Grupa 331, Seminar 9, 03.12.2020, EDDP

1. Se dă sistemul: $\begin{cases} x_1' = -x_1 + x_2 - 2x_3 \\ x_2' = 4x_1 + x_2 + e^{-t} \\ x_3' = 2x_1 + x_2 - x_3 \end{cases}$

a) Scrierea sistemului în formă matricială: $x' = Ax + b(t)$

b) Mulțimea soluțiilor sistemului

c) Soluția care verifică: $x_1(0) = 1$; $x_2(0) = -1$; $x_3(0) = 2$.

a) $$\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \underbrace{\begin{pmatrix} -1 & 1 & -2 \\ 4 & 1 & 0 \\ 2 & 1 & -1 \end{pmatrix}}_{A} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ e^{-t} \\ 0 \end{pmatrix}}_{b(t)}$$

b) * rezolvăm sistemul omogen atașat: $\bar{x}' = A\bar{x}$.

• valorile proprii pt A:

$$\det(A - \lambda I_3) = 0$$

$$\begin{vmatrix} -1-\lambda & 1 & -2 \\ 4 & 1-\lambda & 0 \\ 2 & 1 & -1-\lambda \end{vmatrix} = 0 \Rightarrow$$

$C3 \leftarrow C3 + 2C2$

$$\Rightarrow 0 = \begin{vmatrix} -1-\lambda & 1 & 0 \\ 4 & 1-\lambda & 2(1-\lambda) \\ 2 & 1 & -1-\lambda+2 \end{vmatrix} = (1-\lambda) \begin{vmatrix} -1-\lambda & 1 & 0 \\ 4 & 1-\lambda & 2 \\ 2 & 1 & 1 \end{vmatrix} =$$

$L2 \leftarrow L2 - 2L3$

$$= (1-\lambda) \begin{vmatrix} -\lambda-1 & 1 & 0 \\ 0 & 1-\lambda-2 & 0 \\ \cancel{2} & \cancel{1} & \cancel{1} \end{vmatrix} = (1-\lambda)\, 1 \cdot (-1)^{3+3} \begin{vmatrix} -\lambda-1 & 1 \\ 0 & -1-\lambda \end{vmatrix} =$$

$$= (1-\lambda)(-\lambda-1)(-1-\lambda) = -(\lambda-1)(1+\lambda)^2 =$$

$$= \underbrace{(-1)^3 (\lambda-1)(\lambda+1)^2}$$

$$P_A(\lambda) = (-1)^n (\lambda-\lambda_1)^{m_1} \dots (\lambda-\lambda_k)^{m_k}$$

$$(\lambda-1)(\lambda+1)^2 = 0 \Rightarrow \begin{cases} \lambda_1 = 1, \; m_1 = 1 \\ \lambda_2 = -1, \; m_2 = 2 \end{cases}$$

Pt. $\boxed{\lambda_1 = 1, m_1 = 1}$ $\Rightarrow$ determinăm $u \in \mathbb{R}^3$, $u \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ a.î. $Au = \lambda_1 u \Rightarrow$

$$\begin{pmatrix} -1 & 1 & -2 \\ 4 & 1 & 0 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = 1\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \Rightarrow \begin{cases} -u_1+u_2-2u_3 = u_1 \\ 4u_1 + \cancel{u_2} = \cancel{u_2} \Rightarrow \boxed{u_1=0} \\ 2u_1 + u_2 - \cancel{u_3} = u_3 \end{cases}$$

$$\Rightarrow \begin{cases} u_2 = 2u_3 \\ u_2 = 2u_3 \end{cases} \Rightarrow u = \begin{pmatrix} 0 \\ 2u_3 \\ u_3 \end{pmatrix} = u_3\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \varphi_1(t) = e^{\lambda_1 t}\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix} \Rightarrow \boxed{\varphi_1(t) = \begin{pmatrix} 0 \\ 2e^t \\ e^t \end{pmatrix}}$$

Pt $\boxed{\lambda_2=-1,\ m_2=2} \Rightarrow$ determinăm $p_0, p_1 \in \mathbb{R}^3$ nu amândoi nuli a.i.:

$$\varphi(t) = (p_0 + p_1 t)e^{\lambda_2 t}$$

să verifice sistemul $\bar{x}' = Ax \Rightarrow$

$$\Rightarrow \left((p_0+p_1t)e^{-t}\right)' = A(p_0+p_1t)e^{-t} \Rightarrow$$

$$\Rightarrow p_1 e^{-t} + (p_0+p_1t)e^{-t}(-1) = (Ap_0 + Ap_1t)e^{-t} \quad | : e^{-t}$$

$$\Rightarrow p_1 - p_0 - p_1 t = Ap_0 + Ap_1 t$$

identificăm coef puterilor lui t $\Rightarrow \begin{cases} -p_1 = Ap_1 \\ p_1 - p_0 = Ap_0 \end{cases} \Rightarrow$

$$\Rightarrow \begin{cases} 0_{\mathbb{R}^3} = Ap_1 + p_1 \\ p_1 = Ap_0 + p_0 \end{cases} \Rightarrow \begin{cases} 0_{\mathbb{R}^3} = (A+I_3)p_1 \\ p_1 = (A+I_3)p_0 \quad | (A+I_3) \text{ la stg} \Rightarrow \end{cases}$$

$$\Rightarrow \underbrace{(A+I_3)p_1}_{0_{\mathbb{R}^3}} = (A+I_3)^2 p_0 \Rightarrow (A+I_3)^2 p_0 = 0_{\mathbb{R}^3} \Rightarrow$$

$$\Rightarrow p_0 \in \underbrace{\ker\left((A+I_3)^2\right)}$$

subspațiu în $\mathbb{R}^3$, de dimensiunea 2, pt care determinăm o bază (din 2 vectori) $\Rightarrow$

$\rightarrow$ calculăm $(A+I_3)^2$ și rezolvăm sistemul $(A+I_3)^2 v = 0_{\mathbb{R}^3}$

$$(A+I_3)^2 = \begin{pmatrix} 0 & 1 & -2 \\ 4 & 2 & 0 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & -2 \\ 4 & 2 & 0 \\ 2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 8 & 8 & -8 \\ 4 & 4 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 8 & 8 & -8 \\ 4 & 4 & -4 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 0=0 \\ 8v_1+8v_2-8v_3=0 \ |:8 \\ 4v_1+4v_2-4v_3=0 \ |:4 \end{cases} \Rightarrow \underline{v_1+v_2-v_3=0} \Rightarrow$$

$\Rightarrow v_3 = v_1 + v_2 \Rightarrow$

$$v = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} = \begin{pmatrix} v_1 \\ v_2 \\ v_1+v_2 \end{pmatrix} \Rightarrow v = v_1\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + v_2\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \in \ker\left((A+I_3)^2\right)$$

$$\Rightarrow \ker\left((A+I_3)^2\right) = Sp\,gen\left\{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right\} \Rightarrow$$

$\Rightarrow \left\{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right\}$ este bază în $\ker\left((A+I_3)^2\right) \Rightarrow$

$\Rightarrow$ pt p_0 este suficient să luăm vectorii bazei:

• pt $p_0 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow p_1 = (A+I_3)p_0 = \begin{pmatrix} 0 & 1 & -2 \\ 4 & 2 & 0 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}$

$$\Rightarrow \varphi_2(t) = \left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix} t\right) e^{-t} \Rightarrow \boxed{\varphi_2(t) = \begin{pmatrix} (1-2t)e^{-t} \\ 4te^{-t} \\ (1+2t)e^{-t} \end{pmatrix}}$$

• pt $p_0 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \Rightarrow p_1 = \begin{pmatrix} 0 & 1 & -2 \\ 4 & 2 & 0 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \Rightarrow$

$$\Rightarrow \varphi_3(t) = \left(\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} t\right) e^{-t} \Rightarrow \boxed{\varphi_3(t) = \begin{pmatrix} -te^{-t} \\ (1+2t)e^{-t} \\ (1+t)e^{-t} \end{pmatrix}}$$

Avem $\{\varphi_1, \varphi_2, \varphi_3\}$ sistem fundamental de soluții $\Rightarrow$

$\Rightarrow$ matricea de soluții este

$$\phi(t) = \begin{pmatrix} 0 & (1-2t)e^{-t} & -te^{-t} \\ 2e^{t} & 4te^{-t} & (1+2t)e^{-t} \\ e^{t} & (1+2t)e^{-t} & (1+t)e^{-t} \end{pmatrix}$$

$$\Rightarrow \underset{\substack{\nearrow \\ \text{mulț. sol.} \\ \text{pt } \bar{z}' = A\bar{z}}}{S_A} = \left\{ \bar{x}(t) = \phi(t)c \;\middle|\; c \in \mathbb{R}^3 \right\}.$$

* aplicăm met. variației constantelor: determinăm $c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} : \mathbb{R} \to \mathbb{R}^3$ a.î. $x(t) = \phi(t)c(t)$ să fie sol. a sist. afin $x' = Ax + b$

$\Rightarrow \left(\phi(t)c(t)\right)' = A\phi(t)c(t) + b(t) \Rightarrow$

$$\Rightarrow \underbrace{\phi'(t)}_{A\cdot\phi(t)}c(t) + \phi(t)c'(t) = A\cancel{\phi}(t)c(t) + b(t) \Rightarrow$$

$$\Rightarrow \cancel{A\phi(t)c(t)} + \phi(t)c'(t) = \cancel{A\phi(t)c(t)} + b(t)$$

$$\Rightarrow \underbrace{\phi(t)c'(t) = b(t)}_{\text{sistem liniar } \underbrace{\text{în nec}}_{\text{algebric}}\ c_1', c_2', c_3'} \Rightarrow$$

$$\begin{pmatrix} 0 & (1-2t)e^{-t} & -te^{-t} \\ 2e^{t} & 4te^{-t} & (1+2t)e^{-t} \\ e^{t} & (1+2t)e^{-t} & (1+t)e^{-t} \end{pmatrix} \begin{pmatrix} c_1' \\ c_2' \\ c_3' \end{pmatrix} = \begin{pmatrix} 0 \\ e^{-t} \\ 0 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} (1-2t)e^{-t}c_2' - te^{-t}c_3' = 0 & \Big|:e^{-t} \\ 2e^{t}c_1' + 4te^{-t}c_2' + (1+2t)e^{-t}c_3' = e^{-t} \\ e^{t}c_1' + (1+2t)e^{-t}c_2' + (1+t)e^{-t}c_3' = 0 & \Big|(-2) \end{cases} \overset{\oplus}{\Rightarrow}$$

$$\Rightarrow e^{-t}(\cancel{4t}-2-\cancel{4t})c_2' + (1+\cancel{2t}-2-\cancel{2t})e^{-t}c_3' = e^{-t} \Rightarrow$$

$$\Rightarrow -2e^{-t}c_2' - e^{-t}c_3' = e^{-t} \quad \Big|:e^{-t} \Rightarrow$$

$$\Rightarrow -2c_2' - c_3' = 1 \quad \Big|\cdot(-t)$$

din prima ec. $\Rightarrow (1-2t)c_2' - tc_3' = 0$ $\Rightarrow$

$$\Rightarrow \quad 2tc_2' + tc_3' = -t$$
$$(1-2t)c_2' - tc_3' = 0 \quad (+)$$
$$c_2' \quad / \quad = -t \quad \Rightarrow \quad \boxed{c_2' = -t} \Rightarrow$$

$$\Rightarrow 2t - c_3' = 1 \Rightarrow \boxed{c_3' = 2t-1}$$

din ec a treia:

$$e^{t}c_1' + (1+2t)e^{-t}\cdot(-t) + (1+t)e^{-t}(2t-1) = 0.$$

$$e^{t}c_1' + e^{-t}(\cancel{-t}-\cancel{2t^2}+\cancel{2t}-1+\cancel{2t^2}-\cancel{t}) = 0$$

$$e^{t}c_1' - e^{-t} = 0 \Rightarrow \boxed{c_1' = e^{-2t}}$$

Integrând ec $\Rightarrow$ $c_1 = \dfrac{e^{-2t}}{-2} + K_1$; $c_2 = -\dfrac{t^2}{2} + K_2$; $c_3 = t^2 - t + K_3$

Deci soluția generală a sistemului afin este:

$$\underbrace{x(t) = \begin{pmatrix} 0 & (1-2t)e^{-t} & -te^{-t} \\ 2e^{t} & 4te^{-t} & (1+2t)e^{-t} \\ e^{t} & (1+2t)e^{-t} & (1+t)e^{-t} \end{pmatrix} \begin{pmatrix} -\frac{1}{2}e^{-2t} + K_1 \\ -\frac{t^2}{2} + K_2 \\ t^2 - t + K_3 \end{pmatrix} \; ; \; K_1, K_2, K_3 \in \mathbb{R}}_{S_{A,b}}$$

c) pt $t = 0 \Rightarrow x(0) = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \Rightarrow$

$$\Rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -\frac{1}{2} + K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{cases} \boxed{K_2 = 1} \\ \cancel{-1} + 2K_1 + K_3 = \cancel{-1} \\ -\frac{1}{2} + K_1 + K_2 + K_3 = 2 \end{cases} \Rightarrow \begin{cases} 2K_1 + K_3 = 0 \\ K_1 + K_3 = \frac{3}{2} \quad \Big| \cdot (-2) \end{cases} \quad (+)$$

$$-K_3 = -3$$

$$\Rightarrow \boxed{K_3 = 3} \Rightarrow \boxed{K_1 = -\frac{3}{2}}$$

(2) Fie sistemul: $\begin{cases} x_1' = 5t^4 x_2 \\ x_2' = 5t^4 x_1 \end{cases}$ (1)

a) Arătați că prin schimbarea de variabilă $t^5 = s$ se obține sistemul: $\begin{cases} y_1' = y_2 \\ y_2' = y_1 \end{cases}$ (2)

b) Determinați soluția generală a sistemului 2, apoi soluția generală a sistemului (1). Precizați un sistem fundamental de soluții pt (1).

(3) Fie sistemul $\begin{cases} x_1' = 3t^2 x_2 \\ x_2' = 3t^2 x_1 \end{cases}$ (3)

a) Arătați că $\varphi_1(t) = \begin{pmatrix} e^{-t^3} \\ -e^{-t^3} \end{pmatrix}$ este soluție a sistemului (3)

b) Folosind reducerea dimensiunii sistemului (3), determinați mulțimea soluțiilor pt. (3) și o soluție φ_2, astfel încât $\{\varphi_1, \overline{\varphi_2}\}$ să fie sistem fundamental de soluții pt (3).

(3) $\begin{pmatrix} x_1' \\ x_2' \end{pmatrix} = \begin{pmatrix} 0 & 3t^2 \\ 3t^2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$; $\underline{x' = A(t)x}$

a) φ_1 sol $\Leftrightarrow$ $\varphi_1'(t) = A(t)\varphi_1(t)$ $\Leftrightarrow$

$$\Leftrightarrow \begin{pmatrix} e^{-t^3}(-3t^2) \\ -e^{-t^3}(-3t^2) \end{pmatrix} = \begin{pmatrix} 0 & 3t^2 \\ 3t^2 & 0 \end{pmatrix} \begin{pmatrix} e^{-t^3} \\ -e^{-t^3} \end{pmatrix} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -3t^2 e^{-t^3} = -3t^2 e^{-t^3} \checkmark \\ 3t^2 e^{-t^3} = 3t^2 e^{-t^3} \checkmark \end{cases} \quad \text{Adev.}$$

b) $n = 2$

$m = 1$ nr. de soluții independente

$$\det(\varphi_{11}(t)) = \det(e^{-t^3}) = e^{-t^3} \neq 0, \ \forall t \in \mathbb{R}.$$

Se face schimbarea de variabilă:

$$x = Z(t)\, y$$

unde $Z(t) = \begin{pmatrix} e^{-t^3} & 0 \\ -e^{-t^3} & 1 \end{pmatrix}$

al doilea vector din baza canonică din $\mathbb{R}^2$.

$$\det Z(t) = e^{-t^3} \neq 0 \Rightarrow \exists (Z(t))^{-1}$$

Sistemul:

$$(Z(t)y)' = A(t)Z(t)y \Rightarrow$$

$$\Rightarrow Z(t)y' + Z'(t)y = A(t)Z(t)y \Rightarrow$$

$$\Rightarrow y' = \underbrace{(Z(t))^{-1}\left[A(t)Z(t) - Z'(t)\right]}_{B(t)} y$$

$$Z'(t) = \begin{pmatrix} -3t^2 e^{-t^3} & 0 \\ 3t^2 e^{-t^3} & 0 \end{pmatrix}$$

$${}^T(Z(t)) = \begin{pmatrix} e^{-t^3} & -e^{-t^3} \\ 0 & 1 \end{pmatrix} \Rightarrow (Z(t))^* = \begin{pmatrix} 1 & 0 \\ e^{-t^3} & e^{-t^3} \end{pmatrix} \Rightarrow$$

$$\Rightarrow (Z(t))^{-1} = \frac{1}{\det Z(t)} \cdot (Z(t))^{*} = e^{t^3}\begin{pmatrix} 1 & 0 \\ e^{-t^3} & e^{-t^3}\end{pmatrix} = \begin{pmatrix} e^{t^3} & 0 \\ 1 & 1\end{pmatrix}$$

Se obține:

$$B(t) = \begin{pmatrix} e^{t^3} & 0 \\ 1 & 1\end{pmatrix}\left[\begin{pmatrix} 0 & 3t^2 \\ 3t^2 & 0\end{pmatrix}\begin{pmatrix} e^{-t^3} & 0 \\ -e^{-t^3} & 1\end{pmatrix} - \begin{pmatrix} -3t^2\cdot e^{-t^3} & 0 \\ 3t^2 e^{-t^3} & 0\end{pmatrix}\right] \Rightarrow$$

$$\Rightarrow B(t) = \begin{pmatrix} e^{t^3} & 0 \\ 1 & 1\end{pmatrix}\left[\begin{pmatrix} -3t^2 e^{-t^3} & 3t^2 \\ 3t^2 e^{-t^3} & 0\end{pmatrix} - \begin{pmatrix} -3t^2 e^{-t^3} & 0 \\ 3t^2 e^{-t^3} & 0\end{pmatrix}\right] =$$

$$= \begin{pmatrix} e^{t^3} & 0 \\ 1 & 1\end{pmatrix}\begin{pmatrix} 0 & 3t^2 \\ 0 & 0\end{pmatrix} = \begin{pmatrix} 0 & 3t^2 e^{t^3} \\ 0 & 3t^2\end{pmatrix} \Rightarrow$$

$$\Rightarrow \begin{pmatrix} y_1' \\ y_2'\end{pmatrix} = \begin{pmatrix} 0 & 3t^2 e^{t^3} \\ 0 & 3t^2\end{pmatrix}\begin{pmatrix} y_1 \\ y_2\end{pmatrix} \Rightarrow \begin{cases} y_1' = 3t^2 e^{t^3} y_2 \\ y_2' = 3t^2 y_2 \end{cases} \Rightarrow$$

$\Rightarrow$ ec. liniară omogenă în y_2: $\boxed{y_2(t) = c_2 e^{t^3}} \Rightarrow$

$$\Rightarrow y_1' = 3t^2 e^{t^3} \cdot c_2 e^{t^3}$$

$y_1' = 3c_2 t^2 e^{2t^3}$ ec. de tip primitivei $\Big| \Rightarrow y_1 = \int 3c_2 t^2 e^{2t^3} dt =$

$$= \frac{3c_2}{6}\int \left(e^{2t^3}\right)' dt \Rightarrow$$

$$\Rightarrow \boxed{y_1(t) = \frac{c_2}{2} e^{2t^3} + c_1}$$

Pt sistemul în x $\Rightarrow$

$$\Rightarrow x(t) = \begin{pmatrix} e^{-t^3} & 0 \\ -e^{-t^3} & 1\end{pmatrix}\begin{pmatrix} \frac{c_2}{2} e^{2t^3} + c_1 \\ c_2 e^{t^3}\end{pmatrix} =$$

$$= \begin{pmatrix} c_1 e^{-t^3} + \frac{1}{2} c_2 e^{t^3} \\ -\frac{c_2}{2} e^{t^3} - c_1 e^{-t^3} + c_2 e^{t^3}\end{pmatrix} = c_1 \underbrace{\begin{pmatrix} e^{-t^3} \\ -e^{-t^3}\end{pmatrix}}_{\varphi_1(t)} + c_2 \underbrace{\begin{pmatrix} \frac{1}{2} e^{t^3} \\ \frac{1}{2} e^{t^3}\end{pmatrix}}_{\varphi_2(t)} \Rightarrow$$

$c_1, c_2 \in \mathbb{R}$.

$\Rightarrow$ un sistem fundam. de sol. pt (3) este:

$$\left\{ \begin{pmatrix} e^{-t^3} \\ -e^{-t^3}\end{pmatrix} ; \begin{pmatrix} \frac{1}{2} e^{t^3} \\ \frac{1}{2} e^{t^3}\end{pmatrix} \right\}.$$

Temă: (2)

+ ca temă:

(4) Se dă sistemul: $\begin{cases} x_1' = \frac{1}{t} x_1 - \frac{2}{t} x_2 + \ln t, & t>0 \\ x_2' = \frac{2}{t} x_1 - \frac{3}{t} x_2 \end{cases}$ (4)

a) forma matricială

b) Arătați că prin schimbarea de variabilă $t = e^s$ se obține: $\begin{cases} y_1' = y_1 - 2y_2 + s \\ y_2' = 2y_1 - 3y_2 \end{cases}$ (5)

c) Soluția generală pt (5) și apoi pt. (4).