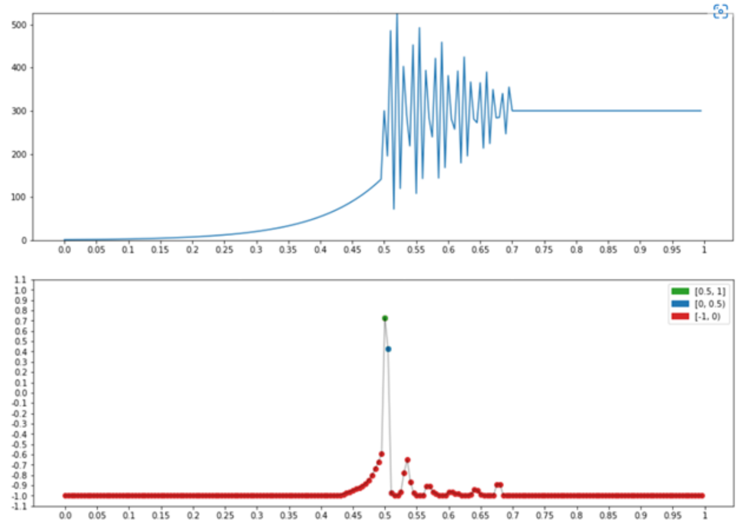


Figure 19. "Right oscillation with left increase": $\mu_x(P) = \mu_x^{(-1,1,1,1)}$.

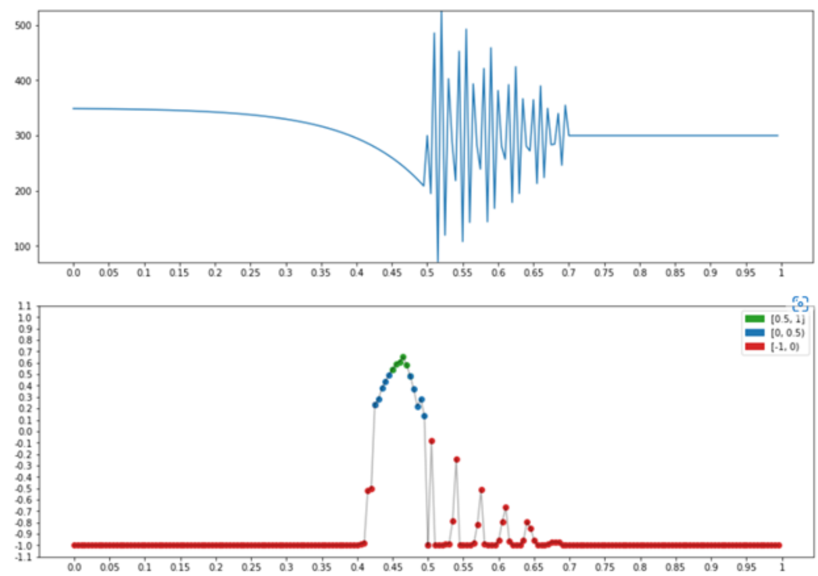


Figure 20. "Right oscillation with left decrease": $\mu_x(P) = \mu_x^{(1,-1,1,1)}$.

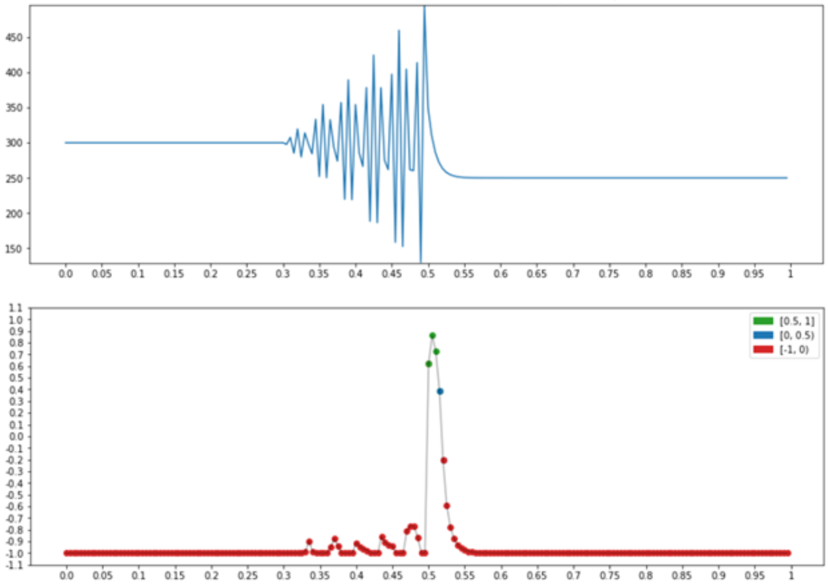


Figure 21. "Left oscillation with right decrease": $\mu_x(P) = \mu_x^{(1,1,-1,1)}$.

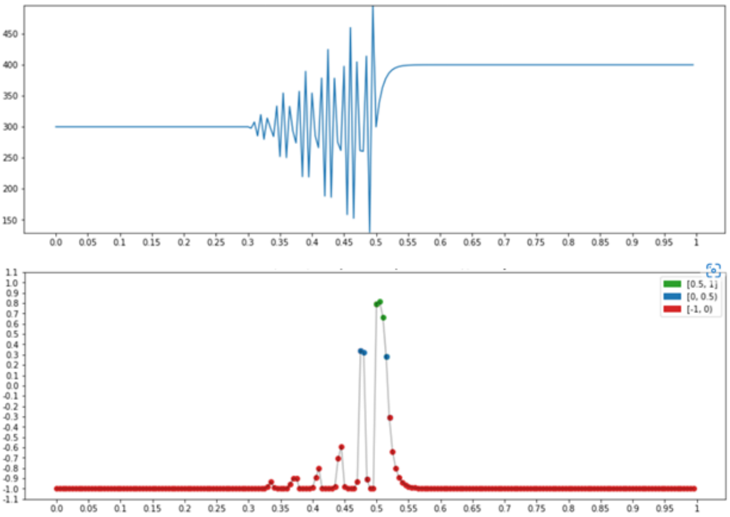


Figure 22. "Left oscillation with right increase": $\mu_x(P) = \mu_x^{(1,1,1,-1)}$.

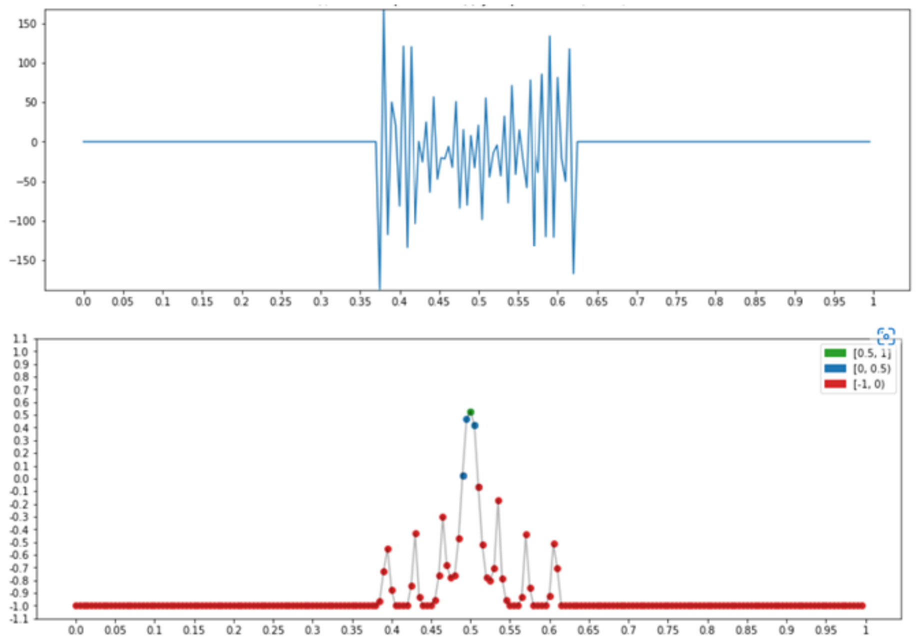


Figure 23. "Two-way oscillation": $\mu_x(P) = \mu_x^{(1,1,1,1)}$.

Figure 25 shows an example of morphological analysis of a fragment of a magnetic storm record using fuzzy disjunction of elementary measures (4).

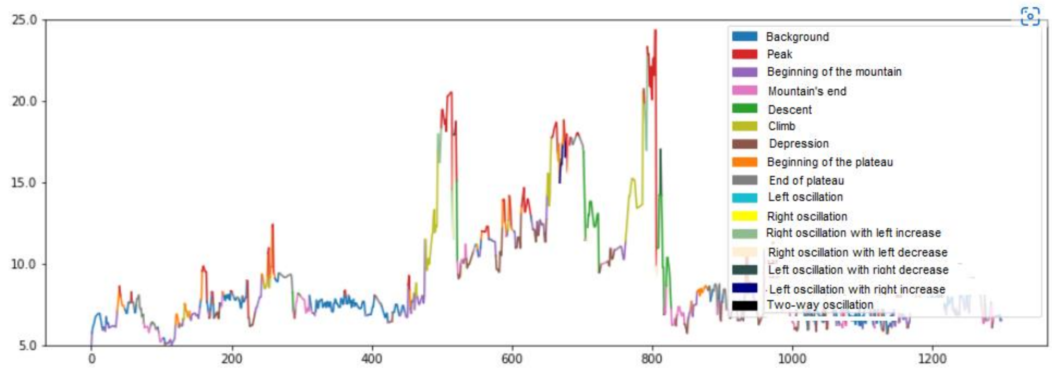


Figure 24. An example of morphological analysis of a fragment of a magnetic storm record using fuzzy disjunction of elementary measures.

The main disadvantage of scenario (5)-(8) is its instability due to the direct dependence on the value $x(t)$. Hence the necessary requirement that the value $x(t)$ is "non-random" for any $t \in T$, that is, the record x is "globally smooth".

The natural way out is ↔ transition from the original record x to some kind of its smoothing Smx and then consider the corresponding morphological measure for Smx as the morphological measure for the x record (this is an example of the second part of the further activity):

$$x \to Smx \to \mu_{Smx}^i \rightleftarrows \mu_x^i \quad (9)$$

3. *R*-morphological analysis

Now the morphological activity of the third kind: we connect the morphological analysis with the logic of the interpreter (see Introduction), and then we will receive and analyze the results of such a connection.

The logic of the interpreter assumes an active attitude of the expert to the record. Moreover, a significant part of DMA study of records - is associated with their study through streightening - non-negative quantitative expressions of properties that are of interest to the expert [1,5].

3.1. *Record straightening*

Let's start with its rigorous definition.

3.1.2. Definition

1. The straightening construction R is a non-negative functional on T, parameterized by T:

$$R: F(T) \times T \to R^{+} \tag{10}$$

2. The straightening of x based on the construction R is a non-negative function $R_x: t \to R(x,t)$.

The value of $R(x,t)$ is denoted by $R_x(t)$ and is understood as a quantitative assessment of the behavior of record x at node t with local view of R at its dynamics locally, so that the straightening construction is always connected to some fixed localization (5).

DMA leaves free understanding for the expert, the choice of straightening as a quantitative expression of the property of interest to him. However, reality has shown the stability of this choice: the range of basic straightenings has been determined, with which most experts want to deal. Behind each of them is a fundamental mathematical concept, which confirms the correctness of the settings when creating DMA.

3.2. *R-morphological measures*

Basic straightenings R have stochastic stability, so that the record $R_x(t)$ can be considered "smooth", and the application of morphological analysis (5) – (8) to it is correct. This is how the DMA-morphological analysis of the record x, begins, that is, through its stable straightening R_x:

$$R_{\mu_x^*} = \mu^*_{R_x}, \quad R_{\mu_x^i} = \mu^i_{R_x} \tag{11}$$

Such an analysis of the set $R^I_{\mu_x}$ is called R-morphological. It makes it possible to understand how the property of the record x behind the straightening is distributed in time on T.

The more fundamental the property behind R, the more interesting R-analysis is. So, for example, if R is the local variance of E, then E-morphological analysis gives a dynamic understanding of the variance in the record x, which is closely related to continuity. If R is a local length L then L-morphological analysis gives a dynamic understanding of the length on the record x, closely related to the local frequency.

On the straightening R_x, the most interesting are the elevations corresponding to R-anomalies on the record x, according to the logic of the interpreter. It is necessary not only to recognize them, but also to understand how they are constructed in order, in particular, to encode and be able to compare with other anomalies.

Below we will solve this problem using morphological analysis on arbitrary non-negative reliefs and, as a consequence, for straightenings of an arbitrary record.

4. Search for Elevations Using Morphological Measures

With the help of morphological measures of the beginning (end) 2.1.3.2 (2.1.3.3) and peak 2.1.3.8, built on elementary measures in the implementation (8), on non-negative "smooth" reliefs, it is possible to determine the elevations and conduct a morphological analysis. The algorithm presented below is empirical, but proven enough to be presented.

The search logic will be consistently refined, and now we will give its first approximation: the mountain is a triad of foothills and the central part, which we recognize using the measures mentioned above.

4.1. Required minimum: designations, definitions, facts.

Let's give the necessary information for a thesis, but a full-fledged story about the search for elevations using morphological measures.

4.1.1. Definition

A segment in T ↔ a subset (sequence) of nodes without gaps.

4.1.2. Statement

Any subset S in T is a disjoint union of its maximal segments.

4.1.3. Definition

System of segments in S ↔ chain of maximal segments in S, following one after another.

4.1.4. Definition

$T_x(i) = \{t \in T: \mu_x^i(t) > 0\}$ ↔ subset of manifestation in T of the morphological measure μ_x^i.

4.1.5. Statement

$$Supp\ \mu_x^I = \vee_{i \in I} T_x(i)$$

The proof follows from the constructions of the measures μ_x^i (2) and the definition of the measure μ_x^I (4).

4.1.6. Notations

$$\mu_x^{(-1,-1,1,-1)} \leftrightarrow \mu_x^b,$$
$$\mu_x^{(1,-1,-1,-1)} \leftrightarrow \mu_x^e,$$
$$\mu_x^{(-1,1,-1,1)} \leftrightarrow \mu_x^p,$$
$$T_x(\mu_x^b) \leftrightarrow T_x(b),$$
$$T_x(\mu_x^e) \leftrightarrow T_x(e),$$
$$T_x(\mu_x^p) \leftrightarrow T_x(p).$$

4.2. Search algorithm: logic and formalization

4.2.2. Construction of elevation

Elevation construction ↔ {initial stage (left foot), left slope, central part, right slope, final stage (right foot)}

Next, we describe all the parts that make up the elevation according to the DMA method: first, the logic of the nonformal part, then its algorithmic formalization.

4.2.3. Logic of the initial stage

The rise at the initial stage can go with interruptions, which should not be significant either vertically or descending. Interruptions are stochastic steps, not very high and not decreasing much.

4.2.4. Formalization of the initial stage

Maximum system of measure segments μ_x^b, between which there are no measure segments of μ_x^e and μ_x^p ↔ maximal in the union $T_x(b) \cup T_x(e) \cup T_x(p)$ the system of segments lying in $T_x(b)$.

4.2.5. End stage logic

Descending at the final stage may come with interruptions, which should not be significant either vertically or ascending. Interruptions are stochastic right-hand steps, not very high and strongly non-increasing.

4.2.6. Formalization of the final stage

Maximum system of measure segments μ_x^e, between which there are no of measure segments μ_x^b and μ_x^p ↔ maximal in the union $T_x(b) \cup T_x(e) \cup T_x(p)$) system of segments lying in $T_x(e)$.

4.2.7. The logic of the central part

The system of peaks enclosed between the left and right foothill.

4.2.8. Formalization of the central part

Maximum in $T_x(p)$ system of segments lying between the left and right foothills.

4.2.9. Left slope

Relief fragment connecting the end of the left foothill with the beginning of the central part.

4.2.10. Right slope

Relief fragment connecting the end of the central part with the beginning of the right foothill.

4.2.11. Elevations and their chains

Let's sum up the intermediate result. The results of activity 4.2.1-4.2.9 will be fragments m on the relief x, defined by the triads $m(b) < m(p) < m(e)$, according to scenarios 4.2.2-4.2.3, 4.2.6-4.2.7, 4.2.4-4.2.5 (backbones m). Fragments m will be called elevations (mountains) on relief x. All of them are disjunctive. Let's denote by $\mathfrak{M}$ their set: $\mathfrak{M} = \{m_1 < m_2 < \cdots < m_N\}$

Among them there may be groups of elevations that are close to each other. Therefore, the set $\mathfrak{M}$ needs, in the general case, further clustering by combining into single groups elevations that are close to each other. We call such groups chains of elevations (mountain ranges) on relief x, denote them by cm, and their totality by $\mathfrak{CM}$:

$$\mathfrak{M} = \left\{\{m_1, \dots, m_{i_1}\}, \{m_{i_1+1}, \dots, m_{i_2}\}, \dots, \{m_{i_{k-1}+1}, \dots, m_{i_N}\}\right\}$$

$$\mathfrak{CM} = \{sm_1, sm_2, \dots, sm_k\}$$

$$sm_1 = \{m_1, \dots, m_{i_1}\}$$

$$sm_2 = \{m_{i_1+1}, \dots, m_{i_2}\}$$

$$\dots$$

$$sm_k = \{m_{i_{k-1}+1}, \dots, m_{i_N}\}.$$

The transition $\mathfrak{M} \to \mathfrak{CM}$ is based on the proximity relation $m_i \sim m_{i+1}$ in $\mathfrak{M}$:

$$m_i \sim m_{i+1} \leftrightarrow \frac{|m_i| + |m_{i+1}|}{|\min m_i\,;\max m_{i+1}|} \geq \frac{3}{4}$$

4.3. Example: Morphological Analysis of a Magnetic Storm Record

Figures 26-29 show an example of a morphological analysis of a magnetic storm that occurred on 3-4 November 2011. The component Y from the WSE (White Sea) observatory for the period 3-5 November (4320 points) was used as the record for study. The “Energy” straightening with Δ = 5 was used, elementary measures were built with Δ=30. Figure 26b presents a morphological analysis of the straightening for the entire considered period, and in Figure 26a - corresponding anomalies. Figure 27a-b shows the morphological analysis of the storm.

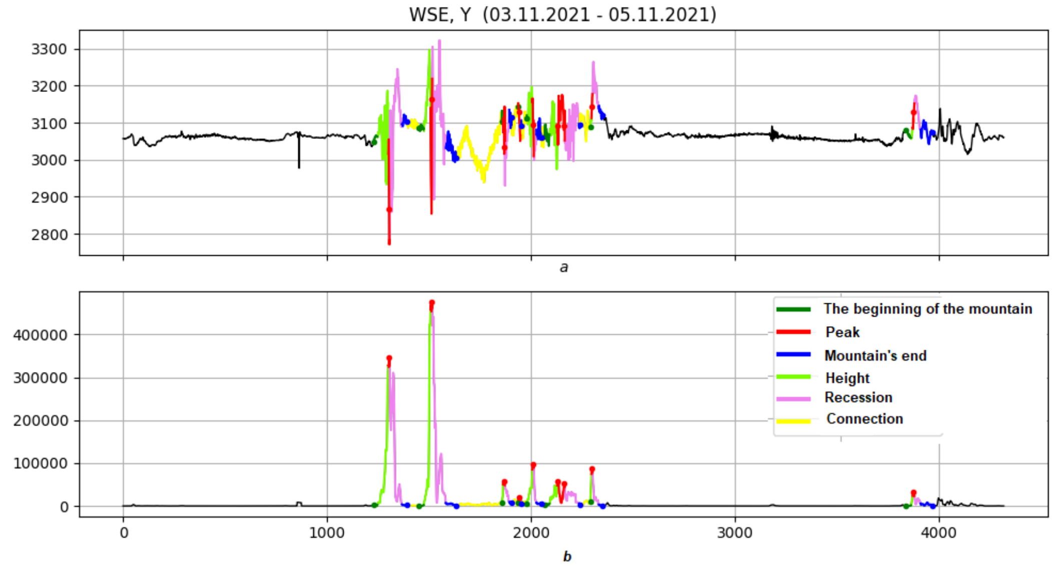


Figure 25. Morphological analysis of straightening for the entire considered period.

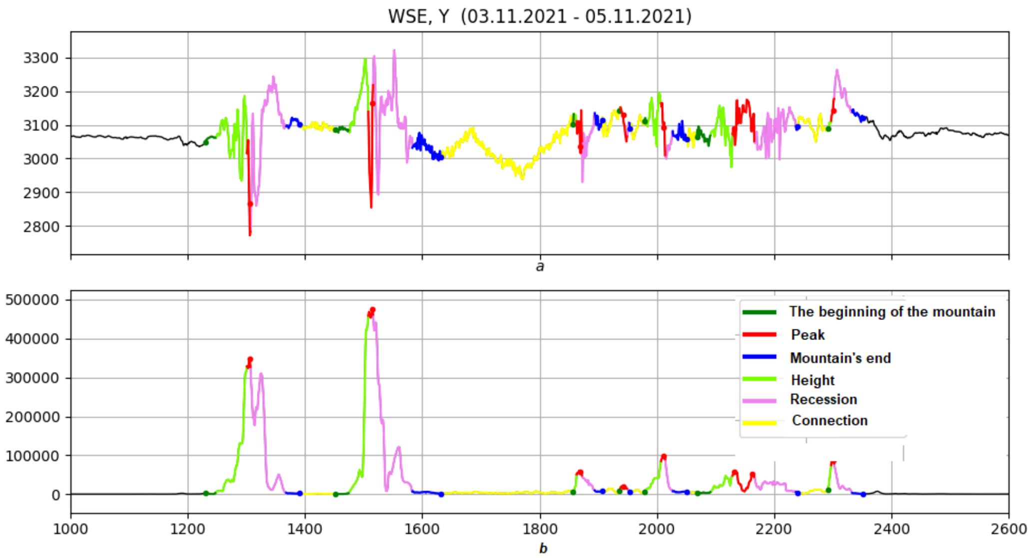


Figure 26. Morphological analysis of the storm for the entire considered period.

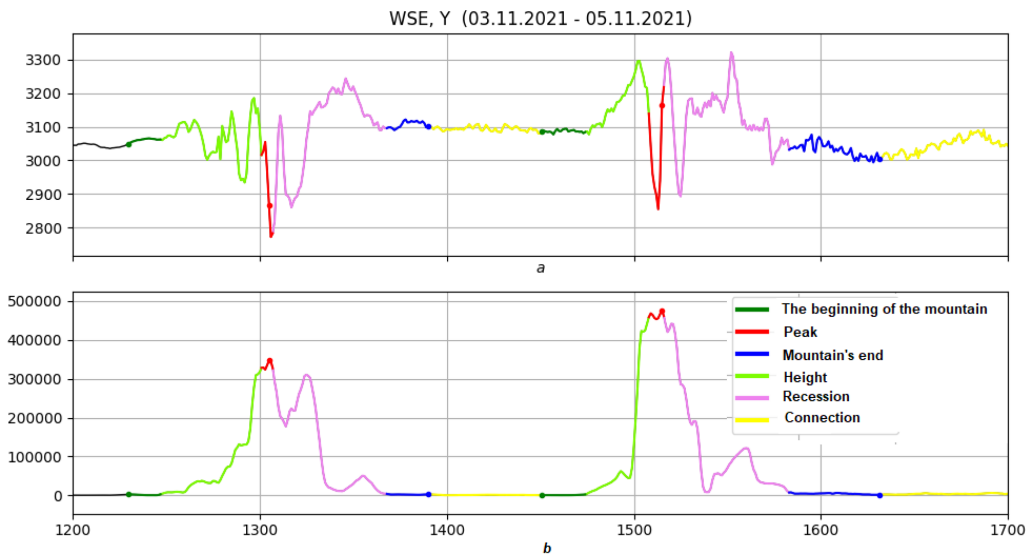


Figure 27. Morphological analysis of the first part of the storm.

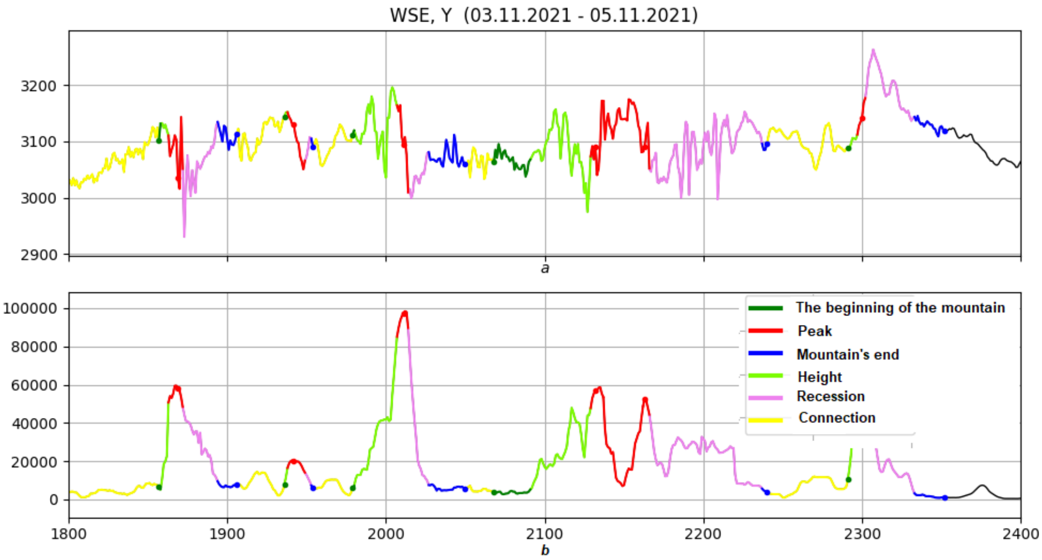


Figure 28. Morphological analysis of the second part of the storm.

5. Conclusions

This section is not mandatory but can be added to the manuscript if the discussion is unusually long or complex.

DMA is a series of algorithms aimed at solving data analysis problems: clustering and tracing in multidimensional spaces, time series analysis, including their smoothing, searching for anomalies and trends on them, studying their morphology, and many more. All DMA algorithms have a universal character and are bound by a single formal basis.

The concept of DMA is modeling a person's ability to analyze data, due to a more natural and stable, compared to mathematical, nature of his perception of form, discreteness and stochasticity. In turn, this is explained by a flexible, adaptive human perception of the fundamental concepts of proximity, continuity, connectedness, and others.

The technical basis of DMA, along with classical mathematics, also includes fuzzy mathematics and fuzzy logic, since they are required as a means of modeling the actions of a person who thinks and operates not with numbers, but with fuzzy concepts [1,2].

All this takes place in this work: the nonformal logic of our approach to the morphological analysis of the record $x(t)$ presented in the first part is one of the possible answers of a person traveling along its graph Γ_x, to sixteen questions 2.1.3.1-2.1.3.16.

The formulation of such answers proposed in the second part of present paper is especially effective for smooth time series, which, in particular, include the basic straightening R_x of record $x(t)$: R-morphological analysis (third part) makes it possible to understand the dynamics of the manifestation of property R on the record $x(t)$. Despite the local nature of the straightening R, this conclusion is no longer local.

The same non-local character is the result of the last fourth part of the paper, concerning the search for mountains on the straightening R_x using the morphological measures of “beginning”, “peak” and “end”. Mountains are recognized with precise indication of boundaries, initial and final stages, explicit indication of slopes and peaks (see Figure 7).

The morphological quality of mountains recognition passes to the R-anomalies corresponding to them on the records and serves as the basis for their further classification and encoding (see Figure 7).

Precisely these requirements for the recognition of extreme geomagnetic events that the MAGNUS analytical complex, with intellectual unit that includes DMA imposes on formalization [1].

The authors suggest the continuation of research, in particular, the construction of new variants of morphological measures 2.1.3.1-2.1.3.16.

6. Patents

This section is not mandatory but may be added if there are patents resulting from the work reported in this manuscript.

Supplementary Materials: The following supporting information can be downloaded at: www.mdpi.com/xxx/s1, Figure S1: title; Table S1: title; Video S1: title.

Author Contributions: All authors contributed to the study conception and design. Conceptualization, original draft preparation: A.S.M., A.A.O.; conceptualization, methodology, review and editing and validation: K.D.A., D.B.V; material preparation, formal analysis, data curation, algorithm development: B.Sh.R., A.A.O. All authors read and approved the final manuscript.

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Data Availability Statement: We encourage all authors of articles published in MDPI journals to share their research data. In this section, please provide details regarding where data supporting reported results can be found, including links to publicly archived datasets analyzed or generated during the study. Where no new data were created, or where data is unavailable due to privacy or ethical restrictions, a statement is still required. Suggested Data Availability Statements are available in section "MDPI Research Data Policies" at https://www.mdpi.com/ethics.

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Conflicts of Interest: The authors declare no conflict of interest.

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